1.1 SETTLEMENT ANALYSIS IN GENERAL

A complete settlement analysis consists of predicting ultimate settlement \( w_\infty \), initial pore-pressure \( \psi_0 \) and settlement \( w \) and porepressure \( \psi \) as functions of time \( t \). A self-consistent consolidation theory must be able to predict independently all these quantities.

Settlement \( (w_t) \) at any time after the application of load is written as:

\[
w_t = w_0 + w_{ct} \tag{1.1}\]

where \( w_0 \) = immediate settlement

\( w_{ct} \) = primary consolidation settlement corresponding to time \( t \).

The ultimate settlement \( (w_\infty) \) is given as:

\[
w_\infty = w_0 + w_{c\infty} \tag{1.2}\]

where \( w_{c\infty} \) = final consolidation settlement.

Progress of consolidation is measured by "Degree of Consolidation Settlement" \( (\bar{w}) \), which is defined as:

\[
\bar{w} = \frac{w_{ct}}{w_{c\infty}} \tag{1.3}\]
Thus (1.1) is rewritten as:

\[ w_t = w_0 + \bar{u}_w w_{\infty} \]  

(1.4)

The various quantities defined in (1.1) to (1.4) are shown schematically in Fig.1.1.

Any settlement analysis must be able to predict all the three quantities on the right hand side of (1.4). A critical review of different settlement analyses currently used in practice is presented in this chapter.

1.2 ONE-DIMENSIONAL SETTLEMENT ANALYSIS

1.2.1 Prediction of Magnitudes of Settlements

One-dimensional settlement analysis assumes that strain and flow of water take place in one direction only. The laboratory test which excellently simulates this assumption is the classical oedometer test. A saturated soil, if prevented from lateral strains, will not undergo any immediate settlement (i.e., \( w_0 = 0 \)). Final consolidation settlement is evaluated from oedometer test results and degree of consolidation settlement corresponding to a given time is
evaluated from the classical one-dimensional consolidation theory (Terzaghi, 1943).

From oedometer tests, one can determine the coefficient of volume compressibility ($m_{v1}$) defined as:

$$m_{v1} = \frac{e_{zz}}{\sigma''_{zz}} \quad (1.5)$$

where $e_{zz}$ is the vertical strain and $\sigma''_{zz}$ is the effective vertical stress (or consolidation pressure). Final consolidation settlement is obtained by integrating $e_{zz}$ from (1.5) as:

$$w_{co} = \int e_{zz} \, dz = \int m_{v1} \cdot \sigma''_{zz} \, dz$$

For a thin laboratory soil sample, the variation in $\sigma''_{zz}$ with depth is negligible and hence it is taken as equal to the applied load intensity itself. However, if (1.6) is used to predict settlements for a field situation, such as a clay layer of considerable thickness, the variation of $\sigma''_{zz}$ with depth cannot be ignored. As such knowledge of the manner of this variation is warranted to predict the final consolidation settlement. For this purpose, Boussinesq's elastic stress distribution theory (1885) is generally
used. This approach has an apparent inconsistency because Boussinesq's theory does not ignore the lateral strains, as assumed in this analysis. As such, use of Westergaard's theory (1938) based on one-dimensional strain approach, appears to be consistent and hence desirable. It is interesting to note that expressions for $\sigma_{zz}'$ in Boussinesq's theory are independent of the two elastic parameters ($R$ and $\nu$), whereas those based on Westergaard's theory include Poisson's ratio. The vertical stress distribution patterns under different types of surface loads as obtained from Westergaard's theory are given by the author (Babu Shanker, 1973).

The ultimate settlements obtained from one-dimensional analyses are thus given by:

$$(w_{\infty})_{\text{conventional 1-D}} = \int m_v l \left( \sigma_{zz}' \right)_B \, dz \quad (1.7)$$

$$(w_{\infty})_{\text{true 1-D}} = \int m_v l \left( \sigma_{zz}' \right)_W \, dz$$

where the subscripts $B$ and $W$ for $\sigma_{zz}'$ represent the elastic theory of Boussinesq and Westergaard respectively.
Based on Hooke's effective stress-strain relations for soil skeleton, one-dimensionality of strain implies the following identities:

\[ u = v = e_{xx} = e_{yy} = 0 \]

\[ \sigma_{xx}' = \sigma_{yy}' = \frac{\nu}{1-\nu} \sigma_{zz}' \quad (1.9) \]

\[ m_{\nu l} = \frac{(1+\nu)(1-2\nu)}{E(1-\nu)} \]

where \( u, v \) are the displacements in \( x \) and \( y \) directions,

\( e_{xx}', e_{yy}' \) are the normal strains,

\( \sigma_{xx}', \sigma_{yy}' \) are the effective normal stresses,

and \( E, \nu \) are the effective elastic parameters of soil skeleton.

