CHAPTER - II

NUCLEAR PARAMETERS FROM PION-\(^4\)He SCATTERING
II.1. Introduction

Several attempts have been made in the past for isolating the complex nuclear amplitude in the forward direction. The imaginary part can be obtained from the total cross section data via the optical theorem. The real parts can be obtained using these imaginary parts. Binon and his group were the first to use a phenomenological fit to the differential cross section data in the Coulomb interference region for $\pi^- - ^4\text{He}$ scattering. They used an expression of the form

$$\frac{d\sigma}{d\Omega}(\theta) = \left| f_c(\theta) + f_N(\theta) e^{-2i\varphi} \right|^2 \tag{2.1}$$

where $f_c(\theta)$ and $f_N(\theta)$ are the pure Coulomb and pure nuclear amplitudes respectively. The relative Bethe phase $2\varphi$ has been taken as

$$2\varphi = -2 \varepsilon \ln \sin \frac{1}{2} \theta + \int_{-\frac{t}{t'-t'}}^0 \frac{dt}{4k^2} \left[ 1 - \frac{f_N(t')}{f_N(t)} \right] \tag{2.2}$$

the coupling parameter $\varepsilon$ being related to the fine structure constant $\alpha$ as

$$\varepsilon = \frac{Z_1 Z_2 \alpha}{\left[ s - (m + M)^2 \right]^{1/2} \left[ s - (m - M)^2 \right]^{1/2}} \tag{2.3}$$
k is the centre of mass momentum; s and t are the usual Mandelstam variables; \( Z_1 \) and \( Z_2 \) are the charges and \( m \) and \( M \) are the masses of the projectile and the target respectively. For evaluating the integral (2.2), an exponential form for the nuclear amplitude has been used. This assumes diffraction scattering ignoring, for instance, the existence of a forward dip. The phenomenological expression used by Binon for \( f^N_N(\theta) \) in Eq. (2.1) is also diffractive and is not of the correct form for all values of \( \cos\theta \) from -1 to +1. Thus the definition of \( f^N_N(\theta) \) makes an explicit use of a model for strong interaction, which may not give an exact representation of the forward elastic scattering. Moreover in a particular model there can be an appreciable residual effect due to Coulomb distortion even in the pure strong interaction amplitude.

A model independent analysis of pion-nucleus scattering data was attempted by Ericson and Locher through the use of forward dispersion relations. Defining the symmetric amplitude for pion scattering from a nucleus \( X \) in the form

\[
\begin{align*}
    f^T(+) (\omega) &= \frac{1}{2} \left[ f^+_\pi X (\omega) + f^-\pi X (\omega) \right] \\
    &= \frac{1}{2} \left[ f^+_\pi X (\omega) + f^-\pi X (\omega) \right]
\end{align*}
\]  

(2.4)

the forward dispersion relation can be written as
The subtraction constants $\text{Re } f^{(+)}(m_\pi)$ are either to be obtained from phase shift analyses or from data on mesic atoms. The latter requires for its calculation an accurate measurement of the width and shift of the 1s levels. Moreover a knowledge of $\text{Im } f^{(+)}(\omega)$ over the entire range $\omega_0 < \omega < \infty$ is essential. $\text{Im } f^{(+)}(\omega)$ in the range $m_\pi < \omega < \omega'$ are obtained from the total cross section data using optical theorem. At higher energies where data are scarce or do not exist at all it is necessary to use some
parametric form of the total cross section. Also in the unphysical region \( \omega_0 < \omega < \frac{m_\pi}{4} \), some parametrization for \( \text{Im} f^{(+)}(\omega) \) has to be made. Similar attempts have been made by Wilkin et al\textsuperscript{14} and Batty et al\textsuperscript{15} with improved total cross section data. The latter group have at places used the semiclassical approach of Faldt and Pilkhun\textsuperscript{69} to get total cross section for \( \pi^+ - ^4\text{He} \) scattering from the experimental data for \( \pi^- - ^4\text{He} \) scattering at energies where the data for positive pions are not available. This could be highly erroneous, particularly in the low energy region. The analysis of Batty et al with two different values of \( \text{Re} f^{(+)}(m_\pi) \) from PSA and from mesic atom data showed that the real parts were not seriously affected (about 10%) due to the change (about 30%) in the subtraction constant. On the other hand, real parts depend strongly on the interpolation of \( \sigma(\omega) \) in the (3,3) resonance region. However, the results were insensitive with respect to the input in the unphysical region. So the total cross section measurement for both \( \pi^+ \) and \( \pi^- \) in the resonance region seems to be absolutely essential.

Thus to obtain the pure strong interaction amplitude in the forward direction one faces either the problem of model dependence or the problem of strong dependence on the knowledge of the total cross section data. Moreover one cannot rely on the phase shift analysis method which involves
too many parameters and often infected with ambiguities.

On the other hand one can define an amplitude in the form

\[
\tilde{f}_N^\pm (\theta) = \sum_{l=0}^{\infty} \frac{2l+1}{2i} \cos \frac{2l+1}{2} \frac{1}{k} (e^{2il\theta} - 1) P_l(\cos \theta), \quad (2.7)
\]

where the Coulomb phases \( \sigma_l \) are explicit. We shall call this the residual nuclear amplitude for \( \pi^\pm \) scattering. Then \( \tilde{f}_N^\pm (0) \) can very easily be evaluated through analysis of pion-nucleus scattering data since this term occurs explicitly in the differential cross section as will be seen in Sec. II.2. We shall see that one can analyse the Coulomb nuclear interference in terms of the real and imaginary parts of \( \tilde{f}_N^\pm (\theta) \) in a model independent way. One can also extract from this the total cross sections for \( \pi^+ \) and \( \pi^- \). The places where the real and imaginary parts of \( \tilde{f}_N^\pm (0) \) go through zero (as a function of both element and energy) will provide interesting physics because these zeroes will be quite sensitive to the cancellation between Coulomb effects and the pion-nucleus strong interaction. Thus the quantity \( \tilde{f}_N^\pm (\theta) \) is a quantity of interest even though it contains residual effects of Coulomb interaction. Moreover for \( N = Z \) nuclei, the pure strong interaction amplitude is related to \( \tilde{f}_N^\pm (0) \) and \( \tilde{f}_N (0) \) in a simple manner and hence can be calculated easily.

The presence of a strong electromagnetic pole in the forward direction suggests that one can apply pole extrapolation
technique to extract the residue of this pole which contains the real part of the residual nuclear amplitude $f_N^+(0)$.

The objective of the present work is, therefore, two fold. The first is to suggest a novel method to obtain the amplitude $f_N^+(0)$ where no specific nuclear model is used. One will then obtain unbiased estimates of the various nuclear parameters involved. The second is to use conformal mapping to achieve accelerated convergence of the series representations so that the goodness of the fits improves optimally and the number of parameters in the fits is reduced considerably.

