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# CHAPTER 1

# INTRODUCTION

*"The scientist does not study nature because it is useful; he studies it because he delights in it, and he delights in it because it is beautiful. If nature were not beautiful, it would not be worth knowing, and if nature were not worth knowing, life would not be worth living."*

*Henri Poincare*

## 1.1 Introduction

Time series is a sequence of measurements at successive times. Study of these time series are important part of scientific studies. Economists, Astrophysicists, meteorologists etc. analyze time series for different types of predictions. Cryptographers and communication engineers analyze the time series to extract a deterministic signal from a noisy background. For a researcher the motive of the analysis is to have better insight and understanding of the underlying dynamics. Linear time series analysis is an established field with very rich background of literature. However in the late half of 20$^{th}$ century it was realized that time series of simple nonlinear systems show very complex behavior and linear methods of analysis may give misleading results. New analysis techniques have been developed specifically to characterize nonlinear systems and availability of powerful computers had brought these methods to the desktop of most scientists and researchers and new tests are constantly being developed.

Linear methods interpret regular structure in a data set, such as a dominant frequency, through linear correlations. The intrinsic dynamics of the linear system are governed by the linear paradigm that small causes lead to small effects. Since linear equations can only lead to exponentially decaying (or growing) or (damped) periodically oscillating solutions, all irregular behavior of the system has to be attributed to some random external input to the system. However nonlinear dynamics has shown that random input is not the only possible source of irregularity in a system's output; nonlinear, chaotic systems can also produce very irregular data with purely deterministic equations of motion in an autonomous way, i.e., without time dependent inputs. Of course, a system which has both, nonlinearity and random input, will most likely produce irregular data as well. There is a broad range of questions that is to be addressed when discussing about nonlinear time series analysis. Various methods have been developed to get the most precise and meaningful results for a clean and clearly deterministic data sets. Many modifications are necessary if the data are not properly described by a linear model with Gaussian inputs? Depending on the data sets and the analysis task, different approaches of analysis are chosen. However, there are a number of general ideas that one should be acquainted with no matter what one's data look like. Nonlinear time series analysis is not as well established and is far less well understood than its linear counterpart.

## 1.2 Review of previous work

The field of economic studies by physicists can trace its roots to Newton and Copernicus, who worked extensively on economic problems, and to a number of others over the centuries who applied to economics the fundamental approach of physics. Some authors consider the works of Walras or Pareto as a first application of physics in economics [1-3, 65]. E.H. Stanley coined the term “Econophysics” to describe efforts to apply physics approaches to various problems of interest in economics [4]. Unlike traditional topics in physics, where collecting data often requires imagination and sometimes years of painstaking labor, in the case of price changes every transaction of every stock is recorded and stored. P. Gopikrishnan andV. Plerou [5-6] analyzed 200 million data of different stocks, and reported a probability density function (pdf) of price changes that was not Gaussian plus outliers, as previously believed. All the data, including data previously termed outliers, conformed to a single pdf encompassing both everyday fluctuations and “once in a century” fluctuations. Instead of a Gaussian or some correction to a Gaussian, a power law pdf with exponent -4 was found. Since the 1970s, a series of significant changes has taken place in the world of finance. In 1973 currencies began to be traded in financial markets and their values determined by the foreign exchange market, a financial market active 24 hours a day all over the world. During that same year, Black and Scholes [7] published the first paper that presented a rational option-pricing formula. Since that time, the volume of foreign exchange trading has been growing at an impressive rate.

A second revolution began in the 1980s when electronic trading, already a part of the environment of the major stock exchanges, was adapted to the foreign exchange market. The electronic storing of data relating to financial contracts - or to prices at which traders are willing to buy (bid quotes) or sell (ask quotes) a financial asset - was put in place at about the same time that electronic trading became widespread. As a result today a huge amount of electronically stored financial data is available. These data are characterized by the property of being high-frequency data - the average time delay between two records can be as short as a few seconds. The enormous expansion of financial markets requires strong investments in money and human resources to achieve reliable quantification and minimization of risk for the financial institutions involved.

