

**CHAPTER-3**  
**DETRENDED FLUCTUATION ANALYSIS OF**  
**FINANCIAL TIME SERIES**

### 3.1 Introduction

Detrended Fluctuation Analysis (DFA) has been established as an important tool for the detection of long range autocorrelations in time-series with non-stationarities, which often affects experimental data. DFA is a method for determining the scaling behavior of data in the presence of possible trends without knowing their origin & shape. It has been applied to diverse field of DNA , heart rate dynamics , human gait , long-time weather records , cloud structure , economical time-series etc.

Consider a time-series  $\tau_i$  , where  $i = 1,2,3,\dots,N$  and  $N$  is the length of the series. To implement DFA method the steps below are followed:

The series is first integrated to obtain the cumulative time series

$$y(k) = \sum_{i=1}^k (\bar{\tau} - \tau_i) \quad \text{where } k = 1,2,3,4,\dots,N \quad (3.1)$$

Here  $\bar{\tau}$  indicates the mean interevent time &  $\bar{\tau} = \sum_{i=1}^k \tau_i$

Breaking up  $y(k)$  into  $T$  non-overlapping time intervals ,  $I_t$  of equal size  $r$ , where  $t = 0,1,2,\dots,T-1$  &  $T$  corresponds to the integer part of  $N/r$ .

The local trend function

$$Y_r(k) = a_t + b_t k \quad \text{for } k \in I_t \quad (3.2)$$

Where the coefficients  $a_t$  &  $b_t$  represents the least-square linear fit of  $y(k)$  in interval  $I_t$ .

The rescaled function

$$F(r) = \frac{1}{S} \left\{ \frac{1}{\text{tr}} \sum_{k=1}^{\text{Tr}} [y(k) - y_r(k)] \right\}^{1/2} \quad (3.3)$$

Where  $s$  = data standard deviation

$$S = \left[ \frac{1}{N} \sum_{k=1}^N (\tau_k - \bar{\tau})^2 \right]^{1/2} \quad (3.4)$$

The Hurst exponent  $H$  is then obtained from the scaling behavior of  $F(r)$

$$F(r) = Cr^H \quad (3.5)$$

Where  $C$  = constant, independent of time lag  $r$

$H$  = Hurst scaling exponent

Here Hurst exponent  $H$  represents the slope of the straight line graph of  $\log F(r)$  to  $\log r$ . Typically  $F(r)$  increases with  $r$ .

In double logarithmic plot of  $\log F(r)$  v/s  $\log r$  graph although the slope of straight line gives the Hurst exponent  $H$ . In practical problem  $H$  depends on the choice of the interval within which the linear fit is performed.

To overcome this problem,  $F(r)$  may be written such that

$$F_H(r) = C_H r^H \quad (3.6)$$

$$\text{Where, } C_H = \left[ \frac{2}{2H+1} + \frac{1}{H+2} - \frac{2}{H+1} \right]^{\frac{1}{2}} \quad (3.7)$$

The above equation gives a one parameter estimator for the Hurst exponent  $H$ , so that we can adjust  $H$  simply to obtain the best arrangement between the theoretical curve predicted by  $F_H(r)$  & the empirical data for  $F(r)$ . The scaling exponent, correlation exponent  $\alpha$  quantify the correlation properties of the signal. It can have different values

$\alpha < 1/2$                       Anti correlated

$\alpha \approx 1/2$	UN correlated
$\alpha > 1/2$	Correlated
$\alpha \approx 1$	1/f noise
$\alpha > 1$	Non stationary random walk
$\alpha \approx 3/2$	Brownian motion

Many physical systems do not obey conventional scaling laws. Broad probability distributions are characteristics for such observables, for instance the growing probability in DLA (Diffusion limited aggregation) the distribution of voltage loss in random resistor network or the probability distribution random walks on fractals [1, 2, 3,].

In all cases the moment of the distribution can't be characterized by a single exponent. So the scaling behavior is non-trivial for different values of  $q = 1, 2, 3, \dots$ . This phenomenon was described for the first time by B.B. Mandelbrot in 1970. Today it is known as multi fractality. The original meaning of multi fractal leads to the question about the processes that create multi fractal structure. The multiplicative cascaded of random processes create multi fractal structure, while additive processes generally produce simple fractals (mono fractals). Multi fractals are used to describe & distinguish varieties of complicated figure, system & processes in nature.

Many algorithms have been developed to measure the multi fractal dimerism. The method to calculate multi fractal spectra was proposed by T. C. Halsey in 1986. In this method multi fractal spectra for quantities of states of a system with a random fractal character can be calculated.

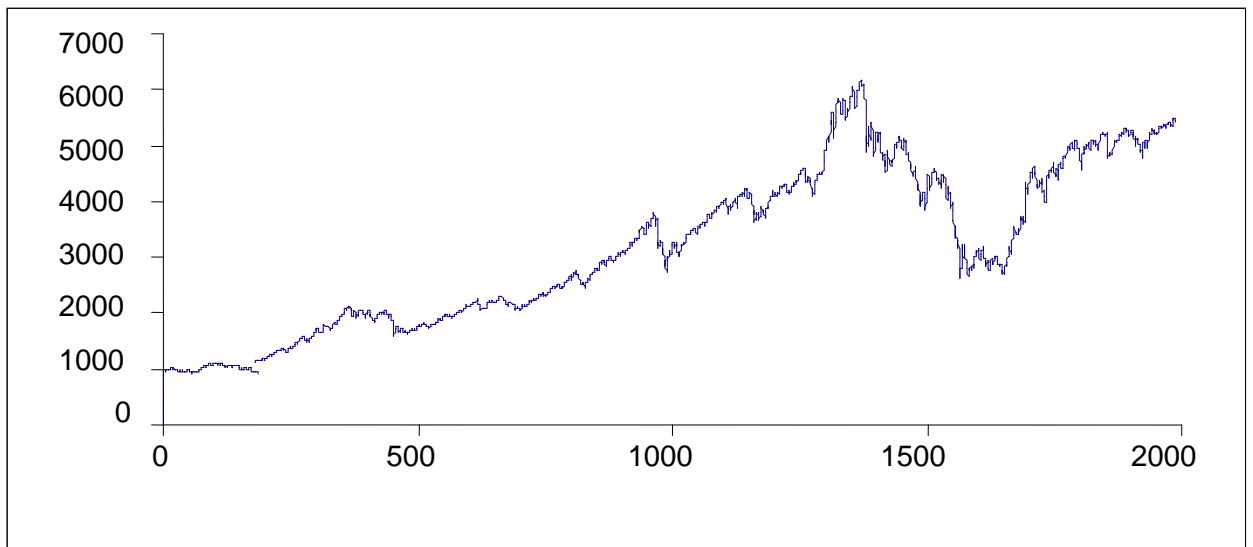
The nature of price fluctuation in stock markets has been of interest to the traders as well as a variety of professionals for a long time. A large numbers of methods have been applied to characterize the time evaluation of stock price and stock market indices. The correlation properties of stock prices can be described using DFA.

### **3.2 Result of Analysis**

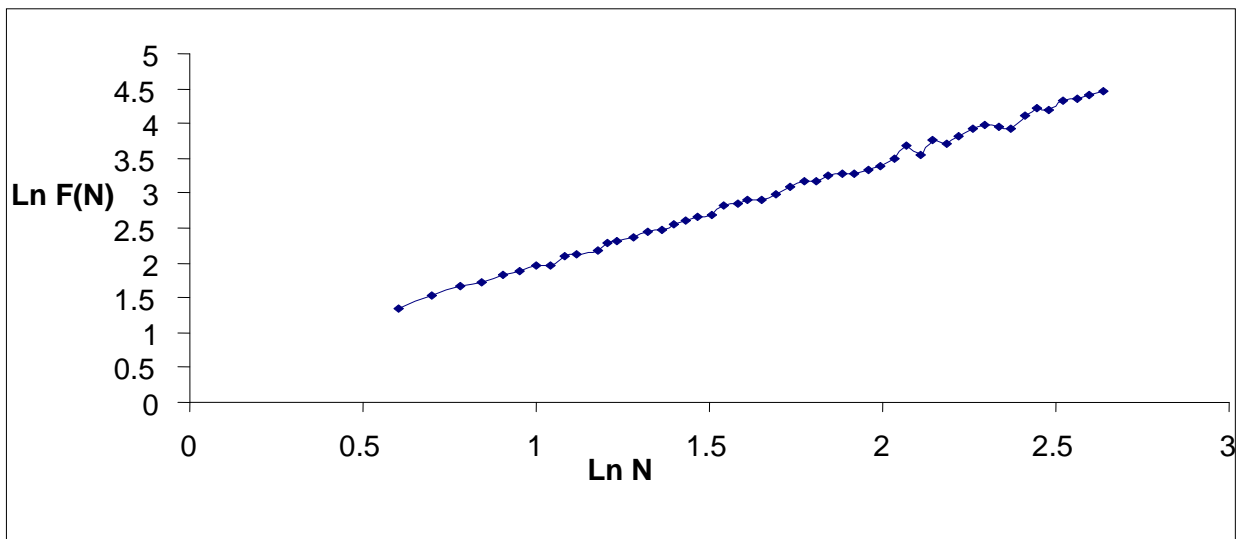
The DFA analysis was performed for the data sets and the results obtained are as follows: The Index values and volumes with its DFA profiles are plotted at different time. Figure 3.1 shows the NSE Index daily closing values from 12.08.2002 to 25.08.2010. Figure 3.2 shows DFA profile for the same NSE Index daily closing values from 12.08.2002 to 25.08.2010. Figure 3.3 is the DFA profile for NSE Index monthly closing values from 12.08.2002 to 25.08.2010. Figure 3.4 shows the DFA profile for NSE Index quarterly closing values from 12.08.2002 to 25.08.2010. Figure 3.5 is the Graph for trading volume of Japanese stock market Index NIKKEI(volume) from 21<sup>st</sup> Jul.2009 to 30<sup>th</sup> Dec.2012 .Figure 3.6 shows the DFA profile for NIKKEI volume. Fig 3.7.shows the Graph of NIKKEI Adjusted closing value  $Y(n)$ . Fig 3.8 shows the DFA profile for NIKKEI Adjusted closing value  $Y(n)$ . Fig 3.9 shows Graph for Taiwan stock market index TWII (Adjusted closing value  $Y(n)$ ). Figure 3.10 is a plot of DFA profile for TWII closing value. Fig 3.11 shows the Graph of TWII (volume)  $X(n)$  from 12<sup>th</sup> Aug.2009 to 1<sup>st</sup> Mar.2012. Figure 3.12 is the plot of DFA profile for TWII volume. Figure 3.13 shows the Graph of STI Adjusted closing value no. of data point  $(n)$  and data point  $Y(n)$ ] from 1<sup>st</sup> Sep.2009 to 1<sup>st</sup> Mar.2012. Figure 3.14 is the plot of DFA profile for STI closing value Figure 3.15 shows the Graph of SEOUL stock market index volume  $X(n)$  from 6<sup>th</sup> Aug.2009 to 1<sup>st</sup> Apr.2012. Figure 3.16 is the plot of DFA profile for Seoul volume. Figure 3.17 shows the Graph of trading volume of German stock market index DAX from 9<sup>th</sup> Apr.2009 to 1<sup>st</sup>

