CHAPTER IX
DYNAMIC RESPONSE STUDIES OF MULTI-AREA SYSTEMS VIA DECOMPOSITION TECHNIQUE

2.1 INTRODUCTION

Study of dynamic response of power system is an important problem that appears in power system operation and control. With stringent requirements imposed on frequency and tie-line power deviation, it will become quite necessary to clearly understand the role played by different control parameters in controlling the time response. Therefore, the dynamic response of total system is vital for load frequency control problem. The dynamic response of the system is studied by representing the system differential equation in a state space model and solving them with the help of a suitable solution technique. Now a days it requires considerable computer time especially for solving multi area power systems. This has motivated the search for faster and numerically stable solution techniques.

The existing conventional techniques are:
(1) Through eigen values and eigen vectors

(2) Runge Kutta Gill method

For computation of the state vectors based on eigen value and eigen vector method, the eigen values and eigen vectors of the coefficient matrix are evaluated. The eigen values are obtained by OR algorithm, which is found to be the most powerful of the methods available at present. The computation of eigen vectors is done through inverse iteration [80]. There are some standard programs [81] available for use. Yet the method is mathematically complicated and requires considerable computer time and core memory.

Among the family of Runge-Kutta methods the most widely used version is that due to Gill. This method is readily applicable to digital computers and is controlling the growth of round off errors. In common with any of the Runge-Kutta family of methods, it is self starting and stable. As indicated in the description of the Runge-Kutta Gill method [82], the solution of a set of differential equations involves the evaluation of the rate of change of variables four times during each step of integration. This method will take a long time to study the response of large systems. Further, the large spread of time constants of
the different elements of the power system makes the system stiff. Therefore any attempt that can be made to reduce the computer time by increasing the integration time interval will lead to erroneous results.

As an alternative to the above schemes, the trapezoidal rule of integration technique is used for calculating the response of the power system [83]. The trapezoidal rule of integration technique is found to be a faster and numerically more stable than the other conventional methods. Dommel and Sato [83] have brought out clearly the benefit of stable implicit integration method in power system application. On the basis of excellent comparative results they have proposed to replace Runge-Kutta method by Trapezoidal rule. The trapezoidal integration method is more tolerant towards larger integration time intervals. However if this method is to be used for studying large systems, this will involve obtaining the inverse of a large matrix which is not attractive from the point of view of core storage and accuracy. Also if the system is non-linear, it becomes more difficult to analyze.

In an attempt to overcome aforementioned difficulties, a method has been proposed using the decomposition technique. The state equations are solved
using trapezoidal integration method along with decomposition technique which avoid the inversion of large matrix and non-linear system is approximated by piece-wise linear system.

2.2 DYNAMIC SYSTEM MODEL

For the study of dynamic response of LFC problem of multi area power systems, the first step is always to obtain a good mathematical model for the system, fairly accurate yet not necessarily complicated. It must also be feasible to have the control schemes realized. With the advance of digital computers sufficiently accurate models representing complex power systems are developed and analyzed. The complete description of the dynamic behaviour of a LFC problem requires consideration of the electrical and mechanical characteristics of governing system, turbine, control area, tie line and the control scheme.

The basis of the model developed here is as per the work reported in literature \([10,14,15,52]\). For the completeness of work, same is briefly explained. The following assumptions have been made for the development of the system model.
a) Automatic generation control does not interact with the voltage control which is true only when the disturbance is small.

b) The individual electric connections within an area are so strong at least in comparison with the ties between adjoining areas, that all generators in the area swing in unison, thus forming a control area.

With the above assumptions the different power system components are modelled as follows.

2.2.1 Governor

The detailed transfer function models of speed governors and turbines are discussed in IEEE Committee report on dynamic models for steam and hydro turbines in power system studies [15]. They have suggested that for all practical purposes the steam turbine governors can be represented by a first order transfer function model and is shown in Fig.2.1. The transfer function of speed governor is given as:

\[
\frac{\Delta X_E(s)}{\Delta P_C(s) - \frac{\Delta P(s)}{R}} = \frac{1}{1 + sT_g} \quad \cdots(2.1)
\]

Where \( T_g \) = Time constant of speed governing mechanism due to movement of linkage arms. It is
FIG. 2.1 GOVERNOR TRANSFER FUNCTION MODEL FOR STEAM TURBINE
normally less than 100 milliseconds.

