CHAPTER 3

STEADY MHD FLOW AND HEAT TRANSFER
OF AN ELECTRICALLY CONDUCTING
VISCO-ELASTIC FLUID BETWEEN
TWO STRETCHED/ SQUEEZED
PLATES IN A POROUS MEDIUM
3.1 INTRODUCTION

The heat and mass transfer of an electrically conducting visco-elastic fluid between horizontal stretched/squeezed plates has a lot of applications in various fields of electro-chemistry, polymer processing and metallurgical operations. In plastic processing (where one deals with the stretching of plastic sheets) and specific metallurgical operations, the principles of hydromagnetic techniques are applied to control the rate of cooling of continuous stretched strips or filaments by drawing them through a quiescent fluid. The application is well marked in case of drawing, annealing and thinning of copper wires. In all these cases, the properties of the final product depend to a great extent on the rate of cooling. By drawing such strips in an electrically conducting visco-elastic fluid subject to a magnetic field, the rate of cooling can be controlled to obtain the final product having desired characteristics under suitable experimental conditions. Another interesting application of hydromagnetics to metallurgy lies in the purification of molten metals from non-metallic inclusions by the application of magnetic field.

In view of its applications in various fields of science, the study of heat and mass transfer of an electrically conducting visco-elastic fluid has attracted the attention of several research workers. The boundary layer flow generated by a continuous moving solid surface has been studied by Sakiadis [1] and the predicted results are experimentally verified by Tsou et al. [2]. Srivastava [3] has studied the flow of second fluids with heat transfer between two plates, one is moving plate and the other one is at rest. The velocity distributions and or, the temperature and concentration profiles for the region surrounding a fibre during wet spinning of a single filament has been discussed by Griffith [4]. Srivastava and Sharma [6] have also extended their works to study the problem in a Newtonian electrically conducting fluid in the presence of a transverse magnetic field.
Erickson et al. [6] have studied the problem of heat and mass transfer on a moving continuous flat plate with suction or injection. Samuel [7] has developed a series solution for laminar boundary layer with stationary origin on a porous surface. The problem relating to the mass transfer in the electro-chemical concentration cell has been studied by Chin [8] and Gorla [9].

The problem, being significantly related to continuous extraction processes (involving squeezing/stretching) used in manufacturing of sheets and fibres in the glass and polymer industries, has drawn the attention of several investigators [10,11]. These studies are based on the assumption that the moving sheet is inextensible. However, the problem relating to stretching of plastic sheets (where the moving boundary is extensible) has been pointed out by Crane [12] and Mc.Cormack and Crane [13]. Gupta and Gupta [14] have studied the case of heat and mass transfer on a stretching sheet with suction or blowing. The flow of an incompressible viscous fluid with constant free stream velocity past an infinite stretching wall has been studied by Danberg and Fansler [15]. Chakrabarti and Gupta [16] have discussed the heat transfer over a stretching sheet. Bookakote and Bharali [17] have studied the heat transfer on the MHD flow between two co-axial non-conducting porous discs, when one rotates and the other is at rest. Borkakoti and Bharali [18] have also discussed the hydromagnetics flow and heat transfer between two horizontal plates, the lower plate being stretched.

Recently, Dash and Ojha [19] have investigated the hydromagnetic flow and heat transfer of visco-elastic fluid over a porous plate in the slip flow regime. Dash and Tripathy [20] have considered the stretching effect of both the plates on the hydromagnetic flow with uniform injection at the upper plate. Ghosh [21] has studied the hall effect on hydromagnetics convection flow of a viscous incompressible electrically conducting fluid between the horizontal and perfectly conducting plates.
The present study aims at the investigation of the hydromagnetic flow and heat transfer of an electrically conducting visco-elastic fluid (Walters' B) between two horizontal, squeezing/stretching plates in a porous medium. Moreover, how the elasticity of the fluid and the permeability of the medium affects the flow phenomena which is the main objective of the analysis presented below.

