Chapter 6

GPS Satellite and Receiver Instrumental Biases Estimation using Linear Adaptive Filter Model

6.1. Introduction

Ionospheric delay is one of the prominent errors in the GAGAN that limit the positional accuracy. The predefined global IGP grid consists of 1808 IGPS. For providing the ionospheric error corrections over the GAGAN service region, 60 IGPS are identified (Sarma et.al., 2000). The ionospheric delay corrections are broadcast as vertical delay estimates at specified Ionospheric Grid Points (IGPs) in the predefined global IGP grid to suitably modify single frequency GPS receiver’s position estimation. However, the estimation of the IGP delay, which is a function of TEC, is limited by instrumental biases. The instrumental bias is the difference of the propagation paths of L1 and L2 signals and is due to the circuitry in the GPS satellite and receiver hardware. Even though the bias error are of the order of ±10nsec, it will become critical in SBAS (Brain, et.al, 1999). Calibration of hardware biases is particularly important in augmented GPS systems where vertical accuracy of 4.5m is required for PA landings. If the differential delay parameters are not calibrated, they propagate into the differential correction through the ionospheric models (Bishop and Mazzella, 1995). The instrumental biases are environment dependent and hence time varying. In the case of hardware calibration, it will be difficult for the master station of the GAGAN located at Bangalore, India to continually monitor all the geographically distributed 20 TEC stations. Therefore, software calibration is the solution and is described in this chapter.

Various techniques are developed for estimating the instrumental biases. One method is Kalman filter design for estimating the biases directly from the receiver’s measurements of the GPS signals. In this method equatorial anomaly effects are not considered and the bias estimation is not accurate for a single receiver (Sardon et. al, 1994). In this chapter, a new model is proposed to estimate the instrumental biases by modelling the TEC using 4th order polynomial.
6.2 Estimation of the satellite and receiver biases

A new model based on Coco et al. (1991) is proposed to estimate the instrumental biases by modelling the TEC using 4th order polynomial. This model is an approximation of the steepest descent model, which uses an instantaneous estimate of the gradient vector of a cost function. In this model, the combined satellite and receiver differential delays are estimated using least-squares method. The vertical TEC at each ionospheric pierce point (IPP) is represented as 4th order polynomial in this model. The inputs to the model are azimuth, elevation angle of each satellite tracked, slant factor, slant TEC, IPP latitude and longitude. The slant TEC measurement (TECd) made on GPS satellite at TEC station is the sum of the real slant TEC, satellite differential delay (bSi) and receiver differential delay (bRk). The differential delay can be modeled as the sum of a receiver bias, a satellite transmitter bias, and a constant times the line-of-sight ionospheric total electron content (TEC) (Gao and Liu, 2002). The following three assumptions are made in implementing this model.

i) The slant and vertical TECs are related by a constant obliquity factor,

ii) Satellite-plus-receiver (SPR) differential delays are assumed to be constant over several hours

iii) the TEC, at the IPP is represented by 4th degree polynomial and is represented as follows (Lao-Sheng Lin, 2001).

\[ TEC_d(\phi, \lambda) = a_0 + a_1 \phi + a_2 \phi^2 + a_3 \phi^3 + a_4 \phi^4 + a_5 \lambda + a_6 \lambda^2 + a_7 \lambda^3 + a_8 \lambda^4 \]

\[ + a_9 \phi \lambda + a_{10} \phi^2 \lambda + a_{11} \phi^3 \lambda + a_{12} \phi^4 \lambda + a_{13} \lambda^2 \]  

(6.1)

Where 

\( a_0, a_1, \ldots, a_{14} \) are the unknown ionosphere model coefficients. \( \phi \) and \( \lambda \) are the IPP latitude and longitudes in geomagnetic coordinate system.

6.2.1 Modelling of instrumental biases

Step 1: Modelling of biases

The biases and the vertical TEC can be modeled as (Ma and Maruyama, 2003)

\[ S(E)_nk \times TEC_{v1} + (b_{R1} + b_{Rk}) = TEC_{sl nk} \]  

(6.2)

where

\( TEC_{sl nk} \) = measured slant TEC from the receiver k to the satellite \( i \),
$E =$ elevation angle from the receiver $k$ to the tracked satellite $i$,

$S(E)_{ik} =$ slant factor

$TEC_{el} =$ vertical TEC at the ionospheric pierce point due to the satellite $i$

$TEC_{t(m-i)} =$ vertical TEC at the ionospheric pierce point due to the satellite $m-i$.

