CHAPTER-IV

MATHEMATICAL PROGRAMMING MODELS FOR MARKETING DECISION SYSTEM : A BRIEF REVIEW

4.1 INTRODUCTION

In a mathematical programming (MP) or optimisation problem, we seek to minimise or maximise a real function of real or integer variables, subject to constraints on the variables. The term mathematical programming refers to the study of these problems: their mathematical properties, the development and implementation of algorithms to solve these problems, and the application of these algorithms to real world problems. The MP techniques helps as a decision tool in the process of decision making in terms of modelling of mathematical programmes and their computational solution. MP has helped many decision makers to solve a variety of business problems. Mathematical programming is the study or use of the mathematical program. It includes any or all of the following:

- Theorems about the form of a solution, including whether one exists;
- Algorithms to seek a solution or ascertain that none exists;
- Formulation of problems into mathematical programs, including understanding the quality of one formulation in comparison with another;
- Analysis of results, including debugging situations, such as infeasible or anomalous values;
- Theorems about the model structure, including properties pertaining to feasibility, redundancy and/or implied relations (such theorems could be to support analysis of results or design of algorithms);
- Theorems about approximation arising from imperfections of model forms, levels of aggregation, computational error, and other deviations;
- Developments in connection with other disciplines, such as a computing environment.

Henderson and Schlaifer (1954) has published earlier application of mathematical programming for better decision making and described shipping schedules for the H.J.Heinz company of this MP techniques. Selective bibliography of earlier MP models and application to marketing problems are: Bass et. al., (1961), Wroe and Shapiro(1963), Buzzell (1962 & 1964), Montgomery and Urban (1963), Day (1964), Langhoff (1965), Sheparovvych, Marcus and Simon (1968), Frank, Kuehn and Massy (1970), and Kotler (1970).

The usage of mathematical programming techniques in marketing management has yielded useful insights into marketing decision-making in areas viz., advertising, media selection decision, advertising budget, new-product development and strategy, warehouse location, territorial allocation of
marketing budget, segmenting market, product pricing, sales-call programming and overall marketing problems.

In the late fifties and early sixties, application of MP techniques to marketing problem has been developed. And the marketing decision models applied MP are on topics viz., advertising budgeting, sales-force assignment and pricing strategy. A number of journals such as management science, The Journal of Marketing Research, The Journal of Advertising Research increasingly featured sophisticated mathematical programming modes for marketing problem showing techniques. There are number of universities offering special short term summer sessions on marketing mathematical programming model research. The courses are about to learn the new mathematical programming techniques and their practical applications. Clark (1962) emphasised on Monsanto chemical company, which developed a large number of computer programs to help in executive analyse a variety of marketing problems. Kotler (1967) described specific application of marketing operations research models developed in the areas of new products pricing, physical distribution, advertising and sales-force management. Learner (1965) described “The DEMON Model : (Decision Mapping via Optimum Go-No Network)” for new product planning. Urban (1968) described “The SPRINTER Model : Specification of Profits with Interaction Under Trial and Error Response” which illustrates decision maker to evaluate profit for the new product and demand and cost interrelationship between the new product and the other products in the line. Green (1963) illustrated application of Bayesian Decision theory in pricing strategy for a large chemical manufacturer. Shycon and Maffei (1960) had developed simulation program for better distribution within warehouses, customers and factories. Kuehn and Hamburger (1963) developed heuristic program which employed for locating warehouses. Vidale and Wolfe (1957) studied about advertising expenditures and response with help of operation research techniques. Kuehn (1961) had developed model for advertising budget using various sets of marketing factors. Kuehn’s (1962) article describes on various marketing factors response of advertising reference. Buzzell (1964) describes model developed at BBD&O with use of linear programming approach in media selection decision. Little et. al., (1963) developed algorithm for travelling salesmen problem. Karg and Thompson (1964) developed heuristic approach for solving travelling salesmen problems. Midler (1961) describes simulation model for sales force development. Magee (1958) discusses the application of operation research methods for allocation of expenditures optimally for promotional effort.