For charged particle scattering there is a Coulomb pole at $t = 0$ and the data from nonforward angles have to be suitably and stably extrapolated to this forward point. In this respect we have preferred to use the elegant method as suggested by Cutkosky and Deo$^{36}$ which can be largely used in nuclear physics for extrapolation to unphysical regions. After proper subtraction of the singular terms the remaining nuclear part is conveniently parametrized in conformally mapped variables dictated by the analyticity of the strong interaction amplitude.

Recent experimental results of negative pion scattering from $^4\text{He}$ nuclei in the $(3,3)$ resonance region$^7$ provide an excellent set of data to carry out the intended analysis
and to assess the merits and workability of the method. The data extend over a wide range of angles from near forward to the very backward region between 110 to 260 MeV. The real parts, both in sign and magnitude, are obtained quite neatly as residues by extrapolation to the forward pole. In fact, the present method provides a direct experimental proof of the existence of a first order Coulombic pole in the forward direction. The total cross section is also calculated from the imaginary part obtained by a method of successive iteration.

In Sec. II.2 the equations for analysing pion–nucleus scattering are presented. Sec. II.3 deals with the actual extrapolation procedure. The methods for obtaining various nuclear data from the measured differential cross sections are also given here. In Sec. II.4 analyticity structure of the nuclear amplitude for \( ^1_\pi - ^4\text{He} \) scattering and the elliptic mapping procedure are discussed. In Sec. II.5, we present the extrapolation procedure for the data of Crowe et al. which are at comparatively lower energy and at wider angles. Sec. II.6 contains a discussion of the results.

II.2. Pion–nucleus Scattering Theory:

For scattering of charged pions from atomic nuclei, Coulomb interaction becomes inevitable in addition to the usual nuclear interaction. Such processes can be explained
with the help of a Coulomb plus a short range nuclear force. The effects of the long range Coulomb field on the nuclear interaction has been dealt with in detail by Goldberger and Watson. For completeness we give the outline of the same.

If we write \( v_n(r) \) for the potential associated with the nuclear interaction, \( v_c(r) \) for the Coulomb potential and \( g_c(r) \) as the cut off for the shielding of the Coulomb interaction, then the Schrodinger equation using the radial wave function \( w_1 \) is

\[
\frac{w_1''}{w_1} - \left[ v_c(r)g_c(r) + v_n(r) + \frac{1}{r^2} + k^2 \right] w_1 = 0 \quad (2.8)
\]

We define

\[
g_c(r) = \begin{cases} 1 & \text{for } r < R \\ 0 & \text{for } r \geq R \end{cases} \quad (2.9)
\]

\( R \) being the 'screening radius' beyond which the two charged particles are effectively shielded from each other. For \( r \gg R \), there is no interaction and \( w_1 \) has the standard asymptotic form

\[
w_1 \rightarrow \sqrt{2/\pi} \sin (kr - \frac{n\pi}{2} + \Delta_1) \quad (2.10)
\]

where \( \Delta_1 \), the phase shift, is the sum

\[
\Delta_1 = \sigma_1 + \nu_1 + \lambda_1 \quad (2.11)
\]
For a short range potential of range $d$

$$v_l \approx 0 \quad \text{for} \quad l \gg kd$$

$$A_l \approx 0 \quad \text{for} \quad l \gg kR$$

and

$$\lambda_l \approx \begin{cases} -\sigma_l & \text{for} \quad l \gg kR \\ -\xi \ln(2kR) = \lambda & \text{for} \quad l \ll kR \end{cases}$$

The latter is a suitable matching phase angle, $v_l$ is the nuclear phase shift and $\sigma_l$ is the Coulomb phase shift written as

$$\sigma_l = \frac{1}{2\pi} \ln \frac{\Gamma(l+1+i\xi)}{\Gamma(l+1-i\xi)} ,$$

$\xi$ being the Coulomb parameter defined by Eq. (2.3).

Then formally the scattering amplitude, when both the incident and target particles have no spin, becomes

$$f(\theta) = \sum_{l=0}^{\infty} \frac{(2i)_{2l+1}}{2i} (e^{i\Delta_l} - 1) P_l(\cos\theta) .$$

This may be reexpressed as

$$f(\theta) = f_{sp}(\theta) + e^{2i\lambda} \left[ f_0(\theta) + \frac{e^{i\lambda}}{f_N(\theta)} \right]$$

where

$$f_0(\theta) = \sum_{l=0}^{\infty} \frac{(2i)_{2l+1}}{2i} (e^{i\sigma_l} - 1) P_l(\cos\theta)$$
The Coulomb amplitude is taken to be

\[ f_c(\theta) = B \exp [2i(\sigma_0 - \frac{\eta}{4} \ln \sin(\frac{1}{2}\theta))] \]  

(2.19)

where

\[ B \equiv \frac{2Jk}{t} \]  

(2.20)

For scattering of strongly interacting particles at angles \( \theta \gg 1/kR \approx 10^{-5} \) radians, \( f_{sp} \) is negligible. Effectively and with much accuracy

\[ \frac{d\sigma}{d\Omega}(\theta) = \left| f(\theta) \right|^2 \]

\[ \approx \left| f_c(\theta) + f_{N}(\theta) \right|^2 \]  

(2.21)

Eq. (2.21) is to be compared with the expression (2.1) used by Biron. Theoretically, they should lead to equal values of the residues at fixed energy to the order \( \mathcal{L}^2 \), if the nuclear model used has been correct.

If we consider the finite charge distributions of the incident and target particles, the above Coulomb amplitude gets
modified and equals
\[
\left\{ f_c(\Theta) - f_c^B \left[ 1 - F(\Theta) \right] \right\}, \tag{2.22}
\]

\( F(\Theta) \) being the electromagnetic form factor. We use the product \( F_\pi(t) F_{4\text{He}}(t) \) for this e.m. form factor \( F(\Theta) \), in which

\[
F_\pi(t) = \exp \left( -\frac{r_\pi^2}{6} |t| \right), \tag{2.23}
\]

\[
F_{4\text{He}}(t) = \exp \left( -\frac{r_{4\text{He}}^2}{6} |t| \right), \tag{2.24}
\]

\( r_\pi \) and \( r_{4\text{He}} \) being the charge radii of pion and helium nuclei respectively. The effects of these terms are found to be very small. The terms are, however, included with acceptable values of the radii: \( r_\pi = 0.8 \text{ fm} \) and \( r_{4\text{He}} = 1.67 \text{ fm} \).

