6.1 Introduction:

Inventory practitioners and industrial engineers have given more importance to three aspects while modelling the inventory models. The first one is utilization which measures the efficiency of the firms that how effectively they use their inventories. The second performance aspect is effectiveness which captures the quality of the process output. This dimension is interpreted broadly to include durability, reliability of supply, and quantity per unit package. As customer judges the goods on the basis of price and quality, the choice of quality is often an important factor to an industry. Third one is productivity which measures the transformation efficiency and is reported as the inventory turnover ratio. The higher turnover causes higher productivity with which an industry uses its inventories. The determination of the most cost-effective production quantity under rather stable conditions is commonly known as the classical Economic Production Quantity (EPQ) inventory problem. Over the last three decades tremendous amount of research effort has been expended on this topic leading to the publication of many interesting results in the literature.

Nowadays, the managers of all manufacturing companies implement the Flexible Manufacturing System (FMS). Volume flexibility is a major component in FMS. It helps us to adjust the rate of production according to the market demand and avoid rapid accumulation of inventories. It is obvious that an increasing rate of production increases the probability of components (machinery parts, labor) failure and thus amount of imperfect items increases. At the start, production process is ‘in-control’ state and the items produced are of good quality and after some time, it may shift to an ‘out-of-control’ state while in process, so in this case items produced are not of good quality. In the Classical Economic Production Lot Size (EPLS) model, the production rate of a machine was regarded to be pre-determined and inflexible (Hax and Candea, 1984). Most of the practitioner assumed that the unit production cost was constant. Khouja (1995) said that the unit production cost depends on the cost of used raw materials, laborers’ wages, electricity and wear and tear costs of machines. As the amount of used raw materials is fixed, it contributes a constant cost part to the unit cost. With the same number of labourers and same amount of resources like energy, if more number of units is produced
per unit time, then total production cost is distributed over the produced units. Therefore unit cost is distributed over the produced units. Thus the unit cost decreases with increased production rate and it is inversely proportional to the number of produced units. Again, during production, different machine parts are damaged and repaired. If the production rate is high, the occurrence of wear and tear of machine parts will be more. Thus the cost due to failure of machine parts and their repair are directly proportional to the rate of production. Hence the machine production rate is very important in the case of production-inventory systems and is a decision variable in the case of volume flexibility. Therefore the unit production cost becomes a function of production rate consisting of (1). a constant, (2). inversely proportional and (3). directly proportional part with respect to production rate. Panda et al. (2008) modeled an Economic Production Lot Size (EPLS) model for imperfect items in which production rate was considered as fixed quantity and the demand rate was probabilistic under certain budget and shortage constraints. They also assumed that the percentage of defective items was stochastic and the nature of uncertainty in the constraints was stochastic or fuzzy. Das et al. (2011) developed an economic production lot size model for an item with imperfect quality by considering machine failure. They developed the model for profit maximization in stochastic and fuzzy-stochastic environments by considering some inventory parameters as imprecise in nature.

In traditional models, algorithms have been developed for solving the inventory problems when inventory parameters like total floor space, production cost, setup cost, holding cost, etc., are precisely known. In real life situations, these parameters may be uncertain. In competitive market, it is not possible to do the business with predefined fixed inventory parameters. Initially, a decision maker (DM) may start with some fixed space and fixed production cost, setup cost etc., but, at a later stage, to meet the sudden increase of demand or to avail the sudden fall in the price of the commodity, he/she is forced to augment some more space and capital as per demand of the situation. Hence, in this case, cost determination is imprecise. Similar may be the case of storage area.

The applications of Geometric Programming (GP) techniques are well documented on inventory control in the literature. Chen (2001) proposed an inventory model under return-on-cost of a product via geometric programming for an intermediate firm to
determine the selling quantity and purchase cost of a product via geometric programming technique. Mandal et al. (2005, 2006) obtained the optimal solution by using Geometric Programming. Islam and Roy (2007) employed the Geometric Programming to solve the model by considering the fuzzy random demand. Liu (2008) obtained the fuzzy profit of the inventory model when demand and cost are fuzzy in nature by using Geometric Programming. Bag et al. (2009) developed the inventory model with fuzzy random demand with flexibility and reliability, considering only single item and solved the problem by Geometric Programming. In that study, they assumed that production rate and production cost per unit item remain constant. Different cost parameters were taken as crisp variable.

It was surprising to observe literature that very few mathematical models that addressed volume flexibility and reliability of production system in fuzzy environment have been developed. Most of the researchers considered different cost parameters associated to inventory as crisp variables and production rate as pre-determined parameter. So, in the present chapter two different production inventory models are being addressed.