4.2 MULTI-OBJECTIVE OPTIMISATION

Multiple criteria decision systems (MCDS) has been an active area of research for almost forty years, beginning with the work of Charnes and Cooper (1961) and evolving through the 1973 conference organised by Cochrane and Zeleny (1973) and beyond. Unlike classical optimisation approaches which emphasise on a single criterion, viz., maximisation of the utility function
minimisation of costs as the objective. As early as 1966, however, authors began to recognise the limitations of single objective models. Bass and Lonsdale (1966) discusses on media selection models based upon several single objective, linear programming techniques and illustrating with utilising three separate objective functions. The single objective model of each of the example objectives was solved and presented separately. Keown and Ducan (1979) describes a number of the limitations of the single objective, linear programming approaches to media planning. In use of single criterion model for decision making purpose can be simplest form as in real world problems are focussed at multi-dimensionality. In reality, the decision environment have multiple, and conflicting objectives and considering this into the decision systems to have optimal solution is referred to as multi-objective decision systems (MCDS). The criteria involved in a MCDS are often conflicting and non-commensurable. The procedure involves generation of multiple alternatives among which a “good alternative” also termed as “compromise solution” has to be singled out. For such a system involving multiplicity of objectives which may also be conflicting viz., maximise sales at minimum cost, the choice of a good marketing strategy from a set of alternative strategies needs to be ranked in order of preference for a satisfactory solution.

According to Hwang and Yoon [1981], the MCDS consists of: A set of quantifiable objectives; A set of well defined constraints and A process of obtaining some trade-off information between the objectives.

A general expression for MCDS is:

Optimise \( G_i(X) : i=1,2,...,k \)

s.t \( X \in S \) ..................................(4.1)

Where \( S \) is the feasible action space and can be represented as:

\[ S = \{ X \in \mathbb{R}^n | A_j(X) \leq B_j : j=1,2,...,m; X \geq 0 \} \] ............. (4.2)

Sometimes, decision-makers (DMs) assign high aspiration levels to objectives and try to achieve these to the extent possible. Assigning suitable aspiration levels \( P_i \) to these objectives \( G_i(X) \), \( V \ i \), the good version of (4.1) can be stated as follows:

Determine \( X \in S \)

s.t \( G_i(X) \geq P_i \), for all \( i \) ............. (4.3)

\( A_j(X) \leq B_j \)

\( X \geq 0 \)

If for an action \( X^* \in S \), criterion \( G_i(X) \), for all \( i \) attain their individual global optimum \( G^*_i \) simultaneously, then \( X^* \) is an optimal solution. The point \( G^* = (G_1^*, G_2^* ,..., G_k^*) \) in the criterion space is also called as the “point of bliss” [Arrow (1964)].
An action $X^i$ is said to dominate another $X^j$, if

\begin{align*}
&\text{(i)} \quad G_h(X^i) \geq G_h(X^j), \text{ for all } h \quad \text{and} \\
&\text{(ii)} \quad G_k(X^i) > G_k(X^j), \text{ for at least one } k
\end{align*}

otherwise non-dominated. In economics, such an action is called "Pareto Optimal". A compromise solution is a non-dominated action which is chosen by generating an extra criterion by synthesising the existing one. This is known as preference structuring (modelling) over the criterion space. The choice of a compromise solution involves some trade-off between the criteria. This solution lies in the efficient frontier. If the MCDS is expressed as in (4.3) then the satisfying action according Simon [1955] is an element of the reduced subset of the feasible action space which satisfies the aspiration levels of each criterion. This action need not be non-dominated.

If in a MCDS, it is not possible to explore a non-dominated compromise action, then satisficing action can also be chosen for implementation. Some of the important multi-objective optimisation method have been discussed as follows:

(i) **Utility Theory**

Utility theory is applicable to deterministic as well as probabilistic problems. The theory dealing with deterministic multi criteria decision making (MCDM) problems is referred to as multiple objective/attribute value theory and that dealing with the probabilistic cases is called multiple attribute utility theory (or simply utility theory). In this approach the multiple objectives are aggregated to a single objective function called value function (utility function) associated in terms of the utility values of each objective. Use of weighted utility models to incorporate the relative importance of the multiple objectives is the best known and widely used method. An extensive literature can be found from Churchman et. al. (1954), Hwang and Masud (1979), Hwang and Yoon (1981), Chakong and Haines (1983). The major difficulty with the weighted utility model is that the DM is required to articulate the preference judgement in terms of relative importance of the criteria subjectively. However, there are some good methods to assess these weights rationally. They are eigen vector method of Saaty (1984), entropy method of Jaynes (1957) and the weighted least square method of Chu et. al., (1979).