Immediately on the application of load, a saturated soil behaves as an incompressible material with \( e_{xx} = e_{yy} = e_{zz} = 0; \gamma = 0.5 = \bar{\gamma} \) (the undrained value of \( \gamma \)). (1.9) yields undrained (or total) stresses \( \sigma_{xx} = \sigma_{yy} = \sigma_{zz} \), which represent an isotropic (or hydrostatic) stress state. The initial porepressure \( (\sigma_0) \) will thus be equal to this value of hydrostatic stress i.e.,

\[ \sigma_0 = \sigma_{zz} \quad (1.10) \]
In fact if the entire surface is normally loaded with an intensity of \( q \), as is done in oedometer tests, one obtains: \( J \circ = \sigma_{zz} = q \).

For a homogeneous, semi-infinite elastic layer, (1.7) yields:

\[
v(\infty) = \int_0^\infty m_{vl}(\sigma_{zz})_B \, dz = m_{vl} q B_f I_f
\]

(1.11)

where \( B_f \) = lateral dimension of loaded area

\( I_f \) = influence factor which depends upon the shape of the loaded area (Bowles, 1968).

1.2.2 Prediction of rate of settlements.

Rates of settlement are predicted from classical one-dimensional consolidation theory of Terzaghi (1923). Terzaghi was the first to formulate a mathematical model for consolidation. In fact his monumental work on consolidation heralded the birth of the Science of Soil Mechanics. He made use of a simplified version of the continuity equation, which is derived in a more general form as:

\[
c \frac{\partial^2 e}{\partial t^2} = \frac{\partial e}{\partial t}
\]

(1.12)

where \( c \) is the coefficient of consolidation,
In fact if the entire surface is normally loaded with an intensity of \( q \), as is done in oedometer tests, one obtains: \( \int \sigma = \sigma_{zz} = q \).

For a homogeneous, semi-infinite elastic layer, (1.7) yields:

\[
\omega = \int_0^\infty m v_1 (\sigma_{zz})_B \, dz = m v_1 q B_f I_f \tag{1.11}
\]

where \( B_f \) = lateral dimension of loaded area
\( I_f \) = influence factor which depends upon the shape of the loaded area (Bowles, 1968).

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\[
c \frac{\partial \epsilon}{\partial t} = \frac{\partial \sigma}{\partial t} \tag{1.12}
\]

where \( c \) is the coefficient of consolidation,
\( c \) is the volumetric strain and 
\[
\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}
\]

In terms of pore pressure (1.13) is written (Gibson and Lumb, 1953) as:
\[
c \nabla^2 p = \frac{1}{2} \frac{\partial \sigma}{\partial t} - \frac{1}{3} \frac{\partial \sigma}{\partial t}
\]

(1.13)

where \( \sigma = \sum_{x} + \sum_{y} + \sum_{z} \) is the sum of total normal stresses. If it is assumed that \( \sigma \) does not vary with time, then (1.13) reduces to:
\[
c \nabla^2 \sigma = \frac{\partial \sigma}{\partial t}
\]

(1.14)

This equation is known as Terzaghi's three-dimensional consolidation equation or simply Terzaghi-Reedulic equation. However, if fluid flow (and hence pore-pressure dissipation) is assumed to be one-dimensional, (1.14) further reduces to:
\[
c \frac{\partial^2 \sigma}{\partial z^2} = \frac{\partial \sigma}{\partial t}
\]

(1.15)

This is the Classical Terzaghi's one-dimensional consolidation equation.

Solution of (1.15) needs an initial condition concerning initial porepressure \( (\sigma_0) \) distribution for which an elastic stress distribution theory has
to be used. The simplest distribution is that which makes \( \sigma_0 = q \) (the applied load intensity) as in an oedometer test. The average degree of porepressure dissipation \( \bar{U}_d \) is obtained by averaging the point porepressure dissipations throughout the depth of the clay stratum:

\[
\bar{U}_d = 1 - \frac{\int_{0}^{z} \bar{U}_d \, dz}{\int_{0}^{z} \bar{U}_d \, dz}
\]

It is tacitly assumed that \( \bar{U}_w = \bar{U}_d \) and thus the consolidation settlement at any time is predicted making use of this identity in (1.4).

Terzaghi's equation (1.15) is in fact similar to the already familiar heat conduction equation (Carslaw and Jaeger, 1947). Because of this analogy, the host of solutions already existing in diffusion theory (Crank, 1956) came in handy to solve variety of consolidation problems (Ingersoll et al., 1969). Further, the recently developed analytical techniques and high speed computers have made it possible to solve the problems of one-dimensional consolidation to simulate a variety of complicated field conditions such as: time-dependent loading and varying permeability of soil (Schiffman, 1958), non-homogeneity
(Schiffman and Gibson, 1964), layered soil (Schiffman and Stein, 1970) and non-linear behaviour of soils (Gibson et al., 1967).