Hence the differential scattering cross section for \( \pi^\pm \) scattered from \( 4\text{He} \) nuclei is

\[
\frac{d\sigma}{d\Omega}(\Theta) = \left[ f_c(\Theta) - f_c^B \left[ 1 - F(\Theta) \right] \right]^2 + \left[ f_N^\pm(\Theta) \right]^2 + 2 \text{Re} f_c(\Theta) f_N^\pm(\Theta) \text{Re} f_N^\pm(\Theta)
+ 2 \text{Im} f_c(\Theta) f_N^\pm(\Theta) \text{Im} f_N^\pm(\Theta) \tag{2.25}
\]

where \( f_N^\pm(\Theta) \) refers to the residual nuclear amplitudes for \( \pi^\pm \).
II.3. Extrapolation Procedure

The scattering cross section, as represented by Eq. (2.25) contains both first and second order e.m. poles at $t = 0$. There is also the e.m. cut from $t = 0$ to $\infty$. The contribution from the pure Coulomb term containing the second order pole is known. The Coulomb nuclear interference terms contain first order $t = 0$ pole and are of significance here. The strength of these singularities at $t = 0$ is proportional to the real and imaginary parts of the forward nuclear amplitude. The pole being on the forward edge of the physical region and the contribution of the interference terms in the forward angles being comparable with the nuclear contribution itself, the pole extrapolation procedure can perhaps be used here for determining nuclear parameter to a high degree of accuracy.

The interference terms having been determined and subtracted out along with the pure Coulomb term, the differential cross section for the strong interaction can be obtained for all scattering angles. However, a word of caution is necessary. Sometimes the extrapolation may not be very stable due to non-negligible contribution from the e.m. cut, especially at low energies.

Details of the extrapolation procedure are outlined below.
With the Coulomb singularities at and cuts from \( t = 0 \), the differential scattering cross section is not analytic and hence, no classical polynomial expansion will converge. We assume that the pole and the cut, to order \( \mathcal{A} \), are given accurately by the generalization of the nonrelativistic theory as given by Eq. (2.19). Only when the singular terms are correctly subtracted from the data, the remainder can be expanded in a convergent series of orthogonal polynomials. The region of convergence will then be the ellipse contained by the nearest singularity in the \( \cos \theta \) plane. To explain this further, let us write the singularity-subtracted cross section \( \gamma^\pm(s, \theta) \) as

\[
\gamma^\pm(s, \theta) = \left. \frac{d\sigma}{d\Omega} (\theta) \right|_{\text{exp}} - \left[ |f_c(\theta) - f_c^B(1-P(\theta))| \right]^2 \\
\pm 2 \left\{ \text{Re } f_c(\theta) - f_c^B(1-P(\theta)) \right\} \text{Re } f^+_N(0) \\
+ 2 \text{Im } f_c(\theta) \text{Im } f^+_N(0) \tag{2.26}
\]

Consider the right hand side of this equation. The first term inside the square bracket is the pure Coulomb contribution and contains the second order pole at \( t = 0 \). The second term is the interference term containing the first order pole at \( t = 0 \). The last term is dominated by the weak electromagnetic cut from \( t = 0 \) to \( \infty \).
The quantity $f(s, \phi)$ (suppressing the $\pm$ sign for convenience) can be expanded in a series such as

$$f(s, \phi) = \sum_{n} a_n(s) p_n(\cos \phi), \quad (2.27)$$

where $p_n$ are polynomials weighted by the experimental errors in the differential cross section data. These are constructed by Schmidt's orthogonalization procedure. From a fit to the data, with a series of $L$ terms, the $\chi^2$ value obtained is

$$\chi^2_{L} = \sum_{i=1}^{N} \left[ \frac{f_i - \sum_{n=1}^{L} a_n p_n(\cos \phi_i)}{\Delta W_i} \right]^2, \quad (2.28)$$

$N$ being the number of data points and $\Delta W_i$ being the error in the differential cross section at $\phi_i$. In case Coulomb singularities are not exactly subtracted, $f(s, \phi)$ will not be analytic and the polynomial expansion (2.27) will not converge. As a result of nonconvergence, $\chi^2$ will be large for any given order of truncation of the series. With correct subtraction of the poles, the $\chi^2$ value will be reduced to a minimum. Thus at the correct subtraction point, $\chi^2$ will show a pronounced dip as the strength of the singularity is varied. The more accurate the data, the sharper the dip. We consider the fit to be good if $\chi^2_{L}/NDF$ (number of degrees of freedom) $\leq 1$. 
Once the Coulomb pole terms are correctly subtracted out, the remaining \( \mathcal{F}(s,\theta) \) is mostly the nuclear differential scattering cross section. However, there are two unknown residues \( \text{Re} \, f_N(0) \) and \( \text{Im} \, f_N(0) \) which are to be found simultaneously. Because of the lack of a distinct pole in the imaginary part of the Coulomb amplitude a substantial variation in the nuclear imaginary part does not appear to affect the extrapolation. So it is possible to use the extrapolation in an iterative manner, taking advantage of the relation

\[
\left| f_N(0) \right|^2 = \frac{d\sigma_N}{d\Omega}(0^\circ)
\]

\[
= \left| \text{Re} \, f_N(0) \right|^2 + \left| \text{Im} \, f_N(0) \right|^2.
\]

(2.29)

For a certain value of \( \text{Re} \, f_N(0) \), the iteration is started by taking some plausible value of the imaginary part in the residue term of Eq. (2.26). \( \mathcal{F}(s,\theta) \) is now extrapolated through the polynomials to give \( d\sigma_N/d\Omega \) at \( \theta = 0^\circ \).

A fresh value for \( \text{Im} \, f_N(0) \) is obtained from relation (2.29). This new value of \( \text{Im} \, f_N(0) \) is now fed back in the residue term and the extrapolation procedure is repeated. This is continued until stable values of \( d\sigma_N(0^\circ) \) and \( \text{Im} \, f_N(0) \) are obtained. Table II.1 shows a typical cycle of iteration at 110 MeV for one value of \( \text{Re} \, f_N(0) \). The fits are carried out for several values of \( \text{Re} \, f_N(0) \) to obtain the minimum of the \( \chi^2 \) curve. It is found that quite accurate values of \( \text{Im} \, f_N(0) \)
### Table II.1: A typical cycle of iteration at 110 MeV for \( \text{Re } f_N(0) = 1.2 \text{ fm} \) and \( L = 7 \), starting with \( \text{Im } f_N(0) = 1.5 \text{ fm} \).

<table>
<thead>
<tr>
<th>No. of Iteration</th>
<th>( \frac{d\sigma_{N(0^\circ)}}{d\Omega} ) (mb/sr)</th>
<th>( \text{Im } f_N(0) ) (fm)</th>
<th>( \chi^2 )</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>45.34</td>
<td>1.759</td>
<td>56.93</td>
</tr>
<tr>
<td>2</td>
<td>45.59</td>
<td>1.766</td>
<td>56.76</td>
</tr>
<tr>
<td>3</td>
<td>45.60</td>
<td>1.766</td>
<td>56.76</td>
</tr>
<tr>
<td>4</td>
<td>45.60 (stable)</td>
<td>1.766 (stable)</td>
<td>56.76</td>
</tr>
</tbody>
</table>
are obtained by this method of iteration with the constraint (2.29).