(ii) **Goal Programming**

Goal programming (GP) is an extension of linear programming to solve the problems involving multiple objectives. In GP it is required to specify aspiration levels for the objectives and the key aim of GP is to minimise deviations from the goal values. Charnes and Cooper (1961) have first
introduced the concept of GP and subsequently a large number of works in theoretical as well as in application fronts are done till now and is one of the widely used techniques for tackling MCDS's.

The first method is due to Charnes and Cooper (1961) and is known as weighted goal programming model. In this model, they have mentioned that the complete goal attainments may not always be possible and the cardinal weights may be used to reflect the relative importance of the goals. Subsequently, Ijiri (1965) suggested a pre-emptive priority structure for the goals where the goals in the same priority level may be distinguished by putting cardinal weights or relative weights. The goals with the higher priority level will be achieved before the lower priority goals. He proposed "generalised inverse" method as a solution procedure. In this procedure, the square root of the sum of squares of goal deviations are to be minimised.

Later on, the GP technique was extensively studies by Ignizio (1976, 1978, 1980a, 1983) and Lee (1971, 1972). The authors developed a modified simplex algorithm as a solution procedure for GP problems. The algorithm is similar to the simplex method of linear programming (LP) to minimise the algebraic sum of goal deviations. Both the authors also discussed the integer format of GP and emphasised their applicability to the real life problems. A generalised GP model where the decision variables can take not only real values but also, mixed-integer, zero-one values which are more appropriate for a variety of real life problems, has also been discussed by Ignizio (1976, 1978). The author also made a study on its non-linearity, duality and its sensitivity analysis. Later he [Ignizio (1980b)] has incorporated the concept of non-dominated solutions by devising an augmented GP technique.

GP with interactive strategies based on the method of Geoffrion et. al. (1972) has been presented by Dyer (1972). Other interactive GP problems have been studied by various authors viz., Mornarhi et. al., (1973), Dauer and Krueger (1977), Nijkamp and Spronk (1979), Steuer (1986) and Lee and Shim (1986). In these processes, the DM has to provide a trade-off information among the objectives during course of optimisation.

A method of solving non-linear GP has been given by Dinkelbach et. al., (1985). Shim and Siegel (1975) proposed a quadratic form of GP. Stochastic GP has been investigated by Contini (1978). A dual simplex technique to solve GP problems has been presented by Kwak and Schniederjans (1985). A comparison of various algorithms for their computational efficiency has been given by David (1984). Hannan (1985) has made an assessment of some criticism of GP.
(iii) Multi-Criteria Simplex Method

This method is also based on the adjacent N-bases generating approach whose details can be found in Evan and Steur [1973] and Yu and Zeleny [1975]. This is a procedure similar in many respects to modified simplex method used in solving lexicographic linear GP problems.

The multi-objective linear programming (MOLP) techniques are based on sound mathematical footing. But, the computational experience reveals that even the moderately sized problem often has an un-workable large numbers of N-extreme points (N-bases). It also consumes substantial amount of computer time and requires a large core storage. These drawbacks limit its application to real-world problems.

The methods of the second category of Class-II of problem formulation come under the category of interactive procedures. In these procedures, the DM supplies to the analyst the implicit or explicit preference (trade-off) information during exploration of a suitable point or points in the criterion space. The trade-off informations depend on the current solution. The underlying concept in the development of interactive procedures is that, the DM is practically unable to specify the global preference structure due to inherent complexity of the MCDM problem; however, preference informations at a particular feasible action on the basis of the attainment levels of the criteria at that action are possible either explicitly or implicitly. As the solution process advances, the DM becomes more and more aware of the problem. This enables him to indicate his trade-off informations at successive actions more rationally. The process continues till a further improvement is no longer necessary. Interactive procedures are applicable only in those cases where the criteria involved are analytic functions defined over the compact set of feasible solution. Interactive procedures are available in various approaches in solving MCDM problems, viz., multi-attribute value theory, GP, methods seeking solution closest to the ideal point and non-dominated solution generating techniques. Interactive procedures based on the multi-attribute value theory (Geoffion et al [1972]), GP (Dyer [1972]), and efficient point generating techniques (Haimes and Hall [1974], Benson [1975], Zinots and Wallenion [1976]) require explicit trade-off information at each iteration. Some iterative procedures require implicit trade-off information. Such trade-off informations are generally supplied by the DM in one of the following two ways:

(1) The DM after examining the level of the various criteria at the current solution point, indicates the amount of the criteria which attained the most satisfactory achievement level to be conceded to improve the achievement levels of others.

(2) He is asked to choose the best compromise or non-dominated action out of a set of good ones presented to him. The choice is based on the outcomes of the actions over the criterion.
In both the ways of preference informations, the trade-off involved is not explicit. The interactive procedures based on the methods of seeking solutions closed to the ideal point or aspiration point (Bnenayoun et al. [1971], Zeleny [1974b]) and those based on the efficient solution generating techniques (Benayoun and Tegny [1970], Steur [1977]), require implicit trade-off informations. Some of the important interactive methods of both the types are discussed below.

(iv) Methods involving Explicit trade-off information

Some important methods which involve explicit trade-off informations are described as follows:

(a) Method of Geoffrion et al.

This method is based on a specific mathematical programming known as Frank-Wolfe algorithm (Frank and Wolfe [1956]). The procedures involves a large step gradient method for solving a vector-maximum problem provided the DM is able to specify an overall value function as an aggregation of multiple-objective (Geoffrion et al [1972]). However, the method does not require this function to be explicitly defined; instead it requires only such local informations as are needed to perform the computation. This method is suitable for linear as well as non-linear problems. It is computationally simple and it's convergence is rapid. However, the articulation of DM's explicit trade-off informations is difficult.

(b) Interactive GP

In this approach, the GP model of a MCDM problem is converted to a single vector-maximum problem (Dyer [1972]) which is solvable by the interactive method of Geoffrion et al. [1972].

(c) Methods of Zinots and Wallenious

This method is applicable when all the objective functions are linear. If the problem involves non-linear concave functions, they are to be first linearised and then subjected to this method. The feasible solution space S should be convex. It is based on the parameter generation of non-dominated actions inter activity (Zinots and Wallenious [1976]). The method guarantees a convergence in a finite number of iterations and the DM's involvement is less. On the other hand, the restrictions that all the objectives and constraints should be linear on amenable to linear approximation limits it's application area. Moreover, it is difficult to assess a reasonable value function to start with.

Some important methods requiring implicit trade-off informations are as follows:
(a) Step Method (STEM)

Benayoun et al. (1971) have proposed interactive methods for solving MOLP problems. These are known as “Step” and related methods. Here, only step method is presented which consists of alternate stages of computation and decision. It allows the DM to learn and recognise good solution. The method have been devised to solve a MOLP problem. If a viable compromise is not reached even after the last iteration, then either some other method has to be used to solve it or it is to be concluded that a compromise solution is not possible.

(b) Method of Displaced Ideal

This is also an interactive procedure suitable for MOLP (Zeleny [1974a, 1977]). The method is based on the generation of efficient solutions and the measurement of the nearness of these from the ideal point using a suitable norm. Usually, the number of N points generated are very large. The reduction of the set of N points is accomplished by the use of Lp metric as a norm. The net result of discarding a subset of the set of N points is equivalent to the displacement of the ideal point.

(c) Interactive MOLP Methods (Method of Steur)

Steur [1977] has discussed an interactive MOLP method which is based on his previous work (Steur [1976a, 1976b, 1976c]). It has been seen that MOLP methods usually generate an unmanageable large set of non-dominated extreme points. In this method, the interval criterion weights are used and gradient cone construction technique is applied to generate a small number set of extreme points. However, this set is not manageable for a large real world problem.