The econophysics studies to financial time series data includes concepts as power-law distributions, correlations, scaling, unpredictable time series and random processes etc. During the past 30 years, physicists have achieved important results in the field of phase transitions, statistical mechanics, nonlinear dynamics, and disordered systems. In these fields, power laws, scaling, and unpredictable (stochastic or deterministic) time series are present and the current interpretation of the underlying physics is often obtained using these concepts. The first use of a power-law" distribution - and the first mathematical formalization of a random walk - took place in the social sciences. Pareto investigated the statistical character of the wealth of individuals in a stable economy by modeling them using the distribution

$$y \sim x^{-\alpha} \tag{1.1}$$

where $y$ is the number of people having income $x$ or greater than $x$ and $\alpha$ is an exponent that Pareto estimated to be 1.5 [8]. This result was quite general and applicable to nations as different as those of England, of Ireland, of Germany, of the Italian cities, and even of Peru.

However the concept of a power-law distribution is counterintuitive, because it may lack any characteristic scale. This property prevented the use of power-law distributions in the natural sciences until the recent emergence of new paradigms (i) in probability theory, through the work of Levy [9] and through the application of power-law distributions to several problems pursued by Mandelbrot [10]; and (ii) in the study of phase transitions, which introduced the concepts of scaling for thermodynamic functions and correlation functions [11].

The other concept ubiquitous in the natural sciences is the random walk. The first theoretical description of a random walk in the natural sciences was performed in 1905 by Einstein [12] in his famous paper dealing with the determination of the Avogadro number. The mathematics of the random walk was made more rigorous by Wiener [13].

However the first formalization of a random walk was given in a doctoral thesis by Bachelier [14]. Bachelier presented his thesis for the degree of *Docteur en Sciences Mathematiques*. His thesis, entitled *Theorie de ta speculation* deals with the pricing of options in speculative markets. The probability density function (pdf) of price changes was studied by Bachelier by proposing the drunkard’s walk model. Bachelier determined the probability of price changes by writing down the Chapman-Kolmogorov equation and recognizing a Wiener process satisfies the diffusion equation. This point was rediscovered by Einstein in his 1905 paper on Brownian motion.

Later on, as more data became available, it became clear that the drunkard's walk model does not satisfactorily describe all the data. The term "fat tail" was used to describe the mathematical counterpart of this statement, that the pdf of price changes contains many more events in the tail than predicted by the Gaussian pdf characterizing the drunkard's walk.

The problem of the distribution of price changes has been considered by several authors since the 1950s, Bachelier's original proposal of Gaussian distributed price changes was replaced by a model in which stock prices are log-normal distributed, i.e., stock prices are performing a geometric Brownian motion. In a geometric Brownian motion, the differences of the logarithms of prices are Gaussian distributed. This model provides only a first approximation of what is observed in real data. A number of alternative models have been proposed with the aim of explaining the empirical evidence that the tails of measured distributions are fatter than expected for a geometric Brownian motion; and (the time fluctuations of the second moment of price changes).

Among the alternative models proposed, [15], is Mandelbrot's hypothesis that price changes follow a Levy stable distribution [16]. Levy stable processes are stochastic processes obeying a generalized central limit theorem. By obeying a generalized form of the central limit theorem, these processes have a number of interesting properties. They are stable (as are the more common Gaussian processes) i.e., the sum of two independent stochastic processes, characterized by the same Levy distribution of index $\alpha$ is itself a stochastic process characterized by a Levy distribution of the same index. The shape of the distribution is maintained (is stable) by summing up independent identically distributed Levy stable random variables.

