3.1. INTRODUCTION

In the last chapter the improvement affected by supplementary signal due to the load frequency controller on the dynamic response of multi area systems is recognized. The value of gain parameter ($K$) of the controller plays a crucial role for the overall performance of the system and hence to achieve the objectives of the LFC problem of multi area power systems. Conventional load frequency controllers[9,10] use control signals based on a linear combination of frequency and tie-line power with simple proportional integral control. The gains of the controller were chosen based on intuition and engineering experience to minimize the frequency and tie-line power deviation under transient conditions and to ensure zero steady state error of these quantities. Elgerd et al [10] have reported results of analog computer simulation for optimum gain setting of two equal areas of thermal
system by minimizing a performance index. Nanda et al have reported [52] the results of digital computer simulation of two area hydro-thermal system by minimizing the same cost function and have concluded that the optimum gain setting of a particular area is not much affected by the gain settings of other interconnected areas. Therefore the value of optimum gain setting of any area considering other areas uncontrolled can be taken as optimum value for all practical purposes.

Recently due to advances in digital computer and optimal control theory, it has become practical to apply linear optimal control methods to power system problems. In this method the system state equations are developed first. The LOC is derived by minimization of the state variable deviations and control effort at the same time by choosing a suitable performance index which is a function of both state variables and control effort. The state equations are appended to the performance index by a co-state variable vector to find the optimal control. The theories are well documented in the literature [18]. The algorithm for LOC design [18] recapitulated briefly as below.

(1) A proper linear model for the electric power
system is selected and the state equations are obtained in the form
\[ X = AX + BU. \]

(2) The performance index is chosen as
\[ PI = \frac{1}{2} \int_0^\infty [X^TQX + U^TRU] \, dt. \]

(3) The weighting matrices \( Q \) and \( R \) of P.I. are selected.

(4) The state and co-state system matrix \( M \) is constructed as
\[
M = \begin{bmatrix}
A & -S \\
-Q & -A^T
\end{bmatrix}
\]

Where \( S = BR^{-T}B^T \).

(5) The eigen values and eigen vectors are computed
\[
\text{Eigen values } \Lambda = \begin{bmatrix}
\Lambda^- & 0 \\
0 & \Lambda^+
\end{bmatrix}
\]
\[
\text{Eigen vector } X = \begin{bmatrix}
X_I & X_{III} \\
X_{II} & X_{IV}
\end{bmatrix}
\]
(6) The Riccati matrix $K$ and the control $BU$ is calculated where

$$K = X^{-1} X_1^{-1}, \quad K U = -S K.$$

(7) Then the eigenvalues of the system with LOC are found out

$$X = AX + BU = (A - SK) X.$$

(8) Then the dynamic response of the system for a given disturbance is found out.

The above technique is not computationally attractive since no direct method is prescribed to select the weighting matrices $Q$ and $R$ of matrix Riccati equation. The choice of weighting matrices $Q$ and $R$ is based on intuition and engineering experiences. Arriving at an optimal controller in the above methods to improve the dynamic response of power system has been presented in references [20,21,22,23]. However these methods have the following shortcomings.

(1) If used for a multi area systems solution of higher order Riccati Matrix equation becomes difficult.

(2) The optimal control scheme so obtained involves feed back of all state variables. This in practice means transmission of state variables of one area to other areas which are physically apart.
(3) Not applicable to non-linear systems.

Therefore attempts have been made by different authors for the design of sub-optimal controllers [36,37,40] and extending the LOC design technique to output feedback using canonical form which does not require the measurement of all states. The sub-optimal controllers, using only the directly measurable states, work with the reduced system model and provide a cheap substitute for the optimal controller. Again the problem is handled as a linear regulator problem. The unavailable states are realized by differentiating the output suitable number of times through dynamic elements in the feedback path and the solution is obtained by eigenvalue grouping technique.