There has been several other developments in the direction of finding compromise solutions interactively. Haimes and Hall [1974] have presented an interactive method based on the aggregate trade-off rate function and the surrogate trade-off functions whose values are the DM’s assessment of how much he prefers trading off \( \delta_g \) marginal units of \( i^{th} \) objective \( g_i \) for one marginal unit of \( j^{th} \) objective, \( g_j \). The method of satisfactory goals (Benson [1975]) is based on the principles of improving the levels of the objectives one by one while keeping others at some satisfactory levels. The above two methods require implicit trade-off information. Some of the important methods which require implicit trade-off informations are goal programming step method (GPSTEM; Fichfet [1976]) which is a good version of STEM, sequential multiple objective problem solving method (SEMOPS; Monarchi et al [1973]) and sequential information generating multiple objective problem solving method (SIGMOPS; Monarchi et al. [1976]). The SEMOPS cyclically uses a surrogate objective function based on goals while SIGMOPS embeds a non linear GP approach within the principal problem. It may be observed that the
Interactive procedures are also based somehow or other on the concept of modelling a global preference structure to generate a solution at each iteration. A compromise solution is attained by gradual understanding of the entire process. Hence, the problem coming under class II in preference modelling results in dynamic processes suitable for tackling MCDM problems in evaluative situations. The DM's involvement in providing explicit trade-off information is more than that in providing implicit ones. The solution obtained has a better prospect of being implemented. This methodology requires that all $G_i(X)$ and $A_j(X)$ should be well-defined functions over the compact feasible set $S$. Discrete cases of $S$ cannot be tackled with interactive procedures. Methodologies of this class do not guarantee a global optima. Also the heterogeneity among various criteria involved in a MCDM problem reflect different orderings of the set of alternatives. The development of an outranking method for the purpose comprises of two distinct stages, viz.,

i) The construction of an outranking relation; and

ii) The exploitation of this relation in ranking the alternatives.


4.3 FUZZY MULTI-OBJECTIVE OPTIMISATION

Most of the real world decision problems arising in socio-economic and other systems usually involve multiple conflicting, non-commensurable and fuzzy criteria for a rational decision. A single objective (criterion) optimisation model is generally a simplified form of reality which sometimes leads to either misleading solution or to no solution.

It is realistic to consider the objectives with some suitable levels of attainment. This gives the concept of goals. Moreover, it is more realistic to assign to objectives flexible attainment level (usually intervals) rather than fixed ones. For example, "the annual profit of a certain company should not be less than around 5 million rupees" is an objective with flexible attainment level. This leads to the concept of fuzzy (imprecise) goals. By fuzziness we mean lack of sharp transition from membership to non-membership of an object belonging to a class. In the above example "the annual profit around 5 million rupees" is a class (fuzzy criterion) where there does not exist a sharp boundary to distinguish those profits (objects) around million rupees which belong to this class from those which do not. In many real decision situations there hardly exists a constraint which is strictly binding. It is more realistic to consider constraints also with flexible right hand sides and/or flexible technological coefficients. Such constraints can be treated as fuzzy goals [Bellman and Zadeh (1970)]. Fuzzy goals and/or fuzzy constraints are regarded as fuzzy criteria.
A fuzzy criteria $g(X)$ with a minimum fuzzy aspiration level $b$ is expressed as $g(X) \geq b$ where $X \in \mathbb{R}^n$ denotes the decision variable vector. This criterion is read as "$g(X)$ should be greater than or equal to an aspiration level around $b$". The wavy bar "~" below the equality sign denotes fuzzification.

Let us consider a decision problem involving $K$ fuzzy criteria stated as follows:

Determine $X \in \mathbb{R}^n \geq 0$ such that

$$g_i(X) \geq b_i, \; i = 1, 2, \ldots, K \tag{4.5}$$

This is the fuzzy mathematical model of the MCDM problem which can not be solved in the form of 4.5. Hence, a crisp substitute is essential. Fuzzy set theory is used very elegantly in deriving such a substitute by first identifying each fuzzy criterion $g_i(X)$ as a fuzzy set $g_i$, defined over the set of feasible solutions ($X$'s) and then aggregating all these fuzzy sets $g_i$ to obtain a single fuzzy set of decision $D$, say. The membership function $\mu_D(X)$ of $D$ serves the purpose of an overall objective function, and the $X \in S$ which maximises $\mu_D(X)$ is the optimal decision. The crux of the decision process lies in defining the decision function $\mu_D(X)$ by aggregating the fuzzy sets $g_i$, for all $i$, using a suitable operator.

