CHAPTER 10 - A FUZZY INVENTORY MODEL FOR INSTANTANEOUS DETERIORATING ITEMS WITH CONTROLLABLE DETERIORATION RATE FOR TIME DEPENDENT DEMAND AND HOLDING COST WITH TRIANGULAR, TRAPEZOIDAL, PENTAGONAL, HEXAGONAL, OCTAGONAL, NONAGONAL, DECAGONAL FUZZY NUMBERS UNDER LINGUISTIC ENVIRONMENT
10.1. Introduction:

Inventory System is one of the main streams of the Operations Research which is essential in business enterprises and industries. Interest in the subject is constantly increasing, and its development in recent years closely parallels the development of operations research in general. The inventory system for deteriorating items has been an object of study for a long time, but little is known about the effect of investing in reducing the rate of product deterioration. So in this chapter, an inventory model is developed to consider the fact that the uses of preservation technology reduce the deterioration rate by which the retailer can reduce the economic losses, improve the customer service level and increase business competitiveness.

Inventory of deteriorating items is first studied by Whitin [90]. He considered the deterioration of fashion goods at the end of prescribed storage period. Ghare and Schrader [27] extended the classical EOQ formula with exponential decay of inventory due to deterioration and gave a mathematical model for inventory of deteriorating items. Dave and Patel [18] developed the first deteriorating inventory model with linear trend in demand. He considered demand as a linear function of time. Goyal and Giri [31] gave recent trends of modeling in deteriorating items inventory. They classified inventory models on the basis of
demand variations and various other conditions or constraints. Ouyang et al., [63] developed an inventory model for deteriorating items with exponential declining demand and partial backlogging of shortages.

. The consideration of PT is important due to rapid social changes and the fact that PT can reduce the deterioration rate significantly. By the efforts of investing in preservation technology we can reduce the deterioration rate. So in this chapter, we made the model of Mishra and Singh [56] more realistic by considering the fact that the use of preservation technology can reduce the deterioration rate significantly which help the retailers to reduce their economic losses.

Fuzzy set theory introduced by Zadeh [96], has been researched widely in many fields and various modifications methods and generalization theories have been appeared in different directions. Roy and Maiti [70], fuzzified the values by trapezoidal fuzzy numbers. Bansal [8] introduced hexagonal fuzzy numbers. Felix et al., [23] introduced nonagonal fuzzy numbers and its arithmetic functions. Felix et al., [24] introduced decagonal fuzzy numbers also.

This paper has been motivated by idea of Zadeh [95] about linguistic variables intended to provide rigorous mathematical modeling of natural language and computing with words. Previously we have developed the models with fuzzy numbers only. Additionally in this study we include Linguistic values in natural language. Linguistic model provides natural way of dealing with problems in which the source of imprecision and vagueness occurs and it can be applied in
many fields. Most of the researchers have focused on uncertain linguistic environment in inventory models.

This chapter is organized as follows: In section 1 introduction is given. In section 2 definitions and preliminaries are given. Assumptions and notations are given in section 3. Model development, conclusion and numerical example are given in sections 4, 5 and 6 respectively.

10.2. Notations

\[ T \quad - \quad \text{the length of the cycle time} \]
\[ T_1 \quad - \quad \text{the time at which the inventory level reaches zero, } T_1 \geq 0 \]
\[ T_2 \quad - \quad \text{length of the period during which shortages are allowed, } T_2 \geq 0 \]
\[ \tilde{D} \quad - \quad \text{Fuzzy demand (Triangular, Trapezoidal, pentagonal, hexagonal fuzzy number)} \]
\[ \tilde{A} \quad - \quad \text{fuzzy setup cost (Triangular, Trapezoidal, pentagonal, hexagonal fuzzy number)} \]
\[ \tilde{H} \quad - \quad \text{Fuzzy inventory holding cost (Triangular, Trapezoidal, pentagonal, hexagonal fuzzy number)} \]
\[ \tilde{c} \quad - \quad \text{Fuzzy purchase cost per unit (Triangular, Trapezoidal, pentagonal, hexagonal fuzzy number)} \]
\( \tilde{x} \) - Fuzzy backorder cost per unit short per unit time

\( \pi \) - The cost of lost sales per unit.

