CHAPTER 10

A COMPARISON BETWEEN WMET AND WHMET SERRATED CATRs

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A COMPARISON BETWEEN WMET & WHMET SERRATED CATRs

The design of the reflector with serrations is the crux of the CATR. A CATR reflector without serrations causes undesirable diffraction that results in fluctuations in the quiet zone field. In a Compact Range, incident plane wave front is generated by a field and one or more reflectors in the immediate vicinity of the test zone. Such ranges bring far field measurements inside a protected laboratory environment. The field of compact range technology is very broad and complex. The offset paraboloid configuration was used in the first commercial compact range developed by Scientific-Atlanta. For this configuration, a trade off exists between the reflector focal length and the feed pattern, which effects the aperture illumination taper, edge illumination, back lobe radiation and the resulting test zone size. Diffraction from the edges of a compact range reflector will degrade the performance of the range because the edge becomes a source of stray radiation. Stray radiation, the shape, dimensions of the Plane Wave Zone (PWZ), efficiency factor, optimum CATR feed design, reflector surface accuracy and operational frequency range are the most important factors that will influence overall performance and CATR compactness.

A CATR is designed to produce a plane wave in a limited region of an anechoic room. Such a range mostly consists of one or two reflectors, which are illuminated by a point source. According to the geometrical optic laws, the reflected near field is constant in phase. The amplitude will be tapered due to space attenuation effects and the feed radiation pattern.
this near field region, a test antenna can be placed so that its radiation properties can be measured under nearly far field conditions.

In this chapter, an attempt is made to compare the quiet zone performance of a plane square aperture reflector employing two different types of serrations, viz., a) Width Modulated Exponential and Width Modulated Triangular here afterwards referred to as WMET and b) Width and Height Modulated Exponential and Width and Height Modulated Triangular here afterwards referred to as WHMET.

The WMET serrations described by the boundary functions \( g'(y') \) and \( g'(y') \) are expressed as Fourier series of Width Modulated Exponential with a rate of rise \( 'a_1' \):

### 10.1 WIDTH MODULATED EXPONENTIAL SERRATIONS

\[
g_\text{WME}(y) = a_0 + (t/2p_0) \{ p_1 - p_2 + p_3 - p_4 + p_5 + p_6 \} + \frac{(\exp(-a_1 p_1) - 1)}{a_1} + \frac{(\exp(-a_2 (p_3 - p_2)) - 1)}{a_2} + \frac{(\exp(-a_3 (p_5 - p_3) - 1))}{a_3} + \frac{2t}{p_6} Z(y')
\]

Where \( Z(y') = \sum_{n=1}^{\infty} \{ (\sin q_1/q - (\exp(-a_1 p_1) b_1 \{ -a_1 \cos q_1 + q \sin q_1 \}) + a_1 b_1 \}

\[
- (1/(p_2 - p_1)) \{(p_2 \sin q_2 - p_1 \sin q_1)/q^2 - (\cos q_2 - \cos q_1)/q - p_1 \sin q_1)/q\}

+ \{(\sin q_3 - \sin q_2)/q - \exp(-a_2 (p_3 - p_2)) b_2 (-a_2 \cos q_3 + q \sin q_3)

+ b_2(-a_2 \cos q_2 + q \sin q_2) - 1/(p_4 - p_3) \{(p_4 \sin q_4 - p_3 \sin q_3)/q + (\cos q_4 - \cos q_3)/q^2

- p_4 (\sin q_4 - \sin q_3)/q\} + \{(\sin q_5 - \sin q_4)/q - \exp(-a_3 (p_5 - p_4)) b_3 (-a_3 \cos q_5 + q \sin q_5)

: 173 :
\[ +b_3(-a_3 \cos q \theta + q \sin q \theta) - \frac{1}{(p_0 - p_2)} \{(p_0 \sin q \theta - p_2 \sin q \theta) / q \} \cos(q y') \]
\[ +p_0(-\sin q \theta + \sin q \phi) / q \} \cos(q y') \]

in which \( q = n \pi / p_0, \) \( q_i = q p_i \) and \( b_i = 1 / (a_i^2 + q^2) \)

The boundary functions \( h^+(x') \) and \( h'(x') \) are described as the Fourier series of Width Modulated Triangular function as below:

