CHAPTER 2

EFFECTS OF VOLUME SOURCES AND SINKS OF MASS ON THE FLOW OF AN ELECTRICALLY CONDUCTING FLUID IN A CHANNEL
2.1 INTRODUCTION

The study of magnetohydrodynamics is receiving considerable interest during recent years because of its importance from the energy point of view. Because of several reasons there has been a search for alternative source of energy throughout the world. One of the sources of energy has been suggested as MHD-generators, for power generation. From an application point of view, MHD-pumps are already in use in chemical engineering technology for pumping electrically conducting fluids in the Atomic Energy Centres.

The steady two-dimensional flow of an incompressible viscous fluid through a porous channel when the fluid is withdrawn from the channel walls at the same rate was studied by Berman [1,2]. Such a study is of importance in the problem of transpiration, cooling, boundary layer control and gaseous diffusion. The similarity solution for the flow obtained by Berman for small suction Reynolds number was extended by Sellars [3] for large suction Reynolds number and by Yuan [4] for large blowing Reynolds number. Yuan and Finkelstein [5], and Donoughe [6] considered different aspects of this problem while Morduchow [7] discussed the same problem by the method of averages. The extension of Berman's problem to magneto-
hydrodynamics was made by Rao [8], who studied the laminar steady flow of an incompressible viscous electrically conducting fluid through a channel with equally porous walls in the presence of a uniform transverse magnetic field. Assuming the Hartmann number and the suction Reynolds number to be small, he obtained a similarity solution of the flow field by neglecting the induced magnetic field. Using the same simplifying assumptions, Terrill and Shrestha [9] solved the above problem for all values of the Hartmann number. In a subsequent paper they [10] extended the above hydromagnetic problem to include large positive and large negative suction Reynolds number corresponding to suction and blowing at the channel walls.

There is a class of problems on flows through a channel, which admit similarity solutions. It was shown by Aladiev and Zaichik [11] that there exist similarity solutions of the Navier-Stokes equations in a non-porous channel when there is a uniform volume distribution of sources or sinks of mass in the flow.

Nanda [12] has studied hydromagnetic flow in a channel with volume sources and sinks of mass in the presence of uniform magnetic field applied transverse to the channel walls.
In this chapter the effect of electric field in conjunction with a uniform magnetic field, applied transverse to the channel walls on the flow with volume sources or sinks of mass has been studied.

The following aspects are considered in course of the present discussion.

I. When the induced magnetic field is negligible (which is justified for the flow of liquid metals), a similarity solution exists for the velocity field.

II. When the Hartmann number is very large, one would expect then Hartmann layers on the channel walls in which there is a balance between the viscous and the Lorentz forces. This case is studied under a singular perturbation problem.

III. When the strength of the volume sources or sinks of mass is small, the determination of the velocity field comes under regular perturbation problem.

It may be mentioned that this type of problem has bearing on magnetohydrodynamic flow in a channel with evaporation or condensation.
2.2 FORMULATION OF THE PROBLEM

We have considered a steady flow of an incompressible viscous electrically conducting fluid in a channel in the presence of a uniform transverse magnetic field $\vec{B}$ and an electric field $\vec{E}$. Assuming that the magnetic Reynolds number for the flow is very small so that we can neglect the induced magnetic field. (The flow of liquid metals under laboratory conditions is one of the various applications of such type of flow) We choose $x$-axis coincident with the central axis of the channel and $y$-axis normal to the plates of the channel. Fig 2.1 depicts the schematic diagram of the problem. The governing equations in usual notation are

$$\begin{align*}
\frac{du}{dx} + \frac{dv}{dy} &= - \frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0}{\rho} [E_0 + uB_0], \\
\frac{dv}{dx} + \frac{du}{dy} &= - \frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}, \quad (2.2.1)
\end{align*}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = - \frac{S}{\rho}, \quad (2.2.3)$$

where $\sigma$ is the electrical conductivity (assumed uniform) of the fluid, $S$ is the capacity of volume sources or sinks of mass. The presence of $S$ ($S>0$ corresponds to sinks, $S<0$ sources) in the equation of conservation of mass is given by (2.2.3) is due to evaporation or condensation taking place in the channel. We shall assume $S$ to be constant in the present analysis.
By group-theoretic methods (Na [13]), it can be shown that the above equations admit similarity solutions of the form.

