CHAPTER – 4

OPTIMUM PLANS FOR MARGINAL FARMERS USING FRACTIONAL PROGRAMMING MODEL FOR MAXIMIZING LAND PROFITABILITY

4.1 INTRODUCTION

Fractional Programming Problem (L.P.P.) arises when the objective function appears as a quotient. A Fractional Programming Problem (L.P.P.) is expressed as :-

\[ \max_{x \in X} \frac{f(x)}{g(x)} \]

Where \( f, g : \mathbb{R}^n \rightarrow \mathbb{R} \neq 0 \), and \( X \) is the set of feasible solutions. When \( f \) and \( g \) are affine linear and the feasible region is convex polyhedron, we get a linear fraction programming problem.

A general L.F.P.P. may be stated as follows:-

\[ \max_{x \in X} \frac{c'x + \alpha}{d'x + \beta} \]

Where \( x = \{ x \in \mathbb{R}^n / Ax \leq b, x \geq 0 \} \), the set of all feasible solution.

\[ c, d \in \mathbb{R}^n ; \ \alpha, \beta \in \mathbb{R} \text{ and } d'x + \beta > 0 \forall x \in X \text{ (The case when the denominator may be zero is considered in Martos (1964).) } \]
Solution method for L.F.P.P. have been developed by (Isbell and Marlow (1956), Charnes and Cooper (1962), Martos (1964) and Kanti swarup (1965). The best known method out of these is the one developed by kanti Swarup (1965).


In the realm of programming models, Fractional Programming technique has potential application in agriculture. Maximization of production and / or profit is no doubt, the aim of every producer but a more desirable objective from economic point of view to maximize productivity i.e. production per unit of resource like land, labour, capital or time, mathematical formulation of such problems generally leads to Fractional Programming type problems.

Agricultural scientists have paid very little attention to applications of Fractional Programming in agriculture. Aggarwal (1969) illustrated an application of L.F.P.P. in agriculture through a small hypothetical example. No empirical studies seems to have been conducted for applications of L.F.P.P. models on real life agriculture data.

In the previous chapter two different optimum plans have been developed for maximizing total net return of the farmers using linear programming approach. Linear programming has been extensively used in agricultural research to maximize production and / profit and minimizing cost of diet etc [Desai (1960), Vande Panne and Popp (1963), Singh et. Al. (1972), Rai and Singh (1982) and Singh and Sharma (1988) etc.]. Maximization of production and or net
return is no doubt the aim of every producer but in a predominantly agricultural economy as ours, a more desirable objective from national point view will be to maximize productivity (production/resource) and/or profitability (profit/resource). Productivity implies a simple ratio of output to input. In agriculture, we generally consider the following type of productivity:

In most of the cases each farmer uses several enterprises like crop farming, dairying, piggery farming, poultry farming etc. which are measured in different units and it is impossible to add their production directly so the production of each activity may be evaluated in terms of its value (i.e. quantity X selling price) and net returns are calculated for each enterprises. Therefore, instead of finding productivity we generally evaluate profitability with respect to land, defined as follows:

There are many situations in agriculture where the usual technique of maximization of production or profit by applying L.P. model is not very suitable because the objective function may be optimization of the ratios of two linear function. If there is a shortage of land then the farmer may be interested to maximize returns per hectare of land (i.e. land profitability). In each of these cases the objective function is the ratio of two linear functions, which directly leads into the use of Linear Fractional programming problem (L.F.P.P.) model in agriculture. However, Aggarwal (1969) has suggested the application of this technique in agriculture only through a simple hypothetical example.

In this chapter an attempt has been made to apply L.F.P.P. in a real agricultural situation and
on the basis of empirical analysis the following optimum plans have been developed.

1. **F.P. Plan – I** : Maximizing land profitability with restriction on livestock enterprises


A comparative study has also been made in other to compare the net returns in various optimum plans so developed. Finally, the results obtained by L.F.P.P. model are compared with the results already obtained using L.P. model in the previous chapter.

### 4.2 COMPUTER PROGRAMME FOR THE SOLUTION OF LINEAR FRACTIONAL PROGRAMMING PROBLEM (LFPP)

Non availability of a good computer programme for solving L.F.P.P. was a great hindrance. So the L.F.P.P, problem was solved by MS Exel.