$b_{si} + b_{sk} =$ satellite-plus-receiver (SPR) differential delay

The signal flow graph of the model (Eq. 6.1) which is known as multiple linear regression model is shown in Fig. 6.1 (Haykin S, 2003).

**Fig. 6.1 signal flow graph of the model**

The model is a linear adaptive filtering model, This, in general, consists of two basic processes (Haykin, 2003);

i) a filtering process, which involves computing the output of a linear filter in response to an input signal and generating an estimation error by comparing this output with a desired response

ii) an adaptive process, which involves the automatic adjustment of the parameters of the filter in accordance with the estimation error.

The combination of these two processes working together constitutes a feedback loop (see Fig. 6.2). The figure shows that a transversal filter, around which the least mean square model is built; this component is responsible for performing the filtering process.

The second component is mechanism for performing the adaptive control process on the tap weights of the transversal filter (Kaliath and Frost, 1968). The detailed structure of the transversal filter (see Fig. 6.3) consists of 3 basic elements, namely, a unit delay element, a multiplier and an adder.
The number of delay elements used in the filter determine the finite duration of its impulse response. The role of each multiplier in the filter is to multiply the tap input by a filter coefficient referred to as a tap weight. The combined role of the adders in the filter is to sum the individual multiplier outputs and produce an overall filter output.

The physical phenomenon is characterized by the two set of variables $TEC_a(i)$ and $S(E)_{ak}(i)$. The variable $TEC_a(i)$ is observed at time $i$ in response to the subset of variables $S(E)_{ak}(i)$, $S(E)_{ak}(i-1)$, $S(E)_{ak}(i-1)$,...$S(E)_{ak}(i-M+1)$, applied as inputs. The $TEC_a(i)$ is a function of the inputs $S(E)_{ak}(i)$, $S(E)_{ak}(i-1)$, $S(E)_{ak}(i-1)$,...$S(E)_{ak}(i-M+1)$.

This functional relation ship is modeled as (Haykin, 2003),

$$TEC_a(i)=\sum_{k=0}^{M-1} a_k b_k (S(E)(i-k))+\Pi(i)$$

$$\Pi(i)=TEC_a(i)-TEC_a(n-1)S(E)(i)$$

$$TEC_V(n+1)=TEC_V(n)+2\mu n S(E)(n)$$

Where $a_0, a_{M-1},$ and $b_k$ are unknown parameters of the model, $TEC_V(n+1)$ is the tap weight vector adoption, $\mu$ is the step size parameter and $(\Pi_i)$ represents the measurement error.

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Fig.6.2 Block diagram of adaptive filter model
Step 2: Modeling of vertical TEC

The vertical TEC at the IPP (TEC_v (t_m, \lambda_{ct})) can be represented by Eq. 6.2. By substituting Eq. 6.2 in 6.1, the biases and the vertical TEC can be represented as

\[ S(E)(a_0 + a_1t_m + a_2\lambda_{ct} + a_3t_m^2 + a_4\lambda_{ct}^2 + a_5t_m\lambda_{ct} + a_6\lambda_{ct} + a_7t_m^2\lambda_{ct} + a_8t_m\lambda_{ct}^2 + a_9\lambda_{ct}^2 + a_{10}t_m^4 + a_{11}t_m^2\lambda_{ct} + a_{12}t_m^2\lambda_{ct}^2 + a_{13}t_m\lambda_{ct}^2 + (b_{32} + b_{31})(... + (b_{22} + b_{21}) \right) = TEC_{stjk} \] (6.6)

A total of 43 coefficients (15 coefficients and the biases for the 28 tracked satellites) can be obtained by applying the Least Squares Method of Solutions (LMS) to Eq. 6.6. In Eq. 6.6 it is assumed that the slant TEC, the slant factors are known while 43 coefficients are the unknowns.