\[ u = U (1 - N\eta/2) f'(\lambda); \quad v = (rS/2\rho) [f(\lambda) - \lambda] \quad (2.2.4) \]

where \( r \) is the distance between the plates of the channel, \( U \) is the average velocity at the inlet \( x = 0 \) and

\[ \eta = 2vx/Ur^2, \quad \lambda = 2y/r, \quad N = Sr^2/\rho\nu. \quad (2.2.5) \]

In equation (2.2.5), \( N \) is a dimensionless quantity expressing the intensity of volume sources or sinks. The two components of velocity in equation (2.2.4) are consistent with equation (2.2.3). Using equation (2.2.4) in equation (2.2.2), it can be shown that \( \partial p/\partial y \) is independent of \( x \) so that

\[ \frac{\partial^2 p}{\partial x \partial y} = 0 \quad (2.2.6) \]

Using equation (2.2.4) in equation (2.2.1), we find that

\[ f''' - M^2 f' + (N/4) [f' - (f - \lambda)f'''] + M^2 e = p^* \quad (2.2.7) \]

where a prime denotes differentiation with respect to \( \lambda \). Further, the Hartmann number \( M \), the dimensionless pressure gradient \( p^* \) and the electric field magnitude factor \( e \) are given by

\[ M = (B_0 r/2) (\sigma/\rho\nu)^{1/2}, \quad (2.2.8a) \]

\[ p^* = \frac{r^2}{4\nu\rho U(1-N\eta/2)} \frac{\partial p}{\partial x}, \quad (2.2.8b) \]
\[ e = \frac{-E_0}{B_0 U (1 - N \eta/2)}. \]  

(2.2.8c)

\( p^* \) being a constant because from equation (2.2.7), it is seen that left hand side is a function of \( \lambda \) but the right hand side is a function of \( \eta \). It can be seen that equation (2.2.8b) is compatible with equation (2.2.6).

As the flow is symmetric about the central axis \( (y=0) \) of the channel, the boundary conditions are

\[ \frac{\partial u}{\partial y} = 0, \quad v = 0 \quad \text{at} \quad y = 0 \]  

(2.2.9)

and the no slip conditions are

\[ u = v = 0 \quad \text{at} \quad y = r/2. \]  

(2.2.10)

Since the induced magnetic field is neglected, there are no explicit boundary conditions for the magnetic field. Equations (2.2.4), (2.2.9) and (2.2.10) then give the following boundary conditions for \( f(\lambda) \):

\[ f(0) = 0, \quad f''(0) = 0, \quad f(1) = 1, \quad f'(1) = 0. \]  

(2.2.11)

### 2.3 Solution of the Singular Perturbation Problem for Large Hartmann Number

Now we wish to solve equation (2.2.7) satisfying equation (2.2.11) for \( M \gg 1 \). One would expect that for large Hartmann number \( M \), viscous forces and the magnetic forces
(Lorentz forces) would be of comparable magnitude in a thin layer near the walls $\lambda = \pm 1$. This would lead to a singular perturbation problem and the method of matched asymptotic expansions has been employed to solve this problem. This method is given in Van Dyke [14]. Although equation (2.2.7) is of third order, $p^*$ is unknown and this is found by solving equation (2.2.7) subject to the boundary conditions (2.2.11).

Now equation (2.2.7) is written as

$$\varepsilon^2 f'' - f' + (N \varepsilon^2 / 4) [f'^2 - (f - \lambda) f''] = p^* \varepsilon^2 - e, \quad (2.3.1)$$

where ($\varepsilon = 1/M$) is much smaller than unity. Outside the Hartmann layer, we take the outer expansion for $f$ as

$$f^{(ou)} = f_0^{(ou)} + \xi f_1^{(ou)} + \xi^2 f_2^{(ou)} + \ldots \ldots \ldots \ldots (2.3.2)$$

with the dimensionless pressure gradient $p^*$ as

$$p^* = (C_0/\varepsilon^2) + (C_1/\varepsilon) + C_2 + C_3 \varepsilon + \ldots \ldots \ldots (2.3.3)$$

where the constants $C_0, C_1, C_2, \ldots$ are to be determined. From equation (2.2.11), the outer boundary conditions are

$$f_1^{(ou)} = 0, \quad f_i^{(ou)}(0) = 0, \quad i = 0, 1, 2, 3, \ldots \ldots \ldots \ldots (2.3.4)$$

