CHAPTER 4 - AN INVENTORY MODEL FOR SOLVING TWO STAGE SUPPLY CHAIN USING FUZZY COSTS WITH SHORTAGE
An Inventory Model for Solving Two Stage Supply Chain Using Fuzzy Costs with Shortage

4.1. Introduction

In the last few years, companies have realized that efficient management of inventories across the different facilities in a supply chain is critical to increase the profits. This efficient management is achieved through better co-ordination and more co-operation between the vendor and the buyer. The supply chain management has enabled numerous firms to enjoy great advantages by integrating all activities associated with the flow of material, information, and capital between suppliers of raw materials and the ultimate customers. The benefits of a properly managed supply chain include reduced cost, faster delivery, greater efficiency and lower price for both the business and its customers.

Nagoor Gani and Sabarinathan [61] developed fuzzy integrated inventory model to determine the relevant profit maximizing decision variable values. They did not allow shortages which is unrealistic. To suit the real life situation we allow shortages for both vendor and buyer. The final demand for the product is assumed to be deterministic but price sensitive.

Production rate, ordering quantity, setup cost, shortage cost and holding cost of the buyer and vendor are taken as fuzzy numbers. The lot delivered from the vendor to the buyer is equal – sized batches. As soon as the on-hand inventory at the buyer drops to reorder point, an order of size $\tilde{Q}$ is released by the buyer. The
vendor manufactures the product at the production rate $\tilde{P}$ and in lot sizes which are a multiple of $\tilde{Q}$. The objective is to determine the number of shipments, the selling price $\tilde{\delta}$ as well as order size by allowing shortages for buyer, so that the total profit of the vendor – buyer are maximized.

4.2. Notations

- $c$: the buyer’s unit purchasing price
- $\delta$: Unit selling price of the buyer
- $S$: maximum inventory level for the buyer
- $\tilde{P} = (P_1, P_2, P_3)$: fuzzy production rate of the vendor
- $\tilde{Q} = (Q_1, Q_2, Q_3)$: fuzzy order quantity of the buyer
- $\tilde{A}_v = (A_{v1}, A_{v2}, A_{v3})$: fuzzy setup cost of the vendor
- $\tilde{A}_b = (A_{b1}, A_{b2}, A_{b3})$: Fuzzy ordering cost of the buyer
- $\tilde{\delta} = (\delta_1, \delta_2, \delta_3)$: fuzzy unit selling price of the buyer
- $\tilde{D} = (D_1, D_2, D_3)$: Fuzzy demand rate as a function of unit selling price
- $\tilde{h}_v = (h_{v1}, h_{v2}, h_{v3})$: Fuzzy inventory holding cost for the vendor per year
- $\tilde{h}_b = (h_{b1}, h_{b2}, h_{b3})$: Fuzzy inventory holding cost for the buyer per year
- $\tilde{n} = (n_1, n_2, n_3)$: number of shipments
\begin{align*}
\tilde{S}_b &= (S_{b_1}, S_{b_2}, \ldots, S_{b_n}) & \text{Fuzzy inventory shortage cost for the buyer per year} \\
T\tilde{P}_v &\quad & \text{Annual profit function for the vendor} \\
T\tilde{P}_b &\quad & \text{Annual profit function for the buyer} \\
A\tilde{I}_v &\quad & \text{Vendor’s average inventory}
\end{align*}

4.3. Assumptions

(i) The model deals with a single vendor and a single buyer for a single product.

(ii) The buyer faces a linear Demand $\tilde{D}(\tilde{d})$ as a function of selling price $\tilde{d}$.

(iii) A finite production rate for the vendor is considered.

(iv) The inventory is continuously reviewed. The buyer orders a lot of size $\tilde{Q}$ when the on-hand inventory reaches the reorder point.

(v) The vendor manufactures a production batch $n\tilde{Q}$ at one setup. However, the size of shipment delivered to the buyer is $\tilde{Q}$.