3.2 MATHEMATICAL FORMULATION OF THE PROBLEM

We consider the flow of an incompressible visco-elastic electrically conducting fluid in a porous medium between two horizontal parallel non-conducting plates which are either solid/stretching/squeezed having porosity on the upper plate. The transverse magnetic field, $B_0$ has been applied along the $y$-axis and $y$-axis is perpendicular to the plates placed at $y = \pm h$. The fluid is injected through the upper-porous plate with constant velocity $v_0$. The induced magnetic field is neglected which is valid for small magnetic Reynolds number. The external electric field is zero and the electric field due to polarization of charges is negligible. With the above assumptions the governing equations of steady flow are given by

$$\begin{align*}
\frac{u}{\partial x} + \frac{v}{\partial y} &= -\frac{1}{\rho} \frac{\partial P}{\partial x} + v \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right] - K_0^* \left[ u \frac{\partial^3 u}{\partial x^3} + u \frac{\partial^3 u}{\partial x \partial y^2} + v \frac{\partial^3 u}{\partial x^2 \partial y} ight. \\
&\quad + v \frac{\partial^3 u}{\partial y^3} - 3 \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial y^2} - 2 \frac{\partial v}{\partial y} \frac{\partial^2 u}{\partial y^2} \\
&\quad - 2 \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y} - 2 \frac{\partial v}{\partial y} \frac{\partial^2 u}{\partial x \partial y} \right] - \frac{\sigma B_0^2 u}{\rho} - \frac{v u}{K^*} \\
&\quad (3.2.1)
\end{align*}$$

$$\begin{align*}
\frac{u}{\partial y} + \frac{v}{\partial x} &= -\frac{1}{\rho} \frac{\partial P}{\partial y} + v \left[ \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right] - K_0^* \left[ u \frac{\partial^3 v}{\partial x^3} + u \frac{\partial^3 v}{\partial x \partial y^2} + v \frac{\partial^3 v}{\partial x^2 \partial y} + v \frac{\partial^3 v}{\partial y^3} \\
&\quad - \frac{\partial v}{\partial x} \frac{\partial^2 u}{\partial x^2} - \frac{\partial v}{\partial x} \frac{\partial^2 u}{\partial y^2} - \frac{\partial v}{\partial y} \frac{\partial^2 v}{\partial x^2} - 3 \frac{\partial v}{\partial y} \frac{\partial^2 v}{\partial y^2} - 2 \frac{\partial u}{\partial x} \frac{\partial^2 v}{\partial y^2} \\
&\quad - \frac{\partial u}{\partial y} \frac{\partial^2 v}{\partial x \partial y} - \frac{\partial v}{\partial y} \frac{\partial^2 v}{\partial x \partial y} \right] - \frac{vv}{K^*} \\
&\quad (3.2.2)
\end{align*}$$
\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]  \hspace{1cm} (3.2.3)

where \( u, v \) are the velocity components along \( x \)- and \( y \) directions; \( \sigma \) the electrical conductivity, \( K^* \) the permeability parameter and \( K_0^* \) is the elastic parameter. All concerned variables are taken as independent of \( z \).

The boundary conditions are

\[
\begin{align*}
    u &= -C_1 x, \quad v = 0 \quad \text{at} \quad y = -h \\
    u &= C_2 x, \quad v = -v_0 \quad \text{at} \quad y = +h
\end{align*}
\]  \hspace{1cm} (3.2.4)

where \( C_1 \) and \( C_2 \) are constants.

We assume

\[
u = Cx f' (\eta) \quad \text{and} \quad v = -C h f (\eta) \quad \eta = \frac{y}{h}
\]  \hspace{1cm} (3.2.5)

where \( C > 0 \), \( u = -C_1 x \) velocity of the lower plate, \( v = C_2 x \), velocity of the upper plate and prime denotes differentiation with respect to \( \eta \).

Substituting equation (3.2.5) in (3.2.1) and (3.2.2) we get,

\[
-\frac{1}{\rho} \frac{\partial P}{\partial x} = C^2 x \left[ f' f'' - f' f''' - \frac{f'''}{R} - Rc (f f'' + f'' f') \right]
\]  \hspace{1cm} (3.2.6)

\[
-\frac{1}{\rho h} \frac{\partial P}{\partial \eta} = C^2 h \left[ f' f'' + \frac{f'''}{R} + Rc (f f'' - 3 f' f') \right]
\]  \hspace{1cm} (3.2.7)

where \( R = \frac{Ch^2}{v} \), the injection Reynolds number \( \cdot \)

\( Rc = \frac{K_0^*}{h^2} \), the elastic number \( \cdot \)

\( M = B_0 h \left( \frac{\sigma}{\sqrt{\nu \rho}} \right)^{1/2} \), the Hartmann number \( \cdot \)
and \( \alpha = \frac{CK}{v} \), the permeability parameter.