Correctly subtracted for the value of the residue when $\chi^2$ is minimum, the remainder

$$\tilde{f}_0(s,\theta) = \sum_n a_n p_n(\cos \theta)$$

is extrapolated to $\theta = 0^\circ$. It is also integrated to obtain the total elastic cross section. The total cross section is calculated from the relation

$$\sigma_{tot} = \frac{4\pi}{k} \text{Im} f_N(0) \quad (2.30)$$

The total inelastic cross section is obtained as the difference of the above two cross sections.

II.4. Analytic Structure of Nuclear Amplitude and Elliptic Mapping:

For a stable extrapolation to the forward pole the expansion (2.27) has to be maximally convergent. The entire domain of analyticity should be the region of convergence. Hence the rate of convergence of the polynomial expansion for $f(s,\theta)$ is determined by the analyticity structure of the strong interaction amplitude. The nuclear part of the $\pi^+ - ^4\text{He}$ scattering amplitude is free from poles both in direct and crossed channels. The $t$-channel singularity begins with the two pion exchange and the corresponding right hand cut in the
\[
x_+ = 1 + \frac{2m^2}{k^2}
\]
(2.31)

to infinity. The u-channel singularity is the left hand cut extending from \(-x_-, \) the threshold for production of \(^3\text{He}(^3\text{H})\) and \(p(n)\), to \(-\infty\).

\[
x_- = 1 + \left[ \frac{m_{^3\text{He}(^3\text{H})} + m_p(n)}{2k^2} \right]^2 - \left[ \frac{m + m_{^4\text{He}}}{2k^2} \right]^2.
\]
(2.32)

The full assumed analytic structure of \(\tilde{f}(s,\theta)\) is shown in Fig. 2.1(a).

The polynomial series (2.27) converges within the Lehmann ellipse\(^{19}\) touching the right hand cut at \(x_+\). To optimize the rate of convergence of this series expansion, the \(\cos\theta\) plane is mapped into the interior of an ellipse\(^{24}\), the cuts lying on the boundary of the ellipse in the mapped plane. The mapping is performed in two steps. First, the cuts \((-\infty, -x_-)\) and \((x_+, \infty)\) are symmetrized to \((-\infty, -w)\) and \((w, \infty)\) respectively (Fig. 2.1(b)), by a transformation

\[
w = \frac{x - x_0}{1 - x x_0},
\]
(2.33)

where

\[
x_0 = \frac{x_- - x_+}{x_+ x_- + x_+ x_- - 1},
\]
(2.34)
FIG. 2.1

(a)

- $x_-$

- $p^3\text{He exchange (}n^3\text{H)}$

- $\pi^+ 3\text{He threshold}$

- $\cos\theta$ - plane

- $\pi^+ 3\text{He threshold}$

- $2\pi^0$ exchange

- $x_+$
and $X_\pm = (x_\pm^2 - 1)^{1/2}$. Next, the symmetrized cuts are mapped onto an ellipse (Fig. 2.1(c)) enclosing the entire analyticity region by a transformation

$$z = \sin \left[ \frac{1}{2} F(\sin^{-1} w, k)/K(k) \right], \quad (2.36)$$

where $k = 1/W$, and $K(k)$ and $F(\psi, k)$ are the complete and incomplete elliptic integrals of the first kind.

Table II.2 shows the size of the new ellipse, which is quite large compared to the normal Lehmann ellipse. Orthogonal polynomials $p_n(z)$ are constructed in the new $z$ variable and the quantity $\mathcal{J}(s, \psi)$ is fitted to a series $\sum b_n p_n(z)$ to obtain the $\chi^2$ values.

$\chi^2$ curves for the real part of the forward amplitude $f_N^-(0)$ at five different energies from 110 to 260 MeV are displayed in Fig. 2.2. The minima of these curves correspond to the extracted values of Re $f_N^-(0)$. The results of extrapolation both in cos$\theta$— and $z$—planes are given in Table II.3.
Table II.2: $T_{lab} = \text{pion kinetic energy in the laboratory}$, $x_+ = \text{semi-major axis of the Lehmann ellipse}$, and $a = \text{semi-major axis of the Cutkosky-Deo ellipse}$.

<table>
<thead>
<tr>
<th>$T_{lab}$ (MeV)</th>
<th>$x_+$</th>
<th>$a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>51</td>
<td>3.389</td>
<td>12.385</td>
</tr>
<tr>
<td>60</td>
<td>2.985</td>
<td>10.812</td>
</tr>
<tr>
<td>68</td>
<td>2.718</td>
<td>9.768</td>
</tr>
<tr>
<td>75</td>
<td>2.532</td>
<td>9.034</td>
</tr>
<tr>
<td>110</td>
<td>1.966</td>
<td>6.748</td>
</tr>
<tr>
<td>150</td>
<td>1.655</td>
<td>5.430</td>
</tr>
<tr>
<td>180</td>
<td>1.517</td>
<td>4.821</td>
</tr>
<tr>
<td>220</td>
<td>1.396</td>
<td>4.259</td>
</tr>
<tr>
<td>260</td>
<td>1.316</td>
<td>3.867</td>
</tr>
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</table>
Table II.3: Results of pole extrapolation from $\pi^-$-$^4$He scattering cross section data of Binon et al; values within the parentheses are calculated from phase shift obtained by Binon et al.