1.2.3 Comments on the Method:
To summarise, the conventional one-dimensional settlement analysis has the following drawbacks:

1) Lateral strains are neglected and this leads to the inference that there are no immediate settlements in saturated clays, which in fact is contrary to field evidence. However, this assumption may be reasonably valid for the following two field situations: (i) loaded area being very large compared to the thickness of clay stratum (e.g., blanket loading) (ii) fairly thin clay stratum being buried deep in the ground so that the surrounding layers constrain the clay layer laterally.

2) Final consolidation settlements are estimated from the result ($m_{v1}$) of one-dimensional laboratory consolidation test and a three-dimensional elastic stress distribution theory. The predicted final consolidation settlements ($w_{cw}$) are usually on the higher side of field observations.
For sake of logical consistency, Westergaard's stress distribution theory with its assumption of no lateral strain has to be used in estimating $w_{so}$ values.

3) Water flow is assumed to be one-dimensional and obviously this is an over simplification leading to largely underpredicted rates of settlement.

4) Rates of settlement are linked up with the average rates of porepressure dissipation through a simple diffusion equation under the assumption that total stresses in soil remain constant with time. In other words redistribution of stresses which generally takes place in a consolidating soil (as manifested by changing elastic parameters) is neglected. The process of porepressure dissipation is treated independent of stress distribution, whereas in fact these two phenomena are essentially inter-related in the process of consolidation.
1.3 PSEUDO THREE-DIMENSIONAL SETTLEMENT ANALYSES

1.3.1 Prediction of Magnitudes of Settlement:

Skempton and Bjerrum (1957) suggested a modification to one-dimensional approach to include the possibility of immediate settlement. They suggested that the immediate settlement be computed from elastic displacement theory which yields the following expression:

\[ w = \frac{qB_f I_f (1 - \nu^2)}{E} \]  \hspace{1cm} (1.17)

where \( \nu \) and \( E \) are the two elastic parameters and undrained (total stress) parameters are made use of for calculation of \( w_0 \), i.e., \( \nu = \nu_u \) which is generally taken as 0.5 for saturated clays under the assumption of constant volume. \( E_u \) value is determined, from undrained shear tests, as the slope of the stress-strain curve corresponding to the stress-level of interest (though usually \( E_u \) is measured at half the Peak stress).

The above formula is strictly valid for a semi-infinite medium. However modification of the formula to take care of inelastic behaviour of soil (D'Appolonia and Lambe, 1970; D'Appolonia et al. 1971) and finite
layer depth and embedment of footing (Janbu et al., 1966) have been proposed.

For estimating $w_{\infty}$ Skempton and Bjerrum use the expression:

$$w_{\infty} = \int_0^L m_v \gamma_o \, dz \quad (1.18)$$

This is strictly valid for a one-dimensional situation. However, a three-dimensional approach is used to calculate initial porepressure based on Skempton's porepressure coefficients $A$ and $B$ (Skempton, 1954).

i.e., $\sigma_0 = B \left[ \gamma_3 + A (\gamma_1 - \gamma_3) \right] \quad (1.19)$

where $\gamma_3$ and $\gamma_1$ are respectively the increase in total minor and major principal stresses. For saturated soils $B = 1$ and substitution of this in (1.19) and (1.19) into (1.18) lead to the final result:

$$(w_{\infty})_{SB} = \mu (w_{\infty})_{conventional \ 1-D} \quad (1.20)$$

where $(w_{\infty})_{SB} = w_{\infty}$ calculated from Skempton and Bjerrum's method.

$$(w_{\infty})_{conventional \ 1-D} = w_{\infty} \text{ calculated from conventional 1-D method (refer (1.7))}$$

$$\mu = A + \alpha (1-A) = \text{a correction factor}$$
and $\alpha$ is a factor which depends upon the shape of foundation and its size relative to the depth of clay layer.

1.3.2 Prediction of rates of settlement.

Rates of settlement in this analysis are predicted from the average rates of porepressure dissipation obtained from Rendulic's consolidation theory (Rendulic, 1936). Rendulic extended Terzaghi's theory to include the effect of three-dimensional flow of water. The governing equation is same as (1.14), which in fact is a three-dimensional diffusion (or dissipation) equation. Initial porepressure $\gamma_o$ needed in the solution of the governing equation is obtained from three-dimensional theories of elasticity as $\gamma_o = \frac{\gamma_{xx} + \gamma_{yy} + \gamma_{zz}}{3}$.

Solution of (1.14) yields $\bar{\gamma}$ which is equated to $\bar{U}_w$. Though Rendulic's extension was merely of mathematical interest, his theory formed the basis for solving the problem of sand drains (Barron, 1948). However, the real, physical extension of Terzaghi's one-dimensional consolidation problem to three-dimensions was achieved later by Biot (1941).