In this chapter a method has been proposed for finding the optimum value of gain parameter so as to minimize the frequency and tie-line deviation considering linear and nonlinear (governor dead band) multi area systems. The control scheme is proposed with only the tie-line power deviation and the frequency deviation as the control signal using a proportional and integral control. The gains are chosen so as to minimize the performance index using the steepest descent method [84,85]. Extensive study has been made
to establish the noninteraction property of integrator
gain as found by other workers [52]. The effectiveness
of the proposed method is illustrated through digital
computer simulation considering different energy
systems with and without governor dead band.

3.2. THEORY

3.2.1. Performance Index (PI)

The problem of determining the optimum value of
controller gain is formulated as a mathematical
optimization of a suitable performance index. A
meaningful measure for the quality or goodness of
transient response can be provided by choosing the
integral of squared error (ISE) criterion [10]. In this
case the performance index to be minimized is expressed
as

\[
P.I. = \int_0^T \left( \sum_{k=1}^M \Delta P_k^2 + \sum_{k=1}^{M-1} \frac{\Delta P_k^2}{T_k e_k} \right) dt \quad \ldots \quad (3.1)
\]

The problem is to choose gain parameter \( K_{I_1} \) of area
1 (say) such that the P.I. is minimized. The P.I. so
obtained by equation 3.1 is clearly a function of \( K_{I_1} \).
Since the system differential equations have been discretised for obtaining the solution by Trapezoidal integration method, the PI can also be discretised and can be written as

\[
P.I. = \Delta t \sum_{i=1}^{L} p_i^t \quad \ldots \quad (3.2)
\]

Where \( L = T/\Delta t \)

And \( p_i^t \) is the performance index at the end of \( t \)th interval of time and is given by

\[
p_i^t = \sum_{k=1}^{M} (\Delta F_k^i)^2 + \sum_{k=1}^{M-1} (\Delta P_{TKE}^i)^2 \quad \ldots \quad (3.3)
\]

At any instant of time dropping subscript \( t \) for brevity one has

\[
p_i = \sum_{k=1}^{M} \Delta F_k^2 + \sum_{k=1}^{M-1} \Delta P_{TKE}^2 \quad \ldots \quad (3.4)
\]

The above performance index is minimized through steepest descent method.

3.2.2 Evaluation of gradient

To use steepest descent method the gradient of performance index i.e. the partial derivative of
performance index with respect to the adjustable gain parameter is required. Therefore explicit expressions for gradient of performance index with respect to control parameter should be available. Since in this case no explicit expression is available the gradient of performance index is obtained numerically in the following way.

\[ \frac{\partial p_i}{\partial p} = 2 \sum_{k=1}^{M} \Delta F_k \frac{\partial \Delta F}{\partial p} + 2 \sum_{k=1}^{M-1} \Delta P_{T,\omega K} \frac{\partial \Delta P_{T,\omega K}}{\partial p} . \quad (3.5) \]

\( \Delta F_k \) and \( \Delta P_{T,\omega K} \) are the values of the state variables \( X^1, X^5 \) of \( K \)th area. So it is required to find the gradient of these state variables with respect to the control parameter \( p \).

Following the method outlined in chapter II of this thesis the system of equations can be solved using the decomposition technique. Accordingly the state-equations for each sub-system can be written as

\[ [X]_K = [A]_K [X]_K + [U]_K \quad \text{for} \quad K = 1, \ldots, M. \quad \ldots(3.6) \]

It should be remembered that \( [U]_K \) will consist of elements which are functions of state variables of the other sub-systems apart from any input given to the
sub-system.

