Chapter 5

Significance of Instrumental Biases in Time Delay Estimation

5.1 Introduction

For a safe and secure navigation and landing through augmented GPS system, availability of satellites, accuracy of the ranging signal, integrity of the incoming signals and continuity of the information are the four most important factors (Enge et al., 1996). For PA landings, ionospheric correction takes precedence over these four factors. The ionospheric time delay error, which is a function of Total Electron Content (TEC) is one of the dominant error that affects the positioning accuracy in SBAS. In dual frequency receivers, ionospheric delay can be estimated precisely by taking the advantage of the dispersive nature of the ionosphere. The delay is estimated by measuring the difference in arrival times of the two GPS frequencies. The measurement accuracy of the time delay estimated from the dual-frequency GPS receiver is limited by instrumental biases. The instrumental bias is the difference of the two dispersive delays introduced by the analog hardware in the L1 and L2 signal paths (Coco et al., 1991). This limitation is due to the circuitry in the GPS satellite and receiver hardware. Calibration of hardware biases is particularly important in augmented GPS systems like WAAS/EGNOS/GAGAN. In GAGAN the ionospheric time delays are estimated from dual-frequency GPS receivers located at fixed TEC stations. If the differential delay parameters are not calibrated out they propagate into the differential correction through the ionospheric models. The Geometric Dilution of Precision (GDOP) magnifies the effect in the position domain when computing a differential navigation fix. The differential delays are environmental dependent and hence time varying. In case of hardware calibration, the master station of the GAGAN cannot continually monitor all the TEC stations and some time varying error is always present, further each TEC station must be visited routinely which may be expensive. Whereas, software calibration can occur continuously. In this chapter, a time delay estimation algorithm using the code and phase range measurements of the dual frequency GPS receiver, IPP location estimation algorithm and the importance of instrumental biases in position accuracy are presented.
5.2 Relation between Ionospheric time delay and TEC

The ionosphere covers the region between approximately 50 km and 1000 km above the earth and is characterized by the presence of free electrons. As the GPS signals pass through the ionosphere strata of the atmosphere it faces two delays. The first one is reduction of the propagation velocity of the signal and the second is from bending of the signal. The bending effect is negligible as it causes a signal delay of one milli meter only. This reduction of the propagation velocity causes many problems to GPS users, especially single-frequency users, as the basic idea of GPS depends on the computation of the exact time taken for the signal to travel from the satellite to the receiver. The effect of signal bending is minimized by using a cut-off elevation angle of 10°-15°, so the remaining factor is the propagation delay of the signal. The ionosphere is a dispersive medium for radio waves implying that the refractive index is a function of the radio wave's frequency and the electron density. In the ionosphere the index of refraction depends on several factors such as electron density and frequency of the GPS signal. The phase and group refractive indexes are given by (Hofmann-Wellenhof, 1994).

\[
n_p = 1 + \frac{40.3N}{f^2} \tag{5.1}
\]

\[
n_g = 1 - \frac{40.3N}{f^2} \tag{5.2}
\]

where,

- \(N\) = electron density \((\text{el/m}^2\))
- \(f\) = frequency of the GPS signal

After integrating the phase and group refractive indices along the path of the GPS signal, the range obtained between the satellite and the receiver is different from the true geometric range by the amount called ionospheric error. The error is negative for the carrier phase pseudoranges (phase advance; that is, the measured range is shorter than the geometric range) and positive for the code pseudoranges (a group delay; that is, the measured range is longer than the geometric range) (Komjathy and Langley, 1997).
The ionospheric delay ($\Delta_{\text{ion}}$) is proportional to the number of electron content (integrated density along the signal path) and inversely proportional to frequency squared used:

$$\Delta_{\text{ion}} = \frac{40.3 \text{TEC}}{f^2} \quad (5.3)$$

where TEC is Total Electron Content measured in TEC Units and is given by

$$\text{TEC} = \int N \, ds \quad (5.4)$$

where $s$ is the path from satellite to the receiver.

The density of free electrons varies strongly with the time of the day and the latitude. Therefore, the change of pseudorange measurement caused by the ionospheric refraction may be restricted to the determination of the TEC. However, the TEC itself is a fairly complicated quantity because it depends on time of day, seasonal, the 11-year solar cycle, elevation and azimuth of the satellite and the position of the observation site (Hofmann-Wellenhof, 1994).