In view of symmetry, we are considering flow only in the upper half of the channel. Substitution of equations (2.3.2) and (2.3.3) in equation (2.3.1) gives on equating different powers of $\varepsilon$:

$$-f_0^{(ou)}' = C_0 - e; \quad -f_1^{(ou)}' = C_1;$$

$$f_0^{(ou)}' - f_2^{(ou)}' + (N/4) [f_0^{(ou)}'' - (f_0^{(ou)} - \lambda) f_0^{(ou)}] = C_2 \quad (2.3.5)$$
The solutions of equation (2.3.5) satisfying equation (2.3.4) are
\[ f_0(\xi) = -(C_0 - \varepsilon)X; \quad f_1(\xi) = -C_1\lambda; \]
\[ f_2(\xi) = N/4 (-C_0 + \varepsilon)^2 - C_1\lambda \]  
(2.3.6)

To derive the inner expansion which will be valid in the Hartmann layer and to avoid the non-linearity in equation (2.3.1) we rescale the variables as follows
\[ \zeta = \frac{1 - f}{\varepsilon}, \quad \xi = \frac{1 - \lambda}{\varepsilon} \]  
(2.3.7)

Substitution of equation (2.3.7) in equation (2.3.1) leads to
\[ \frac{d^3\zeta}{d\xi^3} - \frac{d\zeta}{d\xi} + \frac{Ne^2}{4} [\left(\frac{d\zeta}{d\xi}\right)^2 + (\xi - \zeta) \frac{d^2\zeta}{d\xi^2}] \]
\[ = (C_0 + C_1\varepsilon + C_2\varepsilon^2 + \ldots) - \varepsilon \]  
(2.3.8)

The corresponding inner boundary conditions derived from (2.2.11) are
\[ \zeta(0) = 0, \quad \zeta'(0) = 0 \]  
(2.3.9)

Note that the inner expansion is required to satisfy only the no slip conditions at the channel wall. Let us expand \( \zeta \) as
\[ \zeta = \zeta_0 + \varepsilon \zeta_1 + \varepsilon^2 \zeta_2 + \ldots \]  
(2.3.10)

Substituting equation (2.3.10) in equation (2.3.8), and equating different powers of \( \varepsilon \) we get
\[ \frac{d^3\zeta_0}{d\xi^3} - \frac{d\zeta_0}{d\xi} = C_0 - \varepsilon \]  
(2.3.11)
From equation (2.3.9), the boundary conditions for \( \zeta_i \) are given by

\[
\zeta_i(0) = 0, \quad \zeta_i'(0) = 0 \quad (i = 0, 1, 2).
\]

(2.3.14)

The solutions of equation (2.3.11) satisfying equation (2.3.14) and neglecting higher powers of exponential terms are

\[
\zeta_0 = (C_0 - e)(1 - \xi - e^{-\xi})
\]

(2.3.15)

\[
\zeta_1 = C_1 (1 - \xi - e^{-\xi})
\]

(2.3.16)

Using equations (2.3.15), (2.3.16), (2.3.7) and (2.3.10), the three-term inner expansion for \( f \) is obtained as

\[
f_{[\text{in}]} = 1 - \xi(C_0 - e)(1 - \xi - e^{-\xi}) - \xi^2 C_1 (1 - \xi - e^{-\xi})
\]

(2.3.17)

On the other hand the three-term outer expansion for \( f \) is obtained from equation (2.3.2) and equation (2.3.6) as
To determine $C_0$, $C_1$ and $C_2$ we use the asymptotic matching principle of Van Dyke [14]:

The $m$-term inner expansion of (the $n$-term outer expansion) =
the $n$-term outer expansion of (the $m$-term inner expansion),
where $m$ and $n$ are any two integers such that $m$ is either $n$ or
$n + 1$.