(vi) The inventory holding cost at the buyer is higher than that at the vendor. i.e)
\[ \tilde{h}_b > \tilde{h}_v. \]

(vii) Shortage is allowed for buyer.

(viii) The time horizon is infinite.
4.4. Fuzzy Mathematical Model

The optimal policy of the integrated system is derived. However, for comparative purposes, we first obtain the buyer and the vendor policies, if each party optimizes its profit independently. The policies and profits are then compared to the case of integrated system when they co-operate, particularly in information sharing.

We assume that the buyer faces a linear demand $\tilde{D}(\tilde{\delta}) = a - b\tilde{\delta}$, $(a > b > 0)$ as a function of its unit selling price. As $\tilde{D}(\tilde{\delta}) > 0$, the maximum selling price is $a/b$, i.e. $\tilde{\delta} < \frac{a}{b}$. The buyer's yearly profit is equal to the gross revenue minus the sum of purchasing, order processing, and inventory holding costs. The buyer wishes to maximize his yearly profit function, $TP_B$, through the optimal choice of selling price and order quantity, i.e.)
\[
\bar{T}P_b(\tilde{\delta}, \tilde{Q}) = (a - b\tilde{\delta})(\tilde{\delta} - c)
- \frac{h_b \tilde{S}_b^2}{2\tilde{Q}} - \frac{\tilde{S}_b (\tilde{Q} - \tilde{S})^2}{2\tilde{Q}} - \frac{\tilde{A}_b (a - b\tilde{\delta})}{\tilde{Q}}
\]

\[
\text{where } \tilde{Q} = \sqrt{2\bar{A}_b(a - b\tilde{\delta}) \frac{\tilde{h}_b + \tilde{S}_b}{\tilde{h}_b \tilde{S}_b}}
\]

\[
\bar{S} = \frac{\tilde{Q}\tilde{S}_b}{\tilde{h}_b + \tilde{S}_b}
\]

Substituting equation (4.3) in equation (4.1), we get,

\[
\bar{T}P_b(\tilde{\delta}, \tilde{Q}) = (a - b\tilde{\delta})(\tilde{\delta} - c)
- \frac{h_b \tilde{S}_b^2 \tilde{Q}}{2(\tilde{h}_b + \tilde{S}_b)^2} - \frac{\tilde{S}_b \tilde{Q}(1 - \frac{\tilde{S}_b}{\tilde{h}_b + \tilde{S}_b})^2}{2} - \frac{\tilde{A}_b (a - b\tilde{\delta})}{\tilde{Q}}
\]
Substituting equation (4.2) in (4.1) we get,

\[ T \tilde{P}_b(\tilde{\delta}) = (a \tilde{\delta} - b \tilde{\delta} \tilde{\delta} - ac + bc \tilde{\delta}) \]

\[
\begin{align*}
T \tilde{P}_b(\tilde{\delta}) &= \left( \frac{\tilde{h}_b \tilde{S}_b}{2(\tilde{h}_b + \tilde{S}_b)^2} \right) \\
&+ \left( \frac{\tilde{A}_b}{\sqrt{2 \tilde{A}_b}} \right) \\
&+ \left( \frac{b}{2 \sqrt{(a - b \tilde{\delta})}} \right)
\end{align*}
\]

\begin{align*}
\frac{\partial T \tilde{P}_b(\tilde{\delta})}{\partial \tilde{\delta}} &= a - 2b \tilde{\delta} + bc \\
\frac{\partial T \tilde{P}_b(\tilde{\delta})}{\partial \tilde{\delta}} &= \left( \frac{\tilde{h}_b \tilde{S}_b}{2(\tilde{h}_b + \tilde{S}_b)^2} \right) \\
&+ \left( \frac{\tilde{A}_b}{\sqrt{2 \tilde{A}_b}} \right) \\
&+ \left( \frac{b}{2 \sqrt{(a - b \tilde{\delta})}} \right)
\end{align*}