### 4.3 MAXIMIZATION OF LAND PROFITABILITY

For marginal farmers, land is the most scarce resource therefore maximization of net return per hectare of land is more important objective as compared to simply maximizing the net returns. Therefore, there is a need to search such an optimum combination of crop farming and live stock activities, which may provide maximum net returns to the farmer by using minimum cropped area. It can be done by applying Fractional Programming Model. For maximizing land profitability of marginal farmer, two different optimum plans viz. F.P. Plan I and F.P. Plan II are suggested under the following two situations ;
(i) Maximization of land profitability with restriction on livestock enterprises (F.P.Plan I).

(ii) Maximization of land profitability with no restriction on livestock enterprises (F.P.Plan II)

4.3.1 F.P. Plan I and its Optimum Solution:

This plan was specially formulated under existing situation in the study area. As the farmers do not want to deviate much farm their usual practices of using same crop rotations and by keeping limited number of milch animals, therefore, an attempt was made to search an optimum combination of existing crop rotation and animals this plan (F.P.Plan I), for maximizing land profitability of marginal farmers.

The object function for the F.P.Plan I model is as follows:-

Maximize

\[
\text{Total net return on farm (in Rs.)} = \frac{\text{Net return from crop farming} + \text{net returns from labour Employment outside} + \text{net return from livestock activities.}}{\text{Total land available (in hectare)}}
\]

Where,

\[
\text{Total net return on farm} = \text{Net return from crop farming} + \text{net returns from labour Employment outside} + \text{net return from livestock activities.}
\]

And,

\[
\text{Total land available} = \text{total irrigated land} + \text{total unirrigated land} + \text{land used for keeping animals, agricultural equipments etc.}
\]

During the survey it was observed that, marginal farmers generally keep some fixed land only for maintaining milch animals, bullocks, agricultural equipments, machines manure and fertilizers etc. Information was collected from individual farmers in the sample for such type of fixed land and an estimate of such land was obtained by taking the sample mean. It was observed that on an average .0166 hectare (estimate of fixed land) of land was occupied in this manner. On the basis of the
estimates of different parameters obtained in chapter 111. The objective functions F.P. Plan I model is defined as below:

(a) The objective function of ‘F.P.Plan I’ model:

The objective function of F.P.Plan I may be stated as:

\[ \text{Maximize } Z = \sum_{j=1}^{36} C_j X_j \]

Where,

\[ \sum_{j=1}^{19} C_j X_j \] gives the net return from crop farming

\[ \sum_{j=21}^{32} C_j X_j \] gives the net return from labour employment outside

\[ \sum_{j=1}^{36} C_j X_j \] gives the net returns from livestock activities

\[ \sum_{j=1}^{15} d_j X_j \] gives the total irrigated area

\[ \sum_{j=16}^{19} d_j X_j \] gives the total unirrigated area

\[ \beta = \text{an estimate of fixed land} \]

Thus the objective function is

\[ [24630X_1 + 28188X_2 + 16410X_3 + 24990X_4 + 32040X_5 + 18300X_6 + 18600X_7 + 21300X_8 + 22050X_9 + 22476X_{10} + 14850X_{11} + 11610X_{12} + 30240X_{13} + 25260X_{14} + 9060X_{15} + 7830X_{16} + 11780X_{17} + 12810X_{18} + 10236X_{19} + 0X_{20} + 60X_{21} + 60X_{22} + 60X_{23} + 60X_{24} + 60X_{25} + 60X_{26} + 60X_{27} + 60X_{28} + 60X_{29} + 70X_{30} + 70X_{31} + 70X_{32} + 80X_{33} + 19077X_{34} + 22392X_{35} + 14220X_{36} ] \]
Maximize \( Z = \frac{\sum X_{1} + X_{2} + X_{3} + X_{4} + X_{5} + X_{6} + X_{7} + X_{8} + X_{9} + X_{10} + X_{11}}{X_{12} + X_{13} + X_{14} + X_{15} + X_{16} + X_{17} + X_{18} + X_{19}} + 0.0001 \)

\[ (A1) \]

(b) **Constraints of the F.P. Plan I model:**

Objective function (A 1) defined above is maximized subject to the same linear constraints no. (1) to (27), along with non negativity constraints (28) of the model formulated in L.P. Plan I in chapter III. The above problem is now a standard linear fractional programming problem, with 27 constraints and 36 decision variables. It was solved on computer with the help of computer programme MS-Excel.