Eliminating pressure term from equation (3.2.6) and (3.2.7) we get,

\[
f'''' - R (f'' - ff'^{''}) + R Rc (ff^{'''} + f'^{'''} - 2f'f^{''''}) - M^2 f'' - \frac{Rf'}{\alpha} = A
\]  

(3.2.8)

where \( A \) is an arbitrary constant. Now for small values of \( R \), we can develop a regular perturbation scheme by expanding \( f \) and \( A \) in ascending powers of \( R \) as

\[
f = \sum_{n=0}^{\infty} R^n f_n, \quad A = \sum_{n=0}^{\infty} R^n A_n
\]

(3.2.9)

Substituting equation (3.2.9) in (3.2.8), equating like powers of \( R \) and neglecting higher powers of \( R \) we get,

\[
f''''' - M^2 f''_0 = A_0
\]

(3.2.10)

\[
f_1''''' - M^2 f_1'' = A_1 - (f_0'' - f_0^{''''}) + \frac{f_0'}{\alpha} - Rc (f_0^{'''} + f_0^{''''} - 2f_0'f_0^{''''})
\]

(3.2.11)

The boundary conditions become

\[
\begin{align*}
  f_0 = \lambda, & \quad f_0' = C_4, \quad f_n = f_n' = 0 \text{ for all } n > 0 \text{ at } \eta = 1 \\
  f_0 = 0, & \quad f_0' = C_3, \quad f_n = f_n' = 0 \text{ for all } n > 0 \text{ at } \eta = -1
\end{align*}
\]

(3.2.12)

where \( \lambda = \frac{v_0}{Ch} \), injection parameter

\[
C_3 = -C_1/C \quad \text{and} \quad C_4 = C_2/C
\]

3.3 SOLUTION

Equations (3.2.10) and (3.2.11) are solved with the help of equation (3.2.12). The expressions for \( f_0 \) and \( f_1 \) are
\[ f_0(n) = F + N_1 \eta + N_2 \sinh M \eta + N_3 \cosh M \eta \]
\[ f_1(n) = N_4 + N_5 \eta + N_5 \cosh M \eta + N_4 \sinh M \eta - \frac{7A_0}{4M} \left( \frac{1}{M^2} + Rc \right) \]
\[ (N_2 \cosh M \eta + N_3 \sinh M \eta) - \frac{\eta}{2} \left\{ F(1 + M^2 Rc) + \frac{1}{M \alpha} \right\} \]
\[ (N_2 \sinh M \eta + N_3 \cosh M \eta) + \frac{\eta^2 A_0}{4} \left( \frac{1}{M^2} + Rc \right) \]
\[ (N_2 \sinh M \eta + N_3 \cosh M \eta) \]

\( \therefore f(n) = f_0(\eta) + E_f(\eta) \) (3.3.2)

where

\[ N_1 = \frac{\lambda M \cosh M - (C_3 + C_4) \sinh M}{2(M \cosh M - \sinh M)} \]
\[ N_2 = \frac{C_3 + C_4 - \lambda}{2(M \cos h M - \sin h M)} \]
\[ N_3 = \frac{C_4 - C_3}{2 \sinh M}, \quad F = \frac{1}{2} (\lambda - 2N_4 \cos h M) \]
\[ A_0 = -M^2 N, \]
\[ N_4 = \frac{A_0}{4M} \left( \frac{1}{M^2} + Rc \right) N_3 \sin h M (7 - \frac{7}{M} \cot h M - 5 \cot h^2 M) \]
\[ - \frac{\left\{ F(1 + M^2 Rc) + \frac{1}{M \alpha} \right\}}{2} N_2 \left( \frac{1}{\sinh M} + \frac{\cosh M}{M} \right) \]
\[ N_5 = \frac{FM(1 + M^2 Rc) + \frac{1}{\alpha}}{2(M \cosh M - \sinh M)} N_3 \sin h M - \frac{A_0}{4} \left( \frac{1}{M^2} + Rc \right) \]
\[ \frac{N_2 (M \cosh M - 6 \sinh M)}{2(M \cosh M - \sinh M)} \]

\( (M \cosh M - \sinh M) \)
\[ N_6 = \frac{A_0}{4} \left( \frac{1}{M^2} + Rc \right) \left( \frac{7 \cosh hM}{M} - \sin hM \right) N_2 \]
\[ + \frac{\left\{ F(1 + M^2 Rc) + \frac{1}{M \alpha} \right\}}{2} N_3 \cos hM - N_5 \sin hM \]

\[ N_7 = \frac{A_0}{4 M^2} \left( \frac{1}{M^2} + Rc \right) (7 - M^2)N_3 + \frac{5A_0}{4M} \left( \frac{1}{M^2} + Rc \right) N_3 \cosh M \]
\[ + \frac{\left\{ F(1 + M^2 Rc) + \frac{1}{M \alpha} \right\}}{2} N_2 \left( \frac{1}{M} + \cot hM \right) \]

\[ A_1 = -\frac{A_0^2}{M_4} + M^2 (1 + 2M^2 Rc) (N_3^2 - N_2^2) - M^2 N_6 + \frac{A_0}{\alpha M^2} \]