Levy stable processes define a basin of attraction in the functional space of probability density functions. The sum of independent identically distributed stochastic processes characterized by a probability density function with power-law tails,

$$P(x) \sim x^{-(1+\alpha)} \tag{1.2}$$

will converge, in probability, to a Levy stable stochastic process of index a when *n* tends to infinity [17].

The distribution of a Levy stable process is a power-law distribution for large values of the stochastic variable *x*. The fact that power-law distributions may lack a typical scale is reflected in Levy stable processes by the property that the variance of Levy stable processes is infinite for $\alpha < 2$. Stochastic processes with infinite variance, although well defined mathematically, are extremely difficult to use and, moreover, raise fundamental questions when applied to real systems. In physical systems the second moment is often related to the system temperature, so infinite variances imply an infinite (or undefined) temperature. In financial systems, an infinite variance would complicate the important task of risk estimation.

Another approach in this field is the nonlinear approach. A widely accepted belief in financial theory was that time series of asset prices are unpredictable. This belief was the cornerstone of the description of price dynamics as stochastic processes. Since the 1980s it has been recognized in the physical sciences that unpredictable time series and stochastic processes arc not synonymous.

Specifically, chaos theory has shown that unpredictable time series can arise from deterministic nonlinear systems. The results obtained in the study of physical, biological and other systems triggered an interest in economic systems, and theoretical and empirical studies have investigated whether the time evolution of asset prices in financial markets might indeed be due to underlying nonlinear deterministic dynamics of a (limited) number of variables.

One of the goals of researchers studying financial markets with the tools of nonlinear dynamics has been to reconstruct the (hypothetical) strange attractor present in the chaotic time evolution and to measure its dimension $d$. The reconstruction of the underlying attractor and its dimension $d$ is not an easy task. The more reliable estimation of $d$ is the inequality $d > 6$. For chaotic systems with $d > 3$, it is rather difficult to distinguish between a chaotic time evolution and a random process, especially if the underlying deterministic dynamics are unknown. Financial markets exhibit several of the properties that characterize complex systems. They are open systems in which many subunits interact nonlinearly in the presence of feedback. In financial markets, the governing rules are rather stable and the time evolution of the system is continuously monitored. It has become possible to develop models and to test their accuracy and predictive power using available data, since large databases exist even for high-frequency data.

A growing number of physicists have attempted to analyze and model financial markets and, more generally, economic systems. The interest of this community in financial and economic systems has roots that date back to 1936, when Majorana wrote a pioneering paper on the essential analogy between statistical laws in physics and in the social sciences [18]. This unorthodox point of view was considered of marginal interest until recently. Indeed, prior to the 1990s, very few professional physicists did any research associated with social or economic systems. The exceptions included Kadanoff [19], Montroll [20], and a group of physical scientists at the Santa Fe Institute [21].

Since 1990, the physics research activity in this field has become less episodic and a research community has begun to emerge.

Among the important areas of physics research dealing with financial and economic systems, one concerns the complete statistical characterization of the stochastic process of price changes of a financial asset. Several studies have been performed that focus on different aspects of the analyzed stochastic process, e.g., the shape of the distribution of price changes [22-27], the temporal memory [28-31], and the higher-order statistical properties [32-34]. The attempts are ongoing to develop the most satisfactory stochastic model describing all the features encountered in empirical analyses. One important accomplishment in this area is an almost complete consensus concerning the finiteness of the second moment of price changes. This has been a longstanding problem in finance, and its resolution has come about because of the renewed interest in the empirical study of financial systems.

A second area concerns the development of a theoretical model that is able to encompass all the essential features of real financial markets. Several models have been proposed [35-47], and some of the main properties of the stochastic dynamics of stock price are reproduced by these models as, for example, the leptokurtic 'fat-tailed' non-Gaussian shape of the distribution of price differences. Parallel attempts in the modeling of financial markets have been developed by economists [48-50].