5.3 Ionospheric time delay estimation for single frequency GPS receiver users

The ionospheric models represent the properties of the ionosphere as accurately as possible as functions of geophysical indices, with some statistical description of their variability. The important parameter responsible for ionospheric time delay is the total number of electrons encountered by the radio wave on its path from the satellite to the system user. The TEC is a function of long and short term changes in solar flux, magnetic activity, season, time of the day, user location and viewing direction (Sarma, 1999). Many models are available for single-frequency users, which may be divided in two main categories: i) Firstly, state-of-the-art ionospheric models, such as, International Reference Ionosphere (IRI) and Bent. These models require hundreds of coefficients with a tremendous penalty in computational complexity, but which will fit the monthly average behaviour of ionospheric range delay to within a residual bias of approximately 10%. ii) Secondly, simple-computational model with an ideal description for the ionosphere's average behaviour but with other shortcomings such as, low accuracy in describing the variability of the ionospheric behaviour with
different latitudes and times is the Klobuchar model. Presently Klobuchar model is incorporated in single frequency GPS receivers. Klobuchar model coefficients are send through the GPS navigation message (Klobuchar et.al, 1995).

5.3.1 Electron Density Models

i) Bent Electron Density Model

The Bent model (Llewellyn and Bent, 1973) is an empirical worldwide algorithm, capable of accurately estimating the electron density profile and the associated delay and directional changes of a wave due to refraction. It was designed originally for ground to satellite communications but can be used for ground to ground or satellite to satellite communications. The required inputs to the model are the satellite and station positions, time information, daily values of solar flux, the 12-month running averages of solar flux and Zurich sunspot numbers. In the Bent model, only a few TEC measurements were available in low latitude regions. The whole Asia is represented by the data from two stations in Hawaii and Hong Kong. The model is computationally demanding but it accounts for up to 80% of the total ionospheric effect (Newby and Langely, 1992).

ii) The IRI Model

The International Reference Ionosphere (IRI) was developed by an international project sponsored by the Committee on Space Research (COSPAR) and the International Union of Radio Science (URSI). This formed a working group in the late sixties to produce an empirical standard model of the ionosphere based on all available data sources from the worldwide network of ionosondes (Rawer, 1981). For a given location, time and date, the IRI describes many ionospheric variables, including the electron density, for a valid range of altitudes below 1000 Km. Several improved editions of the model have been released, the latest one is IRI-2001. Tests proved that the IRI-95 model performs better than the IRI-90 model in computing the ionospheric delay (Bilitza, 2001).

5.3.2 Klobuchar time delay model

The Klobuchar model (Klobuchar, 1982) was designed based on Bent’s electron density model and is the most widely used model due to its computational simplicity. The Klobuchar’s model is used to correct the ionospheric effect for single frequency
measurements. It uses the cosine model for the daily variation of the ionosphere, with
the maximum being at 14:00 local time and is described by 8 coefficients (α and β),
which are transmitted as part of the GPS satellite navigation message. There are
currently 370 sets of 8 coefficients, which are selected based on the day of the year
(37 groups) and the average solar-flux value for the previous 5 days (10 groups) (see
Fig. 5.1).

Fig. 5.1 Klobuchar coefficients selection procedure

The 8 coefficients are selected by the GPS master control station and placed in the
satellite navigation message. The coefficients transmitted are updated once every ten
days or sometimes more frequently. This model removes about 50% of the total
ionospheric delay at mid-latitudes and is represented through one set of variables that
are valid for few days.

The algorithm, incorporated in the GPS system for single frequency users, consists of
a cosine representation of the diurnal curve, allowed to vary in amplitude, and in
period with user latitude.

Fig.5.2 shows the four parameters of time delay algorithm for single frequency users.
They are: i) Constant DC term representing nighttime constant (DC), ii) Amplitude of
cosine term (A_m), iii) the phase of cosine term and iv) the period of the cosine term
(P).
The model uses the following expressions to compute the ionospheric delay at day and night times respectively.

\[ t_{Z_{ion}} = DC + A_m \cos(\frac{2\pi(t-t_0)}{P}) \]  \hspace{1cm} (5.5)

\[ t_{Z_{ion}} = DC \]  \hspace{1cm} (5.6)

where

\( t \) = local time at receiver (sec)

\( t_0 \) = local time of maximum ionospheric correction (say 14:00hrs)

\( t_{ion} \) = time delay due to the ionosphere in the zenith direction (sec)

\( DC \) = base ionospheric time delay (constant night time offset) (taken as \( 5 \times 10^{-9} \) s)

\( A_m \) = amplitude of the ionospheric delay function (sec)

\( P \) = period of the ionospheric delay function (sec)