Rendulic's consolidation equation (which in fact is a Fourier Equation) had been solved for a variety of loading
problems with the assumption that the initial porepressure distribution satisfies the Laplace equation. Problems of a semi-infinite clay layer subjected to single concentrated load and loads distributed along a line were solved for the top pervious case by Korotkin (1951) (ref. Florin, 1961) and Zaretsky (1967).

Murayama and Akai (1954) solved the two-dimensional consolidation problems pertaining to semi-infinite and finite depth clay layers subjected to loads applied suddenly or gradually. Parabolic loading and the anisotropy with respect to permeability were also considered.

Verigin (1965) gave solutions for the semi-infinite clay layer with free drainag at the top when the layer is subjected to concentrated load, and loads uniformly distributed along the length of a straight line, a circle and the area of a circle. Varying loads and layers of finite thickness were also considered.

A series of solutions is presented by Davis and Poulos (1971, 1972) for the rate of settlement of circular and strip foundations on a soil layer of finite and infinite thickness. The effect of footing shape, soil anisotropy and boundary drainage are also considered. Good agreement between settlement rates observed from model studies and predicted rates is noted.
1.3.3 Comments on the method:

The analysis which makes use of a three-dimensional flow but one-dimensional strain is generally known as a pseudo three-dimensional approach. Summarising this approach, the following observations are made.

1) This method recognises the possibility of immediate settlement, which is evaluated from three-dimensional elastic theory.

2) Final consolidation settlements are evaluated from initial porepressures obtained from a three-dimensional approach. However, one-dimensionality of strain is still retained since the coefficient of volume compressibility ($m_v$) used is obtained from oedometer test.

3) Multi-dimensional flow used in the analysis predicts faster (and more realistic) rates of settlements than the conventional one-dimensional flow.

4) Rates of settlements are linked up with average rates of porepressure dissipation. However, the simple three-dimensional dissipation equation is valid only when the total stresses remain constant with time. This drawback was also seen in one-dimensional analysis.

5) Apparent inconsistency in the manner of estimating
immediate settlements from three-dimensional elastic theories and consolidation settlements from pseudo three-dimensional approaches still exists in this method. However, this method is a good compromise between the conventional one-dimensional and true three-dimensional analyses.

1.4 THREE-DIMENSIONAL SETTLEMENT ANALYSIS

1.4.1 Prediction of magnitudes of settlements:

Settlements can be estimated based on 3-D elastic theories. Such theories have been used successfully for stress distribution in soils (Turnbull et al., 1961).

Surface settlement is calculated from vertical displacement:

\[ w = \int_0^Z e_{zz} \, dz = \int_0^Z \frac{1}{E} \left[ \sigma'_{zz} - \gamma (\sigma'_{xx} + \sigma'_{yy}) \right] \, dz \tag{1.21} \]

For semi-infinite, homogeneous and isotropic medium, the integrations in (1.21) are performed in the limits of 0 to \( \infty \) and the result is simply written as:

\[ w = \frac{q B_f I_f (1 - \nu^2)}{E} \tag{1.17} \]

The above formula can be used to estimate either the immediate settlements or the final settlements provided suitable choice of the two elastic parameters are used. Undrained elastic parameters (\( E_u \) and \( \nu_u \)) are used to calculate immediate settlements; on the other hand effective parameters (\( E \) and \( \nu \)) have to be used for the purpose
of evaluating ultimate settlements. Determination of second set of elastic (effective) parameters is a difficult task. Davis and Poulos (1963) have suggested a method of triaxial testing to evaluate these parameters, which procedure is of course valid for axi-symmetric conditions.

Thus immediate settlement in this method is given by:

\[ w_0 = \frac{q B_f I_f (1-\nu^2)}{E_u} \]  

\( \nu_u \) is usually taken as 0.5. This method incidentally is identical to that used by Skempton and Bjerrum (1957).

The ultimate settlement is given by:

\[ (w_u)_{3-D} = \frac{q B_f I_f (1-\nu^2)}{E} \]  

Equations (1.22) and (1.23) suggest that of the two elastic parameters \( \nu \) has relatively less influence on settlement than \( E \), because of its narrow range of values (varying from 0.5 to 0.0). Since water cannot take any shear stress, the effective and total stress values of shear modulus are identical i.e.,

\[ G = E_u \quad \text{or} \quad \frac{E}{2(1+\nu)} = \frac{E_u}{2(1+\nu_u)} \]

using \( \nu_u = 0.5 \), one obtains:

\[ \frac{E_u}{E} = \frac{1.5}{(1+\nu)} \]
For a highly compressible material \((\nu = 0.0)\), the maximum value of this ratio is given as 1.5. However, in practice this ratio may be as high as 4 to 5 (Lambe and Whitman, 1969).