Thus

\[
\begin{align*}
    f^{(\text{in})}_{1-\text{term}} &= 1 \\
    &= 1 \text{ (rewritten in outer variable)} \\
    &= 1 \text{ (expanded in powers of $\epsilon$).} \quad (2.3.19)
\end{align*}
\]

\[
\begin{align*}
    f^{(\text{ou})}_{1-\text{term}} &= -(C_0 - e) \lambda \\
    &= -(C_0 - e)(1 - \epsilon \xi) \text{ (rewritten in inner variable)} \\
    &= -(C_0 - e) \text{ (expanding in powers of $\epsilon$ and retaining one term).} \quad (2.3.20)
\end{align*}
\]

Matching equation (2.3.19) and equation (2.3.20), we find

\[
C_0 = -1 + \epsilon \quad (2.3.21)
\]

Similarly using two-term and three-term expansions for $f^{(\text{in})}$ and
$f^{(\text{ou})}$, on using equation (2.3.21) we have

...
Matching equation (2.3.22) and equation (2.3.23), we have
\[ C_1 = -1 \]  
(2.3.24)

\[ f_{\text{in}}^{\text{3-term}} = 1 + \varepsilon(1-\xi-e^\xi) + \varepsilon^2(1-\xi-e^\xi) \]  
(rewriting in outer variable and neglecting transcendentally small term)  
\[ = 1 + \varepsilon\{1-(1-\lambda)/\varepsilon\} + \varepsilon^2\{1-(1-\lambda)/\varepsilon\} \]  
(expanding in powers of \( \varepsilon \) and retaining three terms) (2.3.25)

\[ f_{\text{out}}^{\text{3-term}} = \lambda + \varepsilon\lambda + \varepsilon^2\{(N/4) - C_2\}\lambda \]  
(rewritten in inner variable)  
\[ = 1 - \varepsilon\xi + \varepsilon(1 - \varepsilon\xi) + \varepsilon^2\{(N/4) - C_2\}(1 - \varepsilon\xi) \]  
(rewritten in inner variable)  
\[ = 1 - \varepsilon\xi + \varepsilon - \varepsilon^2\xi + \varepsilon^2\{(N/4) - C_2\} \]
(expanding in powers of $\epsilon$ and retaining terms up to $O(\epsilon^2)$)

$$= \lambda + \epsilon \lambda + \epsilon^2 \{(N/4) - C_1\} \quad (2.3.26)$$

Matching equation (2.3.25) and equation (2.3.26), we find

$$C_1 = (N/4) - 1 \quad (2.3.27)$$

Now from equations (2.3.1), (2.3.2) and (2.3.3), the equation for $f_1$ is

$$f_1^{(ou)} - f_3^{(ou)} + (N/4)[2f_0^{(ou)}f_1^{(ou)} - (f_0^{(ou)}-\lambda)f_1^{(ou)}'' - f_1^{(ou)}f_0^{(ou)''}] = C_3 \quad (2.3.28)$$

Substitution from equations (2.3.6), (2.3.21), (2.3.24) and (2.3.27) in equation (2.3.28) and subsequent integration leads to

$$f_1^{(ou)} = \{(N/2) - C_1\} \lambda, \quad (2.3.29)$$

which satisfies equation (2.3.4).

Further solving equation (2.3.13) together with equation (2.3.15) and equation (2.3.21), we find

$$\zeta_1(\xi) = \xi + e^{-\xi} + (N/8)[\xi + 1]e^{-\xi} - \{(N/8) + 1\}, \quad (2.3.30)$$

which satisfies the boundary conditions (2.3.14).

Thus the four-term inner and outer expansions for $f$ are

$$f^{(in)} = 1 - \epsilon(e^{-\xi} + \xi - 1) - \epsilon^2(e^{-\xi} + \xi - 1)$$

$$- \epsilon^3[\xi + e^{-\xi} + (N/8)[\xi + 1]e^{-\xi} - \{(N/8) + 1\}] \quad (2.3.31)$$
Matching equation (2.3.31) with equation (2.3.32), we get
\[ C_3 = \left( \frac{3N}{8} \right) - 1. \]  

Hence, for large \( M \), the pressure gradient \( p^* \) given by equation (2.3.3) becomes
\[ p^* = -\left(1-e\right)M^2 - M + \{(N/4)-1\} + (1/M)\{(3N/8)-1\} + O(1/M^2) \]  
and the four-term outer expansion for \( f \) is
\[ f^{(ou)} = \lambda + (\lambda/M) + (\lambda/M^2) + \{(N/8) + 1\} \lambda/M^3 + \ldots. \]  

2.4 SOLUTION FOR SMALL \( N \) AND ANY VALUE OF \( M \)

It may be seen from equation (2.2.7) that when \( N \) is small, the solution of equation (2.2.7) satisfying equation (2.2.11) gives rise to a regular perturbation problem unlike the one in section (2.3). To this end we expand \( f \) and \( p^* \) as follows,
\[ f = f_0 + Nf_1 + N^2f_2 + \ldots \]  
\[ p^* = P_0' + NP_1' + N^2P_2' + \ldots \]  