Step 3: Least square error estimation

As per the LMS, if a given function is in the form of \( TEC_{vi} = F(\phi_m, \lambda_{ct}) \), then its residual is given as \( TEC_{vi} - F(\phi_m, \lambda_{ct}) \). The sum of the squares of the residual \( \Pi \) must be minimum and is given as (Mehra, 1972).

\[ \Pi = \sum_{i=1}^{n} [TEC_{sli} - F(\phi_m, \lambda_{ct})]^2 = \text{minimum} \] (6.7)
The coefficients \(a_0, a_1, a_2, \ldots, a_{14}\) are unknown while all the \(TEC_{ij}, \phi_{ij}\) and \(\lambda_{cr}\) are given. To obtain the least square error, the unknown coefficients \(a_0, a_1, a_2, \ldots, a_{14}\) should provide zero first derivatives.

\[
\Pi = \sum_{i=0}^{n} \left[ TEC_{ij} - \left\{ S(E)(a_0 + a_1 \phi_{m} + \cdots + a_5 \phi_{m}^5) + \sum_{j=1}^{m} b_j \right\} \right]^2 
\]

(6.8)

\[
\Pi = \sum_{i=0}^{n} \left[ TEC_{ij} - S(E)a_0 - S(E)a_1 \phi_m - \cdots - S(E)a_5 \phi_m^5 - \frac{28}{\sum_{j=1}^{m} b_j} \right]^2 
\]

(6.9)

Step 4: Determination of the polynomial coefficients

To solve for the coefficients, the computed first order derivative of the residual (\(\Pi\)) with respect to each unknown coefficient must be equated to zero.

Step 4.1: Determination of first order derivative with respect to \(a_0\)

Taking the first order derivative with respect to \(a_0\) and equating it to zero.

\[
\frac{\partial \Pi}{\partial a_0} = 2 \sum_{i=1}^{n} \left[ TEC_{ij} - S(E)a_0 - S(E)a_1 \phi_m - \cdots - S(E)a_5 \phi_m^5 - \frac{28}{\sum_{j=1}^{m} b_j} \right] \frac{\partial}{\partial a_0} S(E) = 0 
\]

(6.10)

Solving Eq.6.10

\[
\sum_{i=0}^{n} \left[ TEC_{ij} - S(E)a_0 - S(E)a_1 \phi_m - \cdots - S(E)a_5 \phi_m^5 - \frac{28}{\sum_{j=1}^{m} b_j} \right] \frac{\partial}{\partial a_0} S(E) = 0 
\]

(6.11)

\[
a_0 \sum_{i=1}^{n} S(E)^2 + a_1 \sum_{i=1}^{n} S(E)^2 \phi_m + a_2 \sum_{i=1}^{n} S(E)^2 \phi_m^3 + a_3 \sum_{i=1}^{n} S(E)^2 \phi_m^5 + a_4 \sum_{i=1}^{n} S(E)^2 \lambda_{cr}^2 + \frac{28}{\sum_{j=1}^{m} b_j} \sum_{i=1}^{n} S(E) = 0 
\]

(6.12)

Step 4.2: Determination of first order derivative with respect to \(a_1\)

Computing the partial derivative of the residual (\(\Pi\)) with respect to second coefficient \(a_1\) and equating it to zero i.e.

\[
\frac{\partial \Pi}{\partial a_1} = 0 
\]
Again, solving the Eq.6.13,
\[ 2\sum_{i=1}^{n} \left[ TEC_{dl} - S(E)\phi_0 - S(E)\phi_1 - \sum_{j=1}^{28} b_j \right] \phi_m - S(E)\phi_m^2 - \sum_{j=1}^{28} b_j \phi_m = 0 \] (6.13)

Similarly, other coefficients, \(a_2, a_3, \ldots, a_{14}\) are computed.