The relative importance of immediate settlement can be seen from the ratio \((w_0/w_\infty)\) which from (1.22), (1.23) and (1.24) is given by

\[
\frac{w_0}{w_\infty} = \frac{1}{2(1-\nu)} \tag{1.25}
\]

This relationship is plotted in Fig.1.2. For \(\nu = 0.0\), the immediate settlement is as high as 50 per cent of the ultimate settlement and \(w_0 = w_\infty\) for \(\nu = 0.5\). For more realistic values of \(\nu\), immediate settlement varies between these two limits but does not become zero as is thought of in one-dimensional approach.

1.4.2 Prediction of Rates of Settlements:

As for rates of settlements are concerned, one has to resort to Biot's theory (1941). This theory attempts to solve simultaneously the equations of equilibrium in terms of total stresses and also the equation of continuity. Thus the three displacements and porepressure are evaluated in one stroke without isolating them as is done in other theories. The strains so determined satisfy the compatibility conditions. The initial and ultimate settlements calculated from Biot's theory coincide with those obtained from classical theory of elasticity (Boussinesq, 1885). Thus this theory treats
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consolidation process as a coupled phenomenon of stress distribution and porepressure dissipation. Some special effects (in respect of porepressures) are brought out by this theory which are not manifested in pseudo three-dimensional theories. For a three-dimensional situation, the rate of settlement is not the same as the rate of porepressure dissipation. For any problem, settlement at a given time is computed knowing directly the degree of settlement obtained from Biot's theory.

1.4.3 Comments on the method:

Three-dimensional approach is the most appropriate for situations wherein the loaded area is much less when compared to the thickness of the clay stratum. Lateral strains do occur in such situations and they are manifested in the shape of immediate settlements. Unlike pseudo three-dimensional approach, in this method both the short term and long term settlements are evaluated on the basis of three-dimensional approach. Rates of settlements are evaluated independently and are not assumed to be equal to rates of porepressure dissipation. Thus the theory is self consistent and is based on more realistic assumptions of dimensionality of strain and flow.

1.4.4 Review of the work on Biot's Theory:

This study is based upon Biot's theory and hence a brief review of previous investigations based on Biot's theory is made in the following paragraphs:
General theory of three-dimensional consolidation was first formulated by Biot (1941) on the premise that soil behaves as an elastic porous medium and hence the theory is known as Poro-elastic theory. Apart from establishing the four governing equations in terms of three displacements and porepressure involving five constants, Biot gave a physical interpretation for these constants. Application of the theory to one-dimensional problem corresponding to the classical oedometer test was demonstrated. Simplified form of the theory for saturated soil was also presented. The operational calculus was shown to be a powerful method of solving consolidation problems.

Through a series of articles, Biot had subsequently demonstrated the application of his consolidation theory to a variety of foundation problems. The simplified theory valid for saturated soils was made use of (Biot, 1941 a) in calculating the consolidation settlements under the edge of an infinitely long strip of constant width subjected to suddenly applied sinusoidal loading, uniform load with discontinuity and rectangular load distribution. All these problems were solved with the assumption that the top (loaded) surface of clay was pervious admitting free drainage of water.

Biot and Clingan (1941) extended the problem of calculating consolidation settlements under an infinite strip of constant width loaded uniformly to the case where the strip rested on an impervious top surface. It was shown that settlement under the load was accompanied
by considerable heaving of the unloaded area on both sides of the load. Biot and Clingen (1943) calculated the settlements and bending moments induced in an elastic (foundation) slab resting on a consolidating soil under the action of line loads. Two cases were considered: first, when the slab was pervious to water, and second, when it was impermeable.

Through a series of articles, Mandel (1950, 1953, 1953 a, 1957 and 1961) brought out the mathematical analysis of consolidation of saturated soils. He gave solutions for the problems of concentrated load and normal loads distributed over a circular area both when the clay layer is either finite or infinite in thickness. He demonstrated that the excess porepressures in the initial stages increase with time beyond their original values before they steadily decrease (Mandel, 1950).

Biot (1955) extended his earlier theory of poroelasticity (1941) valid for isotropic material to the general case of anisotropy. The method of establishing the governing equations was also different and more direct as compared to the previous derivation. The particular cases of transverse isotropy and complete isotropy were also discussed.

Biot (1956) established general solutions of the equations for the isotropic case by means of functions satisfying the Laplace and the Heat-conduction equations. These functions were analogous to the Boussinesq-Papkovich
Biot (1956 a) brought out the complete analogy between thermo-elasticity and poro-elasticity thereby establishing a one-to-one correspondence between these two phenomena. This analogy is important for practical applications since the existing solutions of one system can be borrowed into the second system with a mere identification of suitable variables.

Biot (1956 b) further extended his theory (1941) to the case where the porous solid exhibits the most general properties of linear visco-elasticity and anisotropy. General solutions were developed for the isotropic case. As an example the problem of settlement of a loaded column was solved. The second-order effect of the change of permeability with deformation was also discussed.