<table>
<thead>
<tr>
<th>MeV</th>
<th>Ref $\rho_N(0)$</th>
<th>$\frac{d\sigma_N}{d\Omega}$</th>
<th>Im $f_N(0)$</th>
<th>$\sigma_{tot}$</th>
<th>$\sigma_{el}$</th>
<th>$\sigma_{inel}$</th>
<th>$\chi^2$/NDF</th>
</tr>
</thead>
<tbody>
<tr>
<td>110</td>
<td>1.20±0.06</td>
<td>45.65±1.05</td>
<td>1.769±0.013</td>
<td>225.99±1.69</td>
<td>75.55±0.70</td>
<td>150.44±1.86</td>
<td>7  0.98</td>
</tr>
<tr>
<td>150</td>
<td>0.57±0.35</td>
<td>94.50±3.50</td>
<td>3.021±0.007</td>
<td>317.69±0.79</td>
<td>111.55±2.45</td>
<td>206.14±2.57</td>
<td>8  0.99</td>
</tr>
<tr>
<td>180</td>
<td>0.17±0.12</td>
<td>116.90±2.20</td>
<td>3.415±0.026</td>
<td>319.14±2.43</td>
<td>111.10±1.05</td>
<td>208.04±2.65</td>
<td>9  1.14</td>
</tr>
<tr>
<td>220</td>
<td>0.14±0.56</td>
<td>121.80±3.70</td>
<td>3.487±0.032</td>
<td>285.28±2.62</td>
<td>104.00±2.30</td>
<td>181.28±3.48</td>
<td>9  1.51</td>
</tr>
<tr>
<td>260</td>
<td>-0.96±0.09</td>
<td>113.05±1.30</td>
<td>3.222±0.045</td>
<td>235.39±3.29</td>
<td>84.90±0.55</td>
<td>150.49±3.33</td>
<td>10 0.92</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MeV</th>
<th>Ref $\rho_N(0)$</th>
<th>$\frac{d\sigma_N}{d\Omega}$</th>
<th>Im $f_N(0)$</th>
<th>$\sigma_{tot}$</th>
<th>$\sigma_{el}$</th>
<th>$\sigma_{inel}$</th>
<th>$\chi^2$/NDF</th>
</tr>
</thead>
<tbody>
<tr>
<td>110</td>
<td>1.20±0.06</td>
<td>45.65±0.85 (46.36)</td>
<td>1.767±0.016 (1.86)</td>
<td>225.84±2.04 (2.36)</td>
<td>75.65±0.70 (1.36)</td>
<td>150.19±2.16 (2.76)</td>
<td>5  0.96</td>
</tr>
<tr>
<td>150</td>
<td>0.51±0.37</td>
<td>95.50±3.80 (93.11)</td>
<td>3.048±0.004 (3.004)</td>
<td>320.52±0.04 (3.22)</td>
<td>111.60±2.55 (3.22)</td>
<td>208.92±2.55 (3.22)</td>
<td>6  0.99</td>
</tr>
<tr>
<td>180</td>
<td>0.14±0.13</td>
<td>117.55±2.55 (117.38)</td>
<td>3.426±0.032 (3.426)</td>
<td>320.16±3.02 (3.22)</td>
<td>111.08±1.13 (3.22)</td>
<td>203.08±3.22 (3.22)</td>
<td>7  1.20</td>
</tr>
<tr>
<td>220</td>
<td>-0.45±0.70</td>
<td>128.70±6.20 (118.58)</td>
<td>3.557±0.175 (3.423)</td>
<td>291.06±14.32 (3.423)</td>
<td>106.90±3.10 (3.423)</td>
<td>184.16±14.65 (3.423)</td>
<td>7  1.39</td>
</tr>
<tr>
<td>260</td>
<td>-0.97±0.11</td>
<td>112.90±2.00 (103.34)</td>
<td>3.217±0.062 (3.086)</td>
<td>235.03±4.53 (3.086)</td>
<td>84.98±0.58 (3.086)</td>
<td>153.05±4.57 (3.086)</td>
<td>8  0.88</td>
</tr>
</tbody>
</table>
FIG. 2.2

(a) 110 MeV

\[ \chi^2 \]

Re \( f_N^- (\text{fm}) \)

(b) 220 MeV

\[ \chi^2 \]

Re \( f_N^- (\text{fm}) \)

(c) 260 MeV

\[ \chi^2 \]

Re \( f_N^- (\text{fm}) \)

FIG. 2.2
The method discussed is Sec. II.5 is slightly modified for analyzing the data of Crowe et al at low energies. The experiments have been performed for away from the forward region, i.e. at $\theta \approx 30^\circ - 150^\circ$. Most of the fits required about four to five order polynomials in the expansion and preliminary fits gave very high $\chi^2$ values. So we concluded that extrapolation from $\theta \approx 30^\circ$ to $\theta = 0^\circ$ has become unreliable. Quite fortunately, there are differential cross section data for both $\pi^+$ and $\pi^-$ at the same angles. It is possible to fit the sum and difference of the differential cross sections separately. This results in a much better and more accurate analysis. The sum

$$\frac{d\sigma_N^-}{d\Omega} (\theta) + \frac{d\sigma_N^+}{d\Omega} (\theta) = \left[ \frac{d\sigma_i^-}{d\Omega} (\theta) + \frac{d\sigma_i^+}{d\Omega} (\theta) \right]_{\text{exp}}$$

$$- 2 \left[ f_c (\theta) - f_c^B \left[ 1 - F(\theta) \right] \right]^2$$

$$+ 2 \left[ \text{Re} \ f_c (\theta) - f_c^B (1 - F(\theta)) \right] \left[ \text{Re} \ f_N^0 (0) - \text{Re} \ f_N^+ (0) \right]$$

$$- 2 \text{Im} \ f_c (\theta) \left[ \text{Im} \ f_N^- (0) + \text{Im} \ f_N^+ (0) \right] \quad (2.37)$$
and the difference

\[
\frac{\mathrm{d} \sigma_N^-}{\mathrm{d} \Omega} - \frac{\mathrm{d} \sigma_N^+}{\mathrm{d} \Omega} = \left[ \frac{\mathrm{d} \sigma_N^-}{\mathrm{d} \Omega} - \frac{\mathrm{d} \sigma_N^+}{\mathrm{d} \Omega} \right]_{\exp} + 2 \left[ \Re f_0(\theta) - f_0^B(1-F(\theta)) \right] \left[ \Re f_N^-(0) + \Re f_N^+(0) \right] - 2 \Im f_0(\theta) \left[ \Im f_N^-(0) - \Im f_N^+(0) \right] \tag{2.38}
\]

both contain the pole at \( t = 0 \).

If the poles are correctly subtracted, the sum and the difference can be separately fitted to the set of constructed orthogonal polynomials. Now if we write \( \tilde{f}_\pm(s,\theta) \) for the sum or the difference, it can have the polynomial expansions

\[
\tilde{f}_+(s,\theta) = \frac{\mathrm{d} \sigma_N^-}{\mathrm{d} \Omega}(\theta) + \frac{\mathrm{d} \sigma_N^+}{\mathrm{d} \Omega}(\theta) = \sum_n b_n^+ p_n(z) \tag{2.39}
\]

and

\[
\tilde{f}_-(s,\theta) = \frac{\mathrm{d} \sigma_N^-}{\mathrm{d} \Omega}(\theta) - \frac{\mathrm{d} \sigma_N^+}{\mathrm{d} \Omega}(\theta) = \sum_n b_n^- p_n(z) \tag{2.40}
\]

A variation in the residues of the last terms of Eqs. (2.37) and (2.38) changes the \( \chi^2 \) value insignificantly. This is to be expected since they contain \( \Delta \) in second order. For this reason we conveniently take some approximate values for \( \left[ \Im f_N^-(0) + \Im f_N^+(0) \right] \) and \( \left[ \Im f_N^-(0) - \Im f_N^+(0) \right] \) and carry out the extrapolation with different values for \( \left[ \Re f_N^-(0) - \Re f_N^+(0) \right] \) and \( \left[ \Re f_N^-(0) + \Re f_N^+(0) \right] \).

\( \chi^2 \) value show a sharper minima in the latter case.
The values of the sum of the real parts at forward angles are thus quite accurately determined. The sum and the difference given by Eqs. (2.39) and (2.40), when extrapolated to all angles through the polynomials, yield \[ d\sigma_N^\pm /d\Omega (0^0) \text{ and } \sigma^\pm \text{ el} \].