**Step 5: Determination of satellite plus receiver biases**

Satellite plus receiver biases (\(b_j = b_{si} + b_{ri}\)) are determined by computing the partial derivative of residual (\(\Pi\)) with respect to the combined bias (\(b_j\)) i.e. \[ \frac{\partial \Pi}{\partial b_j} = 0 \]

\[ 2\sum_{i=1}^{n} \left[ TEC_{dl} - S(E)\phi_0 - S(E)\phi_1 - \sum_{j=1}^{28} b_j \right] \phi_m - S(E)\phi_m^2 - \sum_{j=1}^{28} b_j \phi_m = 0 \] (6.15)

on solving the above equation,
\[ a_0 \sum_{i=1}^{n} S(E) + a_1 \sum_{i=1}^{n} S(E)\phi_m + a_2 \sum_{i=1}^{n} S(E)\phi_m^2 + a_3 \sum_{i=1}^{n} S(E)\phi_m^3 + a_4 \sum_{i=1}^{n} S(E)\phi_m^4 \]
\[ + a_5 \sum_{i=1}^{n} S(E)\phi_m^5 + a_6 \sum_{i=1}^{n} S(E)\phi_m^6 + \sum_{i=1}^{n} b_j \phi_m = 0 \] (6.16)

Similarly computing the partial derivative of the residual \(\Pi\) with respect to \(b_2, b_3, \ldots, b_{28}\) will give 27 equations similar to Eq.6.16.
Step 6: System of linear equations

There are total 43 equations and 43 unknowns (15 coefficients + 28 satellite biases). The 43 equations, 43 coefficients and the corresponding stant TEC\textsubscript{w} observed can be written in the form of a matrices A, x and B as

$$\mathbf{Ax} = \mathbf{B} \quad (6.17)$$

By substituting the values of A, B and x, Eq. 6.17 can be written as,

$$
\begin{bmatrix}
\sum_{i=1}^{2} a_{i}^{2} & \sum_{i=1}^{2} a_{i} \phi_{m} & \sum_{i=1}^{2} a_{i} \lambda_{c} & \cdots & \sum_{i=1}^{2} a_{i} \phi_{m} \lambda_{c} & \sum_{i=1}^{2} a_{i} \phi_{m} \lambda_{c} (0_{28}) \\
\sum_{i=1}^{2} a_{i} \phi_{m} & \sum_{i=1}^{2} a_{i} \phi_{m}^{2} & \sum_{i=1}^{2} a_{i} \phi_{m} \lambda_{c} & \cdots & \sum_{i=1}^{2} a_{i} \phi_{m} \lambda_{c}^{2} & \sum_{i=1}^{2} a_{i} \phi_{m} \lambda_{c} (0_{28}) \\
\sum_{i=1}^{2} a_{i} \lambda_{c} & \sum_{i=1}^{2} a_{i} \phi_{m} \lambda_{c} & \sum_{i=1}^{2} a_{i} \lambda_{c}^{2} & \cdots & \sum_{i=1}^{2} a_{i} \phi_{m} \lambda_{c} \lambda_{c} & \sum_{i=1}^{2} a_{i} \phi_{m} \lambda_{c} \lambda_{c} (0_{28}) \\
\sum_{i=1}^{2} a_{i} \phi_{m} \lambda_{c} & \sum_{i=1}^{2} a_{i} \phi_{m} \lambda_{c}^{2} & \sum_{i=1}^{2} a_{i} \phi_{m} \lambda_{c} \lambda_{c} & \cdots & \sum_{i=1}^{2} a_{i} \phi_{m} \lambda_{c} \lambda_{c}^{2} & \sum_{i=1}^{2} a_{i} \phi_{m} \lambda_{c} \lambda_{c} (0_{28}) \\
\sum_{i=1}^{2} a_{i} \phi_{m} \lambda_{c}^{2} & \sum_{i=1}^{2} a_{i} \phi_{m} \lambda_{c} \lambda_{c}^{2} & \sum_{i=1}^{2} a_{i} \phi_{m} \lambda_{c} \lambda_{c}^{2} & \cdots & \sum_{i=1}^{2} a_{i} \phi_{m} \lambda_{c} \lambda_{c} \lambda_{c}^{2} & \sum_{i=1}^{2} a_{i} \phi_{m} \lambda_{c} \lambda_{c} \lambda_{c} (0_{28}) \\
\sum_{i=1}^{2} a_{i} \phi_{m} \lambda_{c} \lambda_{c} & \sum_{i=1}^{2} a_{i} \phi_{m} \lambda_{c} \lambda_{c}^{2} & \sum_{i=1}^{2} a_{i} \phi_{m} \lambda_{c} \lambda_{c} \lambda_{c} & \cdots & \sum_{i=1}^{2} a_{i} \phi_{m} \lambda_{c} \lambda_{c} \lambda_{c} \lambda_{c}^{2} & \sum_{i=1}^{2} a_{i} \phi_{m} \lambda_{c} \lambda_{c} \lambda_{c} \lambda_{c} (0_{28}) \\
\end{bmatrix}
$$