**Table 4.1 : Optimum Values of the Decision Variable**

<table>
<thead>
<tr>
<th>Decision Variables</th>
<th>Value</th>
<th>Decision Variables</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original ( X_{1} )</td>
<td>.129</td>
<td>( X_{32} )</td>
<td>42.600001</td>
</tr>
<tr>
<td>( X_{2} )</td>
<td>.091</td>
<td>( X_{35} )</td>
<td>2.0</td>
</tr>
<tr>
<td>( X_{20} )</td>
<td>1.0</td>
<td>( X_{38} )</td>
<td>0.02</td>
</tr>
<tr>
<td>( X_{21} )</td>
<td>45.58001</td>
<td>Surplus</td>
<td>( X_{41} )</td>
</tr>
<tr>
<td>( X_{22} )</td>
<td>43.06001</td>
<td>Slack</td>
<td>( X_{42} )</td>
</tr>
<tr>
<td>( X_{23} )</td>
<td>44.34001</td>
<td></td>
<td>( X_{43} )</td>
</tr>
</tbody>
</table>
### Optimum value of numerator (profit)

\[ \text{Optimum value of numerator (profit)} = \text{Rs. 96808.07} \]

### Optimum value of denominator (land)

\[ \text{Optimum value of denominator (land)} = \text{Rs. 237500} \]

### Optimum value of objective function (land profitability)

\[ \text{Optimum value of objective function (land profitability)} = \text{Rs. 407612.92} \]

#### 4.4 INTERPRETATION OF THE OPTIMUM SOLUTION

(A) **Net returns from crop rotations:**

The net returns from crop rotations appeared in the optimum solution are presented in table 4.2.
Table 4.2: Net return from Crop Rotations

<table>
<thead>
<tr>
<th>SNo.</th>
<th>Crop Rotation</th>
<th>Area allocated (in ha)</th>
<th>Net return per ha (in Rs.)</th>
<th>Total net return (in Rs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Bajra-Wheat (HYV)</td>
<td>.129 (58.65%)</td>
<td>24630.00</td>
<td>3177.27 (57.91%)</td>
</tr>
<tr>
<td>2</td>
<td>Maize -wheat (HYV)</td>
<td>.091 (41.4%)</td>
<td>25188</td>
<td>2292.10 (42.09%)</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>.22</td>
<td></td>
<td>5469.37</td>
</tr>
</tbody>
</table>

Unused land: Irrigated = .29 ha
Unirrigated = .21 ha
Total = .50 ha

It reveals from table No. 4.2 that only two crop rotation out of 19 entered in the optimum solution and occupied 31.88% of the total available land area. The remaining 68.12% land remained unused. One of the reason for it may be that our F.P. Plan I is formulated in such a way that it will utilize minimum area and the maximum net return may come from other activities, viz. employment of labour outside and keeping of milch animals. It was observed that crop farming is not beneficial for the farmers, it appears in the optimum solution only for the production of material, required for family consumption. The crop rotation 'Bajra-Wheat (HYV), was found to be beneficial for farmers as it occupies maximum area (58.6%) and contributes maximum (57.91%) net returns.

(C) Net returns from labour employment outside the family farm:

Monthwise net returns from family labour employment outside are presented in table 4.3.
Table 4.3: Net Returns from Labour Employment