While constructing the elliptic mapping, we have ignored the small difference between the left hand cuts for positive and negative pions.

The minimum value of \( \chi^2/NDF \) for the fits are given in Tables II.4 and II.5. It can be noticed that these \( \chi^2/NDF \) for the fits of the difference of scattering cross sections are quite acceptable, whereas for the fits of the sum of the cross sections we get large values of \( \chi^2/NDF \). This is due to the inability of the pole terms being visible against the background as the higher order polynomials contribute significantly to the \( \chi^2 \) of the fit. This difficulty has been circumvented by constructing an ideal series from the ansatz that the \( b_n^+ \) decreases exponentially for large \( n \) values. This is explained below.

The ideal series \[ \sum_{n=1}^{N} c_n p_n(z) \], \( N \) being the number of data points, is constructed with the weighted polynomials \( p_n(z) \) such that the coefficients \( c_n \) are close to \( b_n^+ \) of Eq. (2.39) in higher order with \( n \) around three to five, but fall off exponentially with increasing \( n \). This is done by plotting a graph of \( \log_e |b_n^+| \) against \( n \) and taking an average slope.
Table II.4: Results of pole extrapolation from the sum of the differential cross section data of Crowe et al. for $\pi^+$ and $\pi^-$. $\chi^2$ is obtained from the fit after subtraction of the ideal series (see text)

<table>
<thead>
<tr>
<th>$T_{lab}$ (MeV)</th>
<th>Ref$_N^-(0)$ - Ref$_N^+(0)$</th>
<th>$\frac{d\sigma_{-}}{d\Omega} + \frac{d\sigma_{+}}{d\Omega}$</th>
<th>$\sigma_{el} + \sigma_{el}^+$</th>
<th>$L, \chi^2/\text{NDF}$</th>
<th>$L, \chi^2/\text{NDF}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(fm)</td>
<td>(mb/sr)</td>
<td>(mb)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>51</td>
<td>$-0.03 \pm 0.17$</td>
<td>$0.65 \pm 0.70$</td>
<td>$57.2 \pm 1.8$</td>
<td>4, 2.0</td>
<td>4, 0.64</td>
</tr>
<tr>
<td>60</td>
<td>$0.0 \pm 0.30$</td>
<td>$15.80 \pm 1.90$</td>
<td>$71.0 \pm 3.2$</td>
<td>5, 6.35</td>
<td>5, 2.04</td>
</tr>
<tr>
<td>68</td>
<td>$-0.06 \pm 0.26$</td>
<td>$17.55 \pm 1.00$</td>
<td>$77.0 \pm 2.2$</td>
<td>5, 2.32</td>
<td>4, 1.43</td>
</tr>
<tr>
<td>75</td>
<td>$0.02 \pm 0.22$</td>
<td>$25.60 \pm 1.80$</td>
<td>$91.2 \pm 3.6$</td>
<td>5, 4.36</td>
<td>5, 0.88</td>
</tr>
<tr>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>$x$-plane</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$z$-plane</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>51</td>
<td>$-0.05 \pm 0.21$</td>
<td>$10.80 \pm 1.10$</td>
<td>$58.0 \pm 2.6$</td>
<td>4, 2.09</td>
<td>4, 0.64</td>
</tr>
<tr>
<td>60</td>
<td>$0.01 \pm 0.27$</td>
<td>$15.60 \pm 1.60$</td>
<td>$70.9 \pm 2.9$</td>
<td>5, 5.34</td>
<td>5, 2.75</td>
</tr>
<tr>
<td>68</td>
<td>$0.22 \pm 0.34$</td>
<td>$19.35 \pm 1.65$</td>
<td>$77.4 \pm 3.4$</td>
<td>4, 1.85</td>
<td>4, 0.80</td>
</tr>
<tr>
<td>75</td>
<td>$0.11 \pm 0.33$</td>
<td>$26.90 \pm 1.30$</td>
<td>$91.1 \pm 2.7$</td>
<td>4, 3.18</td>
<td>4, 1.31</td>
</tr>
</tbody>
</table>
Table II.5: Results of pole extrapolation from the difference of the differential scattering cross section data of Crowe et al for $\pi^+$ and $\pi^-$. 

<table>
<thead>
<tr>
<th>$t_{lab}$ (MeV)</th>
<th>$[\text{Re}f_N^0(0)+\text{Re}f_N^+(0)]$</th>
<th>$[\frac{d\sigma^-}{d\Omega} - \frac{d\sigma^+}{d\Omega}]_{\theta=0^\circ}$</th>
<th>$[\sigma_e^- - \sigma_e^+]$</th>
<th>$L, \chi^2/\text{NDF}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\text{fm}$</td>
<td>$\text{mb/sr}$</td>
<td>$\text{mb}$</td>
<td></td>
</tr>
<tr>
<td>$x$-plane</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>51</td>
<td>$1.175 \pm 0.055$</td>
<td>$-1.47 \pm 0.09$</td>
<td>$-7.67 \pm 0.29$</td>
<td>2, 1.15</td>
</tr>
<tr>
<td>60</td>
<td>$1.415 \pm 0.035$</td>
<td>$-1.44 \pm 0.06$</td>
<td>$-7.10 \pm 0.14$</td>
<td>2, 1.08</td>
</tr>
<tr>
<td>68</td>
<td>$1.200 \pm 0.100$</td>
<td>$-0.93 \pm 0.11$</td>
<td>$-5.60 \pm 0.35$</td>
<td>2, 0.95</td>
</tr>
<tr>
<td>75</td>
<td>$1.410 \pm 0.085$</td>
<td>$-1.03 \pm 0.09$</td>
<td>$-5.14 \pm 0.26$</td>
<td>2, 1.49</td>
</tr>
<tr>
<td>$z$-plane</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>51</td>
<td>$1.220 \pm 0.060$</td>
<td>$-1.66 \pm 0.12$</td>
<td>$-8.36 \pm 0.36$</td>
<td>2, 1.16</td>
</tr>
<tr>
<td>60</td>
<td>$1.465 \pm 0.040$</td>
<td>$-1.63 \pm 0.06$</td>
<td>$-7.90 \pm 0.16$</td>
<td>2, 1.26</td>
</tr>
<tr>
<td>68</td>
<td>$1.230 \pm 0.100$</td>
<td>$-1.04 \pm 0.14$</td>
<td>$-6.23 \pm 0.40$</td>
<td>2, 0.98</td>
</tr>
<tr>
<td>75</td>
<td>$1.485 \pm 0.105$</td>
<td>$-1.24 \pm 0.12$</td>
<td>$-5.82 \pm 0.34$</td>
<td>2, 1.37</td>
</tr>
</tbody>
</table>
A for suitably chosen $n$ values. The corresponding intercept $B$ on $\log_{e} |b_{n}^{+}|$ axis enables us to construct coefficients of an ideal series,

$$|c_{n}| = \exp (An + B)$$  \hspace{1cm} (2.41)

In this series, $c_{n}$ are required to carry the same signs as those of the corresponding $b_{n}^{+}$. Subtracting this ideal theoretical series $\sum_{n=1}^{N} c_{n} p_{n}(z)$ from the experimental cross sections, a fit to the polynomial expansion is made. The new $\chi^{2}$ values ($\chi^{2}$ say) are much lower as we expected.