Using (2.2.11), the boundary condition for \( f \) are taken as
\[ f_0(0) = f''_0(0) = 0, \quad f_0(1) = 1, \quad f'_0(1) = 0 \]  
\[ f_i(0) = f''_i(0) = 0, \quad f_i(1) = 1, \quad f'_i(1) = 0 \quad (i = 1, 2, \ldots) \]

Substituting equation (2.4.1) and equation (2.4.2) in equation (2.2.7) and equating coefficients of \( N^*, N, N^2 \ldots \), we get
\[ f_0'' - M^2 (f'_0 - e) = P_0' \]  
\[ f_1'' - M^2 f'_1 + (1/4) [f''_0 - (f_0 - \lambda)f_0'] = P_1' \]  
\[ f_2'' - M^2 f'_2 + \frac{1}{2} [2 f_0' f'_1 - f''_1 (f_0 - \lambda) + f_0'' f_1'] = P_2' \]

The solution of equation (2.4.5) satisfying equation (2.4.3) is given by

\[ f_0(\lambda) = \frac{\sinh M \lambda - \lambda M \cosh M}{\sinh M - M \cosh M} \]  

with

\[ p'_0 = M^2 e + \frac{(M^2 \cosh M)}{(\sinh M - M \cosh M)} \]

Now substituting \( f_0(\lambda) \) in equation (2.4.6) and solving the equation (2.4.6) subject to boundary conditions (2.4.4), the expression for \( f_1(\lambda) \) is obtained as

\[ f_1(\lambda) = \frac{A_1}{M} \sinh M \lambda + \lambda \left[ \frac{1 + \cosh^2 M}{4(\sinh M - M \cosh M)^2} - \frac{P_1'}{M^2} \right] \]

\[ + \frac{M \cosh M}{4(\sinh M - M \cosh M)^2} \left[ \frac{\lambda \cosh M \lambda}{M} - \frac{\sinh M \lambda}{M^2} \right] \]

\[ - \frac{M \sinh M}{16(\sinh M - M \cosh M)^2} \left[ \frac{\lambda^2 \sinh M \lambda}{M} - \frac{\lambda \cosh M \lambda}{M} - \frac{\sinh M \lambda}{M^2} \right] \]

where

\[ ... \]
\[ A_1 = \frac{M^2}{4(sinhM-McoshM)^3} \left[ \frac{sinh^2M-2cosh^2M}{2M} \right. \]
\[ \left. + \frac{sinhM coshM}{4M^2} \cdot \frac{3}{4} sinhM coshM + \frac{3 sinh^2M}{4M^3} \right] \] (2.4.11)

and

\[ p_1' = \left[ 4M^2 (2coshM + cosh^2M) - 3M sinh^2M coshM \right. \]
\[ \left. - 3M^2(3sinhM + 2sinh^B_M) \right] / [16(McoshM - sinhM)^2] \] (2.4.12)

Similarly, substituting for \( f_0(\lambda) \) from equation (2.4.8), \( f_1(\lambda) \) from equation (2.4.10) and solving the resulting equation (2.4.7) subject to (2.4.4) the expression for \( f_2(\lambda) \) is obtained as

\[
\begin{align*}
    f_2(\lambda) &= A_{13} \frac{sinh M \lambda}{M cosh M - sinh M} + \frac{A_1}{4M^3 (M cosh M - sinh M)} \\
    &\left[ sinh(M \lambda - M) - sinh M \lambda + M cosh(M \lambda - M) - M cosh M + sinh M \right] \\
    &\left\{ \lambda \left( \frac{A_8}{4M^2} - \frac{P_2}{M^2} \right) - \frac{1}{4} \frac{A_1}{2M^2} \left\{ \lambda \ sinh M \lambda + \frac{2}{3} (\lambda^2 M^2 + 2 \lambda) \right\} \right. \\
    &\left. + \frac{A_8}{2M^2} \lambda cosh M \lambda + \frac{A_9}{2M^2} \left\{ \left( \frac{\lambda^3}{3} + \frac{7 \lambda}{2M^2} - \frac{3 \lambda}{2M} \right) cosh M \lambda \right. \\
    &\left. + \left( \frac{\lambda^2}{2} - \frac{3 \lambda^3}{2M^2} \right) sinh M \lambda \right\} - \frac{A_{10}}{2M^2} \left\{ \left( \frac{\lambda^4}{4} + \frac{21 \lambda^2}{4M^2} \right) sinh M \lambda \right. \\
    &\left. - \left( \frac{3 \lambda^3}{2M} + \frac{45 \lambda}{4M^3} \right) cosh M \lambda \right\} \right. \\
\end{align*}
\] (2.4.13)
where