\( \xi \) - Preservation technology (PT) cost for reducing deterioration rate in order to preserve the product, \( \xi > 0 \)

\( m(\xi) \) - Reduced deterioration rate due to use of preservation technology

\( \tilde{\mathcal{Q}} \) - Fuzzy order quantity (Triangular, Trapezoidal, pentagonal, hexagonal fuzzy number)

\( I_1(t) \) - the level of positive inventory at time \( t \), \( 0 \leq t \leq T_1 \)

\( I_2(t) \) - the level of negative inventory at time \( t \), \( T_1 \leq t \leq T \)

\( T_C \) - Total cost for the period \([0, T]\)

\( \tilde{T_C} \) - Fuzzy total cost for the period \([0, T]\)

10.3. Assumptions

1. The demand rate is time dependent that \( D(t) = a + b t \), where \( a > 0 \), \( b > 0 \).

2. Preservation technology is used for controlling the deterioration rate.

3. Holding cost is linear function of time \( H(t) = \alpha + \beta t \), \( \alpha \geq 0 \), \( \beta \geq 0 \).
4. Shortages are allowed and partially backlogged.

5. The lead time is zero.

6. The replenishment rate is infinite.

7. The planning horizon is finite.

8. The deterioration rate is constant.

9. During stock out period, the backlogging rate is 
   \[ B(t) = \frac{1}{1 + \delta(T - t)} \]
   
   where \( \delta \) is backlogging parameter, \((T-t)\) is waiting time.

10. Purchase cost, Ordering cost, backorder cost, and time are taken as Triangular, Trapezoidal, pentagonal, hexagonal fuzzy number.

10.4. Model Development:

Crisp model:

![Figure 10.1. Inventory model with PT](image-url)
The rate of change of inventory, during positive stock out period \([0, T_1]\) occur due to demand and resultant rate, and in shortage period \([T_1, T_2]\) occur due to demand a fraction of demand is backlogged & backlogging rate is \(B(t)\). Hence, the inventory level at any time is governed by the differential equations

\[
\frac{dI_1(t)}{dt} + (\theta - m(\xi))I_1(t) = -(a + bt); \quad 0 \leq t \leq T_1
\]

\[
\frac{dI_2(t)}{dt} = \frac{-(a + bt)}{1 + \delta(T - t)} \quad T_1 \leq t \leq T_2
\]

---(10.1, 10.2)

**Case 1: Inventory level without shortage**

During the period \([0, T_1]\), the inventory depletes due to the deterioration and demand. Hence it is described by the equation,

\[
\frac{dI_1(t)}{dt} + (\theta - m(\xi))I_1(t) = -(a + bt); \quad 0 \leq t \leq T_1
\]

---(10.3)

The solution of equation (10.3) is,

\[
I_1(t) = \left[ -\frac{a}{\theta - m(\xi)} - \frac{b}{\theta - m(\xi)} \left( \frac{1}{\theta - m(\xi)} - \frac{1}{\theta - m(\xi)} \right) \right] + e^{\theta-m(\xi)T_1-t} \left[ \frac{a}{\theta - m(\xi)} + \frac{b}{\theta - m(\xi)} \left( T_1 - \frac{1}{\theta - m(\xi)} \right) \right]; \quad 0 \leq t \leq T_1
\]

---(10.4)
Case 2: Inventory level with shortage

During the interval \([T_1, T]\) the inventory level depends on demand and a fraction of demand is backlogged. The state of inventory during \([T_1, T]\) can be represented by the differential equation

\[
\frac{dI_2(t)}{dt} = \frac{-(a + bt)}{1 + \delta(T - t)} \quad T_1 \leq t \leq T
\]  

---(10.5)

The solution of equation (10.5) is

\[
I_2(t) = \left[ \frac{a}{\delta} \log \frac{1 + \delta(T - t)}{1 + \delta T_2} + b(1 + \delta(T)) \log \frac{1 + \delta(T - t)}{1 + \delta T_2} - \frac{b(T_1 - t)}{\delta} \right]
\]  

---(10.6)

Therefore the total cost per replenishment cycle consists of the following components:

1. Inventory holding cost per cycle:

\[
HC = \int_0^{T_1} H(t)I_1(t)dt
\]

\[
HC = \left[ \begin{array}{c} -6e^{(\theta-m(\xi))T_1}bT_1c(\theta-m(\xi))^2 - 6ab(\theta-m(\xi))e^{(\theta-m(\xi))T_1} \\
+ 6a\alpha(\theta-m(\xi))^3T_1 + 3a\beta(\theta-m(\xi))^3T_1^2 \\
+ 3b(\theta-m(\xi))^3cT_1^2 + 2b\beta(\theta-m(\xi))^3T_1^3 \\
+ 3b\beta(\theta-m(\xi))^2T_1^2 + 6b\beta e^{(\theta-m(\xi))}T_1 - 6a\alpha(\theta-m(\xi))^2e^{(\theta-m(\xi))}T_1 \\
+ 6b\alpha(\theta-m(\xi))e^{(\theta-m(\xi))}T_1 \\
- 6b\beta(\theta-m(\xi))T_1 e^{(\theta-m(\xi))}T_1 + 6a(\theta-m(\xi))^2c \\
+ 6b\alpha + 6ab(\theta-m(\xi))^2T_1 - 6(\theta-m(\xi))c\alpha - 6b\beta \end{array} \right]
\]

-----(10.7)
2. Backorder cost per cycle:

\[ BC = \int_{t_i}^{\hat{t}} xL_2(t) dt \]

\[ BC = x \left[ \frac{1}{2\delta^3} \left( 2aT_z\delta^2 + bT_z^2\delta^2 + 2bT_z\delta + 2bT_1T_z\delta^2 + 2bT_z\delta \log \left( \frac{1}{1 + T_z\delta} \right) \right) + 2b\log \left( \frac{1}{1 + T_z\delta} \right) + 2a\delta \log \left( \frac{1}{1 + T_z\delta} \right) + 2bT_1\delta \log \left( \frac{1}{1 + T_z\delta} \right) \right] \]

\[ -(10.8) \]

3. Lost sales cost per cycle:

\[ LS = \int_{t_i}^{\hat{t}} \pi I_2(t) dt \]

\[ LS = \pi \left[ \frac{1}{2\delta^3} \left( 2aT_z\delta^2 + bT_z^2\delta^2 + 2bT_z\delta + 2bT_1T_z\delta^2 - 2a\delta \log (1 + T_z\delta) \right) - 2b\log (1 + T_z\delta) - 2bT_1\log (1 + T_z\delta) - 2bT_z\delta \log (1 + T_z\delta) + 2bT_2\delta \right] \]

\[ -(10.9) \]

4. Purchase cost per cycle

\[ PC = c \times Q \]

\[ Q = \left[ \frac{-a}{\theta - m(\xi)} + \frac{b}{(\theta - m(\xi))^2} \right] + e^{\theta - m(\xi)\gamma(T_1)} \left[ \frac{a}{\theta - m(\xi)} - \frac{b}{\theta - m(\xi)} \left( T_1 - \frac{1}{\theta - m(\xi)} \right) \right] - \frac{a}{\delta} \log \left( \frac{1}{1 + \delta T_z} - \frac{b(1 + \delta(T))}{\delta^2} \log (1 + \delta T_z) - \frac{bT_z}{\delta} \right) \]

\[ -(10.10) \]
\[
PC = c + e^{\theta - m(\xi)} \left[ -\frac{a}{\theta - m(\xi)} + \frac{b}{(\theta - m(\xi))^2} \right] e^{\theta - m(\xi)} \left( T_1 - \frac{1}{\theta - m(\xi)} \right) - \frac{a}{\delta} \log \frac{1}{1 + \delta T_2} - \frac{b(1 + \delta(T))}{\delta^2} \log(1 + \delta T_2) - \frac{bT_2}{\delta} \right]
\]

\[\text{(10.11)}\]

5. Ordering cost

\[OC = A\]

Therefore the total cost per time unit is given by,

\[TC(T) = \frac{1}{T} [OC + HC + BC + LS + PC]\]
\[ A = \begin{bmatrix} \begin{align*} -6e^{(\theta-m(\xi))T_1}bT_1\alpha(\theta-m(\xi))^2 - 6a\beta(\theta-m(\xi))e^{(\theta-m(\xi))T_1} \\ + 6a\alpha(\theta-m(\xi))^3T_1^2 \\ + 3a(\theta-m(\xi))^3T_1^2 + 2b\beta(\theta-m(\xi))^3T_1^3 \\ + 3b\beta(\theta-m(\xi))^3T_1^2 + 6b\beta e^{(\theta-m(\xi))T_1} \\ - 6a\alpha(\theta-m(\xi))^2e^{(\theta-m(\xi))T_1} \\ + 6b\alpha(\theta-m(\xi))e^{(\theta-m(\xi))T_1} \\ - 6b\beta(\theta-m(\xi))T_1e^{(\theta-m(\xi))T_1} + 6a(\theta-m(\xi))^2\alpha \\ + 6\beta\theta a + 6a\beta(\theta-m(\xi))^2b\alpha - 6b\beta \end{align*} \end{bmatrix} \]