Apr.2012. Figure 3.18 is the DFA profile for DAX volume. Fig3.19 shows the Graph of DAX (Adjusted closing value)  $Y(n)$  from 1<sup>st</sup> Mar.2012 to 12<sup>th</sup> Aug.2009. Figure 3.20 is the DFA profile for DAX closing value. Figure 3.21 shows the Graph of DOW-JONES Industrial Average (Adjusted closing value) from 1<sup>st</sup> Mar.2012 to 27<sup>th</sup> Aug.2009. Fig 3.22 is the plot of DFA profile for DOW-JONES Industrial average adjusted closing value.

Figure 3.23 shows the Graph of NASDAQ Adjusted closing value  $Y(n)$  from 1<sup>st</sup> Mar.2012 to 29<sup>th</sup> Oct.2009. Figure 3.24 is the plot of DFA profile for NASDAQ Adjusted closing value. Fig 3.25 shows the Graph of NASDAQ trading volume  $X(n)$  from 1<sup>st</sup> Mar.2012 to 29<sup>th</sup> Oct.2009. Figure 3.26 is the plot of DFA profile for NASDAQ volume.



[FIG 3.1 NSE index daily closing values from 12.08.2002 to 12.08.2011]



[FIG 3.2 DFA profile for NSE index daily closing values from 12.08.2002 to 12.08.2011]

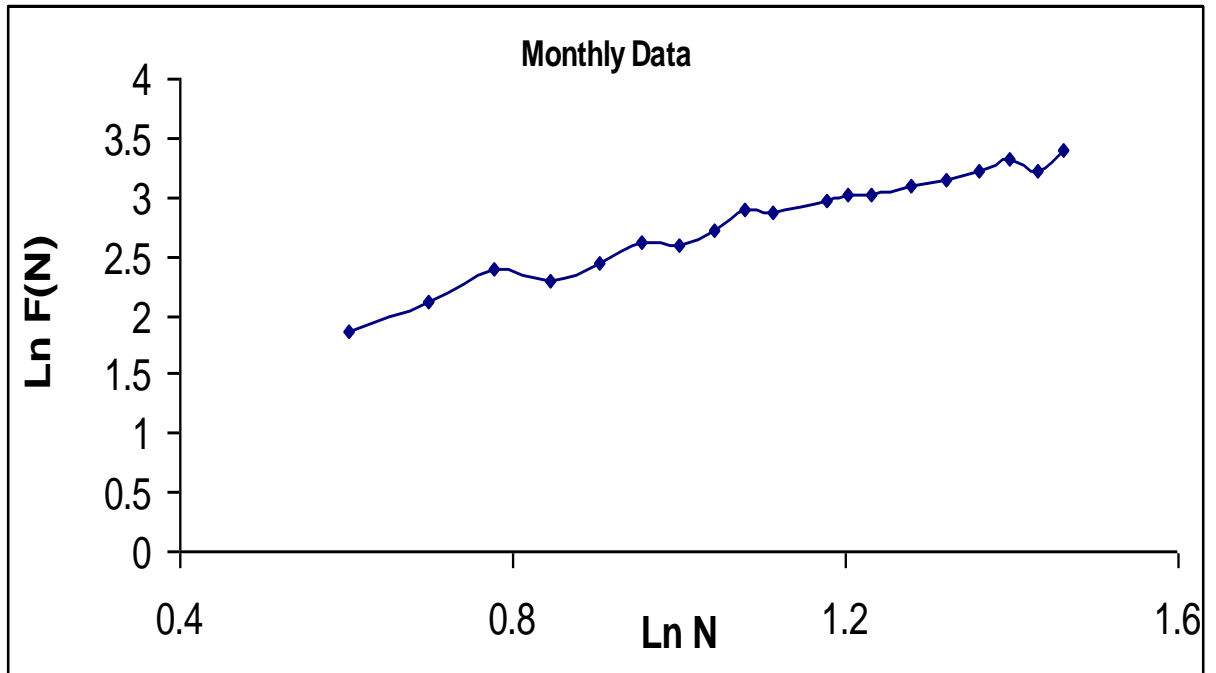
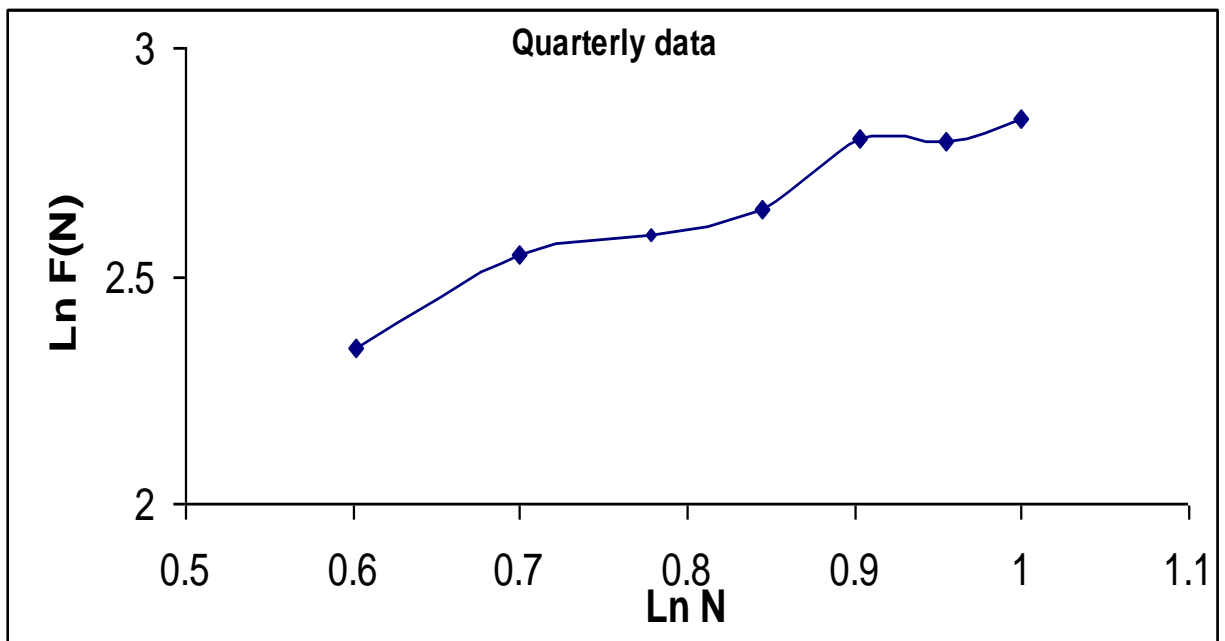
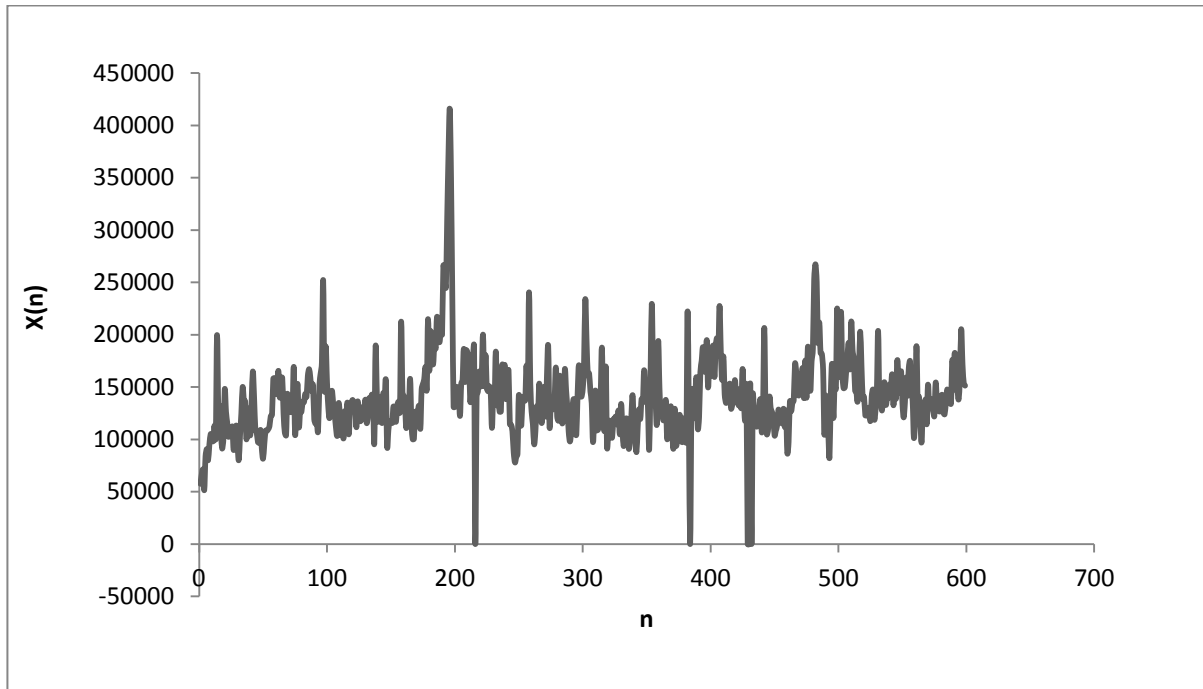