Following the trapezoidal rule, the solution of equation 3.6 at \((t+1)\)th time interval can be written as

\[
[X]^{t+1}_k = [I-A]^{-1}_k \left[ [I+A]_k [X]_k^t + (U_k^t + U_k^{t+1}) \right]
\]

\[
= [I-A]^{-1}_k [I+A]_k [X]_k^t + (U_k^t + U_k^{t+1})
\]

\[
\text{Where} \quad [A] = \begin{bmatrix} \Delta t \end{bmatrix}_k \quad [U] = \begin{bmatrix} \Delta t \end{bmatrix}_k
\]

\[
\frac{\partial [X]^{t+1}_k}{\partial P} = \frac{\partial}{\partial P} [I-A]^{-1}_k \left[ [I+A]_k [X]_k^t + (U_k^t + U_k^{t+1}) \right]
\]

\[
+ [I-A]^{-1}_k \left[ [I+A]_k \frac{\partial [X]_k^t}{\partial P} + \frac{\partial [I+A]_k}{\partial P} [X]_k^t \right]
\]

\[
+ \frac{\partial}{\partial P} [U_k^t + U_k^{t+1}]
\]

\[
\frac{\partial [I-A]^{-1}_k}{\partial P}
\]

is obtained as follows:

Consider the equation

\[
[M] [M]^{-1} = [I]
\]

Where \([M]\) is a non-singular matrix.

Differentiating with respect to a parameter \(P\) one gets

\[
\frac{\partial [M]}{\partial P} [M]^{-1} + [M] \frac{\partial [M]^{-1}}{\partial P} = [0]
\]
Applying this result to equation (3.8) and after some more manipulations one gets

\[ \frac{\partial [X]^{t+1}}{\partial P^k} = [I-A]^{-1}_K \left[ [I+A]_K \frac{\partial [X]^t}{\partial P^k} + \frac{\partial [\Lambda]}{\partial P^k} \left[ X^{t+1}_K + X^t_K \right] + \frac{\partial}{\partial P^k} \left[ U^t_K + U^{t+1}_K \right] \right] \]

\[ K = 1, \ldots M. \quad \ldots (3.10) \]

So the gradient vector \( \frac{\partial [X]^{t+1}}{\partial P^k} \), \( K = 1, \ldots M \) can be obtained by solving the equation (3.10).

**Approximation of equation (3.10)**

If \( [U^{t+1}_t + U^{t+1}_t] \) can be approximated by the expression \( 2 [U]^t \) one has

\[ \frac{\partial [X]^{t+1}}{\partial P^k} = [I-A]^{-1}_K \left[ [I+A]_K \frac{\partial [X]^t}{\partial P^k} + \frac{\partial [\Lambda]}{\partial P^k} \left[ X^{t+1}_K + X^t_K \right] + 2 \frac{\partial}{\partial P^k} [U]^t_K \right] \]

\[ K = 1, \ldots M. \quad \ldots (3.11) \]

Computations were carried out using the exact equation (3.10) and the approximation equation (3.11).

It was found that the value of gradient vector was not
much affected because of the approximation. So in all
studies only equation (3.11) was used for obtaining the
gradient vector.

The solution of equation (3.11) only involves a
repeat solution of equation (3.7) with \([X]_k^l\) being
replaced by \(\frac{\partial [X]}{\partial p}_k^l\) and the input vector is being
replaced by

\[
\left[ \frac{\partial [A]}{\partial p}_k \left[ X^{(l+1)}_k + X^I \right] + 2 \frac{\partial [U]}{\partial p}_k \right]
\]

In terms of computation time, the increase in
computation time amounts to the time required for one
number of iteration in each time interval.

\[
\frac{\partial [X]_0}{\partial p}_k = [0] \quad \text{for } K=1,...,M \text{ since the initial}
\text{conditions} [X]_0^k, K = 1,...,M \text{ are independent of } p. \text{ In the}
\text{present example, the control parameter is } K_I.