The quantities \( A_m \) and \( P \) can be computed using the \( \alpha \) and \( \beta \) coefficients transmitted with navigation message.

\[ A_m = \sum_{i=0}^{4} \alpha_i \phi_{om} \]  \hspace{1cm} (5.7)

\[ P = \sum_{i=0}^{4} \beta_i \phi_{om} \]  \hspace{1cm} (5.8)

where \( \phi_{om} \) is the geomagnetic latitude of the ionospheric subpoint.
\[ \phi_{om} = \phi_i + 11.6 \times \cos (\lambda_i - 291) \]  \hspace{1cm} (5.9)

\( \phi_i \) and \( \lambda_i \) are the user's latitude and longitude in semi-circles. The "\( \alpha \)" terms are the coefficients of a cubic equation representing the magnitude of the vertical delay, and the "\( \beta \)" terms are the coefficients of a cubic equation representing the period of the model.

**Example estimation:**

Using Klobuchar \( \alpha \) and \( \beta \) terms and the receiver location information, ionospheric time delay in the zenith direction is estimated. The \( \alpha \) and \( \beta \) parameters are corresponding to RINEX format navigation message file observed on 15th Dec 2003.

\[
\begin{align*}
\alpha_1 &= 0.01304 \times 10^{-7}, \quad \alpha_2 = -0.1490 \times 10^{-7}, \quad \alpha_3 = -0.596 \times 10^{-7}, \quad \alpha_4 = 0.1192 \times 10^{-6} \\
\beta_1 &= 0.1167 \times 10^{6}, \quad \beta_2 = -0.2458 \times 10^{6}, \quad \beta_3 = 0.00 \times 10^{3}, \quad \beta_4 = 0.9830 \times 10^{-6}
\end{align*}
\]

Receiver position: Latitude \( (\phi_i) = 17.449^0 \), Longitude \( (\lambda_i) = 78.470^0 \), Height = 555m.

Satellite Azimuth \( (A_z) = 327.98^0 \)

Elevation \( (E) = 26.72^0 \)

\( h_{io} \) = altitude of ionospheric shell = 350km, \( r_e \) = earth radius = 6372km

\[ \phi_{om} = \text{geomagnetic latitude} = \phi_i + 11.6 \times \cos (\lambda_i - 291) = 7.66952^0 \]

\[ A_z = (0.01304 \times 10^{-7} ) \times (1) + ( -0.1490 \times 10^{-7} ) \times (7.66952)^1 + ( -0.596 \times 10^{-7} ) \times (7.66952)^2 + 0.1192 \times 10^{-6} \times (7.66952)^3 = 5.01562 \times 10^{-5} \]

\[ P = \sum_{i=0}^{4} \beta_i \phi_{om} = (0.1167 \times 10^{6} ) \times (1) + ( -0.2458 \times 10^{6} ) \times (7.66952)^1 + 0 + (0.9830 \times 10^{-6} ) \times (7.66952)^3 = 441694288 \]

Earth centered angle \( (\psi) = 90 - E - \sin^{-1}[ (r/r_e+h_{io}) \cos E] \)

\[ = 90 - 26.72 - \sin^{-1}[ (6372/6372+350) \cos (26.72) ] = 5.4247^0 \]

Longitude at the pierce point \( = \lambda_i + \sin^{-1}(\frac{\sin \psi \sin A_z}{\cos \phi_i}) = 75.48^0 \)

Longitude at the pierce point \( = \sin^{-1}(\sin \phi_i \cos \psi + \cos \phi_i \sin \psi \cos A_z) = 22.0145^0 \)

Local time at the pierce point \( (t) = \lambda_i/15 \ + UT = 20400 \text{ sec.} \)

\[ t_{ion} = 5 \text{nsec} + 5.01562 \times 10^{-5} \]

\[ = 5 \text{ nsec} + 0.000501562 \text{nsec} = 14.26 \text{ TECU} \]

However, the single frequency Klobuchar algorithm can remove the ionospheric delay up to 50% only. Therefore, for PA landings using GAGAN, a dual frequency receiver is needed to estimate the ionospheric delay precisely.
5.4 Time delay estimation algorithm for dual frequency GPS Receivers

Taking advantage of the ionosphere's dispersive nature, the time delay can be estimated by combining the P-code pseudorange measurements on both L1 and L2 (Eq. 3.12) or the L1 and L2 carrier phase measurements. However, the ionosphere-free linear combination has disadvantages. They are: i) it has a relatively higher observation noise; ii) it does not preserve the integer nature of the ambiguity parameters. To overcome the problems associated with the ionosphere-free linear combination, a new time delay algorithm is proposed, which combines both the code and carrier phase observations.