\[ \text{(4.5)} \]

\[ \text{(4.6)} \]
Differentiating the above equation with respect to \( \tilde{\delta} \),

\[
\frac{\partial^2 \tilde{TP}_b(\tilde{\delta})}{\partial \tilde{\delta}^2} = -2b + \frac{b^2}{4(a - b\tilde{\delta})^3} \left\{ \begin{array}{l}
\tilde{h}_b\tilde{S}_b^2 \sqrt{2\tilde{A}_b} \frac{\tilde{h}_b + \tilde{S}_b}{\tilde{h}_b\tilde{S}_b} \\
2(\tilde{h}_b + \tilde{S}_b)^2 \\
\tilde{S}_b(1 - \frac{\tilde{S}_b}{\tilde{h}_b + \tilde{S}_b})^2 \sqrt{2\tilde{A}_b} \frac{\tilde{h}_b + \tilde{S}_b}{\tilde{h}_b\tilde{S}_b} \\
2 \\
\frac{\tilde{A}_b}{\sqrt{2\tilde{A}_b}} \frac{\tilde{h}_b + \tilde{S}_b}{\tilde{h}_b\tilde{S}_b} \\
\end{array} \right\}
\]

\[----(4.7)\]

Equating equation (4.6) equals to zero and solving the equation we will get the value of \( \tilde{\delta} \), substituting this value of \( \tilde{\delta} \) in equation (4.2) we will get the value of value of \( \tilde{Q} \). Now \( TP_b(\tilde{\delta}) \) is concave.

**Vendor’s Profit Policy**

When the buyer’s order quantity and the selling price are adopted, the orders are received by the vendor at a known interval \( \frac{\tilde{Q}}{D(\delta)} \).

A vendor’s average inventory can then be obtained as follows:

\[
\tilde{AI}_v = \frac{\tilde{Q}}{2} \left[ (\tilde{n} - 1)(1 - \frac{\tilde{D}(\tilde{\delta})}{P}) + \frac{\tilde{D}(\tilde{\delta})}{P} \right]
\]

\[---- (4.8)\]
Hence the vendor’s yearly profit function is,

\[ \tilde{T}_v(\tilde{n}) = ac - bc\tilde{\delta} - \frac{(a - b\tilde{\delta})\tilde{A}}{\tilde{n}\tilde{Q}} - \frac{\tilde{h}\tilde{Q}}{2} \left[ \tilde{n}(1 - \frac{(a - b\tilde{\delta})}{\tilde{p}}) - 1 + \frac{2(a - b\tilde{\delta})}{\tilde{p}} \right] \]

\[ \text{----- (4.9)} \]

such that \( n \) is integer.

It clearly shows that \( T\tilde{P}_v(\tilde{n}) \) is concave in \( n \).

Optimality conditions for \( n^* \),

\[ \tilde{n}^*(\tilde{n}^* - 1) \leq \frac{2(a - b\tilde{\delta})\tilde{P}}{\tilde{h}\tilde{Q}(\tilde{P} - a - b\tilde{\delta})} \leq \tilde{n}^*(\tilde{n}^* + 1) \]

\[ \text{----- (4.10)} \]

If the buyer is free to choose his own marketing and ordering policies \((\tilde{\delta}, \tilde{Q})\), and the vendor is free to choose its number of shipments \( n \), then it is straightforward that the total system profit under individual optimization, \( T\tilde{P}_1(\tilde{\delta}, \tilde{Q}, \tilde{n}) \), is equal to the sum of buyer’s and the vendor’s profits.

\[ T\tilde{P}_1(\tilde{\delta}, \tilde{Q}, \tilde{n}) = T\tilde{P}_y(\tilde{\delta}) + T\tilde{P}_v(\tilde{n}) \]