FIG 3.3 DFA profile for NSE index monthly closing values from 12.08.2002 to 12.08.2011



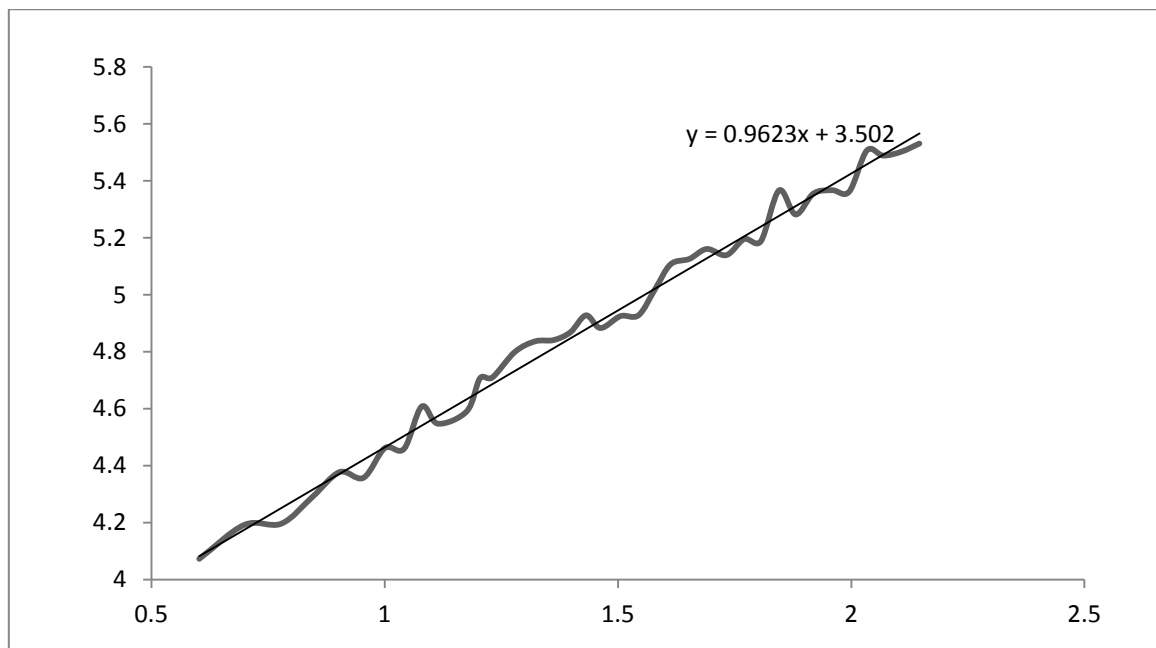
[FIG 3.4 DFA profile for NSE index quarterly closing values from 12.08.2002 to 12.08.2011]



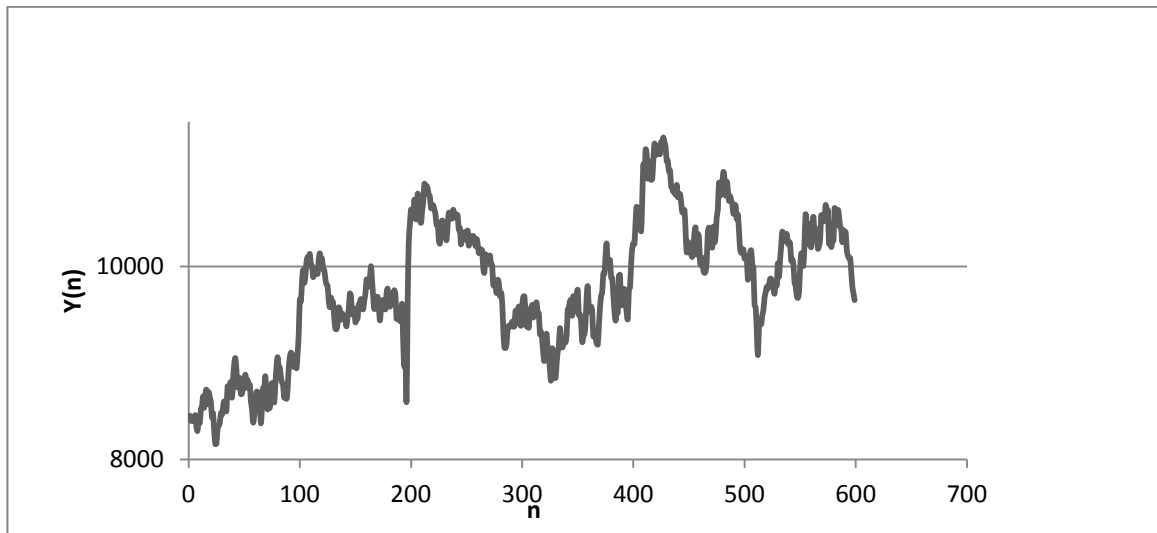


[FIG 3.5 Plot of NIKKEI trading volume  $X(n)$ ]

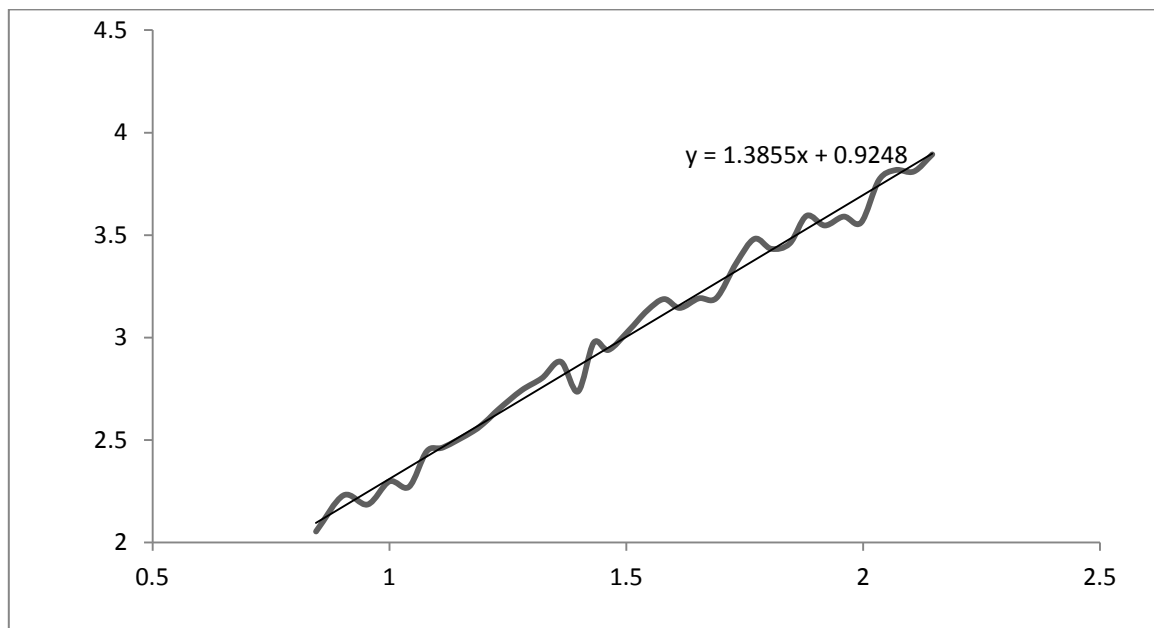
[Date : 21<sup>st</sup> Jul.2009 to 30<sup>th</sup> Dec.2012]