Time delay estimation using code and phase measurements:

To determine the real time ionospheric time delay, dual frequency GPS receiver data from TEC stations are used. It consists of both the code and carrier phase observations on L1 (1575.42 MHz) and L2 (1227.60 MHz) frequencies, denoted as P_i and \phi_i (i = 1, 2). The real time TEC for a satellite k can be estimated by using the following equations (Engler et al., 1996).

\[
P_{R1k} = A_k + \frac{40.3 \text{TEC}_k}{f_1^2} + E_{1k}
\]

\[
P_{R2k} = A_k + \frac{40.3 \text{TEC}_k}{f_2^2} + E_{2k}
\]

\[
\phi_{1k} = A_k + \frac{40.3 \text{TEC}_k}{f_1^2} + \lambda_1 N_1 - i_{1k}
\]

\[
\phi_{2k} = A_k + \frac{40.3 \text{TEC}_k}{f_2^2} + \lambda_2 N_2 - i_{2k}
\]

where,

\( \text{PR}_{1i}, \text{PR}_{2i} \) are the measured pseudoranges on L1 and L2 frequencies in m

\( A_k \) = sum of geometric range, tropospheric error and clock error in m

\( \text{TEC}_k \) = total electron content along the signal path (electrons/m²)

\( E_{1k}, E_{2k} \) are the sum of all errors in m due to instrumental delays and multipath for L1 and L2 code measurements
i_{1k}, i_{2k} are the sum of all errors in \( m \) due to instrumental delays and multipath for L1 and L2 phase measurements

\( N_1, N_2 \) are the phase ambiguities for L1 and L2 signal

\( f_1, f_2, \lambda_1, \lambda_2 \) are the frequency and wavelength of L1 and L2 signals respectively

\( \text{TEC}_k, \text{ionospheric delay} \) and the range measure \( A_{tk} \) are the unknown quantities. \( A_{tk} \) and \( T_{rk} \) can be calculated from the code measurements. \( A_{tk}, T_{tk} \) can be calculated from the phase measurements. Using these four parameters TEC can be estimated.

The factors \( A_{tk} \) and \( T_{rk} \) are given as,

\[
A_{tk} = \frac{f_1^2 PR_{1k} - f_2^2 PR_{2k}}{f_1^2 - f_2^2} \tag{5.14}
\]

\[
T_{rk} = \frac{PR_{2k} - PR_{1k}}{a} \tag{5.15}
\]

where,

\[
a = \frac{40.3 (f_1^2 - f_2^2)}{f_1^2 f_2^2} \tag{5.16}
\]

From Eqs.5.11, 5.12, 5.15 and 5.16

\[
A_{rk} = A_k + \frac{f_1^2 E_{1k} - f_2^2 E_{2k}}{f_1^2 - f_2^2} \tag{5.17}
\]

\[
T_{rk} = TEC_k + \frac{E_{2k} - E_{1k}}{a} \tag{5.18}
\]

Similarly \( A_{4k}, T_{4k} \) are given as,

\[
A_{\phi k} = \frac{f_1^2 \phi_{1k} - f_2^2 \phi_{2k}}{f_1^2 - f_2^2} \tag{5.19}
\]

\[
T_{\phi k} = \frac{\phi_{1k} - \phi_{2k}}{a} \tag{5.20}
\]

From Eqs.5.13, 5.14, and 5.18
From Eqs. 5.17, 5.18, 5.21 and 5.22, it can be observed that all the error effects are in the last terms.

Splitting the Eqns. depending on contributions to $E_{ik}$, Eq. 5.23 can be written for the error term of Eq. 5.17.

$$\begin{align*}
A_{\phi k} & = f_1^2 \lambda_1 N_{1k} - f_2^2 \lambda_2 N_{2k} = A_k - f_1^2 i_{1k} - f_2^2 i_{2k} \\
T_{\phi k} & = \frac{\lambda_1 N_{1k} - \lambda_2 N_{2k}}{a} = TEC_k - \frac{i_{1k} - i_{2k}}{a}
\end{align*}$$

(5.21) (5.22)

where, $b_{ik}$, $m_{ik}$ and $n_{ik}$ are instrumental biases, multipath errors and random noise respectively for satellite $k$ respectively. Similarly, splitting the Eqns. depending on contributions to $E_{ik}$, Eq. 5.24 can be written for the error term of Eq. 5.18.