The functional block diagram and an approximate non-linear model for a mechanical hydraulic speed governing system for a hydro turbine is given in Fig. 2.2 and Fig. 2.3 [52] respectively. Neglecting the nonlinearities such as rate limits, position limits and the small time constants of pilot valve, the transfer function of hydrogovernor may be written as [52].

\[
\frac{\Delta X_e(s)}{\Delta F(s)} = \frac{1}{R} \frac{(1 + T_R s)}{R T_G s^2 + T_G + T_R (\sigma + \delta)} \frac{\sigma}{s + 1} \quad (2.2)
\]

where

- \( \Delta F \) = Frequency deviation, Hertz
- \( \sigma \) = Permanent droop, per unit
- \( \delta \) = Temporary droop, per unit
- \( T_R \) = Despot time constant, seconds
- \( T_G \) = Governor response time, seconds

The hydrogovernor transfer function may be closely approximated by the simpler transfer function

\[
\frac{\Delta X_e(s)}{\Delta F(s)} = \frac{1}{R} \frac{(1 + T_R s)}{(1 + T_R s)(1 + T_G s)} \quad (2.3)
\]
FIG. 2.3 APPROXIMATE NONLINEAR MODEL OF MECHANICAL HYDRAULIC SPEED GOVERNING SYSTEM
Where

\[ T_1 = \frac{T_G + T_R (c + \xi)}{c} \]
\[ T_2 = \frac{T_T T_R c}{T_G T_R (c + \xi)} \]

The above transfer function is found by neglecting \( T_R c \) since it is very small.

2.2.2 Turbine

Fig.2.4 [52] shows the schematic diagram of tandem compound single reheat type steam turbine. Fig.2.5 [52] shows the approximate linear transfer function model of the tandem compound single reheat steam turbine. The time constants \( T_t, T_r \) and \( T_c \) represent delays due to steam chest and inlet piping, reheaters and crossover piping, respectively. The fractions \( F_{HP}, F_{IP} \) and \( F_{LP} \) represent portions of the total turbine power developed in the high pressure, intermediate pressure and low pressure cylinders of the turbine. It may be noted that

\[ F_{HP} + F_{IP} + F_{LP} = 1 \] ...(2.4)
FIG. 2.4 STEAM SYSTEM CONFIGURATION FOR TANDEM-COMPONENT SINGLE REHEAT STEAM TURBINE
FIG. 2.5 APPROXIMATE LINEAR MODEL FOR TANDEM-COMPOUND SINGLE REHEAT STEAM TURBINE
The time delay in the crossover piping $T_C$, being small compared with other time constants, is neglected. The reduced order transfer function is given in Fig. 2.6. The portion of the total power generated in the intermediate pressure and low pressure cylinders

$$\frac{F_{IP}}{F_{LP}} = 1 - \frac{F_{HP}}{F}$$

From Fig. 2.6,

$$\frac{\Delta P_g(s)}{\Delta X_E(s)} = \frac{1 + K \tau s}{(1 + sT_l)(1 + sT_r)} \cdots (2.5)$$

where

$$K_r = F_{HP} = \text{the reheat coefficient, i.e. the fraction of the power generated in the high pressure cylinder.}$$

Ramey and Skoogland [14] and IEEE Committee report [15] have developed the transfer function model for both hydroturbine penstock and hydrogovernors. A detailed transfer function of the hydro turbine penstock is given in literature [14, 15]. The transfer
FIG. 2.6 REDUCED ORDER MODEL FOR TANDEM COMPOUND SINGLE REHEAT STEAM TURBINE
function is given by
\[
\frac{\Delta P(s)}{\Delta X(s)} = \frac{a_{23}}{\frac{1}{1 + \frac{a_{13}}{a_{23}} T \omega s}} \left[ 1 + \left( \frac{a_{31}}{a_{13}} \right) T \omega s \right] 
\]

where \( a_{11} \) and \( a_{13} \) are partial derivatives of flow with respect to head and gate opening. Here the effect of speed deviation on the torque is neglected (since turbine speed changes are small, especially when connected to a system). For an ideal loss less turbine, \( a_{11} = 0.5Z \), \( a_{21} = 1.5Z \), \( a_{13} = 1.0 \) and \( a_{23} = 1.0 \) have been considered [14,15]. \( Z \) represents the initial gate opening (per unit). Substituting the values of \( a_{11}, a_{21}, a_{13}, \) and \( a_{23} \) in equation 2.6 results in

\[
\frac{\Delta P(s)}{\Delta X(s)} = \frac{1 - Z T \omega s}{1 + 0.5Z T \omega s} 
\]

...(2.7)

At full load \( Z = 1.0 \). Therefore

\[
\frac{\Delta P(s)}{\Delta X(s)} = \frac{1 - T \omega s}{1 + 0.5T \omega s} 
\]

...(2.8)

Here it is assumed that the incremental change in turbine torque and incremental change in generation are same.
2.2.3 Control Area

The control area modelling considered here is same as the work of Elgerd and Fosha [10]. When a disturbance $\Delta P_D$ occurs in the area the net power surplus in the area equal to $(\Delta P_D - \Delta P_D)$ MW, and this power will be absorbed by the system in three ways:

1) By increasing the area kinetic energy $W_{kin}$ at the rate:

$$\frac{d}{dt} W_{kin} = \frac{d}{dt} \left( W_{kin}^0 \left( \frac{f}{f^0} \right)^2 \right)$$

$$\approx \frac{d}{dt} \left( W_{kin}^0 \left( 1 + 2 \frac{\Delta F}{f^0} \right) \right)$$

$$= 2 \frac{W_{kin}^0}{f^0} \frac{d}{dt}(\Delta F) \quad \text{(2.9)}$$

2) By an increased load consumption: All typical loads (because of the dominance of motor load) experience an increase $D = \frac{\partial P}{\partial F}$ MW/Hz with speed or frequency. This $D$ parameter can be found empirically.