<table>
<thead>
<tr>
<th>Months</th>
<th>Labour Used (in mandays)</th>
<th>Labour available (in mandays) for outside employment</th>
<th>Total net return (in Rs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>July</td>
<td>46.41999</td>
<td>45.58001</td>
<td>2734.81</td>
</tr>
<tr>
<td>August</td>
<td>48.93999</td>
<td>43.06001</td>
<td>2583.70</td>
</tr>
<tr>
<td>September</td>
<td>48.65999</td>
<td>44.34001</td>
<td>2660.40</td>
</tr>
<tr>
<td></td>
<td>50.47999</td>
<td>34.52002</td>
<td>2071.20</td>
</tr>
<tr>
<td>November</td>
<td>47.67999</td>
<td>37.32001</td>
<td>2239.20</td>
</tr>
<tr>
<td>December</td>
<td>42.75999</td>
<td>42.24001</td>
<td>2534.40</td>
</tr>
<tr>
<td>January</td>
<td>42.29999</td>
<td>42.70001</td>
<td>2562.00</td>
</tr>
<tr>
<td>February</td>
<td>42.14999</td>
<td>49.60001</td>
<td>2976.00</td>
</tr>
<tr>
<td>March</td>
<td>41.99999</td>
<td>48.00002</td>
<td>2880.00</td>
</tr>
<tr>
<td>April</td>
<td>47.13999</td>
<td>44.86001</td>
<td>3140.20</td>
</tr>
<tr>
<td>May</td>
<td>42.59999</td>
<td>42.44002</td>
<td>2970.80</td>
</tr>
<tr>
<td>June</td>
<td>42.39999</td>
<td>42.60001</td>
<td>2982.00</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>32334.70</td>
</tr>
</tbody>
</table>

It is clear from table 4.3 that maximum number of mandays 49.6 are available for outside employment in the month of February, also in each month mandays available are sufficient in number, ranging between 34.52002 to 49.60001. It is therefore, concluded that marginal farmers can earn a total of Rs. 32334.70 by proper outside employment of their family members.

**D) Net returns from milch animals:**

The optimum number of milch animals entered in the optimum plan and corresponding net returns are presented in table 4.4.

Table 4.4: Net returns from Milch Animals
<table>
<thead>
<tr>
<th>Animal bred</th>
<th>No. of Animals</th>
<th>Net return per animal</th>
<th>Total net returns (Rs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buffalo (Murrah)</td>
<td>2</td>
<td>22392.00</td>
<td>44784.00 (75.8%)</td>
</tr>
<tr>
<td>(Sahiwal- jersey )Cow</td>
<td>1</td>
<td>14220.00</td>
<td>14220.00 (24.2%)</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td></td>
<td><strong>59004.00</strong></td>
</tr>
</tbody>
</table>

From table 4.4, it is clear that maximum number of milk animals according to imposed restriction were appeared in the optimum plan. The maximum net returns (75.08%) is from keeping of Buffaloes (Murrah). It indicates that dairy can be adopted as other source of income in order to improve financial position of marginal farmers.

Net Return under F.P. Plan I at a Glance

On the basis of table 4.2, 4.3, 4.4 the total net returns through various activities can be summarized in the table 4.5.

**Table 4.5 : Net Returns Through Various sources in F.P. Plan – I**

<table>
<thead>
<tr>
<th>Source</th>
<th>Net returns</th>
<th>Contribution to total net returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Through Crop planning</td>
<td>5469.37</td>
<td>5.64%</td>
</tr>
<tr>
<td>Through Labour employment outside</td>
<td>32334.7</td>
<td>33.40%</td>
</tr>
<tr>
<td>Through keeping of milk animals</td>
<td>59004</td>
<td>60.96%</td>
</tr>
</tbody>
</table>
The optimum value of land profitability = 407612092 Rs. Per hectare table 4.5 shows that maximum contribution is 60.96% in the total net return is from live-stock activities and the minimum contribution (5.64%) is through crop farming. It is therefore, concluded that F.P. Plan I, can be used efficiently if the land available for crop farming is very small, as it will provide more profit as compared to existing unplanned situation and will least affect the farmers from their existing habits.

4.5 F.P. Plan II AND ITS OPTIMUM SOLUTION

In FP Plan I, there were upper bound restrictions on the number of milch animals, therefore total farm income could not be increased much because in the optimum plan the number of milch animals were at the highest level. With a view to increase farm income still further, the upper bound restriction on the number of milch animals was relaxed in the FP Plan II.

(a) The objective function of F.P. Plan II model:

The objective function of the model in FP Plan II remains same as described in 4.3 (a) for F.P. Plan I.