$$E_{ik} - E_{ik} = b_{2k} - b_{1k} + m_{2k} - m_{1k} + n_{2k} - n_{1k}$$

(5.24)

Eq. 5.24 shows how the different measurement errors are separated. The term $T_{r_k} - T_{\phi_k}$ and $A_{r_k} - A_{\phi_k}$ are given as,

$$A_{r_k} - A_{\phi_k} = \frac{f_1^2 \lambda_1 N_{1k} - f_2^2 \lambda_2 N_{2k}}{f_1^2 - f_2^2} + \frac{f_1^2 b_{1k} - f_2^2 b_{2k}}{f_1^2 - f_2^2} + \frac{f_1^2 m_{1k} - f_2^2 m_{2k}}{f_1^2 - f_2^2}$$

(5.25)

$$T_{r_k} - T_{\phi_k} = \frac{\lambda_1 N_{1k} - \lambda_2 N_{2k}}{a} + \frac{b_{1k} - b_{2k}}{a} + \frac{m_{1k} - m_{2k}}{a} + \frac{n_{1k} - n_{2k}}{a}$$

(5.26)

By replacing the $b_{ik}$ biases with the bias predicted from the post processing and by solving Eqs. 5.24 and 5.25 for the ambiguities $N1$ and $N2$, TEC can be computed as

$$TEC_k = \frac{(\phi_{1k} - \phi_{2k}) + (\lambda_1 N_{1k} - \lambda_2 N_{2k})}{a}$$

(5.27)

The corresponding time delay = $\frac{40.3 \times TEC_k}{c f^2}$

(5.28)

5.4.1 IPP Location Importance

The grid based ionospheric modelling technique has been extensively used for SBAS (FAA-E2892B, 1997). The grid based ionospheric modelling technique assumes the ionosphere to be concentrated on a spherical shell of infinitesimal thickness located at the altitude of about 350 km above the earth’s surface. The implementation of the
single layer grid model requires computation of the intersection of the line of sight between the GPS receiver and the observed satellite on the ionospheric shell as shown in Fig 5.3 (Enge et. al., 1996). The intersection point of the GPS signal path with the ionospheric shell is defined as ionospheric pierce point (IPP) at which the slant ionospheric delay has an elevation angle of E. The IPP are the indicators of the presence of measured GPS data mapped on to the thin-shell surface. The number and locations of IPPs determine the accuracy, availability and integrity of all the grid related parameters considered under GAGAN. They depend on mainly two factors, their location and vertical delays. The IPP location depends on the location of TEC station, azimuth and elevation angles of the SV as viewed from the given TEC station. The density of IPPs is decided by multiplicity of the measurement taken at the distant TEC stations as two TEC stations located nearby may have their same IPPs. For a given azimuth, at 90° elevation the latitude and longitude of the TEC station becomes the location of corresponding IPP, while at cutoff elevation of 5°, the IPP location gets an substantial offset from the station location. An ionospheric grid model consists of grids distributed on the ionospheric shell in preset spacing usually 5° x 5°. The IPP, therefore, will fall within a specific grid defined by its surrounding four grid points.

![Fig 5.3 IPP and slant factor geometry](image)
5.4.2 IPP Location Algorithm

The appropriate geometry for estimating IPP latitude and longitude is presented in Fig.5.3. The IPP latitude ($\phi_{pp}$) and longitude ($\lambda_{pp}$) of IPP can be calculated by using the following equations.

The pierce point is offset from the reference station by an angle $\psi$, as defined from the earth center (Fig.5.3)

\[
\beta + (E+\pi/2) + \psi = \pi
\]

such that $\psi = \pi - (E+\pi/2) - \beta = \pi/2 - E - \beta \tag{5.30}$

where, $E = \text{satellite elevation angle at the ionospheric reference station}$.

The angle $\beta$ may be calculated from the sine rule as

\[
\frac{\sin(E + \pi/2)}{r + h_{io}} = \frac{\sin \beta}{r + h_{rs}} \tag{5.31}
\]

and

\[
\sin \beta = \frac{r + h_{rs}}{r + h_{io}} \sin(E + \pi/2) \tag{5.32}
\]

\[
\beta = \sin^{-1}\left( \frac{r + h_{rs}}{r + h_{io}} \sin(E + \pi/2) \right) \tag{5.33}
\]

where,

- $r_e = \text{mean earth radius}$
- $h_{io} = \text{height of ionospheric reference station}$
- $h_{s} = \text{altitude of ionospheric shell}$.