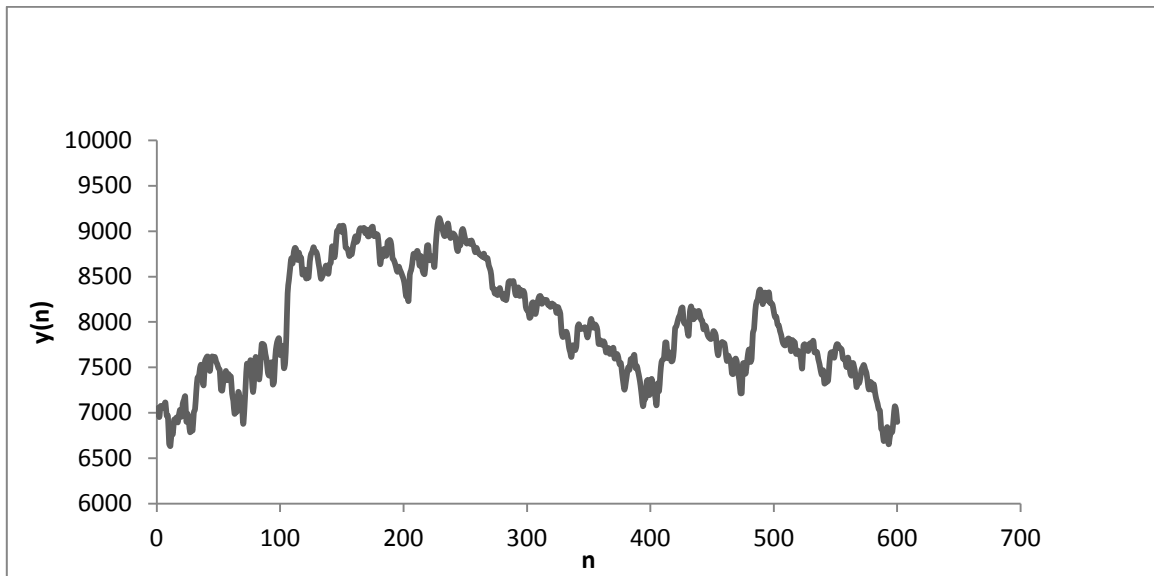
[FIG 3.6 DFA profile of NIKKEI volume]



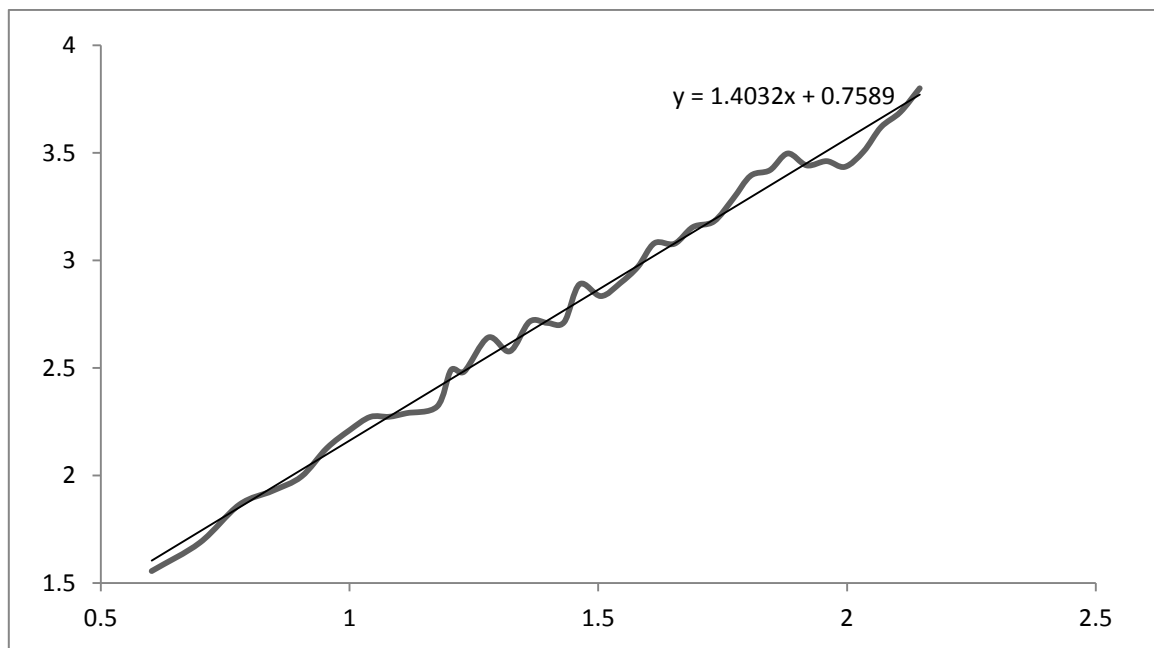
[FIG 3.7 Plot of NIKKEI Adjusted closing value  $Y(n)$   
[Date : 21<sup>st</sup> Jul.2009 to 30<sup>th</sup> Dec.2012]



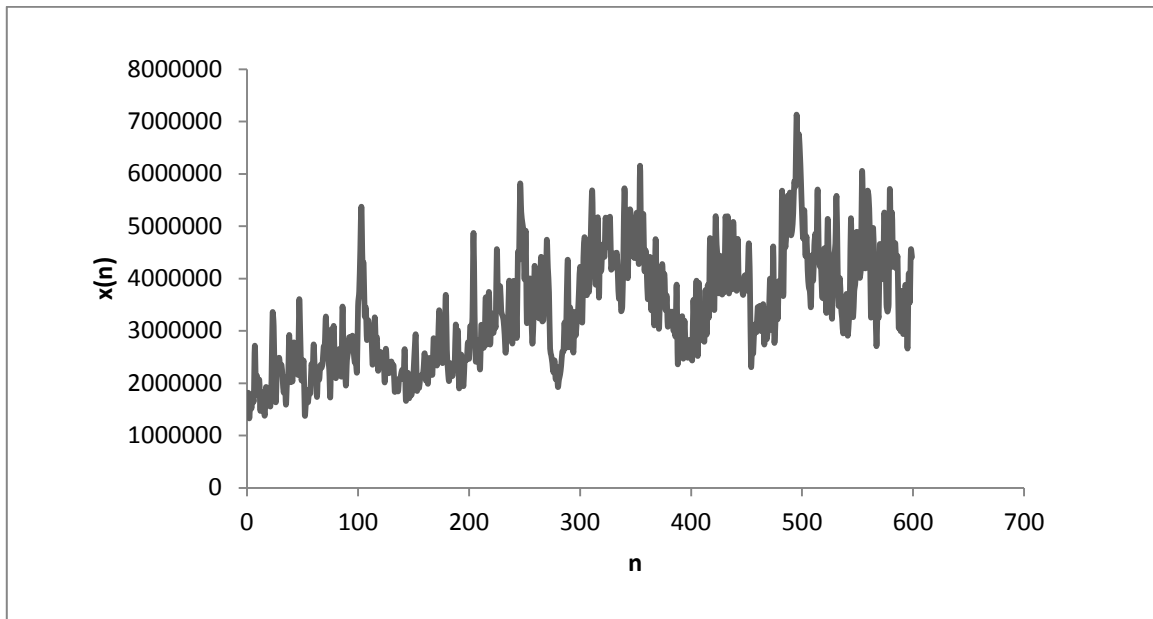
[FIG 3.8 DFA profile of NIKKEI closing value]



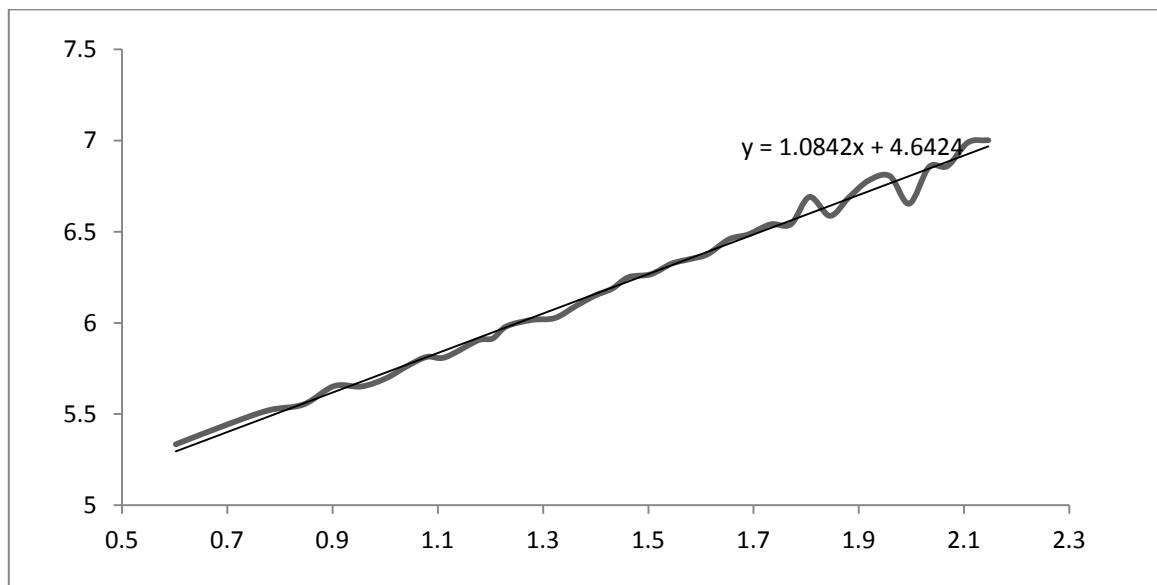
[FIG 3.9 Plot of TWII adjusted closing value  $Y(n)$ .  
[Date : 12<sup>th</sup> Aug.2009 to 1<sup>st</sup> Mar.2012]