3) By increasing the export of power, via tie-lines, with the total amount $\Delta P_{tie}$ MW defined positive out from the area. So the power equilibrium equation is
The above can be written in the form

\[ \frac{\Delta F(s)}{[\Delta P_G(s) - \Delta P_D(s) - \Delta P_{tie}(s)]} = \frac{K_p}{1 + sT_p} \quad \text{(2.11)} \]

where \( T_p = \frac{2H}{f_0} \) sec, and \( K_p = \frac{1}{D} \) Hz/PUMW

The control area can be represented by the block diagram as shown in Fig.2.7. The detail is discussed in reference [10].

2.2.4 Tie-line

The total real power exported from any area, \( P_{tieK} \), equals the sum of all out flowing line powers, in the lines connecting that area with neighbouring areas,

\[ i.e., P_{tieK} = \sum_j P_{tieKJ} \quad \text{(2.12)} \]

The summation shall be extended over all lines that terminate in that area \( k \).
FIG. 2.7 CONTROL AREA TRANSFER FUNCTION MODEL
The total increment in export power from any area $k$ can be symbolized in block diagram form as shown in Fig. 2.8 [10] and can be written in mathematical form as below.

$$
\Delta P_{\text{tie} K} (s) = \frac{1}{s} \sum_{J} T_{KJ} \left[ \Delta F_{K} (s) - \Delta F_{J} (s) \right] \quad \ldots (2.13)
$$

Combining the block diagrams of individual component of power system one can obtain the overall block diagram for thermal and hydro systems as shown in Fig. 2.9 and Fig. 2.10. Fig. 2.11 shows the single area perturbation model of multi-area systems.

2.2.5 Control Scheme

To achieve the basic objective of LFC scheme, i.e. zero steady state error in frequency and tie line power, it is essential to have an integral of area control error as feedback signal. Now-a-days the standard control strategy used in industry is tie line bias control strategy and is given in linear integral form as below.

$$
\Delta P_{cK} = K_{I} \int_{t}^{\infty} (\Delta P_{\text{tie} K} + B_{K} \Delta F_{K}) \, dt \quad \ldots (2.14)
$$
FIG. 2.8 INCREMENTAL TIE-LINE POWER OUT OF ANY AREA
FIG. 2.9 TRANSFER FUNCTION MODEL OF THERMAL SYSTEM
FIG. 2.10 TRANSFER FUNCTION MODEL OF HYDRO SYSTEM
FIG. 2.11 BLOCK DIAGRAM OF SINGLE AREA PERTURBATION MODEL
The constant $K_{I_k}$ is the integrator gain and $B$ is the frequency bias parameter. The control strategy has been indicated by the dotted portions in Fig. 2.11.

This type of model will serve as basis for the development of theory and analysis to follow.

2.3 THEORY

A method is developed to study the load frequency dynamics of multi area electric energy systems using decomposition approach. This method is applicable to all problems where the state variables have a smooth variation in time domain. So that during the entire period of interest the variation of the state variables with respect to time can be approximated by straight line segments.

A typical block diagram of a single area perturbation model [10] is shown in Fig. 2.11 Whenever there is a load disturbance in any area, it will result in change in the frequency and tie-line flows in all areas. If there is no supplementary signal each area will change its output depending on frequency deviation due to governor action. But usually there will also be a time integral supplementary signal which
consists of combination of frequency deviation and tie-line power deviation.

Let us consider a \( m \)-area system of \( N \)-order each. The system can be described by the following state equation.

\[
[ X ] = [ A ] [ X ] + [ U ] \\
\ldots (2.15)
\]

The order of the \( A \)-matrix is \((MN-1) \times (MN-1)\). The structure of \( A \)-matrix of the whole system for an all steam system where \( M = 4 \) and \( N = 6 \) is shown in Fig.2.12. This is a \( 23 \times 23 \) matrix. From the structure one can see that any area \( K \) is connected to other areas by elements appearing in the row of state vector corresponding to tie-line power deviation. Therefore it can be considered as an additional disturbance to that area and the effect can be taken by including a disturbance variable in the row corresponding to tie-line power deviation. With this arrangement the original \( A \)-matrix can be divided into \( M \) separate blocks. Accordingly for \( K \)th area, the system equation can be written as

\[
[ X ]_k = [ A ]_k [ X ]_k + [ U ]_k \\
\ldots (2.16)
\]
Fig 2.12 Structure of $[A]$ for a four area steam system.