As all quantities on the right hand side of Eq.5.32 are known, the value of $\beta$ can be computed. From Eqs 5.32 and 5.29, the earth center angle can be obtained as,

\[
\psi = \pi/2 - E - \sin^{-1}\left( \frac{r + h_{rs}}{r + h_{io}} \sin(E + \pi/2) \right) \tag{5.34}
\]

where, all angles are defined in radians. The latitude and longitude of the pierce point are then derived from known coordinates of ionospheric reference station and the offset angle $\psi$ is given by

\[
\phi_{pp} = \phi_{io} + (\psi \cos(A_o)) \tag{5.35}
\]
\[ \lambda_{pp} = \lambda_m + (\psi \sin(A_x)/\cos(\phi_{pp}) \] (5.36)

where, \(A_x\) is the azimuth angle of the satellite
\(\phi_{pp}, \lambda_{pp}\) are latitude and longitude of the pierce point and
\(\phi_m, \lambda_m\) are latitude and longitude of the reference station

The azimuth (\(A_x\)) and elevation (\(E\)) angles to each satellite are computed as
\[ A_x = \tan^{-1}(y/x) \] (5.37)
\[ E = \tan^{-1}(-z/\sqrt{x^2 + y^2}) \] (5.38)

Where \(x, y\) are the user position coordinates in North-East-Down (NED) frame.

An example estimation is given here to appreciate the physical significance of IPP. Satellite Vehicle with PRN No. 21 is observed on 6th Jan. 2002 from the Bangalore reference station (12.966°N latitude/77.6°E longitude, 880m altitude). With \(r_e = 6372\text{km}, h_e = 880\text{m}, h_0 = 350\text{km}\), the computed values of \(\beta\) and \(\psi\) are 1.2098rad and 1.3635° respectively. Azimuth and elevation angles are calculated using the Eqs. 5.37 and 5.38 as 51.7314° and 9.3193° respectively. The latitude and longitude of the IPP location are computed as 14.2106° and 76.4112° respectively (Eqs. 5.35 and 5.36).

**Slant IPP delay to vertical delay conversion**

The slant ionospheric delay at an IPP, with elevation angle of \(E\) can be converted to the vertical ionospheric delay at the same location by a Slant Factor (SF). The SF for converting the slant delay to vertical delay is given by Eq.5.39

\[
\text{Slant ionospheric delay} = \text{SF} \times \text{Vertical ionospheric delay} \quad (5.39)
\]

\[
\text{Slant factor (SF)} = \left[ 1 - \left( \frac{e \cos E}{r_e + h_0} \right)^2 \right]^{-\frac{1}{2}} \quad (5.40)
\]

The conversion from slant TEC value into the vertical TEC value is realized by dividing the slant TEC with SF is given by

\[
\text{Vertical TEC} = \frac{\text{slant TEC}}{\text{SF}} \quad (5.41)
\]

At 0° elevation angle, the IPP delays is the minimum and it increases with increase in the elevation. The minimum cut off elevation angle of 5° is in general considered in data analysis, where the IPP delay becomes approximately one-third of the slant
delay, while at zenith the vertical delay reaches to maximum value equal to the slant delay.

**Example estimation:** SV PRN No.18 is observed with an elevation angle of 57.938°. The computed slant TEC is 30.095 VTECu. The slant factor is calculated as 1.1428 (Eq.5.30). Vertical TEC and the time delay are calculated as 26.141 VTECu (Eq.5.31) and 9.1627 nsec (Eq.5.30) respectively.

**5.5 Significance of instrumental biases in position accuracy**

The ionospheric time delay error is the dominant error that affects the positioning accuracy in SBAS. In dual frequency receivers, ionospheric delay is estimated precisely by measuring the difference in arrival times of the two frequencies (L1 and L2). GPS signal propagation speed in the ionosphere is characterized by its refractive index. The phase and group refractive indexes are derived to calculate the ionospheric effect on the GPS signal measurement.

The measurement accuracy of TEC thus far estimated from the dual-frequency GPS receiver is limited by instrumental biases. The instrumental bias is the difference of the two dispersive delays introduced by the analog hardware in the L1 and L2 signal paths. This limitation is due to the circuitry in the GPS satellite and receiver hardware. Even though the two signals are supposed to be transmitted from the satellite at the same time, they might be slightly off, due to circuitry differences and the signals of two GPS frequencies (L1 and L2) are not exactly synchronous. The receiver’s bias is more inferior, because they are low cost and do not usually have the atomic clocks that satellites have. The atomic clocks have superior performance compared to receiver clocks, they cost between few thousand dollars for the rubidium clocks to about $20,000 for the cesium clocks. Due to the crystal clocks in the GPS receivers, little clock discrepancies may cause big errors as signals are travelling at the speed of light (Gao and Liu, 2002).

