APPENDIX - I

MODEL STUDIES
The soil used in the model studies is a locally available Black Cotton clay. This soil is chosen so that the theoretical results presented in this study will be of immediate use in the design of foundations and performance studies planned for future in such soils which are abundantly available in this area.

A 1.2 EXPERIMENTAL STUDY

A 1.2.1 Soil type:

The soil used in this study is Black Cotton Clay. The soil is essentially an inorganic clay of high compressibility and has predominant swelling and shrinkage characteristics. The soil has the following index properties:

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific Gravity</td>
<td>2.70</td>
</tr>
<tr>
<td>Liquid Limit</td>
<td>80.0 per cent</td>
</tr>
<tr>
<td>Plastic Limit</td>
<td>26.5 per cent</td>
</tr>
<tr>
<td>Plasticity Index</td>
<td>53.5 per cent</td>
</tr>
<tr>
<td>Shrinkage Limit</td>
<td>13.5 per cent</td>
</tr>
<tr>
<td>Clay Content (fraction finer than 0.002 mm size)</td>
<td>33.0 per cent</td>
</tr>
<tr>
<td>Activity Ratio</td>
<td>1.62</td>
</tr>
</tbody>
</table>

A 1.2.2 Model Studies:

Clay beds are formed in two brick masonry tanks cubical in shape and with inner dimensions of 76 cm. and
90 cm respectively. Three-way drainage is provided for the clay as shown in Fig. 4.7.1. The moulding water content is 45 per cent. Wet soil is taken in small quantities and moulded into the shape of balls which are then evenly dashed into the tanks. Subsequently the clay dump is evenly compacted to a saturated density of 1.73 gm/cc. The entire clay bed is formed by compaction in layers of 10 cm thick.

The model footings are made of mild steel and timber and hence are relatively rigid. Details of footings used are given in Table A 1.1. The settlements are measured by means of dial gauges reading up to 0.002 mm. Some of these gauges are placed directly on the footings and some outside the footings. Settlement readings are taken at closer intervals on the first day of loading and thereafter once in a day up to few weeks.

**TABLE A 1.1**

**DETAILS OF MODEL FOOTINGS**

<table>
<thead>
<tr>
<th>Model No.</th>
<th>Shape of model</th>
<th>Size of the model</th>
<th>Material</th>
<th>Normal Stress</th>
<th>Shear Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Square</td>
<td>100 x 100 mm</td>
<td>Timber</td>
<td>0.50</td>
<td>-</td>
</tr>
<tr>
<td>2.</td>
<td>Square</td>
<td>100 x 100 mm</td>
<td>Timber</td>
<td>0.50</td>
<td>0.17</td>
</tr>
<tr>
<td>3.</td>
<td>Square</td>
<td>100 x 100 mm</td>
<td>Timber</td>
<td>0.50</td>
<td>0.17</td>
</tr>
<tr>
<td>4.</td>
<td>Rectangular</td>
<td>100 x 150 mm</td>
<td>Timber</td>
<td>0.50</td>
<td>-</td>
</tr>
<tr>
<td>5.</td>
<td>Rectangular</td>
<td>100 x 150 mm</td>
<td>Timber</td>
<td>0.50</td>
<td>0.17</td>
</tr>
<tr>
<td>6.</td>
<td>Strip</td>
<td>Width=97 mm or (97x50) mm</td>
<td>Timber</td>
<td>0.26</td>
<td>-</td>
</tr>
<tr>
<td>7.</td>
<td>Strip</td>
<td>Width=97 mm or (97x750) mm</td>
<td>Timber</td>
<td>0.26</td>
<td>-</td>
</tr>
<tr>
<td>8.</td>
<td>Circular</td>
<td>dia.=100 mm</td>
<td>Mild Steel</td>
<td>0.50</td>
<td>-</td>
</tr>
<tr>
<td>9.</td>
<td>Circular</td>
<td>dia.=100 mm</td>
<td>Mild Steel</td>
<td>0.50</td>
<td>-</td>
</tr>
<tr>
<td>10.</td>
<td>Circular</td>
<td>dia.=100 mm</td>
<td>Mild Steel</td>
<td>0.50</td>
<td>0.17</td>
</tr>
</tbody>
</table>
A 1.2.3 SPECIAL TRIAXIAL TESTS

The triaxial test procedure for determining the undrained (total) and drained (effective) elastic parameters needed for the application of poro-elastic theory as suggested by Davis and Poulos (1963) is adopted in this study.

A special triaxial cell with a loading ram of 38 mm $\phi$ is used to apply a constant axial stress through a load hanger and facilitate consolidation of triaxial sample under no lateral strain condition. A back pressure of 2.1 kg/cm$^2$ is maintained in the porepressure lines at all stages of testing to ensure saturation. The complete line of triaxial test assembly is shown in Fig. A 1.2.

The testing procedure involves three distinct stages as indicated in Fig. A 1.3 which closely simulates the field conditions. In the first stage, the sample is consolidated under "no lateral strain (or $k_o$)-conditions" with an axial stress equal to the effective overburden pressure at the sampling depth. Bishop's method (1958) of $k_o$-consolidation is used in which the lateral (cell) pressure is constantly adjusted to yield a no-lateral strain condition. The values of $\mu_1$, $c_1$ (corresponding to one-dimensional consolidation) and $k_o$ can be found from these results.

In the second stage, the sample is subjected to additional loading (with vertical and horizontal load
making use of the soil parameters obtained from laboratory tests.

3.13.1 Soil Parameters:

Stress levels largely affect the various soil parameters. A preliminary study (Ali, 1974) on the effect of stress level on the elastic parameters of soil as determined from special triaxial tests, indicated the following trends:

1) For a given confining pressure, the undrained and drained (or effective) elastic moduli ($E_u$ and $E$) decrease with increasing deviator stress. Similarly, for a given deviator stress, the two elastic moduli decrease with increasing confining pressure.

2) For a given confining pressure, the effective (or drained) Poisson's ratio ($\nu$) increases with increasing deviator stress. However, there is no definite trend as to the nature of dependence of $\nu$ on the lateral stress.

Similar results presented by Davis and Poulos (1963) in respect of kaolin confirm the first conclusion arrived at above.

Keeping in view the stress levels used in the majority of the model footings, the following soil parameters have been chosen, from among the various test results obtained.
(i) Parameters for use in classical one-dimensional analysis:

Coefficient of volume compressibility \( m_v \) = 0.060 \( \text{cm}^2/\text{kg} \)
Coefficient of consolidation \( c_1 \) = 0.035 \( \text{cm}^2/\text{mt} \)

(ii) Parameters for use in Skempton and Bjerrum's method:

Skempton's Porepressure Coefficients \( B = 1.00 \)
\( A = 0.32 \)

Undrained elastic parameters \( E_u = 32.75 \text{ kg/cm}^2 \)
Undrained Poisson's ratio \( \nu_u = 0.50 \) (assumed)

(iii) Additional parameters for use in Biot's theory:

Drained elastic parameter \( E \) = 15.63 \( \text{kg/cm}^2 \)
Drained (or effective) Poisson's ratio \( \nu = 0.45 \)
Coefficient of consolidation \( c_3 \) = 0.018 \( \text{cm}^2/\text{mt} \)

A 1.3.2 Theoretical Predictions:

Most theoretical predictions are valid for flexible footings. However, the model footings used in this study are rigid and not flexible and hence the necessary correction for rigidity is applied to the theoretical values, based on the relative values of influence factors \( I_f \) in (1,17)) for rigid and flexible footings(Bowles, 1968).

A 1.3.2 (a) One-dimensional Theory:

Immediate settlements are assumed to be zero in this method. Ultimate (consolidation) settlements are evaluated using (1,6) with vertical stress distribution obtained from Boussinesq's theory (1885). Alternatively, Westergaard's
<table>
<thead>
<tr>
<th>Method of Prediction</th>
<th>Skempton and Bjerrum's method</th>
<th>3-D method</th>
<th>Average observed Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shape of footing</td>
<td>1-D method</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SQUARE FOOTINGS</td>
<td>$w_0$ 0.000 (-100.0)</td>
<td>0.940 (+1.1)</td>
<td>0.940 (+1.1)</td>
</tr>
<tr>
<td></td>
<td>$w_c$ 1.710 (+42.5)</td>
<td>1.175 (-2.1)</td>
<td>1.150 (-4.2)</td>
</tr>
<tr>
<td></td>
<td>$w_\infty$ 1.710 (-19.7)</td>
<td>4.115 (-0.8)</td>
<td>2.080 (-1.9)</td>
</tr>
<tr>
<td>RECTANGULAR FOOTINGS</td>
<td>$w_0$ 0.000 (-100.0)</td>
<td>1.215 (-17.9)</td>
<td>1.215 (-17.9)</td>
</tr>
<tr>
<td></td>
<td>$w_c$ 2.300 (+74.2)</td>
<td>1.480 (12.2)</td>
<td>1.485 (+12.5)</td>
</tr>
<tr>
<td></td>
<td>$w_\infty$ 2.300 (-17.9)</td>
<td>2.685 (-3.8)</td>
<td>2.700 (-3.8)</td>
</tr>
<tr>
<td>STRIP FOOTINGS</td>
<td>$w_0$ 0.000 (-100.0)</td>
<td>1.115 (+9.9)</td>
<td>1.115 (+9.9)</td>
</tr>
<tr>
<td></td>
<td>$w_c$ 2.180 (+114.8)</td>
<td>1.090 (+23.8)</td>
<td>1.375 (56.2)</td>
</tr>
<tr>
<td></td>
<td>$w_\infty$ 2.180 (+15.0)</td>
<td>2.205 (+16.4)</td>
<td>2.490 (31.4)</td>
</tr>
<tr>
<td>CIRCULAR FOOTINGS</td>
<td>$w_0$ 0.000 (-100.0)</td>
<td>1.010 (-45.5)</td>
<td>1.010 (-45.5)</td>
</tr>
<tr>
<td></td>
<td>$w_c$ 2.290 (+10.1)</td>
<td>1.290 (-38.0)</td>
<td>1.270 (-39.0)</td>
</tr>
<tr>
<td></td>
<td>$w_\infty$ 2.290 (-41.8)</td>
<td>2.300 (-41.6)</td>
<td>2.280 (-42.1)</td>
</tr>
</tbody>
</table>

* The numerical values in parantheses indicate percentages of overpredictions (+ve) or underpredictions (-ve) with respect to observed data from model studies.
curve indicates primary consolidation and the later portion the secondary consolidation. Normally the secondary consolidation portions of the curve are flatter than the primary consolidation portions. However, in the model studies it is generally observed that secondary portion of settlement vs (semi-log) time curve is steeper than the primary portion. For an inorganic clay of the type used in the study, secondary consolidation settlement is normally low and as such the large secondary settlements may only be attributed to three-dimensional strains or breaking up of structural bonds. Large secondary settlements were also observed in model studies conducted by Davis and Poulos (1968) to simulate three-dimensional strain situations. Steep secondary consolidation curves are also noted in one-dimensional consolidation by Lo (1961). In view of this, the ultimate settlement $w_\infty$ was estimated from the point of intersection of the two tangents drawn to the two portions of the observed settlement curves. In some cases, the point of deviation of the initial tangent from the initial portion of the curve is taken as the end of primary consolidation. Immediate settlement ($w_0$) is taken as the settlement corresponding to an elapsed time of 1 minute. Final consolidation settlement ($w_{0\infty}$) is taken as the difference of $w_0$ and $w_\infty$.

The average observed values of $w_0$, $w_{0\infty}$ and $w_\infty$ are shown in Table A 1.2 along with settlements predicted by different theories. The observed time vs consolidation
The larger values of $w_o$ observed in certain model studies may be attributed to lack of proper contact between the model footing and the soil surface and also due to air bubbles present in the compacted soils.

A 1.3.4 (b) Final Consolidation Settlements:

Final consolidation settlements ($w_{o\infty}$) predicted by one-dimensional method are always overestimated. Skempton and Bjerrum's corrected values agree more closely with those predicted from three-dimensional method. It is observed from Table A 1.2 that settlements are overpredicted for strip footings and underpredicted for circular footings. The intensity of normal load used in strip footings is nearly half of that used for other footings. From the effect of stress levels on $E$ and $\lambda$, it is seen that $E$ and $\lambda$ values in strip footings should have been more, which if used, in theoretical predictions would have given less values of settlements. However, the settlements in circular footings are largely underpredicted. Partly this discrepancy may be attributed to the manner in which $w_{o\infty}$ have been determined from the results of model studies and partly to the differences in moulding water contents and other environmental conditions.

A 1.3.4 (c) Ultimate Settlements:

One-dimensional method largely underpredicts the $w_{o\infty}$ values. Skempton and Bjerrum's method yields values which
are more close to those predicted by three-dimensional method. Good agreement between theoretical predictions and observed values are seen for square and rectangular footings and not for strip and circular footings.

A 1.3.4 (d) Rates of Settlements:

The predicted and observed time-settlement (primary consolidation) relations are shown in Figs. A 1.7 to A 1.10. It is seen that observed rates of settlement are faster than those predicted. This difference may be attributed to the accuracy with which coefficient of consolidation is determined. Rates of settlements from Biot's theoretical predictions are faster than those predicted by one-dimensional theory; this difference is essentially attributed to multi-dimensionality of flow assumed in Biot's theory.

A 1.4 CONCLUSIONS

The following conclusions may be drawn from the results presented here.

1) Special triaxial test procedure suggested by Davis and Poulos (1963) for the determination of elastic (consolidation) parameters, though involved, is quite practicable.

2) Stress levels used in the triaxial test affect the soil parameters. Elastic moduli tend to decrease with increasing stresses.
FIG: A1.1 - EXPERIMENTAL SET UP FOR MODEL STUDIES
(a) Stress Distributions

STAGE I
\[ \sigma_1 = P_r \]
\[ \sigma_3 = K_0 P_r \]

STAGE II
\[ \sigma_1 = P_r + \sigma_{2r} \]
\[ \sigma_3 = K_0 P_r + \sigma_{hr} \]

STAGE III
\[ \sigma_1 = P_r + \sigma_{2r} \]
\[ \sigma_3 = K_0 P_r + \sigma_{hr} \]

(b) Triaxial Test Sequence

Fig A.1.3 - Triaxial Test Procedure for Determination of Soil Parameters (Davis and Poulos, 1963)
RESULTS OF SPECIAL TRIAXIAL TESTS ON BLACK COTTON CLAY (STAGE-1)
FIG. A1.5. RESULTS OF SPECIAL TRIAXIAL TESTS ON BLACK COTTON CLAY (STAGE 2)

\[ \Delta \sigma_1 = 0.15 \text{ kPa/cm}^2 \]

\[ \Delta \sigma_3 = 0.75 \text{ kPa/cm}^2 \]

**Pure Pressure Coefficient**

\[ B = \frac{\Delta \sigma_1}{\Delta \sigma_3} \approx 0.1 \]

\[ A = \frac{\Delta \sigma_2}{\Delta \sigma_3} \approx 0.05 \]

**Elongation**

\[ \epsilon_2 = \frac{\Delta L}{L_{\text{mean}}} \approx 0.01 \]

\[ E_L = \frac{\Delta \sigma_1 - \Delta \sigma_3}{\epsilon_2} = 0.2 \text{ kPa/cm}^2 \]
Measured change in length

Measured volume change

Final effective stress conditions

\[ \sigma_1' = 2.325 \text{ kN/m}^2 \]

\[ \sigma_3' = 1.163 \text{ kN/m}^2 \]

Total strain: \( \epsilon = \text{strain during undrained test} + \text{strain during drained test} \)

\[ \epsilon_\text{V} = 0.0152 \]

\[ \epsilon_3 = \frac{1}{2} (\epsilon_\text{V} - \epsilon_1) = -0.00085 \]

\[ \nu = \epsilon_1 \Delta \sigma_3' - \epsilon_3 \Delta \sigma_1' \]

\[ \epsilon_1 \left( \Delta \sigma_1' + \Delta \sigma_3' \right) - 2 \epsilon_3 \Delta \sigma_3' = 0.45 \]

\[ E = \frac{\Delta \sigma_3'}{\epsilon_1} = 15.63 \text{ kN/m}^2 \]

\[ C_3 = \frac{T_{50} R^2}{\epsilon_{50}} = 0.0015 \times (\epsilon_\text{V})^2 \]

\[ = 0.012 \text{ cm}^2/min \]

FIG.A.1.6. RESULTS OF SPECIAL TRIAXIAL TESTS ON BLACK COTTON CLAY (STAGE 3).
FIG A.1.7. TIME-SETTLEMENT RELATIONS FOR SQUARE FOOTINGS.
FIG. A.1.7. TIME-SETTLEMENT RELATIONS FOR SQUARE FOOTINGS.
FIG: A 18. TIME-SETTLEMENT RELATIONS FOR RECTANGULAR FOOTINGS.
FIG. A19: TIME-SETTLEMENT RELATIONS FOR STRIP FOOTINGS
FIG A1.10. TIME-SETTLEMENT FOR CIRCULAR FOOTINGS.
APPENDIX - II

STANDARD INFINITE INTEGRALS
\[ I_3 = \frac{2}{\pi n^2} \int_0^\infty e^{-2tv^2} \frac{v^2}{(v^2+1)(v^2+\delta)} \frac{1}{v^2} \, dv \]
\[ = \frac{1}{(2n-1)} \operatorname{erfc} (\sqrt{t}) \alpha^2 t - \frac{1}{n} \frac{\delta}{(1+n\delta)(1-\delta)} \operatorname{erfc} (\sqrt{t} \delta) e^{2t\delta} \]
\[ - \frac{\delta}{(1+n\delta)(1-n^2\delta)} \operatorname{erfc} \left( \frac{\alpha t}{n\delta} \right) e^{(\alpha^2 t/n^2\delta)} \]

\[ I_4 = \frac{2}{\pi n^2} \int_0^\infty e^{-2tv^2} \frac{v^2 \cos(v\alpha z) + [n(v^2+1)-1]v^2 \sin(v\alpha z)}{(v^2+1)^2} \frac{1}{v^2 + \left( \frac{n-1}{n} \right)^2} \, dv \]
\[ = - \frac{n(n-1)}{(2n-1)^2} \operatorname{erfc} \left( \frac{n-1}{n} \alpha t + \frac{n-1}{2n-1} \right) \exp \left( \frac{(n-1)^2}{(2n-1)} \alpha^2 t + (\frac{n-1}{n})^2 \alpha z^2 \right) \]
\[ + \frac{1}{(2n-1)} \left[ \frac{n}{2} - \alpha^2 t + \frac{1}{4(2n-1)} \right] \operatorname{erfc} (\alpha t - \frac{z^2}{2n-1}). \]
\[ \cdot \exp (\alpha^2 t - \alpha z) + \frac{1}{4} \operatorname{erfc} (\alpha \sqrt{t} + \frac{z}{2n-1}) \exp (\alpha^2 t + \alpha z) \]
\[ + \frac{1}{(2n-1)} \frac{\sqrt{\pi}}{\sqrt{t}} \exp \left( - \frac{z^2}{4t} \right) \]

\[ I_5 = \frac{2}{\pi n^2} \int_0^\infty e^{-2tv^2} \frac{[n(v^2+1)-1]v^2}{(v^2+1)^2} \frac{1}{v^2 + (\frac{n-1}{n})^2} \, dv \]
\[ = - \frac{(n-1)^2}{(2n-1)^2} \operatorname{erfc} \left( \frac{n-1}{n} \alpha t \right) \exp \left[ \frac{(n-1)^2}{n^2} \alpha^2 t \right] \]
\[ + \frac{1}{(2n-1)} \left[ \alpha^2 t + \frac{(n-1)^2}{2(2n-1)} \right] \operatorname{erfc} (\alpha \sqrt{t}) \exp (\alpha^2 t) \]
\[ - \frac{\alpha \sqrt{t}}{(2n-1) \sqrt{\pi}} \]
\[ I_6 = \frac{2}{\pi n^2} \int_0^\infty e^{-\alpha^2 t} v^2 \frac{v^2}{(\sqrt{2+1}) \xi \sqrt{v^2 + (\xi-1)^2}} dv \]

\[ = \frac{1}{(2n-1)} \operatorname{erfc} \left( \alpha \sqrt{t} \right) \exp \left( \alpha^2 t \right) \]

\[ - \frac{(n-1)}{n(2n-1)} \operatorname{erfc} \left( \frac{n-1}{n} \alpha \sqrt{t} \right) \exp \left[ \left( \frac{n-1}{n} \right)^2 \alpha^2 t \right] \]

Certain functions of the above listed integrals are often encountered in the evaluation of settlements and pore pressures and as such are given below for ready reference:

\[ \left[ \frac{1}{(2n-1)} - n e^{-\alpha^2 t} I_6 \right] = \frac{1}{(2n-1)} - \frac{n}{(2n-1)} \operatorname{erfc} \left( \alpha \sqrt{t} \right) \]

\[ + \frac{(n-1)}{(2n-1)} \operatorname{erfc} \left( \frac{n-1}{n} \alpha \sqrt{t} \right) \exp \left[ - \left( \frac{2n-1}{n^2} \right) \alpha^2 t \right] \]

\[ = \operatorname{erf} \left( \alpha \sqrt{t} \right) \quad \text{for } n = 1 \]

\[ = 0 \quad \text{for } n = \infty \]

\[ n e^{-\alpha^2 t} \left[ I_4 - \frac{i}{\alpha^2} \frac{d^2}{dz^2} I_4 \right] = \frac{n}{(2n-1)} \left[ 1 + \operatorname{erf}(\alpha \sqrt{t}) \right] \]

\[ + \frac{(n-1)}{n} \operatorname{erfc} \left( \frac{n-1}{n} \alpha \sqrt{t} \right) \exp \left[ - \left( \frac{2n-1}{n^2} \right) \alpha^2 t \right] \]

\[ - \frac{n}{(2n-1)} \left[ e^{-\alpha^2 t} \operatorname{erfc} \left( \frac{2}{\sqrt{2}} \alpha \sqrt{t} \right) + \frac{(n-1)}{n} \operatorname{erfc} \left( \frac{n-1}{n} \alpha \sqrt{t} \right) \right] \]

\[ \cdot \exp \left[ - \left( \frac{2n-1}{n^2} \right) \alpha^2 t + \left( \frac{n-1}{n} \right)^2 \alpha^2 t \right] \]

\[ = e^{-\alpha^2 t} \left[ \operatorname{erf}(\alpha \sqrt{t} - \frac{2}{\sqrt{2}}) - \operatorname{erfc}(\alpha \sqrt{t}) \right] \quad \text{for } n = 1 \]

\[ = \frac{1}{2} \left[ e^{-\alpha^2 t} \operatorname{erfc} \left( \alpha \sqrt{t} - \frac{2}{\sqrt{2}} \right) - e^{\alpha^2 t} \operatorname{erfc} \left( \alpha \sqrt{t} + \frac{2}{\sqrt{2}} \right) \right] \quad \text{for } n = \infty. \]
\[
\frac{1}{2n-1} - n e^{-\alpha^2 t} \left( I_3 - \frac{2n \sqrt{\delta} e^{-(1-\delta)\alpha^2 t}}{(n+1)^2 - (2-n\delta)^2} \right) = \frac{1}{2n-1} - \frac{n}{2n-1} \text{erfc}(\frac{\alpha \sqrt{t}}{n \delta}) \\
+ \frac{\delta}{(1+n\delta)(1-\delta)} \left( \text{erfc}(\alpha \sqrt{t}) - 2 \right) \exp(-\delta \alpha^2 t - \alpha^2 t) \\
+ \frac{n}{(1+n\delta)(1-\delta)} \text{erfc}(\frac{\alpha \sqrt{t}}{n \delta}) \exp \left[ -\alpha^2 t + \frac{\alpha^2 t}{n \delta} \right] \\
= \text{erf}(\alpha \sqrt{t}) + \frac{\delta}{1-\delta} e^{-\alpha^2 t} \left( \text{erf}(\alpha \sqrt{t}) - 2 \right) \\
+ e^{\alpha^2 t/n^2} \text{erfc}(\alpha \sqrt{t}/n \delta) \right] \quad \text{for } n = 1 \\
= 0 \quad \text{for } n = \infty .
\]

\[
no^{-\alpha^2 t} \left[ -I_1 + \frac{1}{\alpha^2} \frac{d^2}{d^2}(I_1) + e^{-\alpha^2 t} I_3 - \frac{2\delta \delta \alpha^2 t}{(n+1)^2} e^{-(1-\delta)\alpha^2 t} \right] \\
= \frac{n}{2n-1} \left[ 1 + \text{erf}(\alpha \sqrt{t}) - \frac{(2n-1)\delta}{1+n-3n\delta} e^{-(1-\delta)\alpha^2 t} \\
+ \frac{(1-n\delta^2) \alpha^2 t}{e^{n \delta} + \text{erfc}(\frac{\alpha \sqrt{t}}{n \delta})} \right] \\
- \frac{n}{2n-1} \left[ e^{-\alpha^2 t} \text{erfc}(\frac{\alpha \sqrt{t}}{n \delta}) - \frac{(2n-1)\delta}{1+n-3n\delta} e^{-(1-\delta)\alpha^2 t - \alpha^2 t} \right] \\
. \text{erfc}(\frac{\alpha \sqrt{t}}{n \delta}) - \frac{(2n-1)\delta}{1+n-3n\delta} e^{-(1-\delta)\alpha^2 t + \frac{\delta^2}{n \delta}} \right] \\
. \text{erfc}(\frac{\alpha \sqrt{t}}{n \delta} + \frac{\alpha \sqrt{t}}{n \delta}) \\
= \frac{1}{2} \left[ e^{-\alpha^2 t} \text{erfc}(\alpha \sqrt{t} - \frac{\alpha \sqrt{t}}{n \delta}) + \frac{\alpha^2 t}{n^2 \delta} \text{erfc}(\alpha \sqrt{t} + \frac{\alpha \sqrt{t}}{n \delta}) \right] \\
\quad \text{for } n = \infty .
\]
APPENDIX - III

A NOTE ON THE EVALUATION OF DOUBLE INTEGRALS
Settlement and porepressure evaluation in three-dimensional problems concerning distributed loads involve the numerical evaluation of double integrals. This note explains the method of evaluating such integrals using Filon's quadrature method (Tranter, 1962). For example, the general expression for settlement under rectangular shear loads (for \( y = z = 0 \)) is given from (5.17) by:

\[
2GW = \frac{4}{n^2} \int_0^{\infty} \int_0^{\infty} \sin \alpha \sin \lambda \, d\alpha \, d\lambda \quad (\lambda 3.1)
\]

where \( F_s = f(Y, n, t, \text{drainage condition}) = \text{settlement function} \). For example, \( F_s = \text{erf} \left( \frac{Y}{\sqrt{t}} \right) \) for \( n = 1 \) and for pervious boundary case. This functional relationship is evaluated by a SUBROUTINE-'SSP' (Subroutine Settlement Pervious), which in turn calls another subroutine for ERROR function. Asymptotic expansions for error function given in Abramowitz and Stegun (1965) are used in the subroutine.

Quadrature rules are applicable for definite integrals with finite limits. Hence, the above infinite integrals must be written as:

\[
2GW = \frac{4}{n^2} \int_0^{\alpha_{\text{max}}} \int_0^{\alpha_{\text{max}}} \sin \alpha \sin \lambda \, d\alpha \, d\lambda + \epsilon \quad (\lambda 3.2)
\]

where the second term \( \epsilon \) can be made as small as one pleases by choosing proper values for \( \alpha_{\text{max}} \) and \( \lambda_{\text{max}} \). It is assumed that \( \alpha_{\text{max}} = \lambda_{\text{max}} \) and these values are so chosen that the integrand is less than 0.0001. Infact
those limits depend upon "t"; however to keep the limits the same for all values of \( t \), the maximum of all such values are used. For the above problem,

\[
20\omega \approx \frac{4}{\pi} \int_0^{20} \int_0^{20} \frac{\sin \beta \sin \alpha \sin \alpha}{\sqrt{\gamma^2}} \, d\alpha \, d\beta \quad (\text{A} \, 3.3)
\]

There are steep variations in the values of integrand in the above area of integration and hence the zone of integration is divided into four zones as follows:

\[
\int_0^{20} \int_0^{20} \ldots = \int_0^{4} \int_0^{4} \ldots + \int_0^{4} \int_4^{20} \ldots + \int_4^{20} \int_0^{4} \ldots + \int_4^{20} \int_4^{20}
\]

The region 0 to 4 is divided into 60 equal parts and 4 to 20 into 40 equal parts. The integrand in (A 3.3) is finite for \( \alpha = 0 = \beta \). But because of the method, the function to be integrated tends to be infinite, for \( \alpha = 0 = \beta \). Hence the lower limits are taken as 0.0001 instead of 0.

The integrand in (A 3.2) consists of trigonometric functions which oscillate. Use of Simpson's rule, probably the most often used in numerical integration, is not quite satisfactory for such integrals and hence Tranter (1962) suggests the use of Filon's rule instead. This rule for a single integral involving an integrand with sine term for a double strip with 3 ordinates spaced equally at a distance of "h" is given by:

\[
\int_{x_{i-1}}^{x_{i+1}} f(x) \sin (px) \, dx = h \left[ - \frac{\alpha}{2} \left\{ f_{i+1} \cos px_{i+1} + f_{i-1} \cos px_{i-1} \right\} + \frac{\beta}{2} \left\{ f_{i+1} \sin px_{i+1} + f_{i-1} \sin px_{i-1} \right\} + \gamma (f_1 \sin px_1) \right] \\
(A \, 3.4)
\]
The expression in (A 3.4) can be extended for a double integral as follows:

\[
\begin{align*}
&\int_{y_{j-1}}^{y_{j+1}} \int_{x_{i-1}}^{x_{i+1}} f(x, y) \sin(px) \sin(qy) \, dx \, dy \\
&= h k \left[ -A_1 (DF_9 + EF_6 + GF_3) + B_1 (DF_8 + EF_5 + GF_2) \\
&\quad + C_1 (DF_7 + EF_4 + GF_1) \right] \quad (A\ 3.5)
\end{align*}
\]

where \( k \) is the spacing in \( y \)-direction,

\[
\begin{align*}
A_1 &= \alpha \cos px_{i+1} - \frac{a}{2} \sin px_{i+1} \\
B_1 &= \gamma \sin px_i \\
C_1 &= \alpha \cos px_{i-1} + \frac{a}{2} \sin px_{i-1} \\
D &= -\lambda \cos q y_{j+1} + \frac{b}{2} \sin q y_{j+1} \\
E &= \eta \sin q y_j \\
F_1 &= \alpha \cos q y_{j-1} + \frac{b}{2} \sin q y_{j-1}
\end{align*}
\]

and \( F_1, F_2, \ldots, F_9 \) denote the functional values \( f(x, y) \) at various nodal points indicated below:

```
   F7  F8  F9  \\
1-1 1, j+1 1+1, j+1
   F4  F5  F6  \\
1-1 1, j 1+1, j
   F1  F2  F3  \\
1-1 1, j-1 1+1, j-1
<-- h --> h -->
```
α, θ, γ are functions of ph and A, B, G are functions of qk. α, θ, γ are tabulated (Trantur, 1962) or alternatively can be calculated from the following relations:

\[ \alpha = \frac{\theta^2 + \theta \sin \theta \cos \theta - 2 \sin^2 \theta}{\theta^3} \]

\[ s = 2 \left[ \theta (1+\cos^2 \theta) - 2 \sin \theta \cos \theta \right] / \theta^3 \quad (4.3.6) \]

\[ \gamma = 4 (\sin \theta - \theta \cos \theta) / \theta^3 \]

θ = ph

A, B, G can be obtained from α, θ, γ on replacing θ by qk in (4.3.6).

Repeated use of Filon's rule of (4.3.5) enables the evaluation of integral in (4.3.3). Computer Program based on above method is given below along with a typical output.

C RECTANGULAR SHEAR Pervious Settlements (X=B=EDGE, Y=Z=0)
C SQUARE FOOTING, DOUBLE INTEGRAL- Filons Rule
C X=ALPHA, Y=BETA, Q=HORIZONTAL DISTANCE, R=L/B-RATIO
C INTERVAL 0TO20 IS DIVIDED INTO 0AND4TO20
C FIRST ZONE HAS 30 DOUBLE-STRIPS AND SECOND HAS 20
C PRINT 20
20 FORMAT(4OH RECTANGULAR SHEAR Pervious Settlements// 14UHUNDER THE (CENTRAL)EDGE OF SQUARE FOOTING// 270X, 7HN-VALUE, 10X, 7HT-VALUE, 10X, 20IPervious Settlement//)
AP=4.*ATAN(1.)
Q=1.0
R=1.0
\( \Delta x = 0.000018 \)
\( \Delta x = 0.00278 \)
\( B X = 0.96725 \)
\( B X = 0.68704 \)
\( C X = 1.33272 \)
\( C X = 1.31212 \)
AN=1.0
T=0.001
DO 500 I = 1, 5
S=0.0
C
FIRST BLOCK X=0 TO 4, Y=0 TO 4
F=0.0
X1=0.0001
X2=X1+(2./30.)
X3=X2+(2./30.)
DO 100 J=1,30
Y1=0.0001
Y2=Y1+(2./30.)
Y3=Y2+(2./30.)
G11=SQR (X1**1+X1**Y1)
G21=SQR (X2**2+X2**Y2)
G31=SQR (X3**3+X3**Y3)
CALL SSP (AN, G11, T, SP)
F1=SP*SIN(X1)/(Y1**G11**2)
CALL SSP (AN, G21, T, SP)
F2=SP*SIN(X2)/(Y1**G21**2)
CALL SSP (AN, G31, T, SP)
F3=SP*SIN(X3)/(Y1**G31**2)
A=AX1*COS(Q*X3)-BX1*SIN(Q*X3)/2.
B=AX1*SIN(Q*X3)
C=AX1*COS(Q*X1)-BX1*SIN(Q*X1)/2.
DO 80 K=1,30
G12=SQR (X1**1+Y2**Y2)
G22=SQR (X2**2+Y2**Y2)
G32=SQR (X3**3+Y2**Y3)
G13=SQR (X1**1+Y3**Y3)
G23=SQR (X2**2+Y3**Y3)
G33=SQR (X3**3+Y3**Y3)
CALL SSP (AN, G12, T, SP)
F4=SP*SIN(X1)/(Y2**G12**2)
CALL SSP (AN, G22, T, SP)
F5=SP*SIN(X2)/(Y2**G22**2)
CALL SSP (AN, G32, T, SP)
F6=SP*SIN(X3)/(Y2**G32**2)
CALL SSP (AN, G13, T, SP)
F7=SP*SIN(X1)/(Y3**G13**2)
CALL SSP (AN, G23, T, SP)
F8=SP*SIN(X2)/(Y3**G23**2)
CALL SSP (AN, G33, T, SP)
F9=SP*SIN(X3)/(Y3**G33**2)
CALL SSP (AN, G12, T, SP)
F10=SP*SIN(X1)/(Y3**G12**2)
D=AX1*COS(Q*X3)+BX1*SIN(Q*X3)/2.
E=AX1*SIN(Q*X3)
G=AX1*COS(Y1)+BX1*SIN(Y1)/2.
 1)*F2*B*G+F7*C*D+F4*C*B+F1*C*G)
F1=F7
F2=F5
F3=F9
Y1=Y3
Y2=Y3+(2./30.)
Y3=Y2+(2./30.)
X1=X3
X2=X1+(2./30.)
C
SECONDS BLOCK X=0 TO 4, Y= 4 TO 20
F=0,0
X1=0,0001
X2=X1+(2./30.)
X3=X2+(2./30.)
DO 110 J=1,30
Y1=4,0001
Y2=Y1+0.4
Y3=Y2+0.4
G1=SQRT(X1*X1+Y1*Y1)
G2=SQRT(X2*X2+Y2*Y2)
G3=SQRT(X3*X3+Y3*Y3)
CALL SSP (AN, G11, T, SP)
F1=SP*SIN(X1)/(Y1*G11**2)
CALL SSP (AN, G21, T, SP)
F2=SP*SIN(X2)/(Y1*G21**2)
CALL SSP (AN, G31, T, SP)
F3=SP*SIN(X3)/(Y1*G31**2)
A=AX*COS(Q*X3)-BX1*SIN(Q*X3)/2.
B=GX*SIN(Q*X2)
C=AX1*COS(Q*X1)+BX1*SIN(Q*X1)/2.
DO 81 I=1, 20
G12=SQRT(X1*X1+Y2*Y2)
G22=SQRT(X2*X2+Y2*Y2)
G32=SQRT(X3*X3+Y2*Y2)
G13=SQRT(X1*X1+Y3*Y3)
G23=SQRT(X2*X2+Y3*Y3)
G33=SQRT(X3*X3+Y3*Y3)
CALL SSP (AN, G12, T, SP)
F4=SP*SIN(X1)/(Y2*G12**2)
CALL SSP (AN, G22, T, SP)
F5=SP*SIN(X2)/(Y2*G22**2)
CALL SSP (AN, G32, T, SP)
F6=SP*SIN(X3)/(Y2*G32**2)
CALL SSP (AN, G13, T, SP)
F7=SP*SIN(X1)/(Y3*G13**2)
CALL SSP (AN, G23, T, SP)
F8=SP*SIN(X2)/(Y3*G23**2)
CALL SSP (AN, G33, T, SP)
F9=SP*SIN(X3)/(Y3*G33**2)
D=AX3*COS(R*Y3)+BX2*SIN(R*Y3)/2.
E=G13*SIN(R*Y2)
G=AX2*COS(R*Y1)+BX2*SIN(R*Y1)/2.
F=P{0.8/30.,}*(-F9*5*D-F6*5*E-F3*4*0+F8*5*D+F5*5*E+
  F2*5*F7*C*D+F4*5*C*B+F1*5*C*G)
P1=P7
P2=P8
Y3=F9
Y1=Y2
Y2=Y*0.4
81 Y3=Y2+0.4
X1=X3
X2=X1+(2./30.)
110 X3=X2+(2./30.)
S=3+F

THIRD BLOCK X=0 TO 20, Y=0 TO 4
P=0,0
X1=4.0001
X2=X1+0.4
X3=X2+0.4
DO 120 J=1,20
Y1=0.0001
Y2=Y1+(2./30)
Y3=Y2+(2./30)
G1=SQRT(X1*X1+Y1*Y1)
G2=SQRT(X2*X2+Y1*Y1)
G3=SQRT(X3*X3+Y1*Y1)
CALL SSP (AN, G1, T, SP)
F1=SP*SIN(X1)/(Y1*G1**2)
CALL SSP (AN, G2, T, SP)
F2=SP*SIN(X2)/(Y1*G2**2)
CALL SSP (AN, G3, T, SP)
F3=SP*SIN(X3)/(Y1*G3**2)
A=A2*COS(q*X3)-B2*X2*SIN(q*X3)/2.
B=UX2*SIN(q*X2)
C =AX2*COS(q*X1)+BX2*SIN(q*X1)/2.
DO 82 K=1,30
G12=SQRT(X1*X1+Y2*Y2)
G22=SQRT(X2*X2+Y2*Y2)
G32=SQRT(X3*X3+Y2*Y2)
G13=SQRT(X1*X1+Y3*Y3)
G23=SQRT(X2*X2+Y3*Y3)
G33=SQRT(X3*X3+Y3*Y3)
CALL SSP (AN, G12, T, SP)
F4=SP*SIN(X1)/(Y2*G12**2)
CALL SSP (AN, G22, T, SP)
F5=SP*SIN(X2)/(Y2*G22**2)
CALL SSP (AN, G32, T, SP)
F6=SP*SIN(X3)/(Y2*G32**2)
CALL SSP (AN, G13, T, SP)
F7=SP*SIN(X1)/(Y3*G13**2)
CALL SSP (AN, G23, T, SP)
F8=SP*SIN(X2)/(Y3*G23**2)
CALL SSP (AN, G33, T, SP)
F9=SP*SIN(X3)/(Y3*G33**2)
D=-AX1*COS(R*X3)+BX1*SIN(R*X3)/2.
B=0X1*SIN(R*X3)
G=AX1*COS(R*X1)+BX1*SIN(R*X1)/2.
P=F4 (0.8/30.)*(-T9*A*T-F6*4.*E-F3*A*G+F8*E*D+F5*B*E+
F2*B*E+F7*C*D+F4*C*B+F1*C*G)
P1=F7
P2=F8
P3=F9
Y1=Y3
Y2=Y1+(2./30)
82
Y3=Y2+(2./30)
X1=X3
X2=X1+0.4
120
X3=X2+0.4
S=S*F

FOURTH BLOCK X=4 TO 20, Y=4 TO 20
F=0.0
X1=4.0001
X2=X1+0.4
X3=X2+0.4
DO 130 J=1,20
Y1=4.0001
Y2=Y1+0.4
Y3=Y2+0.4
G1=SQRT(X1**2+Y1**2)
G2=SQRT(X2**2+Y2**2)
G3=SQRT(X3**2+Y3**2)
CALL SSF(AN, G11, T, SF)
F1=SF*SIN(X1)/(Y1*G11**2)
CALL SSF(AN, G21, T, SF)
F2=SF*SIN(X2)/(Y2*G21**2)
CALL SSF(AN, G31, T, SF)
F3=SF*SIN(X3)/(Y3*G31**2)
A=AX2*COS(Q*X3)-BX2*SIN(Q*X3)/2.
B=GX2*SIN(Q*X2)
C=AX2*COS(Q*X1)+BX2*SIN(Q*X1)/2.

DO 83 K=1,20
G1=SQRT(X1**2+Y1**2)
G2=SQRT(X2**2+Y2**2)
G3=SQRT(X3**2+Y3**2)
G11=G21=G31
CALL SSF(AN, G11, T, SF)
F1=SF*SIN(X1)/(Y1*G11**2)
CALL SSF(AN, G21, T, SF)
F2=SF*SIN(X2)/(Y2*G21**2)
CALL SSF(AN, G31, T, SF)
F3=SF*SIN(X3)/(Y3*G31**2)
A=AX2*COS(Q*X3)-BX2*SIN(Q*X3)/2.
B=GX2*SIN(Q*X2)
C=AX2*COS(Q*X1)+BX2*SIN(Q*X1)/2.

DO 83 X=4.0001
X2=X1+0.4
X3=X2+0.4

Y1=Y2+0.4
Y2=Y3+0.4
Y3=Y4+0.4

83
X1=X3
X2=X3+0.4
X3=X2+0.4

130
S=SQRT
SETER=4.*S/(API*API)
PRINT6,AN,T,SETER

8 FORMAT(2(10X,F7.3),12X,B15.8)
200 IF(T>10.) 200, 210, 210
T=T*10.0
300 CONTINUE
210 S=ERF
END

SUBROUTINE SSI(AN,G,T,S)
QA=G*T
GB=SQRT(QA)
IF(AN<1.) 10, 10, 12
10 CALL DERF(GB,ERF,ERFC)
S=ERF
GO TO 13
12 DN=2.*AN-1
CALL DERF(GB,ERF,ERFC)
SP=(1.-AN*ERFC)/DN
IF(GB>4.) 14, 14, 15
15 GO TO 13
14 GC=(AN-1.)*GB/AN
GD=(2.*AN-1.)*GL/(AN*AN)
CALL DERF(GC,ERF,ERFC)
S=S+((AN-1.)*ERFC*EXI(-GD)/DN)
13 CONTINUE
RETURN
END

SUBROUTINE DERF(Y,ERF,ERFC)
IF(Y)<6. 25, 25, 26
25 ERF=0.0
ERFC=1.0
GO TO 23
26 X=ABS(Y)
IF(X<6.) 21, 21, 28
21 D1=0.0705230784*X
D2=0.0422820123*X*X
D3=0.009275272*X**3
D4=0.0001520143*X**4
D5=0.00002765672*X**5
D6=0.0000439638*X**6
ERFC=1.0/(1.0+D1+D2+D3+D4+D5+D6)**16
ERFC=1.0-ERFC
GO TO 24
28 ERF=1.0
IF(x-0.0) 22, 23, 23
ERF=1.0*ERF
ERR=-ERR
CONTINUE
RETURN
END

RECTANGULAR SHEAR-barous Setlements
undEr thE (cenTral) Edges of Square FootinG

<table>
<thead>
<tr>
<th>N-VALUE</th>
<th>T-VALUE</th>
<th>Previous Settlement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,000</td>
<td>0.001</td>
<td>0.4228640E-01</td>
</tr>
<tr>
<td>1,000</td>
<td>0.010</td>
<td>0.1211044E 00</td>
</tr>
<tr>
<td>1,000</td>
<td>0.100</td>
<td>0.2845409E 00</td>
</tr>
<tr>
<td>1,000</td>
<td>1.000</td>
<td>0.4746494E 00</td>
</tr>
<tr>
<td>1,000</td>
<td>10.000</td>
<td>0.5335748E 00</td>
</tr>
</tbody>
</table>
APPENDIX - IV

SUGGESTIONS FOR FUTURE WORK
The study reported in this thesis forms only a starting point for further extension of the solutions to many other problems as mentioned below. Some of these problems are being studied presently and are planned for future work by the author.

1) Extension of the present solutions to finite thickness of clay layer.

2) Solutions for loads applied through rigid footings.

3) Solution to problem of finite cylinder subjected to anisotropic loading as in triaxial apparatus.

4) Controlled model studies with measurement of settlements and porepressures.

5) More accurate determination of consolidation soil parameters and the effect of stress level and load history on such parameters.

6) Better numerical methods of evaluating infinite integrals appearing in the present solutions.
APPENDIX - V

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APPENDIX - VI

NOTATIONS
Skempton's porepressure coefficient

Width of the shoulder in embankment type load pattern

Skempton's porepressure coefficient

Lateral dimension of the loaded area

Half-width of the strip

Coefficient of consolidation

Coefficients of consolidation for 1-D, 2-D and 3-D strain conditions

Modified coefficient of consolidation

Displacement function

Effective Young's modulus

Undrained Young's modulus

Exponential integral

Volumetric strain

Average volumetric strain

General strain tensor of soil skeleton

Normal strains

Error function

Complementary error function

Dawson's integral

Effective shear modulus

Total stress (undrained) shear modulus

Influence factor

Modified Bessel function

Imaginary part of \( w(\bar{x}) \)

Bessel function

Load function
k Coefficient of permeability

$k_o$ Coefficient of earth pressure at rest

$l$ Half-length of rectangular footing

$\ln(x)$ Natural logarithmic function

$M(a;b;x)$ Confluent hypergeometric function

$m_v$ Coefficient of volume compressibility

$m_{v1}, m_{v2}, m_{v3}$ Coefficients of volume compressibility for 1-D, 2-D, 3-D strain conditions

N.C. Normally consolidated (clay)

$n$ Auxiliary elastic constant ($= 1-\nu/1-2\nu$)

O.C. Over-consolidated (clay)

$\Gamma$ Magnitude of point load

$p$ Parameter of transformation used in Laplace transforms

$q$ Displacement function

$q_l$ Intensity of load per unit area

$q_l$ Intensity of load per unit length

$R$ Radius of circular loaded area

$\text{Re}(w) \overline{w(x)}$ Real part of $w(x)$

$S$ Displacement function

$s$ Parameter ($= p/r^2$ or $p/\alpha^2$)

$t$ Time

$t_c$ Construction period

$U_i$ Fluid displacement vector

$U_{\sigma}$ Degree of point porepressure dissipation

$U_e$ Degree of average volumetric strain mobilization

$U_w$ Degree of consolidation settlement

$U_{\sigma}$ Degree of average porepressure dissipation
$u, v, w$ Displacements of soil skeleton in $x^-, y^- and z^-$ directions

$w$ Settlement

$w(s)$ Function of Complex Variable $s = e^{-z^2}$ erfc $(-is)$

$w_c$ Primary consolidation settlement

$w_{ct}$ Primary consolidation settlement at time $t$

$w_{c\infty}$ Final Primary consolidation settlement

$w_0$ Immediate settlement

$w_t$ Total settlement at time $t$

$w_{\infty}$ Ultimate settlement

$x$ Space coordinate (horizontal)

$x_j$ Coordinate space vector

$y$ Space coordinate (horizontal)

$z$ Space coordinate (vertical) or depth

$\alpha$ Parameter of transformation (in Fourier transforms)

$\alpha$ Shape factor

$\beta$ Parameter of transformation (in Fourier Transforms)

$\gamma$ Parameter ($= \sqrt{\alpha^2 + \beta^2}$)

$\gamma_w$ Unit weight of porewater

$\nabla^2$ Laplacian operator ($= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}$ - for plane-strain problems)

$= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ - for 3-D problems

$s = \frac{(m^2) - \sqrt{(m^2 + 4n)}}{2n}$
\[ \delta_{ij} \text{ Kronecker delta \((= 1 \text{ for } i = j, = 0 \text{ for } i \neq j)\)} \]

\[ \varepsilon \text{ Fluid dilation} \]

\[ \varepsilon_{ij} \text{ Fluid strain tensor} \]

\[ \lambda \text{ Length to breadth ratio of rectangular loaded area} \]

\[ \lambda', \lambda'' \text{ Effective Lame's constant for soil skeleton} \]

\[ \alpha \text{ Correction factor due to Skempton and Bjerrum} \]

\[ \nu \text{ Effective Poisson's ratio} \]

\[ \nu_t \text{ Total stress Poisson's ratio} \]

\[ \nu_u \text{ Undrained Poisson's ratio} \]

\[ \sigma \text{ Excess porewater pressure} \]

\[ \sigma_{ij} \text{ Total stress tensor} \]

\[ \sigma_0 \text{ Initial excess porepressure} \]

\[ \sigma_v, \sigma_{vo}, \sigma_{v\infty} \text{ Volumetric stresses} \]

\[ \sigma_{ij}^{\text{eff}} \text{ Effective stress tensor} \]

\[ \sigma_\theta \text{ Total bulk stress \((= \sigma_{xx} + \sigma_{yy} + \sigma_{zz} = 3 \sigma_v)\)} \]

\[ \sigma_1, \sigma_2, \sigma_3 \text{ Major, intermediate and minor total principal stresses} \]