* Indicates the non zero value
Here the fifth row will have the additional term to take into account the other areas. The order of the last area will be \((N-1)\). The detailed derivation of the state space equation for linear and nonlinear model is done in article 2.4 of this thesis. \(\Delta F\) the frequency deviation in each area will obviously have only smooth variations with respect to time. So during each time interval the input variables to different areas which are only function of \(\Delta F\) can be approximated by straight line segments. Obviously the smaller the time interval, the greater will be the accuracy.

The state equations of each subsystem are solved by trapezoidal integration method advantageously, then it is combined with decomposition technique which makes the procedure necessarily iterative to arrive at the solution for the whole system. The detailed mathematical derivation for trapezoidal integration method along with decomposition technique is as given below.

Let \(\Delta t = \text{Integration time interval}\).

\[X(t) = \text{Values of state variables at the end of time 't'}.\]

\[X(t+\Delta t) = \text{Values of state variables at the end of time 't+\Delta t'}.\]
Applying trapezoidal integration technique to the equation:

$$[X] = [A] [X] + [U]$$

One gets

$$\frac{[X(t+\Delta t)] - [X(t)\Delta t]}{2} = \frac{[X(t+\Delta t)] + [X(t)]}{2} + \frac{[U(t+\Delta t)] + [U(t)]}{2} \ldots (2.17)$$

$$[I-A \frac{\Delta t}{2}] [X(t+\Delta t)] = [I+A \frac{\Delta t}{2}] [X(t)]$$

$$\ldots (2.18)$$

The above equation can be written as

$$[X(t+\Delta t)] = [SM] [Y] \ldots (2.19)$$

Where

$$[SM] = [I - A \frac{\Delta t}{2}]^{-1}$$

and

$$[Y] = [I + A \frac{\Delta t}{2}] [X(t)] + \frac{[U(t+\Delta t) + U(t)]\Delta t}{2}$$

When equation 2.19 is used with decomposition technique the procedure becomes iterative. The equation 2.19 can be rewritten as
\[ [X^{(i+1)}_{a+1}(t+\Delta t)] = [SM][X_1(t)] + [U^{(i+1)}(t+\Delta t)+U(t)] \frac{\Delta t}{2} \]

where \( X_1(t) \) = \[ I + A \frac{\Delta t}{2} \] \( X(t) \)

and \( i \) denotes the iteration count.

From equation 2.20, it can be seen that \( X^{(i+1)}_{a+1}(t+\Delta t) \) consists of two parts, one is a fixed part and another is a variable part which varies in every iteration.

Let \( X^{(i+1)}_{a+1}(t+\Delta t) = X_2(t+\Delta t) + X_3^{(i+1)}(t+\Delta t) \)

Where \( X_2 \) is the fixed part and given by

\[ [X_2(t+\Delta t)] = [SM][X_1(t)] + [U(t)] \frac{\Delta t}{2} \] \( \ldots(2.21) \)

and \( X_3 \) is the variable part and given by

\[ [X_3^{(i+1)}(t+\Delta t)] = [SM][U^{(i+1)}(t+\Delta t)] \frac{\Delta t}{2} \] \( \ldots(2.22) \)

It is obvious that it will be enough to iterate on \( X_3 \) only. After convergence is obtained, \( X(t+\Delta t) \) can be calculated as \( X_2 + X_3 \).

**Steps for the solution**

1) Let \( \Delta F_k^{(i)} \) \( k = 1, \ldots, M \) be the assumed variations of \( f \) at \( i \)th iteration during a time interval \( \Delta t \), at time \( t \)

2) The state-vectors of each area at the end of \( t + \Delta t \) are obtained using trapezoidal integration method, and
hence $\Delta F^{(t+1)}_k$, $k = 1, \ldots, M$.

3) If $\Delta F^{(t+1)}_k$ is sufficiently close to $\Delta F^{(t)}_k$ for $k = 1, \ldots, M$, one proceeds to next time interval. Otherwise one goes to step (1) with $\Delta F^{(t)}_k$ replaced by $\Delta F^{(t+1)}_k$ for all $k$.

A simplified flowchart for computer application of the above technique is depicted in Fig.2.13.