Other areas that are undergoing intense investigations deal with the rational pricing of a derivative product. The Black & Scholes option-pricing model was published in 1973, almost three-quarters of a century after the publication of Bachelier's thesis. However the Black & Scholes model also needs correction in its application, meaning that the problem of which stochastic process describes the changes in the logarithm of prices in a financial market is still an open one.

Other models are also proposed in which some of the canonical assumptions of the Black & Scholes model are relaxed [51, 52] and with aspects of portfolio selection and its dynamical optimization [53-57]. A further area of research considers analogies and differences between price dynamics in a financial market and such physical processes as turbulence [58] and ecological systems [59].

One common theme encountered in these research areas is the time correlation of a financial series. The detection of the presence of a higher-order correlation in price changes has motivated a reconsideration of some beliefs of what is termed 'technical analysis' [60].

In addition to the studies that analyze and model financial systems, there are studies of the income distribution of firms and studies of the statistical properties of their growth rates [61-63]. The statistical properties of the economic performances of complex organizations such as universities or entire countries have also been investigated [64]

## 1.3 Random Walk Model for stock indices and Stock prices

. The simplest possible model of stock price or stock index $x$ can be given by Random walk. Stock prices and indices are random variables of time and cannot be described using conventional calculus methods. It can be described quantitatively using Ito calculus.

Price change is assumed to be performed with a coin flip. Head corresponds to a change of +1 where as tail correspond to a change of -1. Let the variation of index or price be in units (i.e. 1 Rs or in units of index) then the Return can have only two values ($R_i = \pm 1$) and the cumulative return after $i^{th}$ toss will be

$$S_i = \sum_{j=1}^{i} R_j$$

In this case the price variation follows the Markov property, i.e. the outcomes of the next future event totally depends on the present state but not on previous history i.e. the process has no memory. The process also follows the Martingale properties i.e. the expected outcome in the future are always similar to present outcome, i.e.

$$E\left[\left(S_j \middle| S_i\right), i < j\right] = S_i \quad (1.3)$$

### 1.3.1 Continuous random walk: transition to Brownian motion model

By dividing the total time interval into n steps for

$$R_i = \pm \sqrt{\frac{t}{n}}$$

Value of cumulative return gives

$$S_i = \sum_{j=1}^{i} R_j$$

The expected value of $S_i$, $R_i$, , $R_i^2$ $S_i^2$ ,,$R_i$, $R_i$ etc. are given by

$$E[S_i] = 0$$

$$E[R_i] = 0$$

$$E[R_i^2] = \frac{t}{n}$$

$$E[R_i R_j] = E[R_i] * E[R_j] = 0$$

$$E[s_i^2] = E[(R_i + \ldots..R_i)(R_1 + \ldots..+R_i)]$$

$$= \mathrm{E}[R_1^2 + R_2^2 + \ldots\ldots\ldots..+R_i^2]$$

$$= \mathrm{i}.\frac{t}{n}$$

For i = n $$E[S_n^2] = t \qquad (1.4)$$

With the limit n→ ∞ the process x(t) transforms to what is called Brownian motion given by

$$E[x(t)] = 0$$

$$E[\{x(t)\}^2] = t$$

The process follows the following properties

1. $E[\{x(t)\}^2]$ remains finite for finite t.
2. The process is continuous
3. It is Markovian
4. It follows Martingale property

$$E[x(t)|x(\tau);\ \tau < t] = x(\tau)$$

5. It follows normal distribution i.e. for $\tau < t$

x(t)-x($\tau$) is normal with zero mean and standard deviation

$$\Delta t = \sqrt{t - \tau}\ .$$

The Brownian motion can be simulated in discrete time steps.