**10.2 WIDTH MODULATED TRIANGULAR SERRATIONS**

\[ h^+(x') = a_0 / 2 + t/2 + (2t p_0 / \Pi ) \cdot Z(x') \]

where \( Z(x') = \sum_{n=1}^{\infty} 1/n^2 \left[ \{1 / (p_1) + 1 / (p_2 - p_1)\} \cos q_i \right. \]
\[ - \{1 / (p_2 - p_1) + 1 / (p_3 - p_2)\} \cos q_2 \]
\[ + \{1 / (p_3 - p_2) + 1 / (p_4 - p_3)\} \cos q_3 \]
\[ - \{1 / (p_4 - p_3) + 1 / (p_5 - p_4)\} \cos q_4 \]
\[ + \{1 / (p_5 - p_4) + 1 / (p_6 - p_5)\} \cos q_5 \]
\[ \left. - \{1 / (p_6 - p_5)\} \cos n \Pi - (1/p_1) \} \cos(q y') \right] \]

in which \( q = n \Pi / p_0 \) and \( q_i = q p_i \)
The WHMET serrations described by the boundary functions $g'(y')$ and $g(y')$ are expressed as Fourier series of Width and Height Modulated Exponential serrations as below:

### 10.3 WIDTH AND HEIGHT MODULATED EXPONENTIAL SERRATIONS

$$g^*(y') = \frac{a_0}{2} + \frac{t_1}{p_0} \left[ (p_2 + p_1)/2 + (1/a_1) \left( \exp \left( -a_1 p_1 \right) - 1 \right) \right]$$

$$+ \frac{t_2}{p_0} \left[ (p_4 + p_3 - 2 p_2)/2 + 1/a_2 \left( \exp \left( -a_2 (p_3 - p_2) \right) - 1 \right) \right]$$

$$+ \frac{t_3}{p_0} \left[ (p_6 + p_5 - 2 p_4)/2 + 1/a_3 \left( \exp \left( -a_3 (p_5 - p_4) \right) - 1 \right) \right] + (2/p_0) Z(y')$$

where $Z(y') = \sum_{n=1}^{\infty} t_1 \{(\sin q_1)/q - \exp \left( -a_1 p_1 \right) b_1 (a_1 \cos q_1 + q \sin q_1) + a_1 b_1\}$

$$- t_1/2 (p_2-p_1) \{(1/q^2) (\cos q_2 - \cos q_1) + (\sin q_1/q) (p_2 - p_1)\}$$

$$+ t_2 \{(\sin q_3 - \sin q_2)/q - \exp \left( -a_2 (p_3 - p_2) \right) b_2 (-a_2 \cos q_3 + q \sin q_3)\}$$

$$+ b_2 (-a_2 \cos q_2 + q \sin q_2)\} - t_2/2 (p_4-p_3) \{(\cos q_4 - \cos q_3)/q^2 + (\sin q_4/q) (p_4 - p_3)\}$$

$$+ t_3 \{(\sin q_5 - \sin q_4)/q - \exp \left( -a_3 (p_5 - p_4) \right) b_3 (-a_3 \cos q_5 + q \sin q_5)\}$$

$$+ b_3 (-a_3 \cos q_4 + q \sin q_4)\} - t_3/2 (p_6-p_5) \{1/q^2 (\cos q_6 - \cos q_5)\}$$

$$+ (\sin q_5/q) (p_6 - p_5)\} \cos(q y')$$

in which $q = n 11/p_0$; $q_i = q p_i$ and $b_i = 1/(a_i^2 + q^2)$
The Fourier series representation of the Width and Height Modulated Triangular serrations is given below:

10.4 WIDTH AND HEIGHT MODULATED TRIANGULAR SERRATIONS

\[ h^+(x') = a_0 / 2 + (1/2p_6) [t_1 p_2 + t_2 (p_4 - p_2) + t_3 (p_6 - p_4)] + 2 Z(x') \]

where \( Z(x') = \sum_{n=1}^{\infty} \left[ (t_1 / p_6 p_4) \{p_4 \sinq_4 \} / q + (\cosq_4) / q^2 - 1 / q^2 \right] \)

\[ + \frac{t_1}{p_6 (p_2 - p_4)} \{p_4 \sinq_4 \} / q + (1 / q^2) \{\cosq_4 - \cosq_2\} \]

\[ + \frac{t_2}{p_6 (p_3 - p_2)} \{p_3 \sinq_3 \} / q + (1 / q^2) \{\cosq_3 - \cosq_2\} \]

\[ + \frac{t_2}{p_6 (p_4 - p_3)} \{p_3 \sinq_3 \} / q + (1 / q^2) \{\cosq_3 - \cosq_4\} \]

\[ + \frac{t_3}{p_6 (p_5 - p_4)} \{p_4 \sinq_4 \} / q + (1 / q^2) \{\cosq_4 - \cosq_2\} \]

\[ + \frac{t_3 / p_6 (p_6 - p_5)}{\{p_5 \sinq_5 \} / q + (1 / q^2) \{\cosq_5 - \cosq_4\}\} \cos (q x') \]

in which \( q = n \Pi / p_6 \) and \( q_i = q p_6 \)

10.5 RESULTS

A Compact Range reflector with a square aperture produces ripples in the quiet zone. The ripples can be reduced by adorning the square with serrations. A square aperture of dimension \( 45\lambda \times 45\lambda \) is equipped with WMET and WHMET serrations as shown in Fig 10.1 and
FIG. 10.1: SQUARE APERTURE REFLECTOR EMPLOYING WMET SERRATIONS

FIG. 10.2: DECOMPOSITION OF FIG 10.1
Fig 10.3 respectively. It will be a laborious task to find an analytical expression in a closed form for the Fresnel zone field of an aperture with serrated edges. Hence, recourse is taken to decompose the aperture area ‘S’ into three parts $S_1$, $S_2$ and $S_3$ such that $S = S_1 + S_2 - S_3$ as shown in Fig 10.2 and 10.4 for WMET and WHMET respectively.