$$
\begin{bmatrix}
\mathbf{a}_{0} \\
\mathbf{a}_{1} \\
\mathbf{a}_{2} \\
\mathbf{a}_{3} \\
\mathbf{a}_{4} \\
\mathbf{a}_{14} \\
\mathbf{b}_{17} \\
\mathbf{b}_{18}
\end{bmatrix} = 
\begin{bmatrix}
\sum_{i=1}^{2} \mathbf{TEC}_{i} \phi_{m} \\
\sum_{i=1}^{2} \mathbf{TEC}_{i} \phi_{m}^{2} \\
\sum_{i=1}^{2} \mathbf{TEC}_{i} \phi_{m} \lambda_{c} \\
\sum_{i=1}^{2} \mathbf{TEC}_{i} \phi_{m} \lambda_{c}^{2} \\
\sum_{i=1}^{2} \mathbf{TEC}_{i} \phi_{m} \lambda_{c} \lambda_{c} \\
\sum_{i=1}^{2} \mathbf{TEC}_{i} \phi_{m} \lambda_{c} \lambda_{c}^{2} \\
\sum_{i=1}^{2} \mathbf{TEC}_{i} \phi_{m} \lambda_{c} \lambda_{c} \lambda_{c} \\
\sum_{i=1}^{2} \mathbf{TEC}_{i} \phi_{m} \lambda_{c} \lambda_{c} \lambda_{c} \lambda_{c}
\end{bmatrix}
$$

...(6.18)

Step 7: Solution of \( \mathbf{Ax} = \mathbf{B} \)

The solution of the Eq. 6.18 for the 43 coefficients is as follows (Haykin Simon, 2003)
The first 15 terms of the L.H.S gives the coefficients of vertical TEC and remaining 28 terms gives the combined differential delays. From the combined SPR values, receiver differential delays can be separated. The computed mean SPR differential delays and receiver differential delay can be used to calibrate the measured slant TEC in order to provide precise TEC measurements.

6.3. Data acquisition and processing

As a part of GAGAN setup, a Novatel make dual frequency GPS receiver is located at Hyderabad airport (17.431\(^{\circ}\)N, 78.453\(^{\circ}\)E), India. Several days of navigation and observation data in RINEX format were collected and analyzed. The navigation data file consists of 38 parameters. However, in our calculations only 23 parameters are used. Navigation data is available for every two hours. In between data is generated using standard formulae. Observation data file consists of C/A, P1 and P2 pseudoranges and L1
and L2 phases for all the visible satellites. From this information satellite position, elevation and azimuth angle of satellite, IPP local time, IPP latitude, longitude, geomagnetic latitude, geomagnetic longitude, slant factor, ionospheric time delay and slant TEC for all the visible satellites are estimated.

6.4. Results and discussion

Using the data corresponds to 12 days (1st July 2004 to 12 July 2004), the biases are estimated. Using the satellite elevation and azimuth information, for each satellite the IPP latitude and longitude are estimated. A mesh grid with a square grid spacing of 5° x 5° in latitude and longitude is assumed at an altitude of 350 km above the earth surface. In each 5° square grid, the number of IPPs available are determined. The instrumental biases are assumed to be constant over several hours in a particular mesh grid of 5° square grid size. The differential delay (b_tr+b_k) and the 15 coefficients of the polynomial for all the 29 satellites that were visible from 1st July to 12th July 2005 were estimated for a particular 5° square grid (17.431±2.25°, 78.4530±2.25°). In this particular grid the IPPs are due to 13 visible SVs (PRNs 1, 4, 6, 8, 10, 13, 15, 16, 21, 24, 25, 27, 31).