Suppose that both parties decide to cooperate and agree to follow the jointly optimal integrated policy. The cost stemming from the purchasing price is an internal transfer of money from one supply chain member (the vendor) to another supply chain member (the buyer).
Therefore it is not a cost of the whole supply chain. The total system profit under joint optimization with shortage is,

\[
\tilde{T}_j(\tilde{\delta}, \tilde{Q}, \tilde{n}) = (a\tilde{\delta} - b\tilde{\delta}^2) - \frac{1}{\tilde{Q}} \left[ \frac{\tilde{h}_b \tilde{S}_b}{2(h_b + \tilde{S}_b)^2} \right] + \frac{\tilde{S}_b(1 - \frac{\tilde{S}_b}{h_b + \tilde{S}_b})^2}{2} + \frac{\tilde{h}_b}{2} \left[ \tilde{n}(1 - \frac{(a - b\tilde{\delta})}{\tilde{P}}) - 1 + \frac{2(a - b\tilde{\delta})}{\tilde{P}} \right] \]

\[ - \frac{1}{\tilde{Q}} \left[ \tilde{\Lambda}_b(a - b\tilde{\delta}) + \frac{\tilde{\Lambda}_v(a - b\tilde{\delta})}{\tilde{n}} \right] \]

---- (4.11)

Solution for Joint Model

Differentiating (4.11) with respect to \( \tilde{Q} \) we get,

\[
\frac{\partial \tilde{T}_j}{\partial \tilde{Q}} = - \left[ \frac{\tilde{h}_b \tilde{S}_b}{2(h_b + \tilde{S}_b)^2} \right] + \frac{\tilde{S}_b(1 - \frac{\tilde{S}_b}{h_b + \tilde{S}_b})^2}{2} + \frac{\tilde{h}_b}{2} \left[ \tilde{n}(1 - \frac{(a - b\tilde{\delta})}{\tilde{P}}) - 1 + \frac{2(a - b\tilde{\delta})}{\tilde{P}} \right] + \frac{1}{\tilde{Q}^2} \left[ \tilde{\Lambda}_b(a - b\tilde{\delta}) + \frac{\tilde{\Lambda}_v(a - b\tilde{\delta})}{\tilde{n}} \right] \]

---- (4.12)
Equating (4.12) to zero we will get the value of $\tilde{Q}$

$$
\tilde{Q}^* = \frac{2(a-b\tilde{\delta})\left(\tilde{A}_b + \frac{\tilde{A}_s}{\tilde{n}}\right)}{\tilde{h}_b\tilde{S}_b + \tilde{S}_b\left(1 - \frac{\tilde{S}_b}{\tilde{h}_b + \tilde{S}_b}\right)^2 + \frac{\tilde{h}_b\tilde{S}_b}{(\tilde{h}_b + \tilde{S}_b)^2}}
$$

Substituting (4.13) in (4.11) we get,

$$
\tilde{T}_P(\tilde{\delta}, \tilde{n}) = a\tilde{\delta} - b\tilde{\delta}^2 - \left\{2(a-b\tilde{\delta})\left(\tilde{A}_b + \frac{\tilde{A}_s}{\tilde{n}}\right)\right\} + \frac{\tilde{h}_b\tilde{S}_b}{(\tilde{h}_b + \tilde{S}_b)^2} \left\{\tilde{n}(1 - \frac{\tilde{S}_b}{\tilde{h}_b + \tilde{S}_b}) - 1 + \frac{2(a-b\tilde{\delta})}{\tilde{P}}\right\}
$$