To illustrate the above process, consider the analysis of the data at 68 MeV for which a normal polynomial fit has yielded a minimum $\chi^{2}$/NDF of 1.85 in the $z$-plane for $L = 4$. The plot of $\log_{e} |b_{n}^{+}|$ against $n$ is shown in Fig. 2.3. The average slope $A$ of this for $n \sim 5$ to 14 is $-0.2716$. Its intercept $B$ is 2.447. The ideal series is now constructed with

$$|c_{n}| = \exp (-0.2716n + 2.447)$$

in the series $\sum_{n=1}^{N} c_{n} p_{n}(z)$, the signs of $c_{n}$ being the same as those of $b_{n}^{+}$ with the same $n$ values. This series is subtracted from Eq. (2.37). Then a fit to the polynomial expansion $\sum_{n} b_{n} p_{n}(z)$ is carried out. The subtraction method yields $\chi^{2}$/NDF = 0.8.
FIG. 2.3

Slope $A = -0.2716$
The $\chi^2$ curves so obtained for the lower energies are shown in Fig. 2.4. $\chi^2$ curves for the normal polynomial fit of the difference of differential cross sections of $\pi^+$ and $\pi^-$ are displayed in Fig. 2.5.

II.6. Results and discussions:

The results of our analysis are given in Table II.3-7. The errors correspond to a variation of $\chi^2$ from $\chi^2_{\text{min}}$ to $\chi^2_{\text{min}} + 1$. The elliptic mapping has helped in remarkably improving the convergence rate of the polynomial expansions at energies above 110 MeV. It has reduced the required number of parameters for the fits by two everywhere as indicated by column 8 of Table II.3. But the mapping does not provide much improvement in the lower energy region. Column 6 of Table II.4 shows that only at 75 MeV the required number of parameters decreases from five to four due to mapping. At 68 MeV, even though the number of parameters is not reduced, the $\chi^2/$NDF has fallen from 1.43 to 0.8. But at energies below 60 MeV, there is no improvement at all due to mapping. Similar behaviour is also noted from column 5 of Table IV.5. At 75 MeV, $\chi^2$/NDF has been reduced from 1.49 to 1.37. But below this energy, $\chi^2$/NDF values become worse with mapping. One possible explanation could be that at low energies, far away portions of the cut come closer to the physical region with the mapping and spoil the goodness of the fits.
FIG. 2.4

(a) 75 MeV
L = 4
L = 5

(b) 50 MeV
L = 5
L = 6

(c) 68 MeV
L = 4

Re(\<\bar{\psi}_N - \bar{\psi}_N\> (fm))

\(z\)
\(z\)

\(z\)
\(z\)
Table II.6: Results of pole extrapolation from differential cross section data of Crowe et al., values within the parenthesis are calculated from phase shifts obtained by Binon et al.

<table>
<thead>
<tr>
<th>$T_{\text{lab}}$ (MeV)</th>
<th>Re $f_N^-(0)$ (fm)</th>
<th>Re $f_N^+(0)$ (fm)</th>
<th>$\frac{d\sigma_N^-}{d\Omega}(0^\circ)$ (mb/sr)</th>
<th>$\frac{d\sigma_N^+}{d\Omega}(0^\circ)$ (mb/sr)</th>
<th>Im $f_N^-(0)$ (fm)</th>
<th>Im $f_N^+(0)$ (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>x-plane</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>51</td>
<td>0.572</td>
<td>0.602</td>
<td>4.09</td>
<td>5.56</td>
<td>0.286</td>
<td>0.439</td>
</tr>
<tr>
<td></td>
<td>± 0.089</td>
<td>± 0.089</td>
<td>± 0.35</td>
<td>± 0.35</td>
<td>± 0.117</td>
<td>± 0.082</td>
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<tr>
<td>60</td>
<td>0.707</td>
<td>0.707</td>
<td>7.18</td>
<td>8.62</td>
<td>0.467</td>
<td>0.601</td>
</tr>
<tr>
<td></td>
<td>± 0.150</td>
<td>± 0.150</td>
<td>± 0.95</td>
<td>± 0.95</td>
<td>± 0.125</td>
<td>± 0.097</td>
</tr>
<tr>
<td>68</td>
<td>0.570</td>
<td>0.630</td>
<td>9.31</td>
<td>9.24</td>
<td>0.711</td>
<td>0.726</td>
</tr>
<tr>
<td></td>
<td>± 0.139</td>
<td>± 0.139</td>
<td>± 0.50</td>
<td>± 0.50</td>
<td>± 0.076</td>
<td>± 0.086</td>
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<tr>
<td>75</td>
<td>0.715</td>
<td>0.695</td>
<td>12.29</td>
<td>13.32</td>
<td>0.847</td>
<td>0.921</td>
</tr>
<tr>
<td></td>
<td>± 0.118</td>
<td>± 0.118</td>
<td>± 0.90</td>
<td>± 0.90</td>
<td>± 0.046</td>
<td>± 0.040</td>
</tr>
<tr>
<td><strong>z-plane</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>51</td>
<td>0.585</td>
<td>0.635</td>
<td>4.57</td>
<td>6.23</td>
<td>0.338</td>
<td>0.468</td>
</tr>
<tr>
<td></td>
<td>± 0.109</td>
<td>± 0.109</td>
<td>± 0.55</td>
<td>± 0.55</td>
<td>± 0.108</td>
<td>± 0.090</td>
</tr>
<tr>
<td>(0.639)</td>
<td>(0.637)</td>
<td>(4.94)</td>
<td>(5.20)</td>
<td>(0.290)</td>
<td>(0.337)</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>0.737</td>
<td>0.727</td>
<td>6.99</td>
<td>8.61</td>
<td>0.394</td>
<td>0.576</td>
</tr>
<tr>
<td></td>
<td>± 0.136</td>
<td>± 0.136</td>
<td>± 0.80</td>
<td>± 0.80</td>
<td>± 0.152</td>
<td>± 0.102</td>
</tr>
<tr>
<td>(0.744)</td>
<td>(0.733)</td>
<td>(7.73)</td>
<td>(8.07)</td>
<td>(0.467)</td>
<td>(0.518)</td>
<td></td>
</tr>
<tr>
<td>68</td>
<td>0.725</td>
<td>0.505</td>
<td>9.16</td>
<td>10.20</td>
<td>0.624</td>
<td>0.874</td>
</tr>
<tr>
<td></td>
<td>± 0.186</td>
<td>± 0.186</td>
<td>± 0.83</td>
<td>± 0.83</td>
<td>± 0.149</td>
<td>± 0.060</td>
</tr>
<tr>
<td>(0.733)</td>
<td>(0.714)</td>
<td>(9.35)</td>
<td>(9.73)</td>
<td>(0.630)</td>
<td>(0.680)</td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>0.797</td>
<td>0.687</td>
<td>12.83</td>
<td>14.07</td>
<td>0.805</td>
<td>0.966</td>
</tr>
<tr>
<td></td>
<td>± 0.173</td>
<td>± 0.173</td>
<td>± 0.65</td>
<td>± 0.65</td>
<td>± 0.130</td>
<td>± 0.089</td>
</tr>
<tr>
<td>(0.182)</td>
<td>(0.784)</td>
<td>(13.33)</td>
<td>(13.83)</td>
<td>(0.820)</td>
<td>(0.875)</td>
<td></td>
</tr>
</tbody>
</table>
Table II.7: Total cross section, total elastic and inelastic cross sections obtained by pole extrapolation from the data of Crowe et al.

