CHAPTER II

TOTAL SCATTERING CROSS-SECTION AND NUCLEAR PARAMETERS
II.1 Introduction

The mean lifetime of $\Sigma^+$ particles being very short ($\approx 10^{-10}$ sec), the experimental investigation of the $\Sigma^+p$ interaction is severely handicapped. With the scanty and error-affected data, a few theoretical studies have been attempted. However, in the analysis of total cross-section data of hyperon nucleon scattering, the major efforts have been made to extract estimates of the low energy parameters i.e., the scattering lengths and the effective ranges. Total scattering cross-section is defined as

$$\sigma_t = \frac{4\pi}{k^2} \sum_{l=0}^{\infty} (2l+1) \sin^2 \delta_l$$  \hspace{1cm} (2.1)

One does this by using the relation,

$$kcot \delta_l = \frac{1}{a} + \frac{rk^2}{2}$$  \hspace{1cm} (2.2)

where, $k$ is the centre of mass momentum, $a$, scattering length, $r$, effective range, and $\delta_l$, the phase shift corresponding to ‘$l$’.

As the low energy scattering receives significant contribution only from the $S$-states, one uses the parameters of
the singlet and triplet states to obtain the expression for the total cross-section

\[
\sigma_T = \frac{3\pi}{|\cot \sigma_r - ik|^2} + \frac{\pi}{|\cot \sigma_s - ik|^2}
\]  

(2.3)

Using equations (2.2) and (2.3), the total scattering cross-section is given by

\[
\sigma_T = \frac{3\pi}{(1/a)^2 + (r_s k^2/2)^2 - r_t k^2/a + k^2} + \frac{\pi}{(1/a_s)^2 + (r_s k^2/2)^2 - r_s k^2/a_s + k^2}
\]  

(2.4)

One then tries to obtain estimates of the four parameters \(a_s, a_t, r_s, r_t\) by making a least \(\chi^2\) fit to the data on \(\sigma_T\). The earlier workers are unanimous about the sign of these parameters, but the magnitudes differ. The uncertainties regarding the magnitudes of these parameters could be due to the fact that the effective range calculations are expected to yield reliable results when only very few low energy data are used for the analysis. And the lowest momentum, at which the total cross-section data of \(\Sigma^+p\) and \(\Sigma^-p\) scattering are available, is around 145 MeV/c and 142.5 MeV/c respectively. Thus for all practical purposes, when one tries to obtain estimates of those parameters, one performs
extrapolation through analytic continuation of the data to regions where experimental information is not available. In attempting such an extrapolation, one has to consider a procedure, which has greater information storage capacity and is thus likely to lead to more stable and reliable results. However, in the absence of an universal algorithm for such an analytic extrapolation, one hopes that optimal exploitation of the analytic structure could be a nice tool\textsuperscript{43}. There have been prescription by Cutkosky and Deo\textsuperscript{39} and Ciulli\textsuperscript{40} for providing a relatively stable extrapolation procedure, by optimally exploiting the analytic structure of the scattering amplitude in the energy plane. This technique has been successfully tested by many authors like, Cutkosky and Deo\textsuperscript{39}, Miller et al\textsuperscript{65}, Mohanty and Mohapatra\textsuperscript{68}. Here, we have tried to store the available physical information in the coefficients of an accelerated convergent expansion of $\sigma_T$. We then extrapolate the function, $\sigma_T$, thus constructed to momentum range, 50 MeV/c and use these values as our data to obtain the low energy parameters of $\Sigma^+p$ scattering.

II.2 Scheme of parametrization of $\sigma_T$

The optical theorem states

$$\sigma_T = \frac{4\pi}{k} \text{Im} f(S, 0)$$

(2.5)
where, $S$ is the square of the centre of mass energy $T_0$. To parametrize $\text{Im} \ f(S, 0)$, we note that it is holomorphic in the $s$ plane except for the cuts $S_R \leq S \leq \infty$ and $-\infty \leq S \leq S_L$ (Figure 2.1a) where, for $\Sigma^+p$ scattering,

$$S_R^{\Sigma^+} = (M_A + M_p)^2 \quad (2.6)$$

$$S_L^{\Sigma^+} = 2(M^2_{\Sigma^+} - M_p^2) \quad (2.7)$$

and for $\Sigma^-p$ scattering,

$$S_R^{\Sigma^-} = (M_A + M_p)^2 \quad (2.8)$$

$$S_L^{\Sigma^-} = 2(M^2_{\Sigma^-} - M_p^2) \quad (2.9)$$

$S_R$ and $S_L$ correspond to the opening up of two particle thresholds in the direct and crossed channels respectively. Since data for $\sigma_T$ are available at $S$ values greater than $S_R$, for our analysis, we treat the region $0 \leq S \leq \infty$ as the physical domain. In estimating the low energy parameters using eqn (2.4), the interference terms between $\alpha$ and $r$ are of importance, but can not help in fixing the signs of the individual scattering lengths and effective ranges.
Figure 2.1 (a) Parametrization of the imaginary part of the scattering amplitude, \( \text{Im} f(S,0) \), which is holomorphic in the \( s \) plane, except for the cuts, \( S_L \) and \( S_R \), being the right hand and left hand cuts respectively (b) \( X^- \) and \( X^+ \) depicts the position of the cuts in the \( x \) plane, being symmetrised (c) Square root mapping of the cuts in the \( W = \sqrt{X} \) plane (d) Unit circle of convergence in the mapped \( z \) plane with radius, \( a = \pm 1 \).
As a first step, we symmetrize the cuts on the real axis in the $x$ plane by the mapping (Figure 2.1.b),