Then

\[
\frac{\partial [\bar{A}]}{\partial p}_I = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
-B^I \Delta t & 0 & 0 & 0 & -\Delta t & 0
\end{bmatrix}
\]  

\(\text{(3.12)}\)
\[
\frac{\partial [A]}{\partial p}_K = [0] \text{ for } K = 2, \ldots M. \quad \ldots (3.13)
\]

\[
\frac{\partial [U]}{\partial p}_K = \frac{\Delta t}{2} \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
M \sum_{k=1}^{M} \frac{\partial X_j^k}{\partial p} \\
- 2 \sum_{k=1, k \neq j}^{M} T_j \frac{\partial X_j^k}{\partial p} \\
0
\end{bmatrix} \quad \ldots (3.14)
\]

For \( K = 1, M-1. \)

\[
\frac{\partial [U]}{\partial p}_M = \frac{\Delta t}{2} \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
K \sum_{k=1}^{M-1} \frac{\partial X_j^k}{\partial p} \\
I_M \sum_{k=1}^{M} \frac{\partial X_j^k}{\partial p}
\end{bmatrix} \quad \ldots (3.15)
\]

Where \( X_j^k = \Delta P_{T \circ k} \)

The values obtained from equation (3.11) are substituted in (3.5) to obtain \( \frac{\partial p}{\partial p}. \)

Hence
\[
\frac{\partial \text{PI}}{\partial P} = \Delta t \sum_{i=1}^{L} \frac{\partial p_i}{\partial P}
\]

are obtained.

3.2.3 Minimization of Performance Index

For minimization of performance index, the conjugate gradient method [85] is employed. The conjugate gradient method is a modification of steepest descent method [84]. The method has been described in Appendix B. The conjugate gradient algorithm is stated as follows.

1. Select initial point \( X_i \).
2. Set the first search direction \( S_i = -\nabla f_i \).
3. Estimate the optimal step size \( \lambda_i^* \) using cubic interpolation.
4. Find the improved point given by \( X_i = X_{i-1} + \lambda_i^* S_i \).
5. Set \( i = 2 \) (Iteration count).
6. Compute \( \beta_i \).
7. Compute \( S_i = -\nabla f_i + \beta_i S_{i-1} \).
8. Estimate optimal step size \( \lambda_i^* \) by cubic interpolation.
(9) Locate the new point
\[ X_{i+1} = X_i + \lambda_i S \]

(10) Check if \( X_{i+1} \) is optimum. The process is terminated if optimum is reached otherwise.

(11) Set \( i = i + 1 \) and repeat steps 6, 7, 8, 9 and 10.

A computer program for conjugate gradient method is given in ref. [87] and the same is employed in the present study.

3.3. NUMERICAL EXAMPLES

The following systems are considered in this study.

System 1: Two area hydro-thermal system. The optimum value of thermal area is found out with a disturbance in the same area.

System 2: Two area hydro-thermal system. The optimum value of hydro area is found out with a disturbance in the same area.

System 3: Two equal areas of single stage reheat thermal system.

System 4: Four equal areas of single stage reheat thermal system.

All the systems were tried out with the above
algorithm considering both linear and nonlinear model. The nominal system parameters for thermal system and hydro system are given in Appendix A and is as per IEEE committee [52]. A typical value for governor dead band is taken to be 0.0006 pu frequency [6,10]. A small step load perturbation of 1% of nominal loading in area 1 is considered. Equal weighting factor is given to frequency and tie-line deviation in the performance index.

3.4. RESULTS

The results of choice of optimum gain parameter for area 1 for different gain settings in other areas of the systems under investigation are given in Table 3.1 and Table 3.2. Studies were made for all the systems considering linear model (dead band neglected) and non-linear model (dead band included).

From the result it is seen that variations of $K_{ij}$, $j = 2,..M$ has very little effect on $K_{ii}$ and $P.I_{\text{norm}}$. Therefore the optimum value of $K_{ij}$ achieved at $K_{ij} = 0$, $j = 2,..M$ (i.e. all other areas are uncontrolled) can be considered as more or less optimum for any value of $K_i$ because for all practical cases the difference in performance index (P.I.) can be neglected. This
<table>
<thead>
<tr>
<th>System</th>
<th>K_{II}^{L=1,M}</th>
<th>K_{II}^{initial}</th>
<th>PI</th>
<th>K_{II}^{optimum}</th>
<th>PI</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>0.0</td>
<td>0.2966x10^{-1}</td>
<td>0.621</td>
<td>0.4695x10^{-2}</td>
</tr>
<tr>
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<td>0.4971x10^{-2}</td>
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<td>0.1373x10^{-1}</td>
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<tr>
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### TABLE 3.2
Optimum $K_{II}$ corresponding to several values of $K_{IM}$
(Nonlinear model)