### 1.3.2 Brownian motion

Let x(t) is the Brownian motion then by definition the process w(t) is given by

$$w(t) = \int_0^t F(\tau)\,.\,dx(\tau)$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} F(t_{j-1})(x(t_j)\text{-}x(t_{j-1})) \qquad (1.5)$$

In the expression for w(t) the term F $(t_{j-1})$ is non anticipatory. Thus

$$w(t) = \int_0^t F(\tau).\,dx(\tau) = \lim_{n\to\infty} \sum_{i=1}^{n} F(t_{j-1})\{x\,(t_j)\text{-}x\,(t_{j-1})\} \qquad (1.6)$$

$$dw = f(t).dx \qquad (1.7)$$

The equation (4) can be written as an integral of deterministic part as well as random part

$$w(t) = \int_0^t g(\tau)d\tau \quad + \quad \int_0^t f(\tau).\,dx(\tau) \qquad (1.8)$$

Here the first term is the deterministic term and the second term is the random term. First term can be solved through regular calculus but equation (5) requires stochastic calculus and equation can also be written as

$$dw = g(t).dt + f(t).dx \qquad (1.9)$$

If F(x) is a deterministic function, it can be solved using conventional calculus, but the stochastic function can be differentiated through stochastic integration only.

### 1.3.3 Ito's Lemma

The Taylor's expansion of function F(x) gives

$$F(x+dx) = F(x) + \frac{dF}{dx}.dx + \frac{1}{2}\frac{d^2F}{dx^2}.dx^2 + \ldots\ldots\ldots\ldots\ldots$$

for $dx^2 = dt$

$$F(x+dx)-F(x) = \frac{dF}{dx}.dx + \frac{1}{2}\frac{d^2F}{dx^2}.dt$$

$$\therefore dF = \frac{dF}{dx}.dx + \frac{1}{2}\ \frac{d^2F}{dx^2}.dt \tag{1.10}$$

This is Ito's Lemma

### 1.3.4 Generalization of Ito's Lemma

For a function F which is a function of s

$$F(s).\ ds = a(s)dt + b(s)dx$$

Taylor series expansion of F(s+ds) reveals:

$$F(s+ds) = F(s) + \frac{dF(s)}{ds}ds + \frac{1}{2}\frac{d^2F}{ds^2}ds^2 + \cdots$$

$$dF = \frac{dF(s)}{ds}ds + \frac{1}{2}\frac{d^2F}{ds^2}(a^2).dt^2 + 2a(s)ds.dt.dx + b^2(s).dx^2)$$

With $dt^2 \ll dt$ and $dx^2 = dt,\ \ dx.dt \ll dx$

$$dF = \frac{dF}{ds}.ds + \frac{1}{2}b^2(s).\frac{dF^2}{ds^2}.dt \tag{1.11}$$

Equation (1.11) is the generalized version of Ito's Lemma.

### 1.3.5 Brownian motion with drift

The formula is for the Brownian motion with a drift term is given by

$$ds = \mu.dt + \sigma.dx$$

The first term is the drift term whereas the second term represents the Brownian motion.

$$\therefore s(t) = s(0) + \int_0^t \mu . d\tau + \int_0^t \sigma . dx(\tau)$$

Here $\int_0^t \sigma . dx(\tau) = \lim_{n\to\infty} \sum_{i=1}^n \sigma\,[x(t_j) - x(t_{j-1})]$

$$S(t) = s(0) + \mu . t + \sigma(x(t) - x(0))$$

The simplest model of asset value assumes that asset value of stock prices follows log normal distribution

log normal distribution is defined by the process

$$ds = \mu . s . dt + \sigma . s . dx \qquad (1.12)$$

$$F(s) = \log s$$

$$\frac{dF}{ds} = \frac{1}{s} \quad \frac{d^2F}{ds^2} = -\frac{1}{s^2}$$

$$\therefore dF = \frac{dF}{ds} . ds + \frac{1}{2} b^2(s) . \frac{d^2F}{ds^2} . dt$$

Using relation b(s) = σs

obtains $$dF = \frac{dF}{ds} ds + \frac{1}{2}\sigma^2 s^2 \frac{d^2F}{ds^2} . dt$$

$$dF = (\mu - \tfrac{1}{2}\sigma^2)dt + \tfrac{1}{2}\sigma . dx \qquad (1.13)$$

This is similar to the equation of Brownian motion with drift.