(b) Constraints of the model F.P. Plan I:

(c) The constraints of the model in F.P. Plan II are same as the constraints of the model L.P. Plan II, already described in Chapter III. In this case there is no restriction on the number of milch animals to be maintained. In all, there are 25 constraints along with non-negativity constraint $X_j \geq 0$, $j = 1, 2, \ldots 36$.

The above problem is again a Linear Fraction Programming Problem with 25 constraints and 36 decision variables. It was solved on computer and optimum solution was obtained.
The optimum solution obtained for FP Plan II was found to be exactly same as the optimum solutions of L.P. Plan II already explained in chapter III. This is so because if there is no restriction on live stock enterprise then crop farming is the least profitable activity and is required only for family consumption.

4.6 INTERPRETATION OF THE OPTIMUM SOLUTIONS

As the optimum solution of FP Plan II was found to be same as that of L.P. Plan II for which the detailed interpretation of result with respect to net returns from crop farming, labour employment outside the family farm, and through live stock activities are available in chapter III.

Table 4.6 Net Return under Optimum FP Plan II, at a Glance

<table>
<thead>
<tr>
<th>Source</th>
<th>Net returns</th>
<th>Contribution to total net returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Through Crop planning</td>
<td>4336.52</td>
<td>2.9%</td>
</tr>
<tr>
<td>Through Labour employment</td>
<td>3441.23</td>
<td>2.3%</td>
</tr>
<tr>
<td>outside</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Through keeping of milch</td>
<td>143826</td>
<td>94.8%</td>
</tr>
<tr>
<td>animals</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>156103.75</td>
<td>100.00</td>
</tr>
</tbody>
</table>

The optimum value of land profitability = 407612.92 Rs. / hectare.
reveals from table 4.6 that in F.P. Plan II, is better than FP Plan I as it gives an increase in total net returns over FP Plan 1. Also in this the maximum contribution in net returns (94.8%) is from livestock activities and minimum contribution is through labour employment side. The main advantage in F.P. Plan II is that almost all the family labour is employed on the family farm itself for the maintenance of animals and therefore, it is suggested that if there is possibility of purchasing animals by taking government loans etc., then FP Plan || will be a most profitable optimum plan for marginal farmers.

4.7 COMPARATIVE STUDY OF F.P. PLAN I WITH F.P. PLAN II

(A) Comparison with respect to total net returns:
On the basis of table 4.5, 4.6 and figure 4.1 it reveals that there is an increase in total net-return of F. P. Plan II as compare to F.P. Plan I. In F.P. Plan II, the major contribution in net returns is from keeping of milch animals and it is higher than the contribution of livestock activities in F.P Plan 1. It is therefore, suggested that in the existing situation if the farmer do not want to deviate much from their existing practices like wage earning, F.P. Plan I will provide him much more net returns than what they are getting in unplanned conditions. However if it is possible for them to keep more number of milch animals, then certainty of F.P. Plan II will be a better plan. Hence both these plans can be used in different circumstances for marginal farmers.

(B) Comparison with respect to utilization of family labour on farm:
On the basis of table 4.3 and 3.11 Fig. 4.2 was prepared to compare the pattern of utilization of labour on farm in both the suggested plans. Fig. 4.2 shows that F.P. Plan II is far better than F.P. Plan I as compared to utilization of family human labour. Most of the family labour in F.P. Plan arc involved in maintenance of livestock activities. Finally we may conclude that, as compared to total net returns from farm and with respect to maximum utilization of family
labour F.P. Plan II is a best plan and it is recommended for marginal farmers for improving their economic social status.
Fig. 4.1

COMPARATIVE STUDY OF NET RETURNS UNDER F.P. PLAN-I & F.P. PLAN-II

NET RETURNS (RS.)

F.P. PLAN-I

F.P. PLAN-II
Fig. 4.1

COMPARATIVE STUDY OF NET RETURNS UNDER F.P. PLAN-I & F.P. PLAN-II

NET RETURNS (Rs.)

- CROP PLANNING
- LABOUR
- EMPLOYMENT OUTSIDE
- MILCH ANIMALS
- TOTAL