Appendix - II

A-II.1 Derivation of Equations of Flow Stress [Eqs. Nos. (7.2) and (7.3)]

It is observed from the plot (Fig. 2.12, Chapter II, page 39) of Migaud's data that the flow stress vs. deformation temperature curve for a particular niobium content is composed of two parts: a linear portion and a non-linear portion. However, the graph (Fig. 2.12) did not reveal the temperature at which the nature of the curve changed from linearity to non-linearity. Hence, to know the exact temperature at which such change occurred, it was necessary to replot (Fig. A-II.1) the curves as log(deformation temperature) vs. log(flow stress). The standard method of curve fitting applied to these curves yielded the following equations:

**Non-linear portions**

For 0.098% Nb,
\[
\log(\text{flow stress, MPa}) = -72.717 \log(\text{temp.°C})^2 + 428.47 \log(\text{temp.°C}) - 628.82 \tag{A-II.1}
\]

For 0.032% Nb,
\[
\log(\text{flow stress, MPa}) = -62.942 \log(\text{temp.°C})^2 + 369.79 \log(\text{temp.°C}) - 540.83 \tag{A-II.2}
\]

For 0.0% Nb,
\[
\log(\text{flow stress, MPa}) = -28.717 \log(\text{temp.°C})^2 + 168.37 \log(\text{temp.°C}) - 244.63 \tag{A-II.3}
\]

**Linear portions**

For 0.098% Nb,
\[
\log(\text{flow stress, MPa}) = -4.0986 \log(\text{temp.°C}) + 14.447 \tag{A-II.4}
\]

For 0.032% Nb,
\[
\log(\text{flow stress, MPa}) = -3.9216 \log(\text{temp.°C}) + 13.865 \tag{A-II.5}
\]

For 0.0% Nb,
\[
\log(\text{flow stress, MPa}) = -3.7133 \log(\text{temp.°C}) + 13.182 \tag{A-II.6}
\]
Fig. A-II.1 - Graph of log(deformation temp, °C) vs. log(flow stress, MPa)
The variation of the coefficients of \([\log(\text{temp.})]^2\) in equations (A-II.1) to (A-II.3) was due to the variation in Nb contents in the steels. Hence, these coefficients were required to be expressed in terms of \%\text{Nb}. The same logic was followed for the coefficients of \([\log(\text{temp.})]\) and the constant terms of these equations, i.e., equation Nos. (A-II.1) to (A-II.3). This resulted in the following equation for flow stress (below the \(T_R\) temperature).

\[
\log [\text{Mean Flow Stress (MPa)}] = [9401.4 (\%\text{Nb})^2 - 1370.3 (\%\text{Nb}) - 28.717] . \ [\log(\text{temp.}^\circ\text{C})]^2 + [-55156 (\%\text{Nb})^2 + 8059.4 (\%\text{Nb}) + 168.37] . [\log(\text{temp.}^\circ\text{C})] - [-80848 (\%\text{Nb})^2 + 11843 (\%\text{Nb}) + 244.63] \] ........................................ (A-II.7)

Application of the same reasoning to the linear portion, i.e., equation Nos. (A-II.4) to (A-II.6), yielded the following equation for flow stress above the \(T_R\) temperature.

\[
\log [\text{Mean Flow Stress (MPa)}] = [39.057 (\%\text{Nb})^2 - 7.7592 (\%\text{Nb}) - 3.7133] . \ [\log(\text{temp.}^\circ\text{C})] + [-127.81 (\%\text{Nb})^2 + 25.434 (\%\text{Nb}) + 13.182] \] ......... (A-II.8)

These equations are valid for steels in the same compositional range as that of Migaud’s steel. The solution of these two equations gives the point of intersection of the two curves. The abscissa of this point of intersection is the temperature at which the nature of the curve changes.
A-II.2 Determination of $T_R$ temperature of the present steel analytically

The steel used in the present investigation (GPP steel) had a Niobium content of 0.03\% and was in the similar compositional range (vide Art. 7.2) as that of Migaud’s steel. Hence, equations (A-II.7) and (A-II.8) could be applied to determine the $T_R$ temperature of the steel as follows.

Substituting the value of Nb-content (i.e, 0.03) in equations (A-II.7) and (A-II.8), one obtains,

$$\log \text{[Mean Flow Stress (MPa)]} = -61.368 \times \log(\text{temp.}°\text{C})^2 + 360.5116 \times \log(\text{temp.}°\text{C}) - 527.1568 \quad \text{...(A-II.9)}$$

$$\log \text{[Mean Flow Stress (MPa)]} = -3.9109 \times \log(\text{temp.}°\text{C}) - 13.83 \quad \text{...(A-II.10)}$$

Equating equations (A-II.9) and (A-II.10) and simplifying one obtains,

$$61.368 \times \log(\text{temp.}°\text{C})^2 - 360.5116 \times \log(\text{temp.}°\text{C}) + 540.9868$$

$$\Rightarrow \log(\text{temp.}°\text{C}) = \frac{364.4225 \pm \sqrt{[(-364.4225)^2 - 4 \times 61.368 \times 540.9868]}}{2 \times 61.368}$$

$$\Rightarrow \log(\text{temp.}°\text{C}) = 2.99, 2.948$$

$$\Rightarrow \text{Temp.}°\text{C} = 977, 887$$

The realistic value of 977 °C is taken as the $T_R$ temperature of GPP steel. This may be observed from the graph (Fig. A-II.1) also.