The solution to the equation (1.13) would be

$$F(t) = F_0 + \left(\mu - \tfrac{1}{2}\sigma^2\right) t + \sigma\left(x(t) - x(0)\right)$$

Now

$$\therefore F(s) = \ln s$$

$$S\,(t) = s(0)\ e^{\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma\left(x(t) - x(0)\right)} \qquad (1.14)$$

Equation (1.14) is the solution of equation (1.13) in log normal form.

### 1.3.7 Multidimensional Ito's Lemma

Taking 2-d Taylor expansion of the function

$$\therefore v(s+ds,\ t+dt) = v(s,t) + \frac{\partial v}{\partial s} ds + \frac{\partial v}{\partial t} dt + \frac{1}{2}\left(\frac{\partial^2 v}{\partial x^2} . dx + \frac{\partial^2 v}{\partial t^2} dt + 2\frac{\partial^2 v}{\partial s\,\partial t} ds.dt\right) + \cdots$$

$$dv = \frac{\partial v}{\partial s} ds + \frac{\partial v}{\partial t} dt + \frac{1}{2}\left(\frac{\partial^2 v}{\partial s^2}\right) ds^2$$

$$= \frac{\partial v}{\partial s} ds + \frac{\partial v}{\partial t} dt + \frac{1}{2}\left(\frac{\partial^2 v}{\partial s^2}\right)[a(s,t)ds + b(s,t)dx]^2$$

$$= \frac{\partial v}{\partial s} ds + \frac{\partial v}{\partial t} dt + \frac{1}{2}\left(\frac{\partial^2 v}{\partial s^2}\right)[a^2(s,t)dt^2 + 2a(s,t)b(s,t)dt.dx + b^2(s,t)dx^2]$$

But $dx^2 = dt$

$$\therefore dv = \frac{\partial v}{\partial s} ds + \frac{\partial v}{\partial t} dt + \frac{1}{2}\frac{\partial^2 v}{\partial s^2} . b^2(s,t).dt \qquad (1.15)$$

Equation (1.15) is Ito's Lemma for higher dimension

## 1.4 Black Scholes equation

Equation (12) transforms to Black-Scholes equation subjected to certain constraints. Let s(t) is the asset price and V(s,t) is option price of a given stock or index with the assumption that S is lognormal

Black - Scholes equation for v(s,t) is given by

$$\frac{\partial v}{\partial t}+\frac{1}{2}\sigma^2 s^2\frac{\partial^2 v}{\partial s^2}+rs.\frac{\partial v}{\partial s}-rv=0 \tag{1.16}$$

In the Black- Scholes Equation (1.16)

The term $\frac{\partial v}{\partial t}$ represents the change in option price with time

and the term rv represents the reaction term

$rs.\frac{\partial v}{\partial s}$ represents the time value of the option

$\frac{1}{2}\sigma^2 s^2\frac{\partial^2 v}{\partial s^2}$ is the Diffusive term

For heat flow in a one dimensional object the equation is given by

Q = F.δ(x)

And the corresponding differential equation is given by

$$\frac{\partial Q}{\partial t}=\frac{\partial F}{\partial t}\delta x \tag{1.16}$$

Equation (17) is analogous to heat flow equation from high temperature to low.