### 3.4 HEAT TRANSFER

The energy equation is given by

\[ \rho C_p \left[ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right] = k \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] + \phi + \frac{J^2}{\sigma} \tag{3.4.1} \]

where \( \phi = 2\rho v \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 + \frac{1}{2} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \right] \) is the viscous dissipation term,

\( \frac{J^2}{\sigma} \) is the Ohmic dissipation, \( C_p \) is the specific heat at constant pressure and \( k \) is the thermal conductivity.

Two different cases are considered.

**Case I:** When the plates are maintained at different temperatures.

**Case II:** When the upper plate is at constant temperature and the lower plate is a diabatic.

**Case I:** when the plates are at different temperatures, the boundary conditions are
\[ T = T_1 \text{ at } y = +h \]
\[ T = T_0 \text{ at } y = -h \]

where \( T_0 < T_1 \)

Using the expression
\[
T = T_0 + \frac{C^2 h^2}{RC_p} \left[ \phi (\eta) + \frac{x^2}{h^2} \psi (\eta) \right]
\]

and comparing the co-efficients of \( x^2 \) and the term independent of \( x \), Equation (3.4.1) reduces to
\[
\psi'' = P_y R \left[ 2f' \psi - f' \psi' \right] - (f'' + M^2 f'^2)
\]
\[
\phi'' + 2\psi = -P_y R (4f'^2 + f' \psi')
\]

where \( P_y = \frac{\rho v C_p}{k} \)

Further, expanding \( \phi \) and \( \Psi \) in powers of \( R \) as
\[
\phi = \sum_{n=0}^\infty R^n \phi_n, \quad \psi = \sum_{n=0}^\infty R^n \psi_n
\]

and substituting equation (3.4.6) in (3.4.4) and (3.4.5) and comparing the co-efficients of like powers of \( R \), neglecting the terms \( O(R^2) \), we have
\[
\psi_0'' = 0
\]
\[
\phi_0'' + 2\psi_0 = 0
\]
\[
\psi_1'' = P_y \left[ 2f'_0 \psi_0 - f_0 \psi'_0 - f''_0 - M^2 f'^2_0 \right]
\]
\[
\phi_1'' + 2\psi_1 = -P_y \left[ 4f'^2_0 + f_0 \phi'_0 \right]
\]
The boundary conditions given in equation (3.4.2) reduce to
\[ \begin{align*}
\phi_0 &= s, \quad \psi_0 = 0, \quad \text{at } \eta = 1 \\
\phi_n &= \psi_n = 0 \quad \text{for all } n > 0 \quad \text{at } \eta = \pm 1
\end{align*} \]
(3.4.11)

where \( s = \frac{(T_1 - T_0) RC_p}{C^2 h^2} \)

Solving equations (3.4.7) - (3.4.10) with equation (3.4.11) we get
\[ \psi_0 = 0 \]
(3.4.12)
\[ \phi_0 = \frac{s(1 + \eta)}{2} \]
(3.4.13)
\[ \psi_1 = P_y M^2 \left[ A_3 + A_2 \eta - \frac{N_1^2}{2} \eta^2 - \frac{1}{4} (N_2^2 + N_3^2) \cosh 2M \eta \\
- \frac{2N_1 N_2}{M} \cos h M \eta - \frac{N_2 N_3}{2} \sin h 2M \eta - \frac{2N_1 N_3}{M} \sin h M \eta \right] \]
(3.4.14)
\[ \phi_1 = P_y \left[ A_5 + A_4 \eta - (2N_1^2 + N_2^2 M^2 - N_3^2 M^2 + A_8 M^2) \eta^2 \\
- \frac{A_2 M^2}{3} \eta^3 + \frac{N_1^2 M^2 \eta^4}{12} - \frac{4N_1 N_2}{M} \cosh h M \eta - \frac{3}{8} (N_2^2 + N_3^2) \cosh 2M \eta \\
- \frac{4N_1 N_3}{M} \sin h M \eta - \frac{3}{4} N_2 N_3 \sin h 2M \eta \right] \]
(3.4.15)