<table>
<thead>
<tr>
<th>T_{lab} (MeV)</th>
<th>\sigma^{\text{tot}}^- (mb)</th>
<th>\sigma^{\text{tot}}^+ (mb)</th>
<th>\sigma^{\text{el}}^- (mb)</th>
<th>\sigma^{\text{el}}^+ (mb)</th>
<th>\sigma^{\text{inel}}^- (mb)</th>
<th>\sigma^{\text{inel}}^+ (mb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>x-plane</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>51</td>
<td>57.44 ± 23.70</td>
<td>88.17 ± 16.47</td>
<td>24.77 ± 0.91</td>
<td>32.68 ± 0.91</td>
<td>31.67 ± 23.72</td>
<td>55.49 ± 16.50</td>
</tr>
<tr>
<td>60</td>
<td>85.50 ± 22.89</td>
<td>110.04 ± 17.76</td>
<td>31.95 ± 1.60</td>
<td>39.05 ± 1.60</td>
<td>53.55 ± 22.94</td>
<td>70.99 ± 17.83</td>
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<tr>
<td>68</td>
<td>121.10 ± 12.94</td>
<td>123.65 ± 14.65</td>
<td>35.60 ± 1.11</td>
<td>41.40 ± 1.11</td>
<td>85.50 ± 12.99</td>
<td>82.25 ± 14.69</td>
</tr>
<tr>
<td>75</td>
<td>136.23 ± 7.40</td>
<td>148.13 ± 6.43</td>
<td>43.03 ± 1.80</td>
<td>48.17 ± 1.80</td>
<td>93.20 ± 7.61</td>
<td>99.96 ± 6.68</td>
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<tr>
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<td></td>
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</tr>
<tr>
<td>51</td>
<td>67.88 ± 21.69</td>
<td>93.99 ± 18.08</td>
<td>24.82 ± 1.31</td>
<td>33.18 ± 1.31</td>
<td>43.06 ± 21.73</td>
<td>60.81 ± 18.12</td>
</tr>
<tr>
<td>60</td>
<td>72.14 ± 27.83</td>
<td>105.46 ± 18.68</td>
<td>31.50 ± 1.45</td>
<td>39.40 ± 1.45</td>
<td>40.64 ± 27.87</td>
<td>66.06 ± 18.73</td>
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<tr>
<td>68</td>
<td>106.28 ± 25.38</td>
<td>148.86 ± 10.22</td>
<td>35.59 ± 1.71</td>
<td>41.81 ± 1.71</td>
<td>70.69 ± 25.43</td>
<td>107.05 ± 10.36</td>
</tr>
<tr>
<td>75</td>
<td>129.47 ± 20.91</td>
<td>155.37 ± 14.31</td>
<td>42.64 ± 1.36</td>
<td>48.36 ± 1.36</td>
<td>86.83 ± 20.95</td>
<td>107.01 ± 14.38</td>
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</table>
Elimination of the background contribution from the terms with \( n > L \) by subtraction of an ideal series conspicuously reduces \( \chi^2/NDF \) values as can be seen from columns 5 and 6 of Table II.4, thereby providing a better fit. At 60 MeV, even though \( \chi'^2/NDF \) is a little above the acceptable value, one can notice the substantial improvement the method has provided by diminishing the ratio from 6.35 as obtained by the usual fit to 2.04 in \( x \)-plane and from 5.34 as obtained by the usual fit to 2.75 in the \( z \)-plane.

Fig. 2.6 is a plot of the real parts of the forward amplitude against the laboratory kinetic energy of pions. The values agree well with the results obtained by Biron et al for the strong interaction amplitude. They also agree fairly well with those of dispersion relation calculations made by Wilkin et al and Batty et al. However beyond 220 MeV both Biron's result and our result differ appreciably from the results of dispersion relation calculations. The results for \( \pi^+ \) at lower energies are not shown in the figure to avoid overlapping.

From Table II.5 it is obvious that the cross sections for \( \pi^+ \) and \( \pi^- \) are different. After repeated trials we have observed that this difference is appreciable and is more conspicuous with decreasing energy. We are thus led to believe that there is considerable distortion of the nuclear amplitude at low energies due to Coulomb effects. Fig. 2.7 is a plot of the total cross section, total elastic and total
FIG. 2.6
inelastic cross sections versus pion kinetic energy. The results are in close agreement with those of Binon. At low energies there are large errors in $\sigma_{\text{tot}}$. The total cross section is maximum between 150–185 MeV around the energy region of $\pi N (3,3)$ resonance. The extrapolated cross section $\frac{d\sigma_{\text{N}}(0^\circ)}{d\Omega}$ versus the laboratory kinetic energy is displayed in Fig. 2.8 for both negative and positive pions. The results are in close agreement with those of Binon for $\pi^-$. 

As an additional check, we have calculated Coulomb distorted nuclear data at forward angles from the phase shifts given by Binon et al.\textsuperscript{7} The values are given in Tables II.3 and II.6 within brackets. It can be seen that our results of pole extrapolation agree well with their values. We emphasize that our results should be more reliable, as they are free from any ambiguity arising from phase shift analysis.

More exact values of the magnitude of Coulomb distortion at low energies can be found when accurate data, still closer to the forward region, are available. This method, which has successfully extracted the nuclear parameters without the explicit knowledge of the strong interaction model, can in principle be extended to other light nuclei, which would throw more light on the behaviour of nuclear scattering near the $(3,3)$ resonance region.
FIG. 2.8

\[ \frac{d\sigma_N}{d\Omega} (0^\circ) \text{ (mb/sr)} \]

vs.

\[ T^{lab} \text{ (MeV)} \]

FIG. 2.8