The Fresnel field is evaluated for $Z = 64\lambda$ and for $y = 0$ for the above two cases with width modulation factors given in Table 10.1. The variation of relative power in dB with transverse distance in wavelengths is furnished in Fig 10.5 – 10.8. From Fig 10.5 – 10.8, it is observed that by proper selection of width modulation factors, lesser ripple and enhanced quiet zone width are achieved in WHMET than WMET. From cases 2, 3, 4 & 5, it is observed that a portion of the quiet zone is ripple free in WHMET. In WMET, smoother quiet zone amplitude is achieved through case 4. Also width of the quiet zone is enhanced much in WHMET than WMET [65].

TABLE 10.1 : WIDTH MODULATION FACTORS : WMET SERRATIONS AND WHMET SERRATIONS

<table>
<thead>
<tr>
<th>Case</th>
<th>$P_0$</th>
<th>$P_1/P$</th>
<th>$P_2/P$</th>
<th>$P_3/P$</th>
<th>$P_4/P$</th>
<th>$P_5/P$</th>
<th>$P_6/P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>(a0/2)/27.5</td>
<td>3.5</td>
<td>4.6</td>
<td>11.5</td>
<td>13.3</td>
<td>23.5</td>
<td>27.5</td>
</tr>
<tr>
<td>2.</td>
<td>(a0/2)/25</td>
<td>4</td>
<td>5</td>
<td>13</td>
<td>16</td>
<td>23</td>
<td>25</td>
</tr>
<tr>
<td>3.</td>
<td>(a0/2)/22.5</td>
<td>3</td>
<td>4</td>
<td>10</td>
<td>12</td>
<td>20</td>
<td>22.5</td>
</tr>
<tr>
<td>4.</td>
<td>(a0/2)/15</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>12</td>
<td>15</td>
</tr>
<tr>
<td>5.</td>
<td>(a0/2)/13.5</td>
<td>2</td>
<td>3</td>
<td>6</td>
<td>7.5</td>
<td>11.5</td>
<td>13.5</td>
</tr>
<tr>
<td>6.</td>
<td>(a0/2)/7</td>
<td>0.75</td>
<td>1</td>
<td>2.5</td>
<td>3</td>
<td>6</td>
<td>7</td>
</tr>
</tbody>
</table>
FIG. 10.3: SQUARE APERTURE REFLECTOR EMPLOYING WHMET SERRATIONS

FIG. 10.4: DECOMPOSITION OF FIG 10.3
FIG. 10.5: FRESNEL ZONE FIELD OF 45λ X 45λ SQUARE APERTURE REFLECTOR EMPLOYED WITH WMET SERRATIONS FOR CASES 1, 2 & 3
FIG. 10.6: FRESNEL ZONE FIELD OF $45\lambda \times 45\lambda$ SQUARE APERTURE REFLECTOR EMPLOYED WITH WMET SERRATIONS FOR CASES 4, 5 & 6
FIG. 10.7: FRESNEL ZONE FIELD OF 45λ X 45λ SQUARE APERTURE REFLECTOR EMPLOYED WITH WHMET SERRATIONS FOR CASES 1, 2 & 3
FIG. 10.8: FRESNEL ZONE FIELD OF $45\lambda \times 45\lambda$ SQUARE APERTURE REFLECTOR EMPLOYED WITH WHMET SERRATIONS FOR CASES 4, 5 & 6
10.6 CONCLUSIONS

The Compact Range has emerged as an attractive alternative to the conventional far field range as it has the potential to provide a high quality plane wave in the region of the test antenna. However, great care must be exercised to minimize the reflector edge diffraction. Serrating the edges of the reflector redirects the diffracted field away from the quiet zone. Beeckman's method of producing the Fresnel region field due to square aperture with serrated edges and employing PO has been utilized.

The Fresnel field is calculated for both WMET and WHMET serrated aperture reflectors. Less ripple is observed for specific cases of width modulation factors. Meticulous selection of width modulation factors result in ripple free quiet zone for many cases in WHMET than in WMET.

From the above discussion, it is to be concluded that a reflector with MHMET serrations provide better quiet zone performance than a reflector with WMET serrations. It may be noted that the conventional compact ranges established by Scientific Atlanta also utilize reflectors whose serration shape is closer to WHMET.