$$X = S - \frac{1}{2}(S_R + S_L) \quad (2.10)$$

Taking note of the fact that, for our analysis, negative $S$ will be unphysical, hence, we do the following mapping

$$W = \sqrt{X} \quad (2.11)$$

This symmetrizes separately the left hand and right hand cuts respectively, on the imaginary and real axes of the complex $w$ plane, on both sides. In this $w$ plane, the cuts on the real axis, representing the cut $S_R \leq S \leq \infty$, run from $-\infty \leq W \leq -W_0$ and $W_0 \leq W \leq \infty$ (Figure 2.1.c).

The optimal convergence can now be obtained\textsuperscript{39,40} by mapping the symmetrical cuts of the $w$ plane to form the boundary of an unifocal ellipse with $W = \pm 1$ as foci. But the size of the ellipse is quite large. Therefore, the following circular mapping has been preferred

$$Z = \frac{(W_0 + W^2)^{1/2} - W_0^{1/2}}{W} \quad (2.12)$$
This maps the entire domain of analyticity of $\text{Im} f(S, 0)$ into the interior of the unit circle in the $z$ plane with its radius $a = \pm 1$ (Figure 2.1d). Further, $W = 0$ is mapped onto $Z = 0$, $W = \pm \infty$ to $Z = \pm 1$, and the cuts on the imaginary axis of $W$ form the boundary of the unit circle. Hoping that no region of analyticity is left out of the region of convergence by this mapping, the expansion of $\text{Im} f(S, 0)$ in the mapped variable, $Z$, will converge optimally. So, we construct $\text{Im} f(S, 0)$ as polynomials in $Z$,

$$\text{Im} f(S, 0) = \sum_{n=0}^{\infty} a_n Z^n,$$

(2.13)

and

$$\sigma_T = \frac{4\pi}{k} \sum_{n=0}^{\infty} a_n Z^n.$$

(2.14)

II.3 Results and Conclusion

Using eqn (2.14), we have tried to fit all the $\Sigma^+ p$ and $\Sigma^- p$ scattering total cross-section data\textsuperscript{24,27,28}. After several attempts, it has been found that just two terms in the expansion of eqn (2.14), for both the scatterings, are enough to give a good fit to the experimental data i.e., for analysis of both $\Sigma^+ p$ and $\Sigma^- p$ scatterings.

$$\sigma_T = \frac{4\pi}{k} (a_0 + a_1 Z)$$

(2.15)
However, the truncated series still gives a faithful representation of the actual $\sigma_T$ in so far as the fit to the $\sigma_T$ data is concerned.

For $\Sigma^+ p$ scattering, we get the best fit with $\chi^2 / \text{NDF} = 0.42$. The values of the coefficients are $a_0^{\Sigma^+} = 0.6$, $a_1^{\Sigma^+} = 1.23$. For $\Sigma^- p$, the best fit is also obtained with $\chi^2 / \text{NDF} = 0.42$. The coefficient values for this scattering are $a_0^{\Sigma^-} = 1.05$, $a_1^{\Sigma^-} = 1.50$.

Comparison of the theoretical and experimental values for these scattering is shown in Table 2.1.

Table 2.1. Comparison of the theoretical and experimental values for the selected set of data by Nagels et al. $^{24,27,28}$. The laboratory momenta are given in MeV/c and the cross-sections in mb.