**Model-1: A Fuzzy Multi-item Production Model with Reliability and Flexibility under limited Storage Capacity with Deterioration via Geometric Programming:**

It is often difficult to determine the actual inventory costs while modelling inventory models. Costs fluctuate depending upon different aspects of market situation. So that cost parameters such as production cost, carrying cost and interest cost are imprecise in nature i.e., cost parameters are fuzzy variables. The inventory problem is controlled by some constraints. Restrictions on storage space influence the optimal inventory cost. In real life problems, it is almost impossible to predict the required storage space precisely. This decision can be changes with requirement and market situations. So, the decision-makers may change storage space within some limits as per the demand of the situation. It is also well known that the worth of products gradually decreases with time due to storage and many other conditions. So it affects the optimal solution. As no production system is perfect, consideration of reliability factor is also important. It is also observed from the literature that most of practitioners developed the models considering single item only.
which is not realistic. From the above discussion, it can be concluded that there is a need for the reformulations of the inventory control policy taking into account the entire factor.

In the present work, a multi-item Economic Production Quantity (EPQ) model with deterioration is considered, where unit production cost, holding cost are assumed to be imprecise in nature and the production process is assumed not to be 100% perfect. In the proposed model here, the storage area was considered to be limited due to high rent and scarcity of space in market place. Due to the general nature of the proposed relationship between production setup cost, process reliability and flexibility, optimization by calculus often leads to a system of non-linear equations which, in general, are very hard to be solved explicitly and numerical methods are always needed to obtain approximate solutions. Consequently, closed-form optimal solutions to this EPQ problem are not easily available through calculus-based optimization technique. A Modified Geometric Programming (MGP) approach was employed here to solve the proposed multi-item EPQ model due to posynomial nature of objective function (as MGP is one of the most efficient optimization techniques to deal with posynomials).

Model-2: Volume Flexible Production-Policy with Repairable Defective Product

Under increased competition, inventory related businesses are focused for the coordination between their procurement and marketing decisions to avoid over stocks when sales are low or demand are high. An effective means of such coordination is to conduct the inventory control and manufacturing decision jointly. A manager requires that every employee—operators, analysts, quality inspectors, salesman, purchasing agents, or planners—are thoroughly and strictly disciplined about feeding updates into the system. The main task in doing so is to determine the optimal rate of production and inventory policy.

In this section, a Fuzzy Production Inventory Model (FPIM) with volume flexible production rate has been considered. Different inventory associated costs are taken as trapezoidal fuzzy number. During the production-run-time, the manufacturing process may shift to an ‘out-of-control’ state. In ‘out-of-control’ state, a part of produced items are
defective. The defective items are reworked immediately at a cost. The inventory cost function both in the crisp sense and fuzzy sense are derived. The fuzzy model is defuzzified by using Graded Mean Integration Representation (GMIR) method. Cost function is minimized by considering production rate as decision variables.

Now, systematic developments of two different scenarios are going to be present in the following sections.

6.2 A Fuzzy Multi-item Production Model with Reliability and Flexibility under limited Storage Capacity with Deterioration via Geometric Programming:

A situation was considered where a company produces a multi-item using a conventional production process with a certain level of reliability. The process reliability depends on a number of factors such as machine capability, use of on-line monitoring devices, skill level of the operating personnel and maintenance and replacement policies. Higher reliability means products with acceptable quality are more consistently produced by the process, thereby reducing the costs of scrap and rework of substandard products, wasted materials and labour hours. However, high reliability can only be achieved with substantial capital investment that will increase the cost of interest and depreciation of the production process.

6.2.1 Assumptions:

In this section, development of production inventory model is made under the following assumptions.

(1) Demand rate $D_i$ for the $i^{th}$ item is uniform over time.

(2) Total time horizon is infinite.

(3) Total cost of interest and depreciation per production cycle is inversely related to a setup cost and directly related to production process reliability according to the following equation $f_i(S_i, r_i) = a_i S_i^{-b_i} r_i^{c_i}$, where $a_i, b_i, c_i (\geq 0)$ are shape parameters.
(4) The unit production cost is a continuous function of demand \( D_i \) and takes the following form \( p_i = \alpha D_i^{\beta} \), where \( \beta_i (>0) \) and \( \alpha_i (>0) \) are shape parameters.

(5) The deterioration rate \( \theta_i (<1) \) is uniform over time and shortages are not allowed.

The third assumption is based on the fact that to reduce the costs of production setup, scrap and rework on shoddy products, substantial investment is required in improving the flexibility and reliability of the production process. Consequently the total cost of interest and depreciation per production cycle of the modern flexible production process is much higher than that of conventional inflexible process. In reality, this relationship should be discrete but a continuous function is used as an approximation which is needed to simplify the subsequent mathematical analysis. A similar equation modelling the relationship between setup cost, interest and depreciation cost was suggested by Van Beek and Putten (1987). 

\[
\frac{df_i(S_i,r_i)}{dS_i} < 0 \quad \text{and} \quad \frac{df_i(S_i,r_i)}{dr_i} > 0
\]

which accord with the desired relation between interest and depreciation cost, setup cost and process reliability in a simple but general format. An advantage of the power function is that, by taking the natural logarithm of both sides, the coefficients \( a_i, b_i, c_i \) can be conveniently estimated using statistical linear regression procedures.