---------(4.14)
Let

$$TP_j = \left\{ \begin{array}{c}
2(a - b \bar{\delta}) \\
\tilde{A}_b + \frac{\bar{A}_b}{\bar{n}}
\end{array} \right\} \left\{ \begin{array}{c}
\frac{\bar{h}_b \tilde{S}_b}{(\bar{h}_b + \tilde{S}_b)^2} \\
+ \tilde{S}_b \left(1 - \frac{\tilde{S}_b}{\bar{h}_b + \bar{S}_b}\right)^2 \\
+ \bar{h}_b \left[ \bar{n}(1 - \frac{(a - b \bar{\delta})}{\bar{P}}) - 1 \right] \\
+ \frac{\bar{h}_b}{2} \left[ \frac{2(a - b \bar{\delta})}{\bar{P}} \right]
\end{array} \right\}$$

----- (4.15)

For a given value of $\bar{\delta}$, maximizing $TP_j$ is equivalent to minimizing $TP^j$.

Taking the first and second partial derivatives with respect to $\bar{n}$, and equating first order derivative to zero we obtain

$$\bar{n} = \left\{ \begin{array}{c}
\frac{\bar{A}_b \bar{h}_b}{\bar{h}_b} \frac{\bar{S}_b^2}{(\bar{h}_b + \tilde{S}_b)^2} \\
+ \tilde{A}_b \tilde{S}_b \left(1 - \frac{\tilde{S}_b}{\bar{h}_b + \bar{S}_b}\right)^2 \\
+ \tilde{A}_b \tilde{h}_b \left[ -1 + \frac{2(a - b \bar{\delta})}{\bar{P}} \right] \\
+ \tilde{A}_b \tilde{h}_b \left[ 1 - \frac{a - b \bar{\delta}}{\bar{P}} \right]
\end{array} \right\}$$

----- (4.16)
Defuzzification of this model:

Using Signed Distance method, we will get the crisp value of selling price \((\delta)\), Order quantity \((Q)\), number of shipments \((n)\) for individual and joint model, Total profit for buyer \((TP_B)\), Total profit for vendor \((TP_v)\), Total system profit under individual optimization \((TP_1)\), the joint total profit of vendor \((TP_vj)\), the joint total profit of buyer \((TP_Bj)\), total system profit under joint optimization \((TP_j)\).

For a given value of \(n\), \(TP_j\) can be written as

\[
TP_j(D) = m_1 D + m_2 D^2 - \sqrt{m_3 D + m_4 D^2}
\]

----- (4.17)

where

\[
m_1 = \frac{a}{b}
\]

\[
m_2 = \frac{-1}{b}
\]

\[
m_3 = 2 \left( K_b + \frac{K_v}{n} \right) \left\{ h_b \left( \frac{S_b}{h_b + S_b} \right)^2 + \frac{h_v}{n} \left( \frac{S_v}{S_v + h_v} \right)^2 \right\} + S_b \left( 1 - \frac{S_b}{h_b + S_b} \right)^2 - S_v \left( 1 - \frac{S_v}{h_v + S_v} \right)^2 (n-1)
\]

\[
m_4 = \frac{2}{P} \left( K_b + \frac{K_v}{n} \right) S \left( 1 - \frac{S_v}{S_v + h_v} \right)^2 (2-n)
\]

and \(D(\delta) = a - b\delta\). There is a one to one relationship between price and demand. Therefore, we base our analysis on the identification of the optimal value of demand, rather than the optimal value of price. The first and second partial derivative of \(TP_j(D)\), with respect to \(D\) are as follows.
\[
\frac{\partial TP_j(D)}{\partial D} = m_1 + 2m_2D - \frac{m_3 + 2m_4D}{2\sqrt{m_3D + m_4D^2}}
\]

\[
\frac{\partial^2 TP_j(D)}{\partial D^2} = 2m_2 + \frac{m_3^2}{4}(m_3D + m_4D^2)^{-\frac{3}{2}}
\]

**Case 1: n = 1**

Hence \(m_4 > 0\), therefore there are two saddle points, \(SP_1\) and \(SP_2\). The total profit function is convex when \(SP_1 < D < SP_2\), and is concave when \(D \leq SP_1\) or \(D \geq SP_2\). The optimal value of the demand is then

\(D^* = LO_1\) if \(LO_1 < a\), and it is \(D^* = a\) if \(LO_1 \geq a\).