<table>
<thead>
<tr>
<th>System</th>
<th>$K_{II}$</th>
<th>$K_{II}$ Initial</th>
<th>$P_I$ Initial</th>
<th>$K_{II}$ optimum</th>
<th>$P_I$ optimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0</td>
<td>0.0</td>
<td>$0.3738 \times 10^{-1}$</td>
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<td>$0.5709 \times 10^{-2}$</td>
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<td>$0.6147 \times 10^{-2}$</td>
</tr>
<tr>
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</tr>
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<td>2</td>
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<td>$0.4860 \times 10^{-1}$</td>
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<td>$0.2700 \times 10^{-1}$</td>
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<td>$0.4776 \times 10^{-1}$</td>
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<td>$0.2697 \times 10^{-1}$</td>
</tr>
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<td>0.6</td>
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<td>$0.3875 \times 10^{-1}$</td>
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<td>$0.1744 \times 10^{-1}$</td>
</tr>
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<td>3</td>
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<td>$0.2215 \times 10^{-1}$</td>
<td>1.171</td>
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</tr>
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<td>$0.1715 \times 10^{-1}$</td>
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<td>$0.1698 \times 10^{-2}$</td>
</tr>
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<td>$0.1723 \times 10^{-1}$</td>
<td>1.08</td>
<td>$0.1857 \times 10^{-2}$</td>
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<td></td>
<td>0.6</td>
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<td>$0.1650 \times 10^{-1}$</td>
<td>1.091</td>
<td>$0.2075 \times 10^{-2}$</td>
</tr>
<tr>
<td>4</td>
<td>0.0</td>
<td>0.0</td>
<td>$0.1532 \times 10^{-1}$</td>
<td>0.862</td>
<td>$0.1418 \times 10^{-2}$</td>
</tr>
<tr>
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<td>0.751</td>
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</tr>
<tr>
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<td>$0.1065 \times 10^{-1}$</td>
<td>0.624</td>
<td>$0.1394 \times 10^{-2}$</td>
</tr>
<tr>
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<td>0.6</td>
<td>0.0</td>
<td>$0.1043^{-1}$</td>
<td>0.524</td>
<td>$0.1429 \times 10^{-2}$</td>
</tr>
</tbody>
</table>
inference was derived in earlier literature [52] and the results obtained in this method well compare with it. The optimum values for linear and nonlinear models are taken from the Table 3.1 and Table 3.2 and are shown in Table 3.3. The dynamic response curves for the system under investigation taking $K_{i j} = K_{i i}$ ($K$ is the optimum value considering all other area uncontrolled), $j = 1, \ldots, M$ are illustrated in Fig.3.1 to Fig.3.24. The figures give the behaviour of the frequency deviation in the disturbed area, frequency deviation in other area and tie line power deviation with respect to time. The results presented in Fig.3.1 through Fig.3.24 demonstrate the improvement in dynamic performance in sense of maximum overshoot, steady state error and settling time. It is seen from the response curves that the proposed control improves the interconnected system dynamic performances compared with that obtained without control in both linear and nonlinear case. The dynamic response curves are quite satisfactory and one can choose the gain parameter found in the above method for all practical purposes. Thus it can be concluded that the optimal gain setting of any area is independent of the controller gain setting of other areas.
TABLE 3.3
Optimum gain settings