\[ P_1' = \frac{[\cosh M \sinh M - \sinh M - 32M^2(A_{12} + A_{13})\sinh M](\sinh M - M\cosh M)^2 + 32A_{12}M^2\cosh M(\sinh M - M\cosh M)^2}{32(\sinh M - M\cosh M)^4} \]

\[ A_2 = \frac{1}{\sinh M - M\cosh M} \]

\[ A_3 = \frac{1 + \cosh^2 M}{4(\sinh M - M\cosh M)^2} - \frac{p_1'}{M^2} \]

\[ A_4 = \frac{\cosh M}{4M(\sinh M - M\cosh M)^2} \]

\[ A_5 = \frac{\sinh M}{16M^2(\sinh M - M\cosh M)^2} \]

\[ A_6 = 2A_1A_2M - 2A_3A_2M \cosh M \]

\[ A_7 = 2M^2A_2A_5 \]

\[ A_8 = 2M(A_2A_3 - A_1A_2\cosh M) \]

\[ A_9 = M^6(A_2A_5M\cosh M + A_4A_2M\cosh M + A_4 - A_5) \]

\[ A_{10} = (A_1M - A_2A_4M^2\cosh M - A_2A_5M^2\cosh M + A_1A_2M^2\cosh M + A_4M^2 + A_5M^2 - A_2A_3M^2) \]

\[ A_{11} = A_5^2 A_2^5 M^4 \cosh M + A_5 M^4 \]
\[ A_{12} = \frac{1}{4} \left[ -\frac{A_6}{M^2} + \frac{A_7}{3M^2} (M^2 + 2) + \frac{A_8}{2M^2} \sinh M \right. \\
\quad + \frac{A_9}{2M^2} \left( \cosh M \right. \left. + \frac{A_{10}}{2M^2} \left\{ \cosh M + 7 \cosh M - \frac{3}{2} \sinh M \right\} \right) \]

\[ + \frac{A_{10}}{2M^2} \left\{ \sinh M - \frac{3}{2} \cosh M \right\} - \frac{A_{11}}{2M^2} \left\{ \frac{\sinh M}{4} + \frac{21 \sinh M}{4M^2} \right\} \]

\[ - \frac{3}{2M^2} \left\{ \frac{\cosh M}{4} - \frac{45 \cosh M}{4M^2} \right\} \]

\[ + \frac{A_{12}}{2M^2} \left\{ \frac{\sinh M}{6} - \frac{5 \cosh M + 2 \sinh M}{M} \right\} \]

\[ + \frac{A_{13}}{2M^2} \left\{ \cosh M - \frac{3 \sinh M}{4} - \frac{6 \sinh M}{M^2} \right\} \]

It can be shown from equation (2.4.9) and equation (2.4.12) that

\[ \lim_{M \to 0} P''_0 = -3 \quad \text{(2.4.15)} \]

\[ \lim_{M \to 0} P''_1 = \frac{3}{7} \]

which agrees with Aladiev and Zaichik [11].

Using (2.2.4), the shear stress at the upper wall is

\[ \left[ \mu \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right]_{y = \pi / 2} = \frac{2 \mu U}{I} \left( 1 - \frac{M}{2} \right) F''(1) \quad \text{(2.4.16)} \]
2.5 DISCUSSIONS AND RESULTS

(a) METHOD OF MATCHED ASYMPTOTIC EXPANSIONS

From the equation (2.3.31) for the inner expansion with \( \varepsilon = 1/M \) indicates the growth of boundary layer near the channel wall for large \( M \), the order of the boundary layer is of order \( 1/M \).

From the equation (2.3.34) it is clear that the rate of pressure drop increases with the increase in the value of electric field strength \( e \). Further, the equation shows that when \( N > 0 \) (volume sinks of mass), the rate of pressure drop decreases with decrease in \( N \). But when \( N < 0 \) (volume sources of mass), rate of pressure drop increases with an increase in the value of \( |N| \).