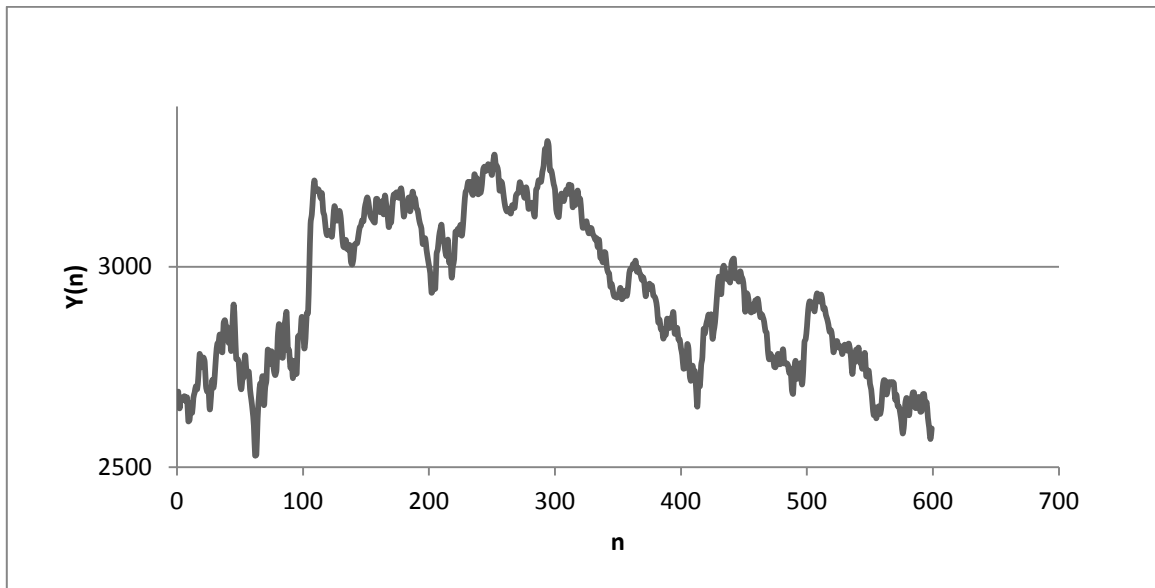
[FIG 3.10 DFA profile of TWII daily closing value]



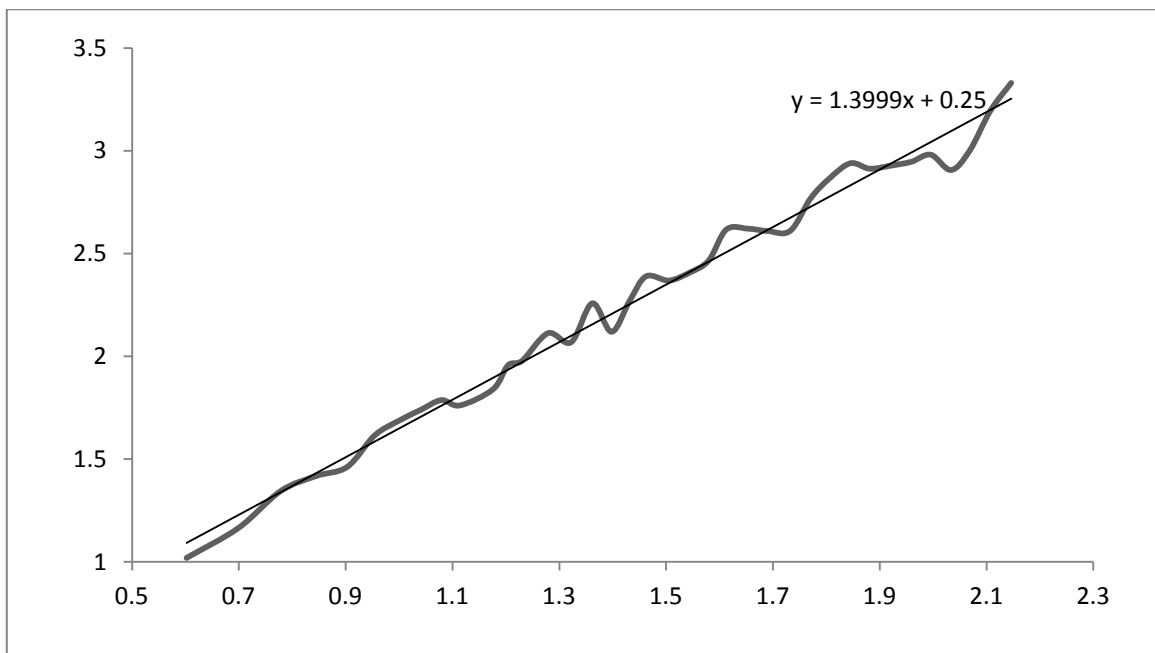
[FIG 3.11 Plot of TWII volume  $X(n)$ .  
[Date: 12<sup>th</sup> Aug.2009 to 1<sup>st</sup> March 2012]



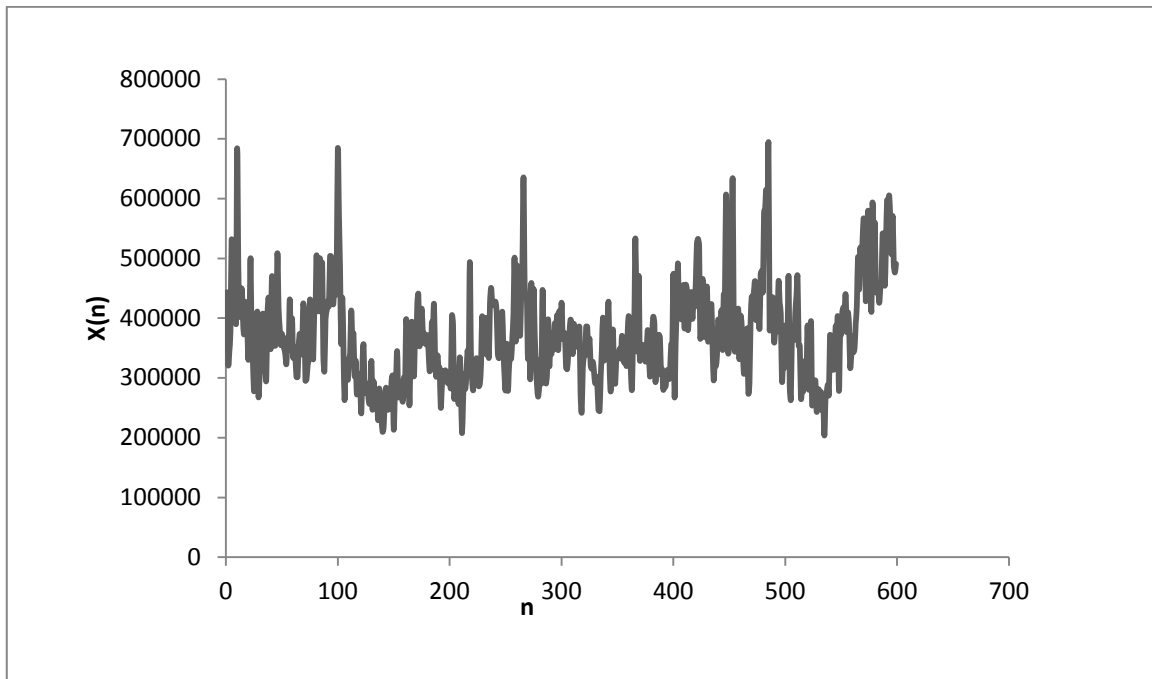
[Fig 3.12 DFA profile of TWII volume]



[Fig 3.13 Plot of STI adjusted closing value  $Y(n)$   
[Date: 1<sup>st</sup> Sep.2009 to 1<sup>st</sup> Mar.2012]