where
\[ A_2 = \frac{2N_1 N_3}{M} \sin h M + \frac{1}{2} N_2 N_3 \sin h 2M \]
\[ A_3 = \frac{N_1^2}{2} + \frac{2N_1 N_2}{M} \cosh M + \frac{1}{4} (N_2^2 + N_3^2) \cosh 2M \]
\[ A_4 = 2 \left( \frac{1}{3} + \frac{2}{M^2} \right) MN_1 N_3 \sin h M + \left( \frac{M}{6} + \frac{3}{4M} \right) MN_2 N_3 \sin h 2M \]
From equation (3.4.3) the non-dimensional expression for temperature is obtained as

\[
A_5 = 2N_1^2 + \left( \frac{5N_1^2}{12} + N_2^2 - N_3^2 \right) M^2 + \left( \frac{4N_1N_3}{M} + 2N_1N_2M \right) \cosh M \\
+ \left( \frac{3}{8} + \frac{M^2}{4} \right) \left( N_2^2 + N_3^2 \right) \cosh 2M
\]

For a moderate distance from y-axis, the second term in equation (3.4.16) is negligible in comparison to other terms. Hence equation (3.4.16) reduces to

\[
\frac{T - T_0}{T_1 - T_0} = \left[ \phi_0 + R\phi_1 + \frac{x^2}{h^2} (\psi_0 + R\psi_1) \right] / s \]

\[
= \left( \frac{1 + \eta}{2} \right) + E\phi_1 + E X^2 \phi_1 \quad (3.4.16)
\]

where \( E = \frac{R}{s} \), \( X = \frac{x}{h} \)

For temperature, instead of equation (3.4.3), we use the expression

\[
T = \frac{C^2 h^2}{RC_p} \left[ \phi(\eta) + \frac{x^2}{h^2} \psi(\eta) \right] \quad (3.4.19)
\]
The boundary conditions become
\[ \phi_0 = s_1, \quad \psi_0 = 0, \quad \phi_n = \psi_n = 0 \text{ for all } n > 0 \text{ at } \eta = 1 \]
\[ \phi_n' = \psi_n' = 0 \text{ for all } n > 0 \text{ at } \eta = -1 \]

where \[ s_1 = \frac{T_1 RC_p}{C^2 h^2} \]

The expressions for \( \phi_0, \psi_0, \phi_1 \) and \( \psi_1 \) are given by
\[ \phi_0 = s_1 \]
\[ \psi_0 = 0 \]
\[ \phi_1 = P_y \left[ A_{10} - A_6 \eta - A_8 \eta^2 - \frac{A_2 M^2 \eta^3}{3} + \frac{M^2 N_1^2}{12} \eta^4 - \frac{4 N_3 N_2}{M} \cosh M \eta \right. \]
\[ - \frac{3}{8} (N_2^2 + N_3^2) \cosh 2M \eta - \frac{4 N_1 N_3}{M} \sinh M \eta - \frac{3}{4} N_2 N_3 \sinh 2M \eta \]
\[ + \frac{1}{2} (N_2^2 + N_3^2) M \sin h 2M \left] \right. \]
\[ (3.4.21) \]
\[ \psi_1 = M^2 P_y \left[ A_7 - A_6 \eta - \frac{1}{2} N_1^2 \eta^2 - \frac{1}{4} (N_2^2 + N_3^2) \cosh 2M \eta \right. \]
\[ - \frac{2 N_1 N_2}{M} \cosh M \eta - \frac{1}{2} N_2 N_3 \sin h 2M \eta - \frac{N_1 N_3}{M} \sinh M \eta \]
\[ \left. \right] \]
\[ (3.4.22) \]

where
\[ A_6 = N_1^2 + 2 N_1 N_2 \sin h M + \frac{1}{2} (N_2^2 + N_3^2) M \sinh 2M \]
\[ - N_2 N_3 M \cosh 2M - 2 N_1 N_3 \cos h M \]
\[ A_7 = A_6 + \frac{N_1^2}{2} + \frac{2 N_1 N_2}{M} \cosh M + \frac{1}{4} (N_2^2 + N_3^2) \cosh 2M \]
\[ + \frac{2 N_1 N_3}{M} \sin h M + \frac{1}{2} N_2 N_3 \sin h 2M \]
\[ A_8 = 2N_1^2 + (N_1^2 - N_3^2 + A_3) M^2 \]