Again, it is observed that an increase in the Hartmann number \( M \) increases the pressure gradient along the channel for fixed mass source/sink strength.

(b) METHOD OF REGULAR PERTURBATION

The following discussions hold good for small \( N \) and any value of \( M \).

Fig. 2.2 shows the variation of \( f(\lambda) \) with \( \lambda \) for several values of \( M \) with \( N = 0.02 \). It is observed that an increase in the value of Hartmann number \( M \) results in decrease of \( f \). This physically means that the magnetic field has the decelerating influence on the normal component \( f(\lambda) \).
Fig. 2.3 shows that an increase in M decreases $f'(\lambda)$ (the velocity component parallel to the channel walls) near the wall ($\lambda=0$) but at the other wall the effect is reversed. It is further seen that over the central portion of the channel variation of $f'$ is very less. Moreover, an increase in M results in a progressive flattening of $f'$. These observations agree with [12].

It is interesting to note that with a moderately large value of M (M=10), $f'$ maintains uniform value upto the middle of the channel then there is a sharp decrease, showing a steep gradient in $f'$. This exhibits the boundary layer behaviour near the walls.

The numerical values of $f''(1)$ are entered in Table 2.1. It is seen that an increase in N (volume sinks of mass) increases the wall shear stress. This means that an increase in the rate of pressure drop increases the wall shear stress. This result has been already derived on the forgoing discussion. Further, for fixed N, the wall shear stress decreases with increase in Hartmann number which is a direct consequence of the flattening of the velocity profile with increase in M.

Values of $f(\lambda)$ and $f'(\lambda)$ have been computed for several values of N with M=10. The numerical values are entered in Table 2.2 and Tab. 2.3. It is seen that both the
velocity components increase with the increase in the value of N for a fixed value of M.

2.5 CONCLUSION

(i) The rate of pressure drop increases with the increase in the value of electric field intensity.

(ii) The magnetic field has decelerating influence on the normal components of the velocity field.

(iii) An increase in N (N > 0) increases wall shear stress.

(iv) The rate of pressure drop decreases with the decrease in the value of N (for N > 0 and N < 0)
Fig. 2.1. SKETCH OF THE PHYSICAL PROBLEM.
Fig. 2. VARIATION OF $f(\lambda)$ WITH $\lambda$ FOR SEVERAL VALUES OF $M$ WITH $N = 0.02$
Fig. 2.3. VARIATION OF $f'(\lambda)$ WITH $\lambda$ FOR SEVERAL VALUES OF $M$ WITH $N = 0.02$
### TABLE 2.1
VALUES OF $f''(1)$

<table>
<thead>
<tr>
<th>N\M</th>
<th>10</th>
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<th>20</th>
<th>25</th>
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<tr>
<td>0.02</td>
<td>-11.110826</td>
<td>-16.071247</td>
<td>-21.052496</td>
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<tr>
<td>0.04</td>
<td>-11.110541</td>
<td>-16.071064</td>
<td>-21.052361</td>
<td>-26.041458</td>
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<tr>
<td>0.06</td>
<td>-11.110257</td>
<td>-16.070883</td>
<td>-21.052227</td>
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</tbody>
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TABLE 2.2  
VALUES OF $f(\lambda)$ FOR $M = 10$

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<th>$\lambda$ \ N</th>
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<th>0.04</th>
<th>0.06</th>
</tr>
</thead>
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<td>0.11110004</td>
<td>0.11110043</td>
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<tr>
<td>0.2</td>
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<td>0.22218718</td>
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<tr>
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<td>0.44417062</td>
<td>0.44417214</td>
<td>0.44417363</td>
</tr>
<tr>
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<td>0.66463363</td>
<td>0.66463566</td>
<td>0.66463763</td>
</tr>
<tr>
<td>0.8</td>
<td>0.87385321</td>
<td>0.87385482</td>
<td>0.87385637</td>
</tr>
<tr>
<td>\lambda</td>
<td>0.02</td>
<td>0.04</td>
<td>0.06</td>
</tr>
<tr>
<td>--------</td>
<td>-----------</td>
<td>-----------</td>
<td>-----------</td>
</tr>
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    and Finkelstein, A.B.
    and Shrestha, G.M.
    and Zaichik, L.T.