The mean SPR differential delay for 13 satellites over the 12 day period is computed and are presented in Table 6.1. The standard deviation (σ) of the mean SPR differential delay for the 12 day period was computed for each satellite and the mean standard deviation (ордин) for 13 satellites are presented in Table 6.2. Fig. 6.4 shows the SPR instrumental biases for the SV PRN 31, 25, 10 and 6. The biases observed are positive values (2 to 7 nsec.) for SVs 6 and 31 and negative values (-3 to -8 nsec.) for SVs 10 and 31. Fig. 6.5 shows the σ of the mean SPR instrumental biases for 13 GPS satellites. Maximum σ is observed for the SV 31 and minimum σ is observed for SV 10. The standard deviation values indicate the day to day variability of the SPR differential delay estimates. Fig. 6.6 compares the TEC estimation for a SV PRN 1 and 31 (12 July 2004) after modelling of instrumental biases. The bias error estimated is -3.638 nsec and -4.71 nsec for satellites 1 and 31 respectively. From the results, it is found that the SPR differential delays of 13 satellites are varying from -6.4060 to 5.4117 nsec. The results indicate that day to day variation of SPR differential delay is small and it is less than 1 nsec. The average value of
the \( \sigma \) of the SPR differential delay estimate is 1.17 nsec, which represents an error estimate of the SPR differential delays.

Table 6.1 SPR instrumental biases for the 12 day period for 13 satellites

<table>
<thead>
<tr>
<th>SV PRN Date</th>
<th>SPR (nsec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>July01</td>
<td>4.16</td>
</tr>
<tr>
<td>July02</td>
<td>-4.76</td>
</tr>
<tr>
<td>July03</td>
<td>-3.56</td>
</tr>
<tr>
<td>July04</td>
<td>-5.06</td>
</tr>
<tr>
<td>July05</td>
<td>-4.11</td>
</tr>
<tr>
<td>July06</td>
<td>-3.66</td>
</tr>
<tr>
<td>July07</td>
<td>-3.53</td>
</tr>
<tr>
<td>July08</td>
<td>-2.99</td>
</tr>
<tr>
<td>July09</td>
<td>-2.70</td>
</tr>
<tr>
<td>July10</td>
<td>-4.07</td>
</tr>
<tr>
<td>July11</td>
<td>-5.24</td>
</tr>
<tr>
<td>July12</td>
<td>-3.34</td>
</tr>
</tbody>
</table>

Table 6.2 Mean standard deviation (\( \bar{\sigma} \)) of the mean SPR instrumental biases for 13 GPS satellites for the period 1st to 12 July 2004

<table>
<thead>
<tr>
<th>SV PRN</th>
<th>SPR Value (nsec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-4.71</td>
</tr>
<tr>
<td>( \bar{\sigma} )</td>
<td>2.91</td>
</tr>
</tbody>
</table>

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Fig. 6.4 SPR instrumental biases for the 12 day period for 4 satellites at Hyderabad(17.431°N, 78.453°E)

Days starting from 1st July to 12th July 2004
Satellite Nos. 1,4,6,8,10,13,15,16,21,24,25,27,31, Period: 1st - 12th July, 2004

Fig. 6.5 σ instrumental biases for 13 GPS satellites for the 12-day period (1st to 12th July 2004)

Fig. 6.6 Comparison of TEC estimation after modeling of instrumental biases at Hyderabad (17.43°N, 78.430°E)
6.5 Conclusions

A new model is proposed to estimate the instrumental biases by modelling the ionospheric TEC using 4\textsuperscript{th} order polynomial. This model is an approximation of the steepest descent model, which uses an instantaneous estimate of the gradient vector of a cost function. The estimate of the gradient is based on sample values of the tap-input vector and an error signal. The model iterates over each coefficient in the filter, moving it in the direction of the approximated gradient. The model can be used to calibrate the dual frequency GPS receivers for precise TEC measurement even when the receiver internal hardware calibration is not available. The experimental results from the 12 day period indicate that the estimation precision of the satellite and receiver differential delay is of the order of ±0.17nsec. It is found that the error in the TEC estimation for the SV PRN 1 and 31 are −3.638nsec and −4.71nsec respectively. It is also found that the results are consistent over the period and the method is accurate and faster for real time applications like GAGAN.