Note that the production process reliability, \( r_i \), is constant in order to simplify the mathematical analysis of the problem, and that the interest and depreciation cost may also be modeled in terms of the unreliability, \( (1-r_i) \), of the production process, i.e., 

\[
f_i(S_i,r_i) = a_i S_i^{-b} (1-r_i)^{-c_i}, \text{where } a_i, b_i, c_i (\geq 0). \]

So that \( f_i(S_i,r_i) \rightarrow \infty \) as \( r_i \rightarrow 1 \) to reflect the fact that the process will never be 100% reliable.

6.2.2 Notations:
To construct the model for this section, the following parameters and variables are considered:

\[ TC(D_i, S_i, q_i, r_i) = \text{total average cost of production and inventory holding cost per unit time.} \]

Parameters for the \( i \text{th} \) \((i=1, 2, 3, \ldots, n)\) item are

- \( S_i \) Setup cost per batch (decision variable)
- \( D_i \) Demand rate (decision variable)
- \( q_i \) Production quantity per batch (decision variable)
- \( r_i \) Production process reliability (decision variable)
- \( H_i \) Inventory holding cost per item per unit time
- \( f_i(S_i, r_i) \) Total cost of interest and depreciation for a production process per production cycle
- \( p_i \) Unit demand dependent production cost
- \( \theta_i \) Deterioration rate

### 6.2.3 Mathematical Formulation of Inventory Model:

First of all a model is developed in which all parameters are considered as deterministic in nature i.e., these parameters are crisp in nature.

### 6.2.4 Crisp Model:

Let the amount of the stock for the \( i \text{th} \) item \((i=1, 2, \ldots, n)\) be \( r_i q_i \) at time \( t=0 \). In the interval \((0,T')\), the inventory level gradually decreases due to demand and deterioration.
By this process the inventory level reaches zero level at time $T_i$. This process repeats itself in each cycle. If $q_i'(t)$ is the inventory level of the $i^{th}$ item at time $t$ over the time period $(0,T_i)$, then

$$q_i'(t)+\theta q_i(t)=-D_i \quad \text{for} \ 0 \leq t \leq T_i$$

#### (6.1)

With initial and boundary conditions:

$$q_i(0)=r_q, \quad q_i(T_i)=0$$

On solving the above differential equation, one can get

$$q_i(t)=r_q e^{\theta t}+\frac{D_i}{\theta_i} (e^{\theta t} - 1)$$

#### (6.2)

and using the condition $q_i(T_i)=0$

$$T_i = \frac{r_q}{D_i}$$

#### (6.3)

Now, inventory holding cost = $H_i \int_0^{T_i} q_i(t)\,dt = H_i r_q$ 

#### (6.4)

Total inventory related cost per cycle of the $i^{th}$ item = setup cost + production cost + inventory holding cost + interest as well as depreciation cost

$$= S_i + p_i q_i + H_i r_q + f_i(S_i, r_i)$$

#### (6.5)

The total average cost of the inventory system consists of the setup cost, production cost, inventory holding cost and interest and depreciation cost for the $n$ items is given by

$$TC(D_i, S_i, q_i, r_i) = \sum_{i=1}^{n} \left[D_i S_i q_i^{-1} r_i^{-1} + \alpha_i D_i^{1-\theta_i} r_i^{-1} + H_i r_i + a_i D_i S_i^{-h} q_i^{-\theta} r_i^{-1}\right]$$
There are some restrictions on available resources in inventory problem that cannot be ignored to derive the optimal total cost. One of the restrictions is that there is a limitation on the available floor space where the items are to be stored. Then

\[ \sum_{i=1}^{n} w_i q_i \leq w \]

Hence the inventory problem for \( n \)-items is as follows:

The problem is to find demand levels, setup cost, production quantity per batch, and production process reliability so as to minimize the total average cost function subject to the total space constraints. It may be written as

\[
\text{Min } TC(D_i, S_i, q_i, r_i) = \sum_{i=1}^{n} \left[ D_i S_i q_i^{-1} r_i^{-1} + a_i D_i S_i^{-1} q_i^{-1} r_i^{-1} + \alpha_i D_i S_i^{-1} q_i^{-1} r_i^{-1} + \beta_i D_i S_i^{-1} q_i^{-1} r_i^{-1} \right] \\
\text{s.t } \sum_{i=1}^{n} w_i r_i q_i \leq w \\
D_i, S_i, q_i, r_i > 0 \quad (i = 1, 2, 3, \ldots, n)
\]