**Case 2: n=2**

Hence \(m_4 = 0\), and therefore there is a saddle point, \(SP = \frac{b^2}{4m_3}\) the total profit function is convex when \(D < SP\), and is concave when \(D \geq SP\). Because the total profit function is zero at \(D = 0\), there is no more than one local optimal amount for the demand. The optimal value of the demand is then

\(D^* = LO_2\) if \(LO_2 < a\), and it is \(D^* = a\) if \(LO_2 \geq a\).

**Case 3: n \geq 3**

Hence, \(m_4 < 0\), and therefore there are two saddle points, \(SP_1\) and \(SP_2\). The total profit function is concave when \(SP_2 < D < SP_1\), and it is convex when \(D \leq SP_2\) or \(D \geq SP_1\). Moreover, \(m_3, t > 0\) and thus \(SP_1 > 0\) and \(SP_2 > 0\) The optimal value of the demand is then
\[ D^* = \text{LO}_3 \text{ if } \text{LO}_3 < a, \text{ and it is } D^* = a \text{ if } \text{LO}_3 \geq a. \]

As no closed form solution exists for the local optimal values of the demand, we use numerical method to find \( \text{LO}_i \), \( i=1, 2, 3, \).

### 4.5. Numerical example:

We consider an example with the following data:

\[ \tilde{P} = (3100, 3200, 3300) / \text{year} \quad \tilde{A}_r = (\text{Rs.} 300, \text{Rs.} 400, \text{Rs.} 500) / \text{setup} \]

\[ \tilde{A}_b = (\text{Rs.} 20, \text{Rs.} 25, \text{Rs.} 30) / \text{order} \quad \tilde{h}_v = (\text{Rs.} 3, \text{Rs.} 4, \text{Rs.} 5) / \text{unit} / \text{setup} \]

\[ \tilde{h}_b = (\text{Rs.} 4, \text{Rs.} 5, \text{Rs.} 6) / \text{unit} / \text{year} \quad a = 1500, b = (10, 20, 30, 40), \]

\[ c = \text{Rs.} 5 / \text{unit} \quad \tilde{S}_b = (\text{Rs.} 8, \text{Rs.} 9, \text{Rs.} 10) / \text{unit} \]

We analyze the effect of demand’s price sensitivity. The effect is evaluated by the impact on the benefits of vendor-buyer coordination as well as impact on the decision variables.

\( TP_j \) and \( TP_1 \) represent the total system profit under joint and individual optimization. Joint total profit allocated to the buyer and the vendor as follows

\[ TP_{uj} = \frac{TP_j(n)}{TP_1} TP_j \quad \text{and} \quad TP_{bj} = \frac{TP_j(n)}{TP_1} TP_j \]
Table. 4.1. Decision Variables under individual optimization (fuzzy environment)

<table>
<thead>
<tr>
<th>b</th>
<th>$\delta$</th>
<th>Q</th>
<th>n</th>
<th>TPv</th>
<th>TP_B</th>
<th>TP_I</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>(77.3647, 77.5463, 77.7284)</td>
<td>(69.397, 85.088, 104.102)</td>
<td>(5, 5, 5)</td>
<td>(1708.596, 2381.424)</td>
<td>(51507.994, 52162.444)</td>
<td>(53216.59, 54543.868)</td>
</tr>
<tr>
<td>40</td>
<td>(12.8106, 12.8118, 12.8118)</td>
<td>(88.883, 123.720, 172.126)</td>
<td>(5, 5, 5)</td>
<td>(2475.053, 3538.403)</td>
<td>(12806.413, 13160.015)</td>
<td>(9281.156, 10854.418)</td>
</tr>
</tbody>
</table>

Table. 4.2. Decision Variables under joint optimization (fuzzy environment)