\[ TC(T) = \frac{1}{T} \left[ \begin{align*} & 2aT_2\delta^2 + bT_2^2\delta^2 + 2bT_2\delta + 2bT_1T_2\delta^2 + \\ & 2b\log\left(\frac{1}{1+T_2\delta}\right) + 2\log\left(\frac{1}{1+T_2\delta}\right) + 2bT_1\delta\log\left(\frac{1}{1+T_2\delta}\right) \right] \\ & + \pi \frac{1}{2\delta^2} \left[ 2aT_2\delta^2 + bT_2^2\delta^2 + 2bT_2\delta + 2bT_1T_2\delta^2 - 2a\delta\log(1+T_2\delta) - \\ & 2b\log(1+T_2\delta) - 2bT_1\delta\log(1+T_2\delta) - 2bT_1\delta \log(1+T_2\delta) + 2bT_2\delta \right] \\ & + \frac{a}{T_1} + \frac{b}{(\theta-m(\xi))^2} + \\ & e^{(\theta-m(\xi))T_1} \left[ \frac{a}{\theta-m(\xi)} - \frac{b}{\theta-m(\xi)} \left( T_1 - \frac{1}{\theta-m(\xi)} \right) \right] - \\ & \frac{a}{\delta} \log \frac{1}{1+\delta T_2} - \frac{b(1+\delta(T_1))}{\delta^2} \log(1+\delta T_2) - \frac{bT_2}{\delta} \right] \]

---(10.12)

### 10.5. Fuzzy Model:

From the above crisp model, ordering cost, time, backorder cost, purchase cost are considered as triangular, trapezoidal, pentagonal and hexagonal fuzzy numbers.

Then the total cost equation with fuzzy parameter is follows.
\[
\tilde{A} = \begin{bmatrix}
-6e^{(\theta-m(\xi))\tilde{t}_1}b\tilde{T}_1\alpha(\theta-m(\xi))^2 - \\
6a\beta(\theta-m(\xi))e^{(\theta-m(\xi))\tilde{t}_1} + \\
6a\alpha(\theta-m(\xi))^3\tilde{T}_13a\beta(\theta-m(\xi))^3\tilde{T}_1^2 + 3b(\theta-m(\xi))^3\alpha\tilde{T}_1^2 + 2b\beta(\theta-m(\xi))^3\tilde{T}_1^3 + 3b\beta(\theta-m(\xi))^2\tilde{T}_1^2 + 6b\beta e^{(\theta-m(\xi))}\tilde{t}_1 - \\
6a\alpha(\theta-m(\xi))^2 e^{(\theta-m(\xi))}\tilde{t}_1 + \\
6b\alpha(\theta-m(\xi)) e^{(\theta-m(\xi))}\tilde{t}_1 - \\
6b\beta(\theta-m(\xi))\tilde{T}_1 e^{(\theta-m(\xi))}\tilde{t}_1 + 6a(\theta-m(\xi))^2\alpha + \\
6b\alpha\alpha + 6a\beta(\theta-m(\xi))^2\tilde{T}_1 - 6(\theta-m(\xi))b\alpha - 6b\beta
\end{bmatrix}
\]