<table>
<thead>
<tr>
<th>System</th>
<th>Optimum gain</th>
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</thead>
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<td>Linear model</td>
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</tr>
<tr>
<td>2</td>
<td>0.509</td>
</tr>
<tr>
<td>3</td>
<td>1.181</td>
</tr>
<tr>
<td>4</td>
<td>0.59</td>
</tr>
</tbody>
</table>
FIG. 3.1 TEST SYSTEM 1 (LINEAR MODEL) (FREQUENCY DEVIATION IN AREA 1)
FIG. 3.2 TEST SYSTEM 1 (LINEAR MODEL)
(FREQUENCY DEVIATION IN AREA 2)
FIG. 3.3 TEST SYSTEM 1 (LINEAR MODEL)  
(TIE-LINE POWER DEVIATION)
FIG. 3.4 TEST SYSTEM 2 (LINEAR MODEL) 
(FREQUENCY DEVIATION IN AREA 1)
FIG. 3.5 TEST SYSTEM 2 (LINEAR MODEL)  
(FREQUENCY DEVIATION IN AREA 2)
FIG. 3.6 TEST SYSTEM 2 (LINEAR MODEL) (TIE-LINE POWER DEVIATION)

- SUB OPTIMAL
- UNCONTROLLED
FIG. 3.7 TEST SYSTEM 3 (LINEAR MODEL)  
(FREQUENCY DEVIATION IN AREA 1)
FIG. 3.8 TEST SYSTEM 3 (LINEAR MODEL) (FREQUENCY DEVIATION IN AREA 2)
FIG. 3.9 TEST SYSTEM 3 (LINEAR MODEL)
(TIE-LINE POWER DEVIATION)
FIG. 3.10 TEST SYSTEM 4 (LINEAR MODEL)
(FREQUENCY DEVIATION IN AREA 1)
FIG. 3.11 TEST SYSTEM 4 (LINEAR MODEL)  
(FREQUENCY DEVIATION IN AREA 2)
FIG. 3.12 TEST SYSTEM 4 (LINEAR MODEL)
FIG. 3.13 TEST SYSTEM 1 (NONLINEAR MODEL)
(FREQUENCY DEVIATION IN AREA 1)
FIG. 3.14 TEST SYSTEM 1 (NONLINEAR MODEL)  
(FREQUENCY DEVIATION IN AREA 2)
FIG.3.15 TEST SYSTEM 1 (NONLINEAR MODEL)
(TIE-LINE POWER DEVIATION)
Fig. 3.16 Test System 2 (Nonlinear Model)
(Frequency Deviation in Area 1)
FIG. 3.17 TEST SYSTEM 2 (NONLINEAR MODEL)
(FREQUENCY DEVIATION IN AREA 2)
Fig. 3.18 Test System 2 (Nonlinear Model) (Tie-Line Power Deviation)
FIG. 3.19 TEST SYSTEM 3 (NONLINEAR MODEL)
(FREQUENCY DEVIATION IN AREA 1)
FIG. 3.20. TEST SYSTEM 3 (NONLINEAR MODEL)
(FREQUENCY DEVIATION IN AREA 2)
FIG. 3.21 TEST SYSTEM 3 (NONLINEAR MODEL) (TIE-LINE POWER DEVIATION)
FIG. 3.22 TEST SYSTEM 4 (NONLINEAR MODEL) (FREQUENCY DEVIATION IN AREA 1)
FIG. 3.23 TEST SYSTEM 4 (NONLINEAR MODEL)
(FREQUENCY DEVIATION IN AREA 2)
FIG. 3.24 TEST SYSTEM 4 (NONLINEAR MODEL)
(TIE-LINE POWER DEVIATION)
The results of system 2, system 3, and system 4 in Table 3.3 give the optimum gain setting of thermal area connected to systems of different configurations. Although the parameters of the thermal area are taken same for all the cases in both linear and nonlinear model the gain settings are different. So gain setting of any area depends on the configuration of the system to which it is connected.