<table>
<thead>
<tr>
<th>b</th>
<th>$\delta$</th>
<th>Q</th>
<th>n</th>
<th>TP_vj</th>
<th>TP_Bj</th>
<th>TP_j</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>(37.9611, 37.9519, 38.9807)</td>
<td>(102.720, 149.507, 212.866)</td>
<td>(4, 4, 4)</td>
<td>(1254.494, 2308.349)</td>
<td>(22332.316, 24573.557)</td>
<td>(25620.695, 26881.907)</td>
</tr>
<tr>
<td>40</td>
<td>(11.8106, 11.8118, 12.8118)</td>
<td>(120.9099, 179.988, 262.026)</td>
<td>(4, 4, 4)</td>
<td>(2032.913, 3491.239)</td>
<td>(5590.270, 9218.4995)</td>
<td>(9641.316, 10709.739)</td>
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</tbody>
</table>
Table. 4.3. Decision Variables under individual optimization (after defuzzification)

<table>
<thead>
<tr>
<th>b</th>
<th>$\delta$</th>
<th>Q</th>
<th>n</th>
<th>TP$_v$</th>
<th>TP$_B$</th>
<th>TP$_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>116.3196</td>
<td>128.463</td>
<td>7</td>
<td>3525.3803</td>
<td>78198.668</td>
<td>81724.048</td>
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<tr>
<td>20</td>
<td>59.9374</td>
<td>159.197</td>
<td>7</td>
<td>3314.074</td>
<td>36187.613</td>
<td>39501.687</td>
</tr>
<tr>
<td>30</td>
<td>28.898</td>
<td>182.597</td>
<td>7</td>
<td>4765.3775</td>
<td>19085.019</td>
<td>23850.397</td>
</tr>
<tr>
<td>40</td>
<td>19.2174</td>
<td>188.972</td>
<td>7</td>
<td>5205.399</td>
<td>10903.862</td>
<td>16109.261</td>
</tr>
</tbody>
</table>

Table. 4.4. Decision Variables under joint optimization (after defuzzification)

<table>
<thead>
<tr>
<th>b</th>
<th>$\delta$</th>
<th>Q</th>
<th>n</th>
<th>TP$_{vj}$</th>
<th>TP$_{Bj}$</th>
<th>TP$_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>118.2647</td>
<td>201.8913</td>
<td>6</td>
<td>3916.3218</td>
<td>83424.2335</td>
<td>87340.555</td>
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<tr>
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<td>57.1874</td>
<td>228.4035</td>
<td>6</td>
<td>3417.808</td>
<td>36976.133</td>
<td>40297.874</td>
</tr>
<tr>
<td>30</td>
<td>27.648</td>
<td>263.872</td>
<td>6</td>
<td>4897.203</td>
<td>19382.156</td>
<td>24087.617</td>
</tr>
<tr>
<td>40</td>
<td>17.9674</td>
<td>275.722</td>
<td>6</td>
<td>5380.976</td>
<td>13101.897</td>
<td>16178.072</td>
</tr>
</tbody>
</table>
Sensitivity analysis

From tables 4.1 & 4.2, we see that when \( b \) increases ordering quantity also increases and total profit for both buyer and vendor decreases.

From tables 4.3 & 4.4, we observe that when \( b \) increases the ordering quantity also increases. Under individual optimization vendor’s total profit increases and buyer’s total profit decreases.

From tables 4.3 & 4.4, it is clear that the individual profit for buyer and vendor and the joint profit with co ordination are much greater than those without coordination.

4.6. Conclusion

This chapter addresses an inventory problem under fuzzy cost with allowable shortage for buyer. Our venture of this chapter is to get more profit for buyer and vendor in co - ordination comparatively with their individual profits without co – ordination. We have developed the model which is more suitable for real life situations. We have achieved this by obtaining more profit for the required model consequently our optimum profit for the joint model is more than the profit of the model by [61].