\[
\tilde{C}(T) = \frac{1}{T}\left[\frac{1}{2\delta^2} \begin{bmatrix}
2a\tilde{T}_z\delta^2 + b\tilde{T}_z^2\delta^2 + 2b\tilde{T}_z\delta + 2b\tilde{T}_1\tilde{T}_2\delta^2 + \\
2b\log\left(\frac{1}{1+\tilde{T}_z}\right) + \\
2b\log\left(\frac{1}{1+\tilde{T}_2}\right) + 2a\delta\log\left(\frac{1}{1+\tilde{T}_2}\right) + 2b\tilde{T}_1\delta\log\left(\frac{1}{1+\tilde{T}_2}\right)
\end{bmatrix} + \\
\pi\left[\frac{1}{2\delta^2} \begin{bmatrix}
2a\tilde{T}_z\delta^2 + b\tilde{T}_z^2\delta^2 + 2b\tilde{T}_z\delta + 2b\tilde{T}_1\tilde{T}_2\delta^2 - 2a\delta\log(1+\tilde{T}_2) - \\
2b\log(1+\tilde{T}_2) - 2b\delta\tilde{T}_1\log(1+\tilde{T}_2) - 2b\tilde{T}_1\delta\log(1+\tilde{T}_2) + 2b\tilde{T}_2\delta)
\end{bmatrix} + \\
-\frac{a}{\theta-m(\xi)} + \frac{b}{(\theta-m(\xi))^2} + \\
e^{\theta-m(\xi)\tilde{t}_1}\left[\frac{a}{\theta-m(\xi)} - \frac{b}{\theta-m(\xi)} \left(\tilde{T}_1 - \frac{1}{\theta-m(\xi)}\right)\right] - \\
\frac{a}{\delta}\log\left(\frac{1}{1+\tilde{T}_2}\right) - \frac{b(1+\delta(\tilde{T}_1))}{\delta^2}\log(1+\delta\tilde{T}_2) - \frac{b\tilde{T}_1^2}{\delta}
\end{bmatrix}
\]

---(10.13)
10.6. Numerical Examples:

Example 1.1: Given $\tilde{A} = (200, 250, 300)$, $\alpha = 0.2$, $\beta = 0.1$, $\theta = 0.5$, $m(\xi) = 0.3$,

$\tilde{T}_1 = (0.8, 0.9, 1.0)$, $b = 2$, $a = 10$, $\tilde{x} = (8, 10, 12)$, $\delta = 8$,

$\tilde{T}_2 = (0.323, 1.323, 1.823)$, $\pi = 8$, $\tilde{c} = (30, 40, 50)$,

then the fuzzy total cost is $T\tilde{C} = (4181.51, 2503.07, 2290.86)$.

Crisp Total cost for Triangular fuzzy number is, $TC = 2991.81$.

Example 1.2: Given $\tilde{A} = (150, 250, 300, 350)$, $\alpha = 0.2$, $\beta = 0.1$, $\theta = 0.5$,

$m(\xi) = 0.3$, $\tilde{T}_1 = (0.7, 0.8, 0.9, 1.0)$, $b = 2$, $a = 10$, $\tilde{x} = (6, 8, 12, 14)$, $\delta = 8$,

$\tilde{T}_2 = (0.123, 0.523, 1.323, 1.823)$, $\pi = 8$, $\tilde{c} = (20, 30, 40, 60)$,

then the fuzzy total cost is $T\tilde{C} = (4725.95, 3560.25, 2507.61, 2294.20)$.

Crisp Total cost for Trapezoidal fuzzy number is, $TC = 3272.1$.

Example 1.3: Given $\tilde{A} = (150, 200, 300, 350, 400)$, $\alpha = 0.2$, $\beta = 0.1$, $\theta = 0.5$,

$m(\xi) = 0.3$, $\tilde{T}_1 = (0.6, 0.7, 0.8, 0.9, 1.0)$, $b = 2$, $a = 10$, $\tilde{x} = (5, 6, 8, 10, 12)$, $\delta = 8$,

$\tilde{T}_2 = (0.323, 0.823, 1.323, 1.823, 2.123)$, $\pi = 8$, $\tilde{c} = (20, 25, 30, 40, 50)$,

then the fuzzy total cost is $T\tilde{C} = (3409.37, 2570.03, 2231.99, 2054.56, 2079.18)$.

Crisp Total cost for Pentagonal fuzzy number is, $TC = 2588.88$. 
**Example 1.4:** Given $\tilde{A} = (100, 150, 200, 250, 300, 350)$, $\alpha = 0.2$, $\beta = 0.1$, $\theta = 0.5$, $m(\xi) = 0.3$, $\tilde{T}_1 = (0.3, 0.4, 0.8, 0.9, 1.0, 1.1)$, $b = 2$, $a = 10$, $\tilde{x} = (5, 6, 8, 10, 12, 14)$, $\delta = 8$, $\tilde{T}_2 = (0.123, 0.323, 0.523, 0.823, 1.323, 1.823)$, $\pi = 8$, $\tilde{c} = (30, 40, 50)$, then the fuzzy total cost is $T\tilde{C} = (2834.6, 2882.09, 3552.45, 3223.77, 2779.34, 2533.6)$.