2.4 SYSTEM EQUATION

The block diagram of single area perturbation model is shown in Fig.2.11 and the detailed transfer functions of single stage reheat type thermal area and hydro area are shown in Fig.2.9 and Fig.2.10. The system equations are derived for both thermal area and hydro area considering small load change.

a) Thermal area

\[
\begin{align*}
X_1 &= \Delta F, \\
X_2 &= \Delta X_E, \\
X_3 &= \Delta P_{0e}', \\
X_4 &= X_3, \\
X_5 &= \Delta P_{lpe}', \\
X_6 &= \Delta P_c.
\end{align*}
\]

Referring to Fig.2.9 for $K$th area one can write

\[
X_1 = -\frac{1}{T_p} X_1 + \frac{k}{T_p} X_3 - \frac{k}{T_p} X_5 - \frac{k}{T_p} \Delta P_D(t) ...(2.23)
\]
FIG. 2.13 FLOW CHART

START

READ SYSTEM DATA

DO K = 1, M

EVALUATE
\[ \frac{\Delta t}{[1-A_k^{-2}][1+4A_k^{-2}]} \]

EVALUATE
\[ \frac{\Delta t - 1}{(S\eta_e)_{e_2} + (1 - A_k)^{-2}} \]

K

DO K = 1, M

WRITE T, [X]

EVALUATE
\[ U(t+\Delta t) \]

EVALUATE STATE VECTOR
BY TRAPEZOIDAL METHOD
\[ X^{i+1}(t+\Delta t) \]

REPLACE
\[ U_k(t) \] BY \[ U_k(t+\Delta t) \]
FOR \( K = 1, M \)

REPLACE
\[ X_k(t) \] BY \[ X_k(t+\Delta t) \]
FOR \( K = 1, M \) AND \( T < T_{\text{max}} \)

STOP

FIG. 2.13 FLOW CHART
The values of A's are given in their respective linear and nonlinear model.

The above can be written as below

\[
\begin{align*}
X_2 &= A_{21} X_1 - \frac{1}{T_0} X_2 + \frac{1}{T_0} X_5 \quad \ldots (2.24) \\
X &= A X + A X + A X + A X + A X \quad \ldots (2.26)
\end{align*}
\]

\[
\begin{align*}
X_5 &= 2\pi \sum_{Jk}^M T_{jk} X_k - 2\pi \sum_{Jk}^M T_{jk} \Delta F_j \quad \ldots (2.27)
\end{align*}
\]

\[
\begin{align*}
X_5 &= -K_{BX} - K X \quad \ldots (2.28)
\end{align*}
\]
b) Hydro system

Referring to Fig. 2.10 one can write the state equation for hydro system as below:

\[
X = \begin{bmatrix}
\frac{-1}{T} & 0 & \frac{KP}{TP} & 0 & \frac{-KP}{TP} & 0 \\
A_{21} & -\frac{1}{T} & 0 & \frac{T - TR}{TP} & 0 & \frac{TR}{TP} \\
A_{31} & 2(T + TW) & -2 & -2(T - TR) & 0 & -2TR \\
A_{41} & 0 & 0 & \frac{-1}{T} & 0 & \frac{1}{T} \\
B_{Jk} & 2\pi \sum_{J \neq k} T_{Jk} & 0 & 0 & 0 & 0 \\
-K & 0 & 0 & 0 & -K & 0
\end{bmatrix}
\]

\text{(2.29)}
The values of A's are given in their respective linear and nonlinear model.

System equation for last area

Since \( \sum_{j=1}^{M} \Delta P \text{T.e. } J = 0 \)

\[
\Delta P \text{T.e. } M = - \sum_{j=1}^{M-1} \Delta P \text{T.e. } J \quad ... (2.31)
\]

Making this substitution one gets the A matrix for last area (i.e. Mth area) to be of the order \((N-1) \times (N-1)\). The state equation for last area is given below:
In a similar manner the system equations for hydro area can be obtained.

2.5 VALIDITY OF THE ALGORITHM

To test the validity of the proposed decomposition algorithm, two worked out examples from literature [1,34] are tried out. For comparison of the results both the examples are solved with the proposed decomposition algorithm and Trapezoidal integration.
method. It is established in literature [83] that the trapezoidal integration method is more accurate and efficient. Therefore no other method is tried out for comparison of the results of the proposed decomposition algorithm. The response curves for the above two examples are shown in Fig. 2.14 to 2.19. Both the responses found in different methods are well comparable. Further the curves found are compared with the respective curves given in the literature [1,34]. From the comparison it is seen that the proposed decomposition algorithm is very accurate.