\[ A_9 = 2A_8 - \left( \frac{N_1^2}{3} + A_2 \right) M^2 + 4N_1N_2 \sin hM + \frac{3}{4} (N_2^2 + N_3^2) M \sin h2M \]

\[ - 4N_1N_3 \cosh M - \frac{3}{2} N_2N_3M \cos h2M \]

\[ A_{10} = 3A_8 - \left( \frac{5N_1^2}{12} + \frac{2A_2}{3} \right) M^2 + 4N_1 \left( N_2 + \frac{N_3}{M} \right) \sin hM \]

\[ + \frac{3}{4} (N_2^2M + N_3^2M + \frac{N_2N_3}{2}) \sin h2M + 4N_1 \left( \frac{N_2}{M} - N_3 \right) \cos hM \]

\[ + \frac{3}{8} (N_2^2 + N_3^2 - 4N_2N_3M) \cos h2M \]

From equation (3.4.17) the non-dimensional expression for temperature is obtained as

\[ \frac{T}{T_1} = \left[ \phi_0 + R\phi_1 + \frac{x^2}{h^2} (\psi_0 + R\psi_1) \right] / s_1 \]

\[ = \left[ 1 + \frac{R}{s_1} \phi_1 + \frac{x^2}{h^2} \frac{R}{s_1} \psi_1 \right] \]

\[ = \left[ 1 + E\phi_1 + E X^2 \psi_1 \right] \quad (3.4.23) \]

where \( E = \frac{R}{s_1} \) and \( X = \frac{x}{h} \)

At a moderate distance from y-axis, the second term is negligible in comparison to other terms. Hence equation (3.4.23) reduces to

\[ \frac{T}{T_1} = 1 + E X^2 \psi, \quad (3.4.24) \]

**Numerical Integration by Runge-Kutta Method**

For solving the equation (3.2.8) exactly, we shall adopt step-by-step integration method of Runge-Kutta. For carrying out numerical integration, the equation in question is reduced to a set of first order differential equations. With
In order to carry out the step-by-step integration, we shall use Gill's procedure. To start the integration, it is necessary to know all the values of \( y_1, y_2, y_3 \) and \( y_4 \), at \( \eta = -1 \) from which point we start our forward integration. But from the boundary conditions it is seen that the values of \( y_3(-1) \) and \( y_4(-1) \) are not known. So we try to supply such values of \( y_3(-1) \) and \( y_4(-1) \) along with the known values of the other functions at \( \eta = -1 \), as would satisfy the boundary conditions at \( \eta = 1 \) to a prescribed accuracy, after the step-by-step integrations are performed. This is done by a "corrective procedure". For this a set of auxiliary equations are to be derived denoting by \( e \), any one of the values \( y_3(-1), y_4(-1) \) and differentiating the primary equation w.r.t. \( e \) we have \( \frac{\partial y_i}{\partial e} = P_i, i = 1, 2, \ldots \).

So the results of Runge-Kutta method is agreeable to our results obtained by perturbation method. However, the graphs and tables are presented based upon the numerical values obtained by perturbation method.

### 3.5 RESULTS AND DISCUSSION

The effects of Hartmann number \( (M) \), injection parameter \( (\lambda) \), permeability parameter \( (\alpha) \), elastic parameter \( (Re) \), the Reynolds number \( (R) \) on the primary velocity \( (\text{f}') \), transverse velocity \( (f) \) and the temperature field are detailed under the following two conditions.

1. Plates are at different temperatures
2. Lower plate is adiabatic.

Further, we have studied the four special cases relating to the stretching/squeezing of the plates.
Case A: When \( C_3 = C_4 = 0 \) (both plates are solid)

Case B: when \( C_3 = 1, C_4 = 0 \) (only the lower plate being a stretching sheet), the case of Borkakoti and Bharali [18].

Case C: when \( C_3 = -1, C_4 = 1 \) (lower plate is squeezed and the upper plate is stretched at the same rate)

Case D: \( C_3 = C_4 = 1 \) (both the plates being stretching sheets), the case of Dash & Tripathy [20]

The present problem is primarily devoted to study the effect of permeability parameter due to flow through porous medium.