The GPS satellite clocks, although highly accurate, are not perfect. Their stability is about 1 to 2 parts in $10^{13}$ over a period of one day. This means that the satellite clock error is about 8.64 to 17.28 nsec. per day. The corresponding pseudorange error is 2.59 m to 5.18m (Rabban, 2002). The pseudorange is a measure of the distance between the satellite at the time of the GPS signal emission and the GPS antenna at
the time of GPS signal reception. The transmitting time is measured through maximum correlation analysis of the receiver code and the GPS signal. The receiver code is derived from the clock used in the receiver. The GPS signal is generated by the clock used in the GPS satellite. The measured pseudorange is different from the geometric distance between the satellite and the receiver’s antenna because of the errors of the both clocks and influences of the signal transmitting medium. Code range measurements are obtained by measuring the transmitting time of the signal and multiplying it with velocity of light (c). A clock error δt cause a path length error of cδt. Similarly, a clock error of δt cause a phase error of cδt/λ. Because of the factor c, a small clock error may cause a very large code and phase error. Therefore, clocks on the satellite and receivers play a very important role in determining the position accuracy.

The instrumental biases comprises of two components. They are: hardware bias in the GPS receiver known as inter-frequency bias (bRF) or receiver differential delay and differential delay in the RF paths of the GPS satellite transmitter known as group delay (bgd).

Typical bias values of the Turbo Rogue receiver are usually in the range of +10 nanoseconds (nsec) of differential delay with an uncertainty of 0.2 nsec and the satellite transmitter biases lie in the range of 3 ns or 9 TECU (1 nsec of differential delay at L band = 2.85 TECU) (Brian et.al, 1999). Therefore, obtaining absolute measurements of TEC from GPS data requires the simultaneous estimation of the satellite biases (or the sum of the biases for uncalibrated receivers). Estimation of instrumental biases is also required to calibrate the calibrations of ionospheric modeling. Calibration of these biases is particularly important in GAGAN implementation, where the vertical position accuracy of 4.5m is required for precision approaches.

5.5.1 Significance of biases estimation in the GPS augmented systems
The receiver differential delay can be determined by internal calibration, though only few receivers provide this option. Moreover, most receivers show a poor stability of their differential group delays. The differential delays of the GPS satellites are determined during pre-launch factory tests. These values are included in the satellite broadcast message, but they do not agree with those values obtained from ground
measurements. Therefore, the GPS measurements are used to estimate combined satellite and receiver differential delays together with the TEC (Lanyi and Roth, 1988).

Calibration of hardware biases is particularly important in augmented GPS systems like WAAS/EGNOS/GAGAN. The WAAS serves better the requirements of USA, which comes in mid latitude region than it does any other part of the world. As India comes under equatorial region, where ionospheric time delay behavior is highly dynamic, more care is to be taken into account for developing ionospheric time delay model for GAGAN. A dedicated and modified WAAS system is necessary for India. In GAGAN the ionospheric corrections are estimated from dual-frequency receivers located at fixed TEC stations. If the satellite and receiver instrumental biases are not calibrated out, they propagate into the differential correction through the ionospheric models like grid based inverse distance weighted algorithm (Conker et. al, 1995). Although this is a second order effect in the pseudo-range domain, the geometric dilution of precision (GDOP) magnifies the effect in the position domain when computing a differential navigation fix.

Both software and hardware calibration techniques have been developed for estimating these errors. Hardware calibration is very reliable and generally more accurate in an instantaneous sense. However the $b_{SI}$'s and $b_{R}$'s are environmentally dependent and hence time varying. This is especially true for receiver $b_{R}$'s subject to large temperature variations. Since the INMMC of the GAGAN cannot continually monitor all reference receivers, some time varying error is always present, further each ionospheric TEC must be visited routinely, which may be expensive. Software calibration can occur continuously by taking the real time data of GPS observations from the dual frequency receivers. For this purpose a suitable bias estimation model must be developed to remove the ionosphere's contribution to the $L_1$-$L_2$ difference measurement.