2.6 APPLICATION TO LINEAR SYSTEM

2.6.1 Modelling

For analysis of linear system the equations developed with the aforementioned algorithm are used. For linear system the values of A's and U's are as below.

a) THERMAL SYSTEM

\[
A_{21} = -\frac{1}{RT} g, \quad A_{41} = -\frac{K}{RT T_t} g, \quad A_{42} = \frac{1}{TT_t} \left( -\frac{1}{T_T} - \frac{K}{T_g T_t} \right),
\]

\[
A_{43} = -\frac{1}{TT_t} r, \quad A_{44} = -\frac{1}{TT_t} r, \quad A_{46} = \frac{K}{TT_t} g.
\]
FIG 2.14 EXAMPLE 1 (FREQUENCY DEVIATION IN AREA 1)
FIG. 2.15 EXAMPLE 1 (FREQUENCY DEVIATION IN AREA 2)
FIG. 2.16  EXAMPLE 1 (TIE LINE POWER DEVIATION)
FIG. 2.17 EXAMPLE 2 (FREQUENCY DEVIATION IN AREA 1)
FIG. 2.18 EXAMPLE 2 (FREQUENCY DEVIATION IN AREA 2)
FIG. 2.19 EXAMPLE 2 (TIE LINE POWER DEVIATION)
\[ U_2(t) = U_4(t) = 0 \]

a) HYDRO SYSTEM

\[ \begin{align*}
A_{21} &= \frac{-T}{RT_1 T_2}, \quad U_2(t) = 0 \\
A_{31} &= \frac{2T}{RT_1 T_2}, \quad U_3(t) = 0 \\
A_{41} &= \frac{-1}{RT_1}, \quad U_4(t) = 0
\end{align*} \]

2.6.2 Numerical example and result

The above mentioned algorithm is tried out to study the dynamic performance of the following systems.

1. Two area Hydro-Thermal system
2. Two equal areas of Thermal system
3. Four equal areas of Thermal system

The nominal system parameters for the thermal system and hydro system are given in Appendix-A and is as per the recommendations of IEEE committee report on power plant response [52]. The physical connection of four area system is shown in Fig. 2.20. The improvement in dynamic performance due to supplementary signal is
Fig. 2.20 Physical Connection of Four Area System
investigated. Therefore the investigation is done with and without supplementary signal in all the cases. The value of controller gain for all the system is arbitrarily chosen to be 0.05. A small disturbance of 0.01 pu step load change in area one is considered.

Both one piece and multi piece studies are made for a duration of 40 seconds. For one piece method, trapezoidal rule of integration was performed on the whole system. The results those are obtained by different methods are found to agree up to third decimal place. But the advantage in decomposition technique is that for any change in the parameter of a sub-system the solution matrix of only that sub-system has to be recalculated.

The following dynamic response curves with 1% step load perturbation in area 1 for all the systems under investigation are shown in Fig.2.21 to Fig.2.29.

a) Time-Frequency deviation in area 1.

b) Time-Frequency deviation in area 2.

c) Time-Tie line power deviation in area 1
   (Sum of tie line power deviations of all tie lines terminating at area 1)

Comparing the frequency deviation in area 1 and frequency deviation in area 2 for all the test systems,
FIG. 2.21 TEST SYSTEM 1.
(FREQUENCY DEVIATION IN AREA ONE)
FIG. 2.22 TEST SYSTEM 1.
(FREQUENCY DEVIATION IN AREA TWO)
FIG 2.23 TEST SYSTEM 1
(TIE-LINE POWER DEVIATION)

TIE-LINE POWER DEVIATION IN P.U.

Without ss
With ss

TIME IN SEC.

0.00 10.00 20.00 30.00 40.00

0.000 0.005

-0.005

-0.010

-0.015
FIG. 2. Test System 2.
(Frequency Deviation in Area One)
FIG. 2.25 TEST SYSTEM 2.
(FREQUENCY DEVIATION IN AREA TWO)
FIG 27S TEST SYSTEM 2. (TIE-LINE POWER DEVIATION)

TIME IN SEC

TIE-LINE POWER DEVIATION IN P.U.
FIG. 2.2.7 TEST SYSTEM 3.
(FREQUENCY DEVIATION IN AREA ONE)
FIG. 2.28 TEST SYSTEM 3.
(FREQUENCY DEVIATION IN AREA TWO)
FIG. 2 Test System 3.
(Tie-Line Power Deviation)
it is seen that the maximum overshoot in area 1 is more than that of area 2. The settling time in both the area are same. The rate of frequency deviation in area 2 is slow during the first cycle of the transient. From the tie line power deviation curves for all the systems it is seen that the settling time is same as that of frequency deviation curve.

Comparing the response curves of test system 1 (Hydro Thermal system) and test system 2 (Two equal areas of Thermal system) the following observations are made.

1) Settling time is more in test system 1.
2) Maximum overshoot in frequency is more in test system 1.
3) Maximum overshoot in tie line power deviation is almost same for both the systems.

Comparing the response curves of test system 2 (two equal areas of thermal system) and test system 3 (Four equal areas of thermal system) the following observations are made.

1) Settling time is more in test system 3
2) Maximum overshoot in frequency is less in test system 3
3) Maximum overshoot in tie line power deviation is more in test system 3.