Suppose for the above system, the DM specifies a boundary point $p_i < g_i$ for the fuzzy goal "$g_i(X) > g_i$". Three cases arise:

i) any alternative $x^1$ gives full satisfaction to the goal if "$g_i(x^1) \geq g_i$";

ii) $x^1$ gives partial satisfaction if "$g_i(x^1) \geq g_i$" is moderately violated, i.e., $p_i \leq G_i(x^1) < g_i$ and

iii) $x^1$ gives no satisfaction at all if $G_i(x^1) \geq g_i$" is strongly violated, i.e., "$G_i(x^1) \leq p_i$".

If no satisfaction and full satisfaction correspond to real numbers 0 and 1 respectively, then the levels giving partial satisfaction lying in the interval $(p_i, g_i)$ correspond to points in the open interval $(0,1)$. Then the membership function of the $i$th fuzzy set $G_i$ may be defined linearly Zimernann(1976, 1979) as follows:

$$\mu_{G_i}(x) = \frac{(G_i(x) - p_i)}{(g_i - p_i)} \quad \text{for } G_i(x) > g_i$$

$$\mu_{G_i}(x) = \frac{(G_i(x) - p_i)}{(g_i - p_i)} \quad \text{for } G_i(x) \in [p_i, g_i] \quad \ldots \ldots \quad (4.6)$$

$$\mu_{G_i}(x) = 0 \quad \text{for } G_i(x) < p_i$$

Here $\mu_{G_i}(x)$ represents the degree of satisfaction of the DM for the fuzzy goal "$g_i(x) > g_i$". Here, $\mu_{G_i}(x)$ is linear in $G_i(x)$ and therefore the marginal satisfaction of the DM is assumed to be constant. Similarly, for the fuzzy goal "$G_i(x) < g_i$", the linear membership function can be expressed as,
Here, $p_i$ represents the lower tolerance limit for fuzzy goal $G_i(x) > g_i$ and $b_i$ represents the upper tolerance limit for the fuzzy goal $G_i(x) < g_i$.

Having identified all other criteria as fuzzy sets defined over $S$, they are aggregated with a suitable operator, depending on the nature of criteria involved in the decision process, to give a fuzzy set of decision $D$. The membership function $\mu_D(X)$ (also termed as decision function) establishes an overall ordering of the alternative feasible solutions. The solution $X \in S$ which maximises $\mu_D(X)$ is the optimal solution.

The definition of various decision functions $\mu_D(X)$ for a MCDM problem with $k$-fuzzy criteria $g_i(X)$ depends on the operator used in aggregating the fuzzy sets $g_i$’s of criteria. They are enumerated as follows:

1. Based on intersection operator,

   $$\mu_D(X) = \mu_{g_1}(X) \wedge \mu_{g_2}(X) \ldots \ldots \mu_{g_k}(X) = \bigwedge_{i=1}^{k} \mu_{g_i}(X) \ldots \ldots (4.8)$$

2. For union operator,

   $$\mu_D(X) = \mu_{g_1}(X) \vee \mu_{g_2}(X) \ldots \ldots \mu_{g_k}(X) = \bigvee_{i=1}^{k} \mu_{g_i}(X) \ldots \ldots (4.9)$$

3. $\mu_D(X)$ based on algebraic product operator is given as

   $$\mu_D(X) = \mu_{g_1}(X) \pi \mu_{g_2}(X) \ldots \ldots \mu_{g_k}(X) = \pi_{i=1}^{k} \mu_{g_i}(X) \ldots \ldots (4.10)$$