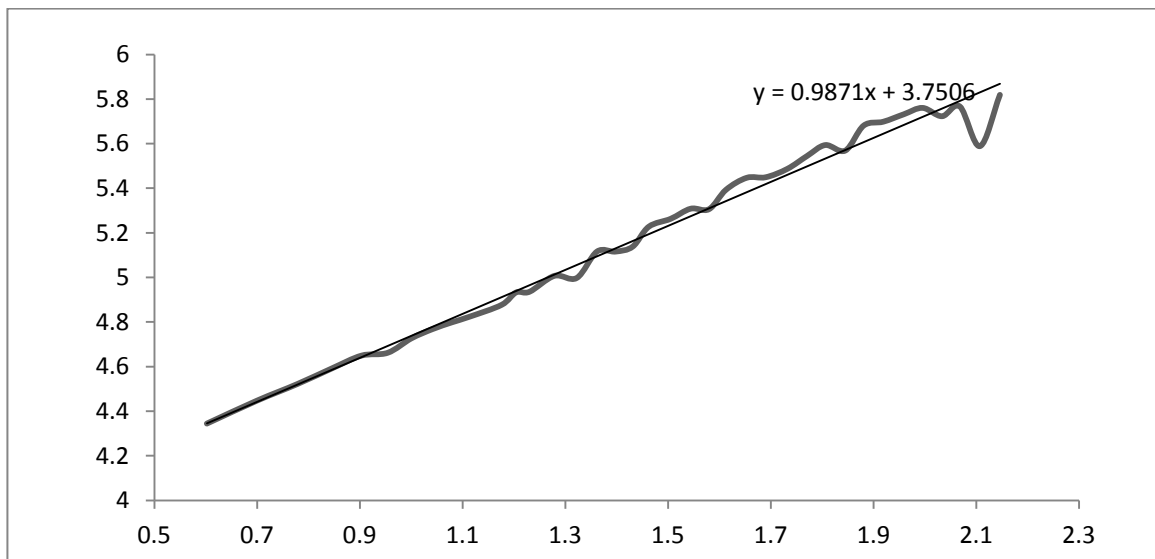


[Fig 3.14. DFA profile of STI closing value]

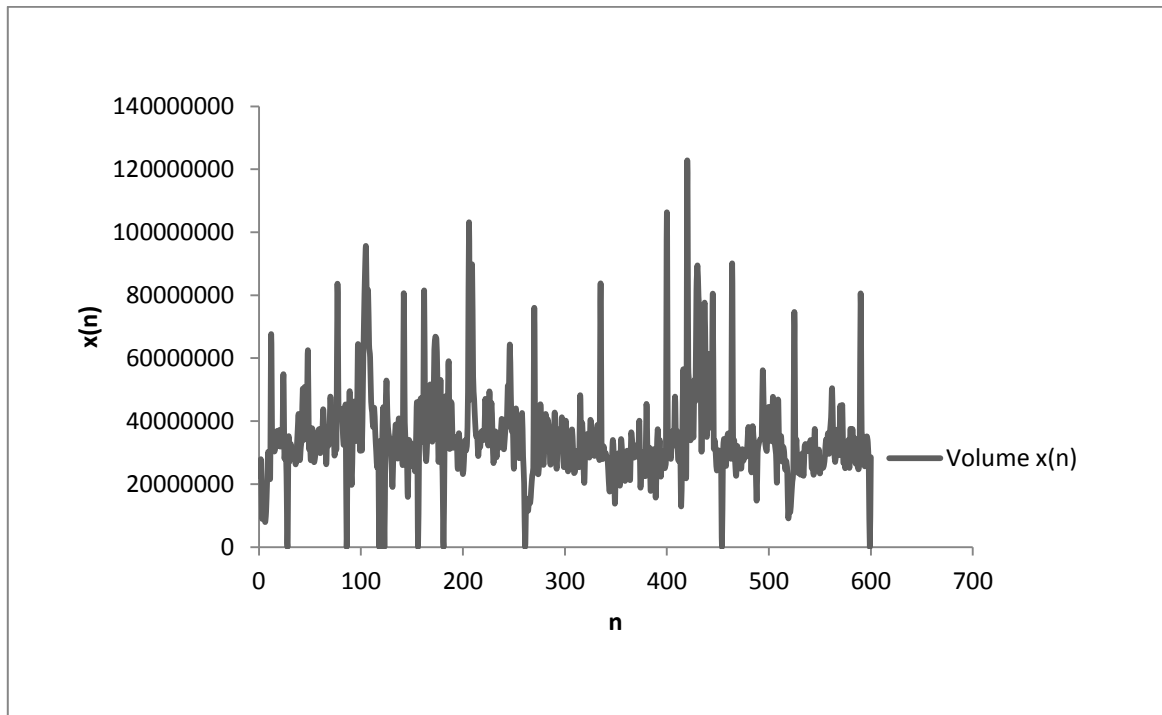


[Fig 3.15 Plot of SEOUL trading volume  $X(n)$ ]

[Date : 6<sup>th</sup> Aug.2009 to 1<sup>st</sup> Apr.2012]

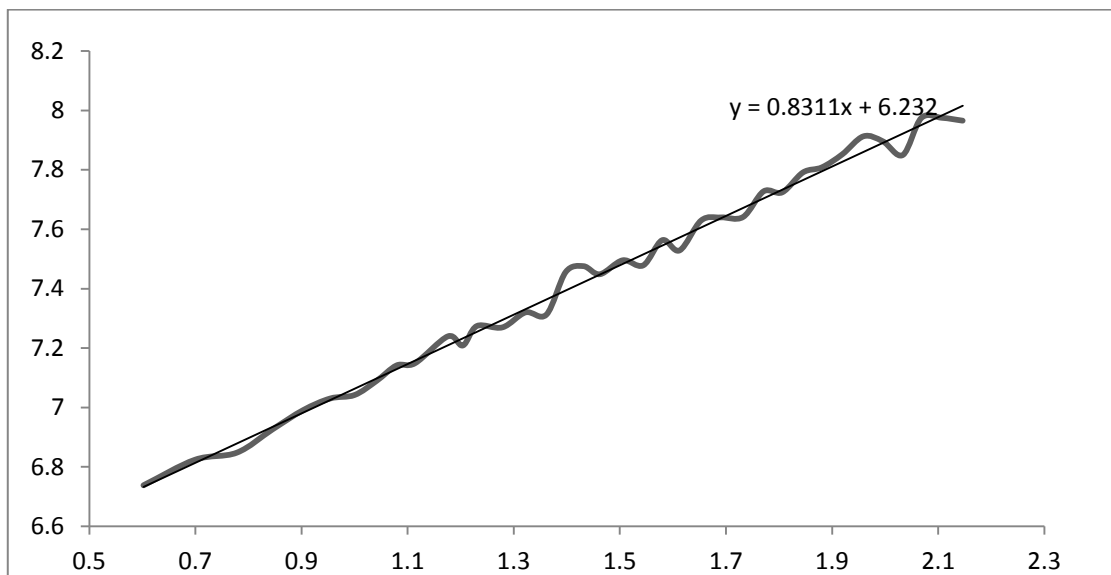


[Fig 3.16 DFA profile of Seoul volume]

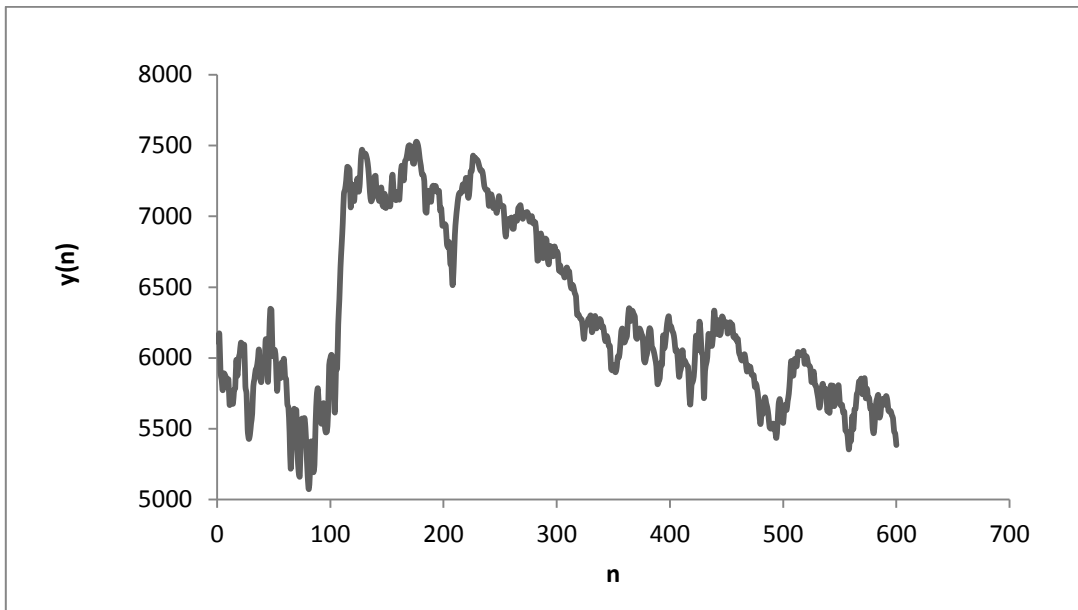


[Fig 3.17 Plot of DAX trading volume  $X(n)$ ]

[Date : 9<sup>th</sup> Apr.2009 to 1<sup>st</sup> Apr.2012]