Solutions were presented by Freudenthal and Spillers (1962) for an infinite visco-elastic consolidating layer subjected to either uniform surface stresses or uniform surface displacements. The problems of elastic and visco-elastic consolidating half-space subjected to concentrated load were also solved. The analogous coupled thermo-elastic problems and applications in the field of secondary consolidation were discussed.

Biot and Willis (1957) suggested few experimental procedures to determine the various elastic coefficients made use of in the theory of consolidation. However,
Davis and Poulos (1963) proposed a more practical method of determining $E$ and $V$ (the two fundamental elastic parameters used in consolidation) by conducting special triaxial tests.

Biot (1962) summarised his linear theory of porous media and extended it to finite deformations (1972) and also to non-linear bodies (1973). These extensions are possible because of the recent developments in irreversible thermodynamics and visco-elasticity.

Following Biot's general solutions (1956), quite a few consolidation problems were solved by different authors using either the stress or displacement function formulation. Two stress functions of Biot type (1956) were successfully employed by Pariia (1958) for axi-symmetric problems of isotropic semi-infinite soil medium and also for transversely anisotropic medium (1958 a).

Biot's stress functions were used by Sanyal (1972) for the analysis of two-dimensional deformation of semi-infinite soil medium.

De Jong (1957) suggested three stress functions for the unique solution of axi-symmetric consolidation problems. He solved the problems of circular load resting on semi-infinite body and that of rigid sphere embedded in soil.

The problem of consolidating spherical bodies was treated by Pariia (1958 b), Jana and Sanyal (1971), Cryer (1963), Gibson et al. (1963) and De Jong and Verruijt (1965).
Instead of the two displacement functions (one a scalar function and the other a vector function) suggested by Biot (1956), McNamee and Gibson (1960) proposed two scalar displacement functions which are particularly useful for semi-infinite body or infinite layer when stresses or displacements are prescribed on the surface. Problems of plane strain and axial symmetry are closely related when expressed in terms of these functions.

McNamee and Gibson (1960 a) had successfully demonstrated the use of their displacement functions with the help of repeated integral transformation technique for the solution of the semi-infinite body subjected to (i) a uniform pressure over an infinitely long strip and (ii) a uniform pressure over a circular area. Settlements and dilations were derived when the surface was either fully permeable or completely impermeable to the flow of pore-water.

Gibson and McNamee (1957 and 1963) successfully employed their two displacement functions for the solution of the problem of semi-infinite body subjected to a uniform normal pressure without any tangential (or shear) load over a rectangular area when the surface was fully pervious.

Schifman and Fungaroli (1965) used three displacement functions to solve the problem of a unit tangential load uniformly distributed over a circular area. The excess pore-pressures at a point were analysed both for an impermeable and pervious top surface.

Verruijt (1971) showed that McNamee and Gibson’s
displacement functions (1960) can be derived as a special case of Biot's functions (1956). He generalised the McNamee and Gibson's functions and also the Schiffman and Fungaroil's functions to include the effect of compressibility of pore-fluid.

The general solutions of three-dimensional consolidation equations were directly obtained by De Leeuw (1966) using Laplace transforms for problems of cylindrical bodies satisfying the conditions of plane strain and axial symmetry. As an example, the solution was obtained for a vertical sand drain. It is interesting to note that for such a simplified problem, Biot's theoretical predictions reduce to those of Terzaghi-Rendulic.

Most of the solutions discussed above relate to cases where the thickness of the consolidating layer is very large compared to the dimensions of the loaded area. However, Gibson et al. (1970) solved for the first time the problem of the clay layer of finite thickness (though of infinite lateral extent) to the surface of which a uniform pressure was applied over an infinite strip or over a circular area. One of their important conclusions, which has some relevance to the present study, is that if the clay layer thickness is greater than five times the width of the footing the clay layer may be treated as a semi-infinite medium. In their paper, the authors considered that the clay layer rested on a smooth rigid base.

Booker (1973) extended this work to the case of clay layer resting on a rough rigid base. He has presented
direct and elegant solutions not only for strip and circular footings but also for square footings.

The problem of plane strain consolidation was also solved by finite element method (Christian and Boshun, 1970). Schiffman and Fungaroli (1973) presented solutions to the problem of consolidation under slowly moving loads.

Ever since the publication of Biot's theory (1941), many investigators tried to critically examine its generality over the earlier theories due to Terzaghi (1923) and Rendulic (1936). Gibson and Lumb (1953) established clearly the link between Biot's and Terzaghi-Rendulic theories. The continuity equation of Biot (one of the four governing equations) reduces to Rendulic equation as a special case when it is assumed that the internal total (volumetric) stresses do not change with time. Since Terzaghi's theory is a one-dimensional version of Rendulic theory, one can say that these two theories (or Terzaghi-Rendulic theory as is known sometimes) isolate the two otherwise interconnected phenomena of consolidation and stress distribution.