The results of system 3 and system 4 give the optimum gain setting of thermal area in two area systems and four area systems. It is seen that increase in number of areas decreases the gain settings in both linear and nonlinear model. It is also seen from the table 3.3 that the optimum gain settings obtained from the linear model are quite at variance with what it is obtained from the nonlinear model (realistic model).

3.5. COMPUTATIONAL ASPECTS

The conjugate gradient subroutine [87] was used to obtain the optimal values of gain parameter.

The gradient of the P.I. was evaluated by the numerical method as explained in article 3.2.2 and P.I. was evaluated using the decomposition technique. The computer program was developed in FORTRAN.
The speed of convergence in conjugate gradient method for minimizing P.I. was examined. The trends of the values of performance index during optimization process for both linear and nonlinear model are shown in Table 3.4 and Table 3.5. The values of parameter and corresponding values of objective function are given as per their respective iteration count. It is found that most of the reduction in P.I. is obtained at the end of first iteration itself and the reduction obtained further is insignificant. Hence the parameter value obtained at the end of first iteration itself may be satisfactory. Since this method was found to perform satisfactorily no other method was tried out.

3.6. CONCLUSION

A method has been developed for optimum choice of gain parameters in L.F.C. problems. Since the decomposition technique is used there is no limitation to the size of the system that can be studied and also the non-linearities like dead band can be included. The method is simple and accurate. The versatility of the technique claims its superiority over other methods.
<table>
<thead>
<tr>
<th>System</th>
<th>Iteration count</th>
<th>Value of parameter $K_i$</th>
<th>Value of P.I.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0.0</td>
<td>$0.2966 \times 10^{-1}$</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.351</td>
<td>$0.5412 \times 10^{-2}$</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.621</td>
<td>$0.4695 \times 10^{-2}$</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0.0</td>
<td>$0.3126 \times 10^{-1}$</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.359</td>
<td>$0.1398 \times 10^{-1}$</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.509</td>
<td>$0.1373 \times 10^{-1}$</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0.0</td>
<td>$0.1575 \times 10^{-1}$</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>1.172</td>
<td>$0.1362 \times 10^{-2}$</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.181</td>
<td>$0.1363 \times 10^{-2}$</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0.0</td>
<td>$0.1035 \times 10^{-1}$</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.352</td>
<td>$0.1519 \times 10^{-2}$</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.593</td>
<td>$0.1343 \times 10^{-2}$</td>
</tr>
</tbody>
</table>
TABLE.3.5
Rate of convergence for nonlinear system.

<table>
<thead>
<tr>
<th>System</th>
<th>Iteration count</th>
<th>Value of parameter $K_1$</th>
<th>Value of P.I.</th>
</tr>
</thead>
<tbody>
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<td>0</td>
<td>0.0</td>
<td>$0.3738 \times 10^{-1}$</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.349</td>
<td>$0.5806 \times 10^{-2}$</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.455</td>
<td>$0.5881 \times 10^{-2}$</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0.0</td>
<td>$0.1066$</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.345</td>
<td>$0.6956 \times 10^{-1}$</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.131</td>
<td>$0.5625 \times 10^{-1}$</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0.0</td>
<td>$0.2210 \times 10^{-1}$</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>1.1009</td>
<td>$0.1584 \times 10^{-2}$</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.1716</td>
<td>$0.1588 \times 10^{-2}$</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0.0</td>
<td>$0.1532 \times 10^{-1}$</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.779</td>
<td>$0.1424 \times 10^{-2}$</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.862</td>
<td>$0.1418 \times 10^{-2}$</td>
</tr>
</tbody>
</table>
From the above investigations on different systems in this study, the following conclusions may be drawn

(1) Optimum gain setting of any area is independent of the controller gain setting of other areas which has been concluded in ref. 52.

(2) Optimal gain setting of controller of any area depends on the configuration of the whole system to which it is connected.

(3) The increase in number of areas decreases the gain setting in both linear and nonlinear case.

(4) The optimum gain settings obtained from the linear model are not really the optimum gain settings for nonlinear (realistic) model.