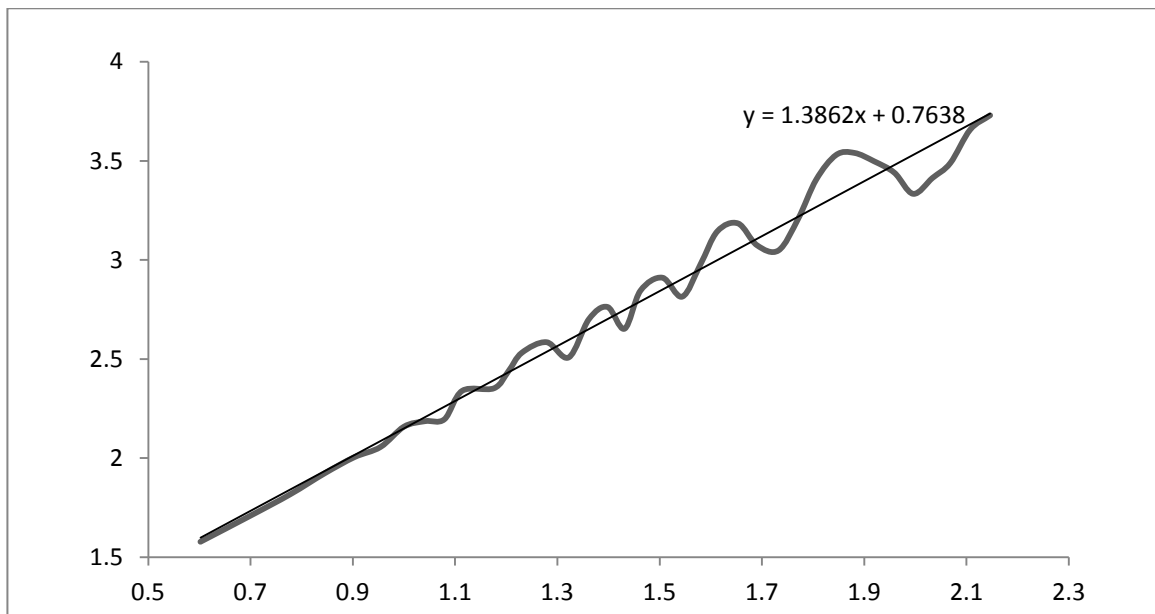


[Fig 3.18 DFA profile for DAX volume]



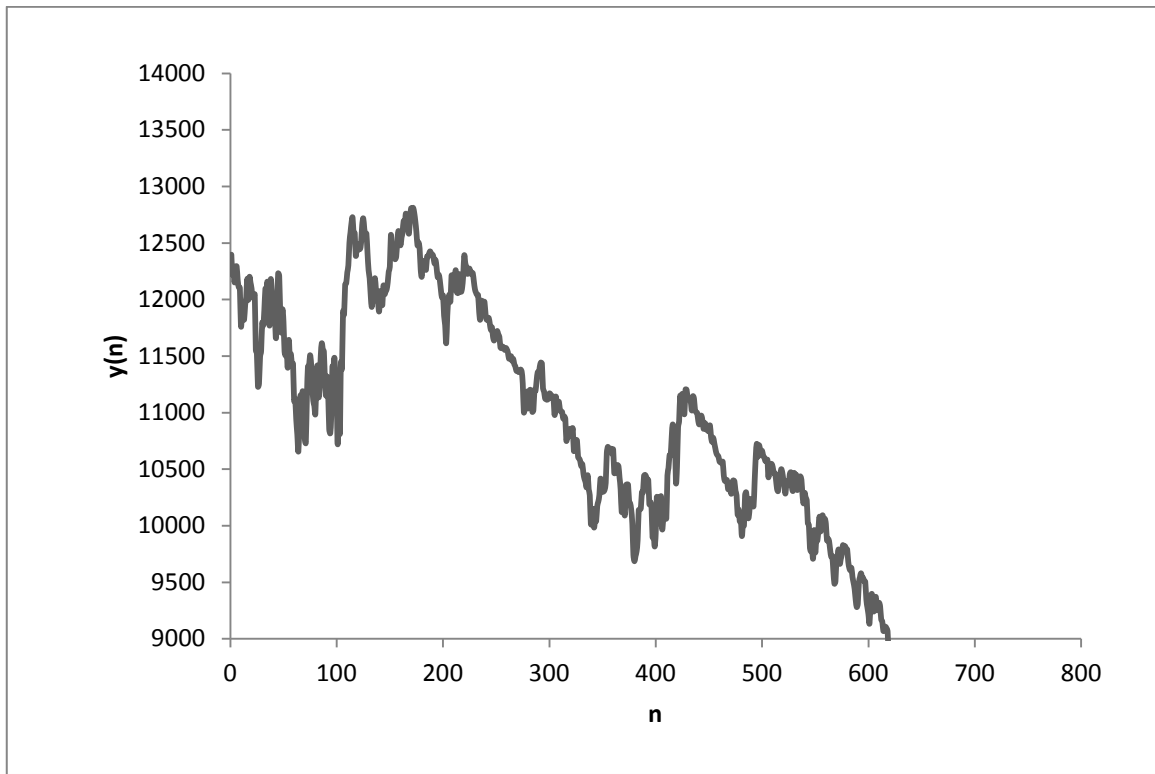
[Fig3.19. Plot of DAX Adjusted closing value  $Y(n)$ ].

[Date: 12<sup>th</sup> Aug.2009 to 1<sup>st</sup> Mar.2012]



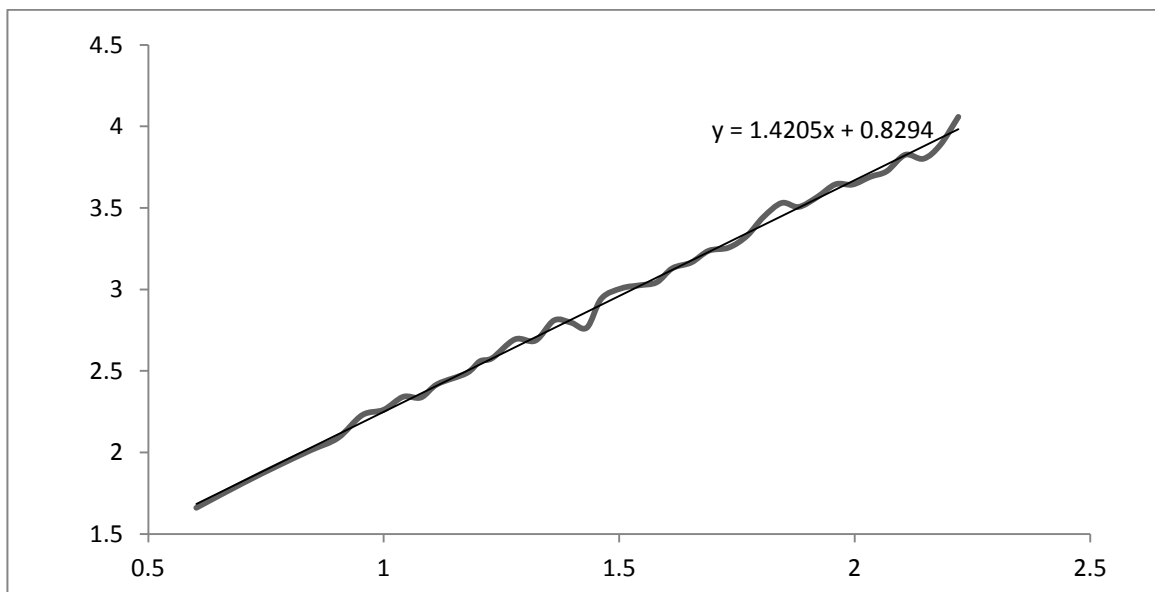
[Fig 3.20 DFA profile of DAX closing value]



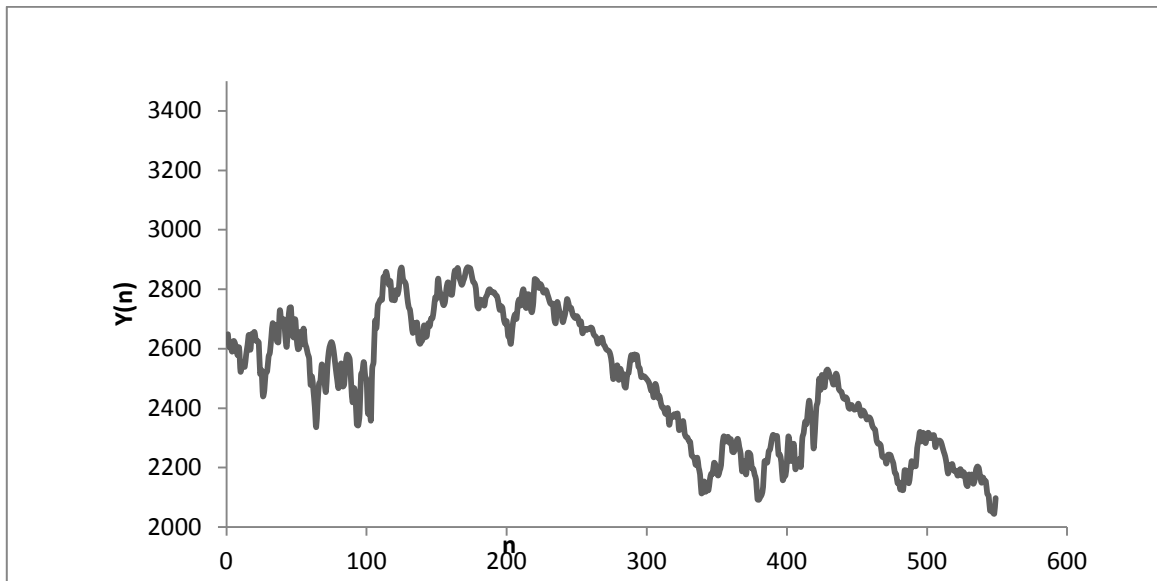


[ Fig 3.21 Plot of DJIA adjusted closing value  $Y(n)$ ].

