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The present thesis is divided into two distinct parts, Part I and Part II. In part I, we consider higher order meson equations and the effect of such an equation on some nuclear phenomena. In part II, we make a study of elementary particles in curved space-time in order to include the general theory of relativity.

Field theory at present has failed to give definite answers to most physical problems outside the realm of quantum electrodynamics. This is particularly attributed to the failure of perturbation theory in meson physics. Our defence for undertaking researches of Part I is that, in absence of better mathematical techniques, it is useful to seek the possible success of some entirely different meson theory to explain these nuclear phenomena, inspite of the fact that these theories may have difficulties of their own. In this attempt, we have met with limited success as regards experimental agreement. But our efforts have given rise to many points of theoretical interest regarding these higher order meson equations.

In Part II, we have considered the elementary particles and their interactions in curved space-time. It is often noted that general theory of relativity is neglected in the study of elementary particles and their interactions. In the author's opinion, the success of the concepts of general theory of relativity in the macroscopic world points to its general validity in the physical world, and hence should not be forgotten when we study the behaviour of elementary
particles. Here we have given a scheme for writing down quantum mechanical equations in curved space-time with the help of a fairly physical postulate.

**NOTATIONS**

Most of the notations have been explained in appropriate places in the text. However, the following broad features may be noted.

We have throughout used natural units, such that

\[ h = c = 1. \]

\( \eta^{\mu\nu} \) always denotes the metric tensor of the space-time world, which, when the space is flat, is given as

\[ \eta_{11} = \eta_{22} = \eta_{33} = \eta_{00} = 1. \]

We have throughout assumed summation convention. Regarding tensor indices, in Part I, where only special theory of relativity is taken into account, we have written all the tensor indices as covariant, with the summation convention here as

\[ A_{\mu}B_{\nu} = A_{1}B_{1} + A_{2}B_{2} + A_{3}B_{3} - A_{0}B_{0} \]

\[ = A_{\mu}B_{\nu} - A_{0}B_{0} , \]

and similarly for summation of tensors of higher rank. In Part II, the distinction between contravariant and covariant tensors is important, and hence here the appropriate tensor indices are written.

We particularly note the difference in notation
in Part I and Part II. In Part I, $\delta_{kl}$ denotes the Kronecker delta throughout, whereas in Part II, this denotes the flat space metric. Also, in Part I, $x_0$ is the time coordinate, whereas in Part II, $x^0$ is the time coordinate.

**SUMMARY OF THE PRESENT WORK**

**Part I:** Here we have mainly considered a pseudo-scalar meson theory that satisfies a fourth order equation. Such a theory was first proposed by Bhabha (1950) and Thirring (1950) and has the distinct advantage of giving rise to finite results for many physical processes where we previously had infinities. It has also been shown by Bhabha that the second order potential obtained from this does not have a $r^{-3}$ singularity, making bound states more probable. Because of these advantages, we have carried out a more detailed application of this to some nuclear phenomena.

In Section 2, §2, we have first generalised Schwinger's Action Principle to the case when the Lagrangian contains derivatives of field operators up to an arbitrary finite order. The commutation relations for space-like separated points have been obtained from this action principle for this general case.

In Section 3, §1, we have applied this technique for the case of the Lagrangian containing second order derivatives of the field operators, which gives rise to the fourth order equations. The commutation relation of the field operators for arbitrary separation of space-time points has been obtained. The result is not new, (see
Thirring (1950)), but the method is an illustration of the generalisation given in Section 2, § 2. In Section 3, § 2, we have defined a symmetrical energy-momentum tensor for this Lagrangian, again applying Schwinger's Action Principle. It is noted that here we require a technique similar to the case of particles with spin, although actually we are dealing with particles without spin.

In Section 4 we have obtained an expression for the anomalous magnetic moments of the nucleons up to the second order in meson nucleon coupling constant. It is noted that the values thus obtained are in comparatively excellent agreement with experiments. The coupling constant independent ratio of the anomalous neutron moment to the proton moment is 1.6 as compared with the experimental value of 1.07. Such an agreement has not been obtained in conventional meson theories without bringing in parameters other than the coupling constant, such as in cut-off methods, or bringing in interactions of strange particles which provides an additional coupling constant. It may also be noted that in the calculations of the present Section we do not have any infinities that would be present in conventional meson theory.

To obtain the anomalous magnetic moment in this case, we have also defined current in the particular case when the Lagrangian contains second order derivatives of the field operators.

In Section 5, we have evaluated the S-matrix elements for the interaction of two nucleons in this theory.
upto the fourth order in the coupling constant. The second order S-matrix element has been applied to the case of neutron-proton scattering. The result so obtained is not in agreement with experiments, and the coupling constant here becomes different from what we have for anomalous magnetic moments. In the fourth order calculations, Thirring (1950) has noted earlier that in this theory the self-mass of the nucleon is small and finite. We further note that the vertex diagram also gives rise to a finite contribution for the charge renormalisation which is also small for a reasonable value of the coupling constant. The physically important part after the renormalisation has been carried out has been calculated.

Since in this theory we only change the meson propagator with the nucleon propagator remaining the same, the divergent expression for the vacuum polarisation or meson self-energy graph here remains unaltered. The renormalisation of this term after a covariant separation of infinities has to be carried out here in the standard manner, and this gives rise to serious difficulties with interesting consequences. We note in Appendix, Section 5, that this renormalisation can be carried out only if the free field operator has two suitably different bare masses, so that finally we can have one physical mass. On the other hand, if we had a particle with a single bare mass, such an interaction would result in two different physical masses. Such a splitting of the mass would be very interesting if the renormalisation terms were finite; unfortunately this is not so.
In Appendix, Part I, we have considered the interpretation of the fourth order meson field as consisting of two interacting fields satisfying Klein-Gordon equation, one of which is hermitian and the other, anti-hermitian. The interpretation of the hermitian field operator is usual. On the other hand, quantisation of the anti-hermitian field is to be carried out as in Pauli (1943). This requires indefinite metric, and with this interpretation, is the cause of negative energy or probability associated with these higher order equations (Pais and Uhlenbeck (1950)). The hermitian fields represent real particles, and, with the present usage, the anti-hermitian fields represent "ghosts".

Part II:- Firstly, we have set up here the equations for elementary particles when the space is curved by substituting the curved space metric for the flat space one in the algebraic relationship that determines the matrices describing these fields. It is observed that for Dirac or Duffin-Kemmer matrices, the same set of equations are obtained in the linear approximation if we assume that the flat space equations are true in the local frame of reference at any point. The above is taken as a fundamental postulate. The approximate corrections when the curvature is small have been obtained. It is observed that this procedure with its covariant generalisation means a definition of 'natural' affine relationship, which gives us teleparallelism or the concept of parallel vectors at a finite distance in space-time. This corresponds to results used by Green (1958).