A critical examination of the above theories of primary consolidation was attempted by Cryer (1963), Schiffman (1967), Schiffman et al. (1969).

Cryer (1963) had shown that for the case of a sphere of soil which is hydrostatically loaded, the predictions for the excess porepressures developed at the centre of the sphere from the Biot (True) theory and the Terzaghi-Rendulic (Pseudo) theory differ very much. While the
pseudo theories predict a steadily decreasing porepressure, the Biot theory predicts that the porepressure will increase initially before decreasing steadily. These peculiarly seeming theoretical predictions were however subsequently verified in laboratory by Gibson et al. (1963). Such predictions were also made earlier by Mandel (1950) and hence this peculiar consolidation effect is now known as "Mandel-Cryer effect".

Two possible reasons for these peculiar local porepressure effects (as given by Christian and Boëthius, 1970) are (i) the Mandel-Cryer effect which occurs when the total stresses increase essentially at constant effective stresses and (ii) porepressure redistribution takes place wherein total stresses remain constant but effective stresses change.

Similar effects were in fact observed during many field pumping tests carried out at the village of "Noordbergum" in the Netherlands. While pumping out from wells located in the top layer of a two-storied leaky aquifer (artesian), it was observed that the porepressures in a certain region of the lower aquifer initially increase before steadily decrease. Verruijt (1969) successfully explained this "Noordbergum Effect" with the help of Biot's theory.
1.5 COMPARISON OF DIFFERENT SETTLEMENT ANALYSES

1.5.1 Analytical Comparison:

The various approaches discussed so far for settlement analyses can be compared on the premise that soil behaves as a homogeneous, isotropic, linear elastic and semi-infinite mass. The validity of the assumption of elasticity for a soil is questionable. Nevertheless, in the absence of a better theory, elastic theory does give a fair estimate of settlements provided the soil parameters obtained under simulated field stress levels are used in theoretical predictions.

The different settlement analyses are compared from a study of settlement predictions for the centre of a flexible circular footing of radius \( R \) subjected to a surface (normal) load intensity of \( q \). The vertical displacement obtained from three-dimensional analysis using (1.17) or (1.21) is given by:

\[
(w_{oc})_{3-D} = \frac{2qR(1-\nu^2)}{E}
\]  

(1.26)

If Westergaard's stress distribution theory (1938) is used instead of Boussinesq's in (1.17), one obtains:

\[
\frac{(w_{oc})_{true} \text{ 1-D}}{(w_{oc})_{3-D}} = \sqrt{\frac{1 - 2\nu}{2(1 - \nu)^3}}
\]

\[\frac{1 - 2\nu}{2(1 - \nu)^3}\]

This ratio is plotted as a function of \( \nu \) in...
which shows the error in true one-dimensional approach. It is seen that for \( \nu = 0.5 \) the settlement (immediate) from one-dimensional analysis is zero as is expected. Fig. 1.3 shows that one-dimensional analysis always underpredicts the ultimate settlements, the underprediction is 30 per cent for \( \nu = 0.0 \).

However, in conventional one-dimensional analysis (1.7) is used with Boussinesq's theory for \( \sigma^{zz} \) distribution. It is to be noted that final effective stresses will be same as the final total stresses since (finally) porepressures reduce to zero on complete consolidation. For the problem under study, one obtains:

\[
(w_{\infty})_{\text{conventional 1-D}} = m v \int_0^\infty (\sigma_{zz})_B \, dz
\]

\[
= m v \int_0^\infty q \left[ 1 - \frac{1}{1+(R/z)^{2/3}} \right] \, dz
\]

\[
= 2 \, m v \, q \, R
\]

\[
\frac{(w_{\infty})_{\text{conventional 1-D}}}{(w_{\infty})_{3-D}} = \frac{m v \frac{E}{1-\nu^2}}{1 - \nu^2} = \frac{1 - 2\nu}{(1-\nu)^2}
\]

using the result in (1.9).

This ratio is also shown in Fig. 1.3. For \( \nu = 0.5 \), the settlement (immediate) predicted by (1.29) is zero. Thus one-dimensional settlement analysis (whether conventional or not) disregards immediate settlement.

Fig. 1.3 shows that for \( \nu = 0.0 \) the conventional settlement analysis predicts results identical to those
obtained from three-dimensional analysis. However, for
more realistic range of $0 < \nu < 0.5$, one-dimensional
approach underpredicts the ultimate settlements. Similar
comparisons were made by Davis and Poulos (1968) for finite
layers of soil.

Fig.1.3 also indicates that for $\nu > 0.33$,
Westergaard's true one-dimensional approach gives more
realistic values of settlements than the conventional one-
dimensional approach. And for $\nu < 0.33$, the latter
approach (though logically inconsistent) gives better
results. However, when the elastic parameters for the
soil can be determined reliably, a three-dimensional
approach is the best.