4. Algebraic sum operator gives $\mu_D(X)$ as

   $$\mu_D(X) = 1 - \pi_{i=1}^{k} (1 - \pi \mu_{g_i}(X)) \ldots \ldots (4.11)$$

Tanaka et al., (1974), Negoita and Relescu (1975), Zimmermann (1976, 1978), Negoita (1981), Yager (1977, 1980) have discussed fuzzy mathematical programming (FMP) for single criterion decision making (SCDM) as well as multiple criteria decision making (MCDM) problems using $\mu_D(X)$ as in 4.8. Zimmermann (1978), Yager (1977, 1980) and Hannan (1979) have discussed
FMP with $\mu_D(X)$ as in 4.10. the reason for wide use of $\mu_I(X)$ based on intersection or algebraic product operator is that it reflects competition among the criteria involved and ensures a compromised minimum level of satisfaction of each of them. Neither the model 4.9 nor 4.11 has found much attention in devisiting any FMP because it reflects full non-competition (compensation) among the criteria and it is quite possible that at the optimal solution some of the criteria have zero attainment levels. Thus, decision functions $\mu_D(X)$ based on intersection and algebraic product operators reflect only competition, whereas, that based on union and algebraic sum operators reflect only compensation, but in real situations there is hardly any decision which does not exhibit competition and compensation simultaneously in varying degrees [Zimmermann and Zysno (1980)]. The present investigation derives its chief motivation because of the above mentioned compensatory - competitive characters of many a real decision situations.


Apart from the multi-objective optimisation one of the important application of fuzzy programming is in the area of multi-objective de-Novo programming problem. De-Novo programming deals with design of an optimal system rather than optimisation of given system. This is a promising tool for optimal design system with multiple objective and hence, can be termed as multi-objective de-Novo programming. In the special case where the number of decision variable equal to number of decision criteria the non dominated feasible solution can be obtained by solving a set of linear algebraic equation. However, there exist no algorithm for solving general multi-objective de-Novo programming which can be stated as : [Zeeleny (1981)]

$$\begin{align*}
\text{Max}_{X_j} & \sum_{k=1}^{n} C_{kj} X_j \quad k = 1, 2, \ldots, l \\
\end{align*}$$
Subject to
\[ \sum_{j=1}^{n} a_{ij} X_j - b_j \leq 0 : i = 1, 2, \ldots, m \] (4.12)
\[ \sum_{j=1}^{m} p_i b_j = B \]
\[ X_j \geq 0 \]

The concept of fuzzy decision making helps if formulating the problem in term of ideal and negative ideal solutions. So that original problem is converted into multi-objective de-Novo problem. A fuzzy methodology of such an approach [Li and Lee (1990)] has been developed for multi-objective de-Novo programming problem. An interactive approach of such a problem is also due to Mangaraj (1995, 1997).

Although various methodologies in the area of fuzzy multi-objective programming has been developed this cases of problem has been not so popular in the application front. But application of such problems has tremendous scope in various application areas as it involves a role of decision maker in the decision making process. Though, decision making problem can be broadly classified as structured decision making process and unstructured decision making process, fuzzy multi-objective programming comes in the later process. The knowledge base of the decision maker is incorporated in the process of decision making in the form of construction of membership functions and in the judgement stages of interactive approaches. In the subsequent chapters some of the algorithms of fuzzy multi-objective programming have been employed to solve multi-objective marketing decision problems. This algorithms have utilised interactive as well as global approaches, linear and piecewise linear membership functions, “min” and/or “addition” aggregation operators and have given results in the solution format for real as well as 0-1 variables. It has been observed that the solutions obtained from all this real life applications are quite satisfactory from implementation point of view.

4.4 CONCLUSION

Compared to various quantitative tools applied to the marketing problems, mathematical programming models have very less application eventhough it has sufficient scope in the area of marketing decision making. When multi-objective optimisation remains hard truth for only sort of development programs, marketing management area demands more application of such tools. The problem involves simultaneous optimisation of various developmental objectives within the available resources. This objectives may have various features viz., conflicting and non-commensurable even then the desired solution can be obtained by compromising there objectives to a certain
degree in comparable with decision maker. In the area multi criteria decision making (MCDM) literature, various techniques are available which can be implemented to a variety of marketing problems. With the introduction of fuzzy set thereby a new class of methodology have evolved taking the concept of fuzzy logic in the MCDM literature. These methods have an advantage over other methods in a sense that role of the decision maker is sufficiently highlighted in the decision making process unlike other MCDM process. Hence, the methodologies can be termed as the problem of satisfaction programming, while the satisfaction of decision maker remains as important variable. As, not much work has been dome in the area of application of fuzzy mathematical programming to marketing problems, this thesis has presented the application of fuzzy mathematical programming in terms of five algorithms applied to five different market segmentation decision problems.