Or

$$\frac{\partial Q}{\partial t}=k(-\frac{\partial F}{\partial t}(x,t)+\frac{\partial F}{\partial x}(x+\delta x,t)) \tag{1.17}$$

From (1.16) & (11.17)

$$\frac{\partial F}{\partial t}.\delta x=k(\frac{\partial F}{\partial x}(x+\delta x)-\frac{\partial F}{\partial x}(x,t))$$

$$\therefore \frac{\partial F}{\partial t} = k[\frac{\frac{\partial F}{\partial x}(x + \delta x) - \frac{\partial F}{\partial x}(x, t)}{\delta x}]$$

At the limit $\delta x \to 0$

The equation (17) transforms to Heat conduction equation.

$$\frac{\partial F}{\partial t} = k\frac{\partial^2 F}{\partial x^2}$$

## 1.5 Objectives of the present investigation

Present models of finance assume that a net return follows normal distribution and asset prices follow log normal distribution. In Gaussian distribution the probability of catastrophic events are very low; But in probability distribution with fat tail such probability is very high. Fat Tail distribution is a probability distribution, whose outcomes are represented as thick ends or “tails” that form towards the edges of the distribution curve, indicating on irregularity high likely hood of catastrophic event.

When the probability distribution curve of daily returns of NIFTY is plotted, the actual curve does not fit to this normal curve. According to normal distribution the probability of extreme events of 20% drift is around once every $2\times 10^{76}\ years$ however for an actual curve it is of the order of once every 5 years. The crashes observed in stock markets show that the returns in stock prices do not follow normal distribution and Brownian motion is not an appropriate model for stock price dynamics.

1. One choice is to use some other distribution. But still no distribution is known to be satisfactorily followed by price or index variations.

2. Another choice is to empirical study the variation of financial time series. This approach is the basic assumption or Technical analysis. The approach assumes that the time series data of such a complex system contains all the signatures of the system, and all the information are stored in it.

Time domain measures of variation represent an evaluation of overall, short-term or long term variation, and are proven as a means of identifying significant alterations in signals. Frequency domain analysis is also an important tool. Power law analysis contributes an analysis of fractal, long range correlations, allowing distinction between the data signals with the slope and intercept of the power law. Detrended fluctuation analysis also represents a means of detecting long range correlations, and is less bound by the stationarity assumption inherent to the other techniques. By measuring the degree to which sequences of data repeat themselves within a signal, approximate entropy provides a measure of signal irregularity, related to the rate of production of new information. Attempts to characterize time series signals should incorporate the 'toolkit' of techniques discussed in this review as well as the publication of raw data and code to facilitate comparison and development of this still young, exciting science.

The science of complex systems is intimately related to variation analysis. Taking a broad system based interpretation, the financial system is a complex system or, more accurately, it is a complex system of complex systems. Every complex system has 'emergent' properties, which define its very nature and function. Variability or patterns of change over time (in addition to connectivity or patterns of interconnection over space) represent technology with which to evaluate the emergent properties of a complex system.

The multi-system nonlinear analysis offers an effective monitoring tool that is capable of improving prognostication of time series data. Intuitively, there is additional information in this analysis. Such variation analysis tracks specific patterns of change in individual parameters over time (akin to calculating the first derivative or velocity in calculus). Monitoring patterns of change in variations continuously over time offers an additional dimension of analysis (akin to a second derivative evaluation or acceleration). Just as monitoring individual system variations offers an evaluation of the underlying individual system producing those dynamics, evaluating multisystem fluctuations provides an evaluation of the whole, namely the systemic response. By using variation analysis at different time points or, more powerfully, continuously over time, it is theoretically possible to track the 'system state' over time. Then, by selecting strategy according to observed patterns the outcomes may be improved.

The techniques of analyzing nonlinear time series has undergone tremendous growth over the past decade, with the development of advanced computational methods that characterize the variation, oscillation, complexity and regularity of signals. These methods were developed in response to theoretical limitations of the others; however, all appear to have clear significance. There is no consensus that any single technique is the single best means of characterizing and differentiating time series; rather, investigators agree that multiple techniques should be performed simultaneously to facilitate comparison between methods, techniques and studies. Nonlinear analysis represents a novel means to evaluate and treat complex financial data, suggesting a shift from simple linear analytical investigation to rather more sophisticated nonlinear techniques. Existing literature documents the value of measuring variations to provide diagnostic, prognostic information.

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