Thus it is established theoretically that one-dimen-
sional approach in general results in underprediction of
final settlements under normal loads particularly for soils
having higher $\nu$ values. If a fair estimate of $\nu$ can be
made, then Fig.1.3 can be used to predict more realistic
settlements based on three-dimensional approach merely from
the simple oedometer tests. This may be a simpler approach,
since of the two parameters $E$ and $m_\nu$, probably the latter
could easily be determined in the laboratory than the former.

1.5.2 Comparison Based on Field and Laboratory Data:

Terzaghi's one-dimensional theory has been used for
many practical situations in predicting magnitude and rate
of settlements. However, the predictions will be no more
accurate than the amount of agreement between the assumptions made in the theory and actual conditions obtained in the field. Though one-dimensional predictions may be nearer the field values when one-dimensional strain and flow situations are obtained in the field, they will not agree with the observed field data for a three-dimensional situation. It is worthwhile to know the percentage error committed by one-dimensional theory while predicting for three-dimensional situations. Such studies may be possible from either the actual field observations or the laboratory data.

Some of the field and laboratory data available for a comparative study is reviewed in the following paragraphs. Skempton and Bjerrum (1957) presented an authentic field data regarding observed settlements in 8 cases, 4 in normally consolidated clays and 4 in over consolidated clays. On an average, the conventional one-dimensional analysis under-predicted settlements by 19 per cent for structures on N.C. clays and overpredicted settlements by 25 per cent in O.C. clays. This in fact led them to modify the one-dimensional analysis to include immediate settlements. This modification had unexpectedly improved the accuracy of theoretical predictions to near 99 per cent of the observed values for both N.C. and O.C. clays. Ever since the publication of this article, the method has come to be treated as one of the best practices of predicting settlements in clays.

Davis and Pulos (1968) have shown theoretically that
(for normal load problems) one-dimensional theory under-predicts settlement as compared to three-dimensional analysis. They claim that "the authors have considered all the available reliable published comparisons between observed and predicted foundation behaviour. There are many reasons why, in individual cases, the predicted settlement differs appreciably from the observed settlement but, from a total of 16 cases which allowed a comparison between the conventional one-dimensional prediction and the observed total final settlement, there were nine cases in which the observed was 7-27 per cent greater than the predicted. It is significant that this is the direction of the error that would be expected from three-dimensional theory".

Mooro and Spencer (1969) compared observed settlement of a building on deep compressible clay with those predicted by different theories. The total settlement predicted by conventional one-dimensional theory was 45 cm as against the observed value of 79.5 cm (which thus represents an underprediction of 42 per cent), however, the three-dimensional analysis improved the prediction to a value of 75 cm (an underprediction of 10 per cent).

Paulos (1972) compared the predicted and measured settlement, rate of settlement for two embankments constructed on deep deposits of clay. It is found that despite the apparent one-dimensional geometry of the embankments, a one-dimensional analysis underestimates
the settlement whereas three-dimensional analyses give reasonable agreement between measured and predicted settlements and rates of settlement.

Davis and Pulos (1968) conducted model studies on rigid footings resting on two different clays to compare the observed settlements and their rates with those predicted by different methods. It is found that one-dimensional method invariably underpredicts (sometimes the per centage of underprediction being 50 per cent) settlements whereas Skompton and Bjerrum's modified method and true three-dimensional method overpredict. However, the per centage of overprediction being a maximum of 15 per cent in three-dimensional analysis and much higher in Skompton and Bjerrum's method. The agreement between experimental and theoretical (Pseudo three-dimensional theory) rates of settlement were found to be quite good.

Burland (1970) describes the laboratory experiments on the consolidation settlements of model footings on deep beds of homogeneous, normally consolidated clays. The observed vertical displacements at various locations are shown to be in good agreement with conventional one-dimensional predictions. However, lateral displacements which were found (maximum values being less than 10 per cent of vertical displacements) in model studies could not be predicted from one-dimensional theory but were in close agreement with those predicted from Burland's theory.
FIG: 1.1. GENERAL TIME-SETTLEMENT RELATION FOR CLAYS.
Fig. 12. Relative Importance of Immediate Settlement in Three-Dimensional Analyses.
FIG. 13. ERRORS IN ONE-DIMENSIONAL SETTLEMENT ANALYSES.

\[
\frac{\omega_{1D}}{\omega_{3D}} = \frac{1 - 2\nu}{(1 - \nu)^2}
\]

\[
\frac{\omega_{1D}}{\omega_{3D}} = \sqrt{\frac{1 - 2\nu}{2(1 - \nu)^3}}
\]

NORMAL LOAD OVER CIRCULAR AREA ON SEMI-INFINITE LAYER

CONVENTIONAL 1-D THEORY

WESTERGAARD'S THEORY

RATIO \( \frac{\omega_{1D}}{\omega_{3D}} \)

EFFECTIVE POISSON'S RATIO \( (\nu) \)