Crisp Total cost for Hexagonal fuzzy number is, $TC = 2967.31$.

**Example 1.5:** Given $\tilde{A} = (100, 125, 150, 200, 250, 300, 350, 400)$, $\alpha = 0.2$, $\beta = 0.1$, $\theta = 0.5$, $m(\xi) = 0.3$, $\tilde{T}_1 = (0.2, 0.3, 0.4, 0.7, 0.9, 1.0, 1.1, 1.2)$, $b = 2$, $a = 10$, $\tilde{x} = (4, 5, 6, 7, 8, 10, 11, 12)$, $\delta = 8$, $\tilde{T}_2 = (0.093, 0.123, 0.223, 0.523, 0.823, 1.323, 1.523, 1.823)$, $\pi = 8$, $\tilde{c} = (10, 15, 20, 30, 40, 50, 60, 65)$, then the fuzzy total cost is

$T\tilde{C} = (3209.69, 2846.67, 2882.09, 3196.24, 3223.45, 2779.6, 2820.15, 2775.57)$.

Crisp Total cost for Octagonal fuzzy number is, $TC = 2966.31$.

**Example 1.6:** Given $\tilde{A} = (100, 125, 150, 200, 250, 300, 350, 375, 400)$, $\alpha = 0.2$, $\beta = 0.1$, $\theta = 0.5$, $m(\xi) = 0.3$, $\tilde{T}_1 = (0.1, 0.2, 0.3, 0.4, 0.5, 0.7, 0.8, 0.9, 1.0)$, $b = 2$, $a = 10$, $\tilde{x} = (3, 4, 5, 6, 7, 8, 9, 10, 11)$, $\delta = 8$, $\tilde{T}_2 = (0.093, 0.123, 0.223, 0.253, 0.453, 0.823, 1.123, 1.323, 1.523)$, $\pi = 8$, $\tilde{c} = (10, 15, 20, 25, 30, 40, 50, 60, 70)$,
then the fuzzy total cost is

\[ T\tilde{C} = (3518.84, 2238.77, 2316.13, 2500.62, 2675.34, 2584.59, 2467.09, 2514.72, 2568.49). \]

Crisp Total cost for Nonagonal fuzzy number is, \( TC = 2598.39. \)

**Example 1.7:** Given \( \tilde{A} = (100, 125, 175, 200, 250, 300, 375, 400, 425, 450), \)

\( \alpha = 0.2, \beta = 0.1, \theta = 0.5, m(\xi) = 0.3, \)

\( \tilde{T}_1 = (0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.5), b = 2, a = 10, \)

\( \tilde{x} = (2, 3, 4, 8, 9, 10, 11, 12, 13, 14), \delta = 8, \)

\( \tilde{T}_2 = (0.323, 0.423, 0.523, 0.823, 0.923, 1.123, 1.223, 1.323, 1.523, 1.823), \pi = 8, \)

\( \tilde{c} = (20, 25, 30, 35, 40, 45, 50, 55, 60, 65), \)

then the fuzzy total cost is \( T\tilde{C} = (1038.14, 1171.17, 1486.75, 1487.75, 2089.5, 
2162.17, 2348.3, 2517.44, 2571.27, 3498.73). \)

Crisp Total cost for Decagonal fuzzy number is, \( TC = 2037.82. \)
Table 10.1: Total cost for fuzzy numbers

<table>
<thead>
<tr>
<th>Fuzzy numbers</th>
<th>Total cost</th>
</tr>
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<tbody>
<tr>
<td>Triangular</td>
<td>2991.81</td>
</tr>
<tr>
<td>Trapezoidal</td>
<td>3272.1</td>
</tr>
<tr>
<td>Pentagonal</td>
<td>2588.88</td>
</tr>
<tr>
<td>Hexagonal</td>
<td>2967.31</td>
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<td>Octagonal</td>
<td>2966.31</td>
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<tr>
<td>Nonagonal</td>
<td>2598.39</td>
</tr>
<tr>
<td>Decagonal</td>
<td>2037.82</td>
</tr>
</tbody>
</table>

Table 10.2. Linguistic terms and fuzzy numbers

<table>
<thead>
<tr>
<th>Linguistic terms</th>
<th>Fuzzy numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very Optimistic</td>
<td>Decagonal fuzzy number</td>
</tr>
<tr>
<td>Optimistic</td>
<td>Pentagonal, nonagonal fuzzy number</td>
</tr>
<tr>
<td>Pessimistic</td>
<td>Octagonal, triangular, hexagonal fuzzy number</td>
</tr>
<tr>
<td>Most Pessimistic</td>
<td>Trapezoidal fuzzy number</td>
</tr>
</tbody>
</table>

Example 2.1.