**Primary velocity**

Case A:

Figure 3.1 shows the effects of the parameters on primary velocity. Comparing the curves IA with VA in Fig.3.1 it is seen that the permeability parameter decreases the primary velocity in the lower half of the channel but the reverse effect is observed in the upper half. The effect of the magnetic parameter, \( M \), on the primary velocity is shown by the curves IA and IIA. The magnetic parameter increases the primary velocity near both the plates but opposite effect is observed at the middle of the channel with a crest at the centre. Comparing IA and IIIA, it is observed that an increase in injection velocity contributes substantially for the growth of primary flow at all points of the channel. The Reynolds number increases the primary velocity upto the middle of the channel then decreases the velocity afterwards. Thus we may conclude that the effect of permeability parameter is opposite to that of Reynolds number of the flow. The presence of elastic elements in the flow has no significant contribution (Comparing the curves IA and VIA).
Case B:

The effect of magnetic parameter is shown by the curves, IB and IIB when the lower plate is stretched. The magnetic parameter increases the primary flow near the upper plate and opposite effect is observed near the lower plate. The effect of permeability when the lower plate is stretched, is to decrease the velocity near the lower plate and to increase it near the upper plate (IB-VB). Thus it may be inferred that the combined effect of permeability and stretching retards the fluid flow near the stretching surface. The effects of the injection parameter, \( \lambda \) and the Reynold number, \( R \), are the same in case of A. This is the case of Borkakoti and Bhareli [18].

Case C:

In the presence of constant injection on the upper plate which is subjected to a stretching force, the effect of magnetic parameter is opposite to that of case B. From the above two cases it is concluded that the stretching force plays a dominant role over Lorentz force, \( M \), on the flow near the plate. The effect of permeability parameter is the same as before i.e. as in case of B. Thus we may conclude that stretching of upper plate in the presence of injection has the same effect as that of the stretching of the lower plate (curves IC and VC).

Case D:

The effect of Lorentz force on the primary flows of visco-elastic liquid is to decrease the velocity near both the stretching sheets but the reverse affect is observed at the centre of the channel (Curves ID and IID). A comparative study of the curves (ID & IID) for \( \lambda=1 \) and \( \lambda=3 \), reveals that the suction parameter significantly changes the primary flow. The primary flow for \( \lambda=3 \) is 8 times that of the primary velocity corresponding to \( \lambda=1 \) at the centre of the channel. The same result has been observed by Dash and Tripathy [20].
The effect of permeability parameter which is of our present interest is to increase the velocity in the lower half of the channel but the opposite effect is observed near the upper half when both the plates are stretched. This is an interesting result in view of the opposite effect being exhibited by permeability parameter in comparison with the above three cases. The primary velocity rapidly changes over the entire flow field due to the stretching effect of both the plates where as no stretching or squeezing of either of the plate is to decrease the primary velocity in the lower half and to increase in the upper half of the channel.

Transverse Velocity

Figure 3.2 depicts the effect of \( M, \lambda, R \) and \( \alpha \) on the transverse component of the velocity \( f \) when there is no stretching/squeezing of the plates (Case A) and in case of one of the plates being stretched/squeezed included in case of B and C. It is seen that magnetic parameter increases the transverse velocity of the fluid near the lower plate and decreases it near the upper plate (curves IA & IIA, IB and IIB, IC and IIC) but when both the plates are being stretched the reverse effect is observed. Thus, it is to note that transverse velocity like primary velocity is also greatly affected due to stretching of both the plates in the presence of magnetic field. Hence, it is concluded that stretching of both the plates reversely affects primary velocity as well as transverse velocity in the presence of permeability parameter or magnetic parameter. The effect of the permeability when both the plates are solid (Case A) and the lower one is squeezed and the upper one is stretched (Case C) the transverse velocity decreases throughout the flow field with an exception in Case C, where flow reversal occurs near the lower plate. Considering the case B it is seen that the permeability increases the velocity near the lower plate but it decreases near the upper plate (Curve IB and VB). In Case D, when both the plates are stretched, transverse velocity component increases throughout (curve ID and VD).
It is interesting to note that due to an increase in suction parameter, $\lambda$, the primary velocity and transverse velocity increases at all points irrespective of stretching or squeezing of the plates. The effect of the Reynold's number, over the flow field in Case A and C, is to increase the transverse velocity at all points except one peculiarity is marked in case C where the flow reversal occurs for the layers adjacent to squeezing plate. This phenomena is due to the dominating effect of squeezing force over the viscosity or the elastico viscosity effect of the fluid. The numerical values of the primary velocity and transverse velocity for different values of the permeability parameter, $\alpha$, are entered in Table 1 and Table 2, respectively. After careful observation of Case A (two plates are solid) and Case D (both plates being stretched equally), it is marked that the fluid layers at the centre of the channel remains unaffected in the presence of squeezing/stretching of the plates (Table 1). But from Table 2, it is seen that the transverse velocity increases in all cases at all points of the channel due to permeability parameter except case C (lower plate is squeezed and upper plate is stretched) where the flow reversal occurs near the lower plate.