2.7. APPLICATION TO NONLINEAR SYSTEM

The speed governor dead band has a great effect on the dynamic performance of power system. For more realistic analysis the governor dead band has to be included which makes the system nonlinear. The proposed method is applied to investigate the effect of dead band on the system dynamic performance. The description of speed governor dead band is illustrated in literature [5, 6, 86]. The block diagram including speed governor [86] is shown in Fig. 2.30. The magnitude of dead band is taken as 0.0006pu or 0.06 percent [86]. For cases presented here the initial position of dead band is selected so that the entire dead band of each area has to be traversed before a response is secured. Actually the system can be anywhere within the dead band [6].

2.7.1 Nonlinear model (Considering dead band)

From the response curves shown in Fig. 2.21 to
FIG. 2.30 BLOCK DIAGRAM OF SINGLE-AREA PERTURBATION MODEL WITH GOVERNOR DEADBAND
Fig. 2.29 it is seen that $\Delta F$ varies rather slowly with time and so if the integration time interval is chosen sufficiently small, one can reasonably assume that during any particular time interval each area operates either entirely inside the dead band region or outside it. Therefore the system to be considered becomes essentially a piece-wise linear system when governor dead band is included. For each area there will be two $A$ matrices, one for operation inside the dead band region and the other for operation outside it. They are derived as follows.

a) Thermal system

(I) Operation inside Dead band:

There will be no signal proportional to frequency deviation. Therefore values of $A$'s and $U$'s are as below:

$$A_{21} = 0, \quad A_{41} = 0, \quad U(t) = 0, \quad U(t) = 0$$

other $A$'s remain as in linear model.

(II) Operation outside the Dead band:

When the frequency deviation is greater than the DB, the signal will be proportional to $(|\Delta F| - DB)$ sign $(\Delta f)$. Therefore the values of $A$'s and $U$'s are as below:

$$A_{21} = -\frac{1}{RT}, \quad A_{41} = -\frac{K}{RTT}.$$
The other A's remain as in linear model.

b) Hydro system

(I) Operation inside dead band.

\[ A_{21} = 0 , \quad U_2(t) = 0 \]
\[ A_{31} = 0 , \quad U_3(t) = 0 \]
\[ A_{41} = 0 , \quad U_4(t) = 0 \]

(II) Operation outside dead band.

\[ A_{21} = \frac{-T}{RT} \quad U_2(t) = \frac{T}{RT} \quad \text{DB Sign(ΔF)} \]
\[ A_{31} = \frac{2T}{RT} \quad U_3(t) = \frac{-2T}{RT} \quad \text{DB Sign(ΔF)} \]
\[ A_{41} = \frac{-1}{RT} \quad U_4(t) = \frac{1}{RT} \quad \text{DB Sign(ΔF)} \]

2.7.2 NUMERICAL EXAMPLES AND RESULTS

The proposed method was tried with the same three systems considering nonlinear model developed above to study the dynamic response. The solution matrix is
calculated once for both mode of operation and stored.
For example in case of a four area system one has to store six \(6\times6\) and two \(5\times5\) solution matrices. In the program instructions are given to choose either of the matrix for a particular area in a particular time interval depending on whether the area is operating inside the dead band or outside it.

If one piece solution is tried out including the dead band one has to calculate the solution matrix for the whole system whenever the mode of operation of any area changes which will be quite time consuming or else one should store sixteen \((23\times23)\) solution matrices. Therefore in case of 'M' area system one has to store \(2(M-1)\) no. of \(6\times6\) matrices and 2 no. of \(5\times5\) matrices using decomposition technique, whereas with one piece solution one has to store \(2^M\) no. of \((6M-1)\times(6M-1)\) matrices.

The following dynamic response curves with 1\% step load perturbation in area 1 for all the systems under investigation are shown in Fig.2.31 to Fig.2.39.

a) Time-Frequency deviation in area 1.

b) Time-Frequency deviation in area 2.

c) Time-Tie line power deviation in area 1

(Sum of tie line power deviations of all tie lines
FIG 2.31 TEST SYSTEM 1
(FREQUENCY DEVIATION IN AREA ONE)
FIG. 2.32 TEST SYSTEM 1.
(FREQUENCY DEVIATION IN AREA TWO)
FIG 2.3 TEST SYSTEM 1 (TIE-LINE POWER DEVIATION)
FIG. 2.35 TEST SYSTEM 2.
(FREQUENCY DEVIATION IN AREA TWO)
FIG 2.36 TEST SYSTEM 2,
(TIE-LINE POWER DEVIATION)
FIG 2.37 TEST SYSTEM 3.
(FREQUENCY DEVIATION IN AREA ONE)
FIG 2.38  TEST SYSTEM 3.
(FREQUENCY DEVIATION IN AREA TWO)
FIG. 2 TEST SYSTEM 3.
(TIE-LINE POWER DEVIATION)
Comparing the frequency deviation in area 1 and frequency deviation in area 2 for all the test systems it is seen that the maximum overshoot in area 1 is more than that of area 2. The magnitude of maximum overshoot of frequency deviation in area 1, frequency deviation in area 2 and tie line power deviation in area 1 for both linear and nonlinear case are same. Comparing these response curves with the corresponding response curves in linear model it is seen that in both linear and nonlinear model the initial responses are almost same but in linear case the transient settles down to a steady state whereas in nonlinear model the system continues to oscillate.