Let \( T = 6 \),

\[ H \text{ (octagonal fuzzy number)} = (8, 10, 11, 12, 13, 14, 15, 16), \]

\[ A \text{ (octagonal fuzzy number)} = (15, 17, 19, 21, 23, 24, 25, 27) \]
K = any three points between 0 and 1. Here k = 0.5.

Table 10.3. Order quantity and total cost for different demand rates

<table>
<thead>
<tr>
<th>D</th>
<th>Q</th>
<th>TC</th>
</tr>
</thead>
<tbody>
<tr>
<td>450</td>
<td>22.68</td>
<td>1428.71</td>
</tr>
<tr>
<td>500</td>
<td>27.88</td>
<td>1523.7</td>
</tr>
<tr>
<td>550</td>
<td>24.89</td>
<td>1579.54</td>
</tr>
<tr>
<td>600</td>
<td>25.99</td>
<td>1649.78</td>
</tr>
<tr>
<td>650</td>
<td>27.06</td>
<td>1717.14</td>
</tr>
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</table>

Table 10.4. Linguistic terms for different demand rates

<table>
<thead>
<tr>
<th>Demand</th>
<th>Linguistic terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>450</td>
<td>Very less</td>
</tr>
<tr>
<td>500</td>
<td>Less</td>
</tr>
<tr>
<td>550</td>
<td>Average</td>
</tr>
<tr>
<td>600</td>
<td>High</td>
</tr>
<tr>
<td>650</td>
<td>Very high</td>
</tr>
</tbody>
</table>

Table 10.5. Linguistic terms for different quantities

<table>
<thead>
<tr>
<th>Order quantity</th>
<th>Linguistic terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.68</td>
<td>Very low</td>
</tr>
<tr>
<td>27.88</td>
<td>Low</td>
</tr>
<tr>
<td>24.89</td>
<td>Medium</td>
</tr>
<tr>
<td>25.99</td>
<td>High</td>
</tr>
<tr>
<td>27.06</td>
<td>Very high</td>
</tr>
</tbody>
</table>
Table 10.6. Linguistic terms for different total cost values

<table>
<thead>
<tr>
<th>Total cost</th>
<th>Linguistic terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>1428.71</td>
<td>Least minimum</td>
</tr>
<tr>
<td>1523.7</td>
<td>Minimum</td>
</tr>
<tr>
<td>1579.54</td>
<td>Fair</td>
</tr>
<tr>
<td>1649.78</td>
<td>High</td>
</tr>
<tr>
<td>1717.14</td>
<td>Maximum</td>
</tr>
</tbody>
</table>

Observations:

- In examples 1.1 to 1.7, ordering cost, holding cost, purchase cost, backorder cost are considered as triangular, trapezoidal, pentagonal, hexagonal, octagonal, nonagonal, decagonal fuzzy numbers respectively and we get the corresponding total costs. We list all these costs in table 10.1.

- From table 10.1 among all the fuzzy numbers, decagonal number gives the minimum total cost. We present these facts in table 10.2 using linguistic terms.

- From table 10.3, demand increases then ordering quantity and total cost also increases.

- From table 10.4, 10.5 and 10.6, Very high demand gives very high ordering quantity and also the maximum total cost.
Conclusion:

The purpose of this study is to present an inventory model involving controllable deterioration rate to extend the traditional EOQ model. The products with high deterioration rate are always crucible to the retailer’s business. So in this study we reduce the deterioration rate using preservation technology. Time, Ordering cost, backorder cost, purchase cost are taken as Triangular, trapezoidal, pentagonal, hexagonal, octagonal, nonagonal and decagonal fuzzy numbers. Centroid method is used to defuzzify the fuzzified inputs. Obtaining the optimum total cost by comparing various fuzzy numbers under uncertain linguistic environment. Linguistic model provides natural way of dealing with problems in which the source of imprecision and vagueness occurs and it can be applied in many fields. Numerical examples have presented to illustrate the model.