**Temperature field (plates are at different temperature)**

Figure 3.3 depicts the temperature distribution for $M=1,3$, $\lambda=1,3$ and $P_{yEX^2}=5,10$ and 20 in four different cases when the plates are at different temperatures.

**Case-A:**

Curves I to V represent the temperature profiles when the plates are neither squeezed nor stretched. It is observed that the temperature profiles are symmetric about the middle of the channel. It is further observed that with the increase in magnetic parameter $M$, suction parameter $\lambda$ and $P_{yEX^2}$, the temperature increases at all points of the fluid flow but the increase of temperature is escalated with higher values of suction parameter having larger thermal boundary layer.
Case-B:

Curves VI to X represent the temperature profiles when only the lower plate being a stretching sheet. It is observed that the temperature of the fluid at all points increases with the increase in values of magnetic parameter \( M \), suction parameter \( \lambda \) and \( P_yE^2 \) but the suction parameter increases the temperature more than the magnetic parameter.

Case-C:

Curves XI to XV depict the temperature distribution when the upper plate is stretched and the lower one is squeezed at the same rate. It is observed that the temperature of the fluid at all points increases with increase of magnetic parameter \( M \), suction parameter \( \lambda \) and \( P_yE^2 \) but the suction parameter increases the temperature more than the magnetic parameter as it was observed in Case B. The above observations remain the same for Case-D also i.e. when both the plates are stretched (Curves XVI-XX).

Temperature field (lower plate is adiabatic)

Figure 3.4 shows the temperature profiles for different values of \( M \), \( \lambda \) and \( P_yE^2 \) when the lower plate is adiabatic.

Case-A:

Curves I to V depict the temperature distribution when the plates are neither squeezed nor stretched. It is observed that an increase in the values of \( M \), \( \lambda \) and \( P_yE^2 \) results in an increase in temperature at all points of the temperature field. The suction parameter \( \lambda \) increases the temperature more than the other parameters.

The temperature profiles are almost similar for other three cases. But it is interesting to note that there is a significant rise in temperature near both the
plates when the lower plate is squeezed and the upper plate is stretched. This agrees with the physical phenomenon that when a fluid is squeezed adiabatically, it heats i.e., there is an increase in internal energy of the fluid which leads to a rise in temperature of the fluid.

3.6 CONCLUSIONS

The following conclusions are established in respect of primary velocity, transverse velocity and temperature field.

(i) The visco-elastic elements in the flow do not contribute significantly to the velocity field.

(ii) The suction parameter, $\lambda$, increases the primary velocity and transverse velocity at all points irrespective of stretching/squeezing of the plates.

(iii) The fluid layers at the centre of the channel remains unaffected due to permeability effect of the medium when both the plates are solid.

(iv) The flow reversal occurs near the lower plate which is stretched in the presence of the permeability of the medium.

(v) The stretching of the plates affect the primary velocity to a greater extent than the transverse velocity. It is due to the fact that the primary velocity is considered along the direction of the stretching of plates.

(vi) The velocity field becomes negative for the layers adjacent the squeezing plate. This is due to the squeezing force over rides the viscosity and visco-elastic effect of the fluid.

(vii) The fluid temperature increases with the increase of suction parameter, $\lambda$, magnetic parameter $M$ and $PyEX^2$. The electric current generated in the field increases the strength of the magnetic field and this causes an increase in temperature of the fluid. The increase in temperature is more significant near the adiabatic wall subject to a squeezing force.
Fig. 3.1 Primary velocity profiles ($f'(\eta)$) against $\eta$
with parameters $M$, $\lambda$, $R$, $R_c$ and $\alpha$.
Fig. 3.2 Transverse velocity profiles, $f(\eta)$ against $\eta$ with parameters $M$, $\lambda$, $R$, $R_c$ and $\alpha$. 
Fig. 3.3 Temperature distributions

(Plates at different temperature)
Fig. 3.4 Temperature distributions

(Adiabatic lower plate)
Table 3.1: Primary velocity component $f'(\eta)$ for different values of $\alpha$ ($M=1$, $\lambda=1$, $R=0.05$, $Re=0.01$)

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Table 3.2: Transverse velocity component $f(n)$ for different values of $\alpha$
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