Comparing the response curves of test system 1 and test system 2 the following observations are made.

1) Test system 1 is more damped.
2) Maximum overshoot in frequency is more in test 1.
3) Maximum overshoots in tie line power deviation are almost same for both the systems.

Comparing the response curves of test system 2 and test system 3 the following observations are made.

1) Maximum overshoot in frequency is less in test system 3.
2) Maximum overshoot in tie line power deviation is more in test system 3.

3) Test system 3 is more oscillatory.

2.8 COMPUTATIONAL ASPECTS

Computer programs are developed in FORTRAN. For validation of the program and the technique, different test systems, as considered by other workers [10,34], are tried out. The results found in this method compare well with the earlier results. For both the linear and non-linear models; runs were made for different $\Delta t$. It was not found necessary to go below $\Delta t = 0.05$ sec. for very accurate results. However for $\Delta t = 0.25$ sec., sufficiently accurate results were obtained. But it took only 60% of time taken for $\Delta t = 0.05$. For $\Delta t < 0.25$, computation time was increasing due to large number of iterations required. So all the studies were made using the trapezoidal integration method and $\Delta t = 0.25$ sec. The condition for convergence was taken as

$$\frac{|\Delta F^{(t+1)} - \Delta F^{(t)}|}{|\Delta F^{(t)}|} \leq 0.001$$
Attempts were made to improve the convergence rate using an acceleration factor as follows:

$$\Delta F^{(l+1)} = \Delta F^{(l)} + ACC(\Delta F^{(l+1)} - \Delta F^{(l)})$$

for $ACC$ varying from 1.5 to 0.5. But $ACC = 1.0$ was consistently found to be the optimum value, thus implying that no acceleration factor is required.

In respect of computer time the proposed decomposition algorithm is compared with trapezoidal integration method [83]. All the test systems in linear and nonlinear models are tried out with the proposed algorithm and trapezoidal integration method. The computer time required in both the methods for all the test systems are given in table 2.1. From the results it is found that for linear models the decomposition method takes more computer time. This is because of the iterations made for solution of whole system. But in case of nonlinear models the computer time is more in case of trapezoidal method. Comparing the time in two area and four area cases in nonlinear model it is found that the computer time in trapezoidal method increases sharply as the number of area increases. This is where
### TABLE 2.1

Computer time for linear and nonlinear model

<table>
<thead>
<tr>
<th>System</th>
<th>Linear model</th>
<th>Nonlinear model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Time in sec Decomposition method</td>
<td>Time in sec Trapezoidal Method</td>
</tr>
<tr>
<td>1</td>
<td>1.65</td>
<td>1.27</td>
</tr>
<tr>
<td>2</td>
<td>1.65</td>
<td>1.27</td>
</tr>
<tr>
<td>3</td>
<td>1.65</td>
<td>1.27</td>
</tr>
<tr>
<td>4</td>
<td>3.02</td>
<td>2.64</td>
</tr>
</tbody>
</table>
the decomposition method decidedly scores over the single piece trapezoidal method. Thus decomposition method is the most suitable method when governor dead band is considered in multi area systems.

2.9. CONCLUSION

In this chapter a method using decomposition technique is developed to study the dynamic response of a multi area electric energy systems. The mathematical model in the form of state equations for linear multi area systems suitable for the proposed method has been developed. An approach is made to overcome the complexity of nonlinearity due to governor dead band and a suitable mathematical model is proposed for implementation of the aforementioned technique for dynamic response study of multi area systems considering governor dead band.

The effectiveness and accuracy of the technique have been illustrated by simulating different systems for both linear and nonlinear model. A comparative study of the responses with different values of integration time interval ($\Delta t$) has been made for all the systems and a suitable value of $\Delta t=0.25$ seconds is
chosen for further studies in this thesis. Based on the numerical results obtained in the studies, the proposed method seems to have a reasonable accuracy compared to that of any other methods and it is easy to implement in computer with less core storage and less computation time. Using decomposition technique there is no limitation to the size of the system that can be studied. Further the nonlinearity like dead band can be included. This method is quite useful for studying the effect of control parameters on the performance of the system since the solution matrix of only the concerned sub-system has to be recalculated.