5.6 Results and discussion

The GPS data required for calculation of TEC values are acquired from a dual frequency GPS receivers located at Hyderabad and Bangalore reference stations. Fig. 5.4 shows the GPS receiver data processing and various parameters estimation. To estimate the time delay, the data corresponding to 18th April 1998 and 6th January 2000 are considered. The vertical TEC values are calculated corresponding to each
GPS satellite by using the Eqs. mentioned in the previous section. The Azimuth and elevation angles are calculated using Eqs. 5.36 and 5.37. The estimated TEC values along with satellite azimuth, elevation angles and the IPP location of the satellites are given in Table 5.1. For example, satellite vehicle with SV 2 is observed with an azimuth and elevation angles of 289.333° and 31.90° respectively. The IPP latitude and longitude estimated for Hyderabad reference station are 18.858° and 74.055° respectively. The slant factor (SF) and vertical ionospheric delay calculated using Eqs.5.40 and 5.41 are 1.72 m and 26.19 m respectively. Slant factor is inversely proportional to the satellite elevation angle. When the elevation angle is 90° i.e., when the satellite is at zenith of the Hyderabad reference station and SF is 1. For example, at elevation angle of 57.93° the slant factor (SF) SV 18 is 1.15 and at an elevation angle of 21.54°, the SF on satellite SV 15 is 2.13. Also, it is observed that TEC is maximum at lower elevation angles and is minimum at the zenith point due to shortest propagation path. However, at 40.81° elevation angle of SV 13 a value of 11.62 TECu is observed, which is smaller than the TEC at elevation angle of 58.65° (SV 19), because the satellite signal path is different and passes through active ionospheric region. Fig. 5.5 presents the time delay estimation of SV 1. The standard deviation of the time delay observed in TECu is 18.62 at L1 frequency are 6.524 m and 3.023 nsec. This data are used to estimate the satellite and receiver biases. Fig. 5.6 shows the vertical TEC variation in cilm² observed for the GPS satellites (SVs. 2, 7, 5 and 15). To reduce the noise in the data, the TECu data are smoothed over a period of 300 sec. This duration of 300 sec. corresponds to the 10 measurement epochs (30 sec. interval) in the RINEX format observation data.

![Flowchart](image-url)

**Fig. 5.4 GPS receiver data processing and various parameters estimation**
### Table 5.1 Ionospheric time delay estimation

<table>
<thead>
<tr>
<th>SV</th>
<th>Elevation (deg.)</th>
<th>Azimuth (deg.)</th>
<th>IPP Lat (deg.)</th>
<th>IPP Long (deg.)</th>
<th>SF</th>
<th>VTEC (TECU)</th>
<th>Time Delay (nsec.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>27.304</td>
<td>212.754</td>
<td>12.934</td>
<td>75.608</td>
<td>1.889</td>
<td>25.491</td>
<td>8.935</td>
</tr>
<tr>
<td>13</td>
<td>40.813</td>
<td>306.884</td>
<td>19.401</td>
<td>75.718</td>
<td>1.461</td>
<td>11.619</td>
<td>4.072</td>
</tr>
<tr>
<td>18</td>
<td>57.938</td>
<td>190.335</td>
<td>15.598</td>
<td>78.206</td>
<td>1.151</td>
<td>26.141</td>
<td>9.162</td>
</tr>
<tr>
<td>27</td>
<td>35.701</td>
<td>334.194</td>
<td>20.973</td>
<td>76.705</td>
<td>1.601</td>
<td>20.611</td>
<td>7.224</td>
</tr>
</tbody>
</table>

![Time delay observations of SV 1](image)

- **Time delay observations of SV 1:**
  - Jan 06 2000
  - Time delay (TECU)

- **Standard deviation of time delay:** 3.023 nsec.
- **Mean:** 13.9 nsec., **Minimum value:** 5.883 nsec., **Maximum value:** 17.91 nsec.

- **Time delay (meters):**

- **Time delay (nano seconds):**

- **Local time in seconds**

*Fig. 5.5 Time delay estimation of SV 1*
5.7 Conclusions

Ionospheric time delay effect on GPS signal measurements are presented. Various ionospheric error correction models available for single frequency and dual frequency GPS receiver users are discussed. Klobuchar time delay model is widely used model in single frequency GPS receivers due to its simplicity. Any GPS user may access Klobuchar model coefficients in the broadcast navigation message. The Klobuchar model has shortcomings regarding its low accuracy and less detailed description of the ionosphere's behaviour. Due to the disadvantages associated with the time delay algorithms of ionosphere-free linear combination of P-code pseudorange measurements on both L1 and L2 or the L1 and L2 carrier phase measurements, new time delay estimation algorithm using dual frequency measurements for code and phase measurements has been presented. Typical results are also presented. Significance of instrumental biases in the position accuracy of augmentation system are highlighted.