[Date: 27<sup>th</sup> Aug.2009 to 1<sup>st</sup> March 2012]

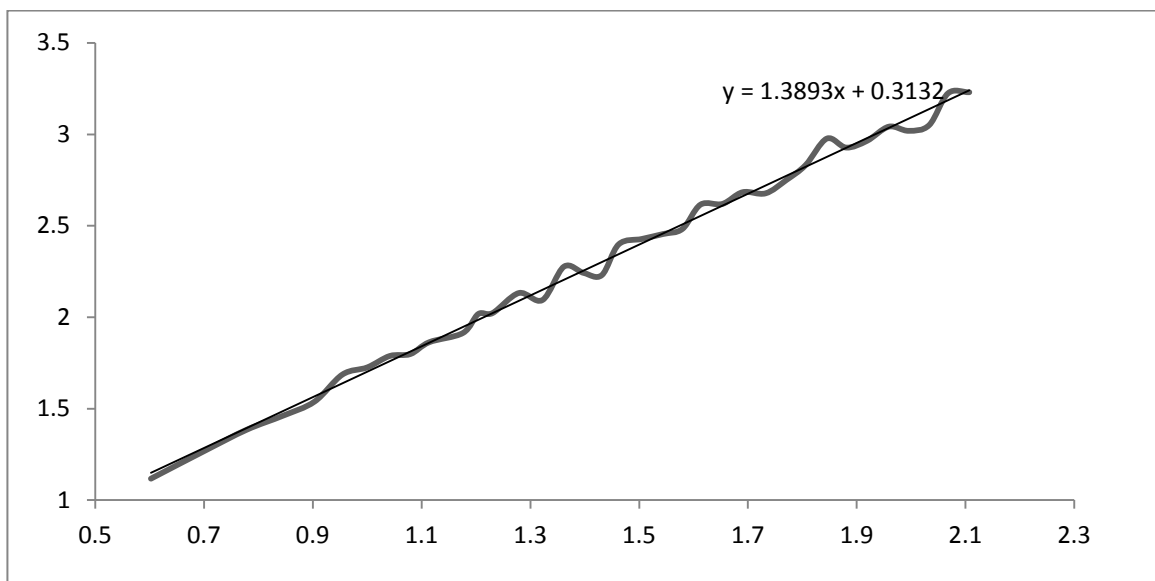


[Fig 3.22 DFA profile of DJIA closing value]

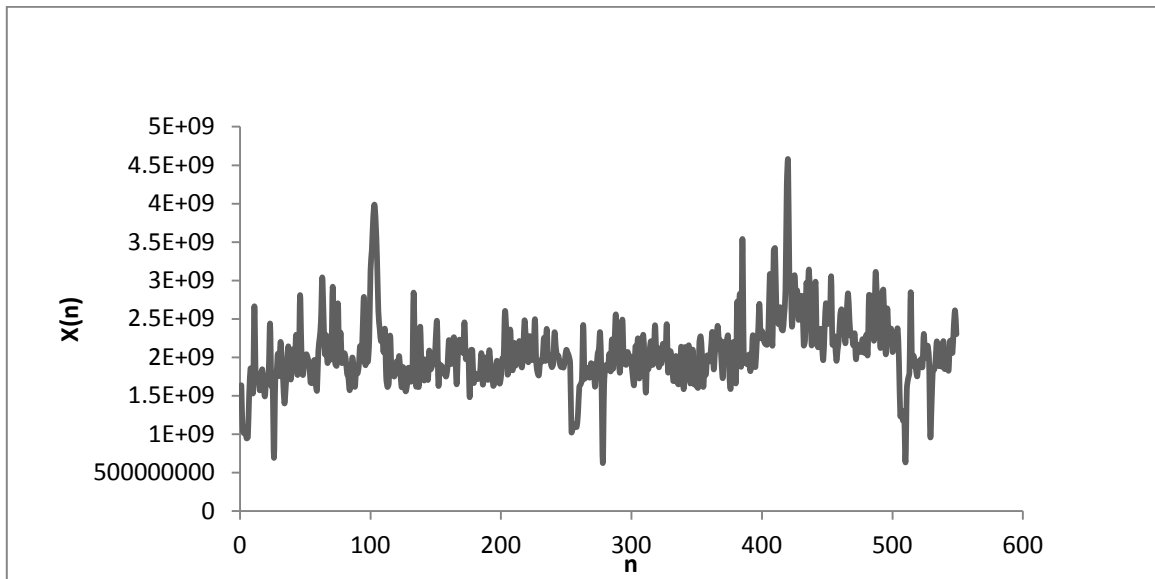


[ Fig 3.23 Plot of NASDAQ daily closing value  $Y(n)$ ].

[Date :- 29<sup>th</sup> Oct.2009 to 1<sup>st</sup> Mar.2012]

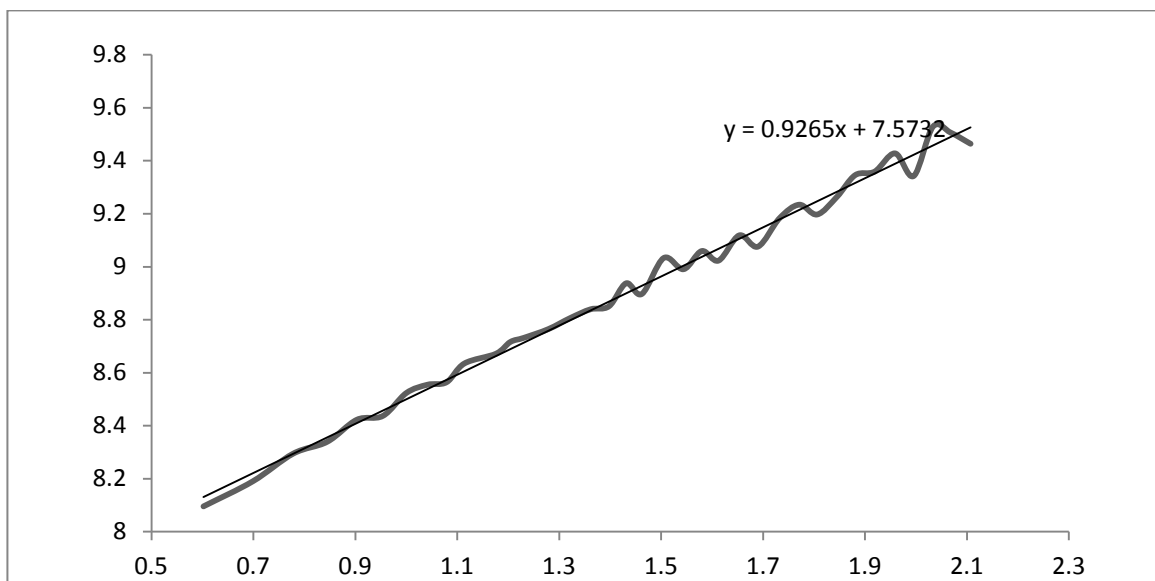


[Fig 3.24 DFA profile of NASDAQ closing value]



[ FIG 3.25 plot of NASDAQ trading volume  $X(n)$ ]

[Date: 29<sup>th</sup> Oct. to 1<sup>st</sup> Mar.2012]



[FIG 3.26 DFA profile of NASDAQ trading volume]

### 3.3 Discussion:

The scaling properties of different time series were calculated using detrended fluctuation analysis. The daily closing value of indices were considered till 26th sep. 2010. Dataset of NIFTY contains 2015 data points whereas DAX data contains 4991 and DJIA data contains 5262 data points. The week-ends and holidays are not considered. The data were collected from the website of yahoo finance [9].

By using DFA analysis, the fractal dimension of NSE index for daily, monthly, and quarterly closing values are calculated. The variation of DFA function values of NIFTY index with  $n$  shows that data follows simple scaling behavior. Almost same result is obtained for daily closing values of DAX and DJIA indices. Since the value of slope is found to be near to 1.5, for all types of data sets with small variance, the market behavior shows nearly classical Brownian random walk. But it is important to note that we have used closing values of indices only. It will be interesting to look for mono/multifractal features in short term (single day data, but intra-day behavior).

This study offers the advantage of a means to investigate long range correlations within a financial signal due to the intrinsic properties of the system producing the signal, rather than external stimuli unrelated to the properties of the system. In addition, the calculation is based on the entire data set and is 'scale free', offering greater potential to distinguish signals based on scale specific measures. Theoretically, the scaling exponent varies from 0.5 (random numbers) to 1.5 (random walk). A scaling exponent greater than 1.0 indicates a loss in long range scaling behavior and an alteration in the underlying system. The technique was initially applied to detect long range correlations in DNA sequences but has been increasingly applied to financial

time signals. [5,10,11]. DFA is not very much affected due to nonstationarity of data. Although DFA represents a novel technological development in the science of variation analysis and has proven its significance, whether it offers information distinct from traditional spectral analysis is debated [11]. It is inappropriate to simply 'run' the DFA algorithm blindly on data sets. Finally, although appealing in order to simplify comparison, the calculation of two scaling exponents (one for small and one for large  $n$ ) represents a somewhat arbitrary manipulation of the results of the analysis. The assumption that the same scaling pattern is present throughout the signal remains flawed, and therefore techniques without this assumption are being developed and are referred to as multifractal analysis.

### 3.4 References

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