<table>
<thead>
<tr>
<th>$P_{\Sigma^-}$ (MeV/c)</th>
<th>$\sigma_{\text{theo}}$ (mb)</th>
<th>$\sigma_{\text{exp}}$ (mb)</th>
<th>$\chi^2 / \text{NDF} = 0.42$</th>
<th>$P_{\Sigma^-}$ (MeV/c)</th>
<th>$\sigma_{\text{theo}}$ (mb)</th>
<th>$\sigma_{\text{exp}}$ (mb)</th>
<th>$\chi^2 / \text{NDF} = 0.42$</th>
</tr>
</thead>
<tbody>
<tr>
<td>142.5</td>
<td>152 ± 38</td>
<td>151.6</td>
<td>145.0</td>
<td>123 ± 62</td>
<td>99.8</td>
<td>147.5</td>
<td>146 ± 30</td>
</tr>
<tr>
<td>152.5</td>
<td>142 ± 25</td>
<td>141.7</td>
<td>155.0</td>
<td>104 ± 30</td>
<td>93.5</td>
<td>157.5</td>
<td>164 ± 32</td>
</tr>
<tr>
<td>162.5</td>
<td>138 ± 19</td>
<td>133.1</td>
<td>165.0</td>
<td>92 ± 18</td>
<td>87.9</td>
<td>167.5</td>
<td>113 ± 16</td>
</tr>
<tr>
<td>175.0</td>
<td>81 ± 12</td>
<td>83.0</td>
<td>175.0</td>
<td>81 ± 12</td>
<td>83.0</td>
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</tbody>
</table>
To obtain estimates for the low energy parameters, we extrapolate our theoretical curves for $\sigma_T$ to lower incident hyperon momentum regions and compute values of $\sigma_T$ for $0.05 \text{ GeV/c} \leq P_Y \leq 0.16 \text{ GeV/c}$ at a bit of 5 MeV/c. Here, $Y$ designates $\Sigma^+$ or $\Sigma^-$. These 23 values (theoretically extrapolated) of $\sigma_T$ at corresponding $P_Y$ then serve the purpose of data points to determine the low energy parameters. Although at high $P_Y$ values the error in the experimental values are as high as 20% to 30%, we in the extrapolated theoretical values, assume an error of only 10% at each point so as to induce some amount of stability into the computed values of $\sigma_s$, $\sigma_t$, $\alpha_t$ and $r_t$. We then make a least $\chi^2$ fit to these 23 values of $\sigma_T$ using equations (2.2) and (2.3). Our best values for the low energy parameters thus obtained, are given in Tables 2.2 and 2.3. Results for $\Sigma^+p$ scattering along with the values of other workers Fast et al$^{20}$, de Swart$^9$, Nagels et al$^{24,27,28}$ in Table 2.2 give a clear idea that there is unanimity regarding the sign of these parameters, except for de Swart$^9$. They have a positive value for $r_t$. As regards the results of Fast et al$^{20}$, although their $\alpha_s$ and $r_t$ match in sign, the values are of a high order. However, our values more or less match well with those of Nagels et al$^{24,27,28}$, except that our $\alpha_t$ value is slightly high. The low $\alpha_t$ values by Nagels et al$^{27,28}$ are primarily for two reasons; (i) they have used data around the momentum value of 150 MeV/c and equations (2.2) and (2.3) are not very accurate at such high values of the momentum; (ii) even at this momentum...
their theoretical values of \( \sigma_T \) are greater than the experimental \( \sigma_T \) values because of the strong SU(3) constraint in their model. Our analysis is independent of such constraints.

Table 2.2 Values of scattering lengths and effective ranges of \( \Sigma^-p \) scattering in fermi from the present and earlier analysis (Fast et al\(^{20}\), de Swart\(^{96}\), Nagels et al\(^{24,27,28}\)).

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Present analysis</th>
<th>Analysis of other workers</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_s )</td>
<td>2.52 ± 0.33</td>
<td>-6 ± 1</td>
</tr>
<tr>
<td>( r_s )</td>
<td>3.34 ± 0.5</td>
<td>2.1 ± 0.3</td>
</tr>
<tr>
<td>( a_t )</td>
<td>1.54 ± 0.02</td>
<td>-0.2 ± 0.05</td>
</tr>
<tr>
<td>( r_t )</td>
<td>-0.61 ± 0.05</td>
<td>-0.40</td>
</tr>
</tbody>
</table>

Table 2.3 Values of scattering lengths and effective ranges of \( \Sigma^-p \) scattering in fermi from present and earlier analysis (Nagels et al\(^{24,27}\)).

<table>
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<tbody>
<tr>
<td>( a_s )</td>
<td>2.99 ± 0.06</td>
<td>-2.77 ± 0.45</td>
<td>-4.6 ± 0.6</td>
</tr>
<tr>
<td>( r_s )</td>
<td>3.39 ± 0.07</td>
<td>3.55 ± 0.32</td>
<td>3.69 ± 0.27</td>
</tr>
<tr>
<td>( a_t )</td>
<td>2.31 ± 0.04</td>
<td>0.63</td>
<td>0.32 ± 0.01</td>
</tr>
<tr>
<td>( r_t )</td>
<td>-0.63 ± 0.01</td>
<td>-0.76</td>
<td>-6.01 ± 0.12</td>
</tr>
</tbody>
</table>

Further we note that, because of this SU(3) constraint and consequent small \( a_t \) values, their zero energy \( \sigma_T \) is much smaller than our zero energy value for \( \sigma_T \).
Similarly for $\Sigma p$ scattering, analysing the results given in Table 2.3 we see that our values match well with those of earlier workers (Nagels et al.$^{24,27}$) in magnitude and sign, except that our values are slightly high for $a_t$. 

Figure 2.2 Effect of changes in $a_s$, $r_s$, $a_t$ and $r_t$ for $\Sigma p$ scattering.
We have presented a parametrization, which faithfully reflects the analytic structure of $\sigma_T$ and hopefully best represents $\sigma_T$. As the fit is quite stable, in the sense that small changes in the parameters produce large noises in $\chi^2$, as shown in Figures 2.2 and 2.3, we hope that our values are reliable and stable. Also, the sensitivity of these fits towards changes in $a_0$, for both the scatterings, are much sharper than those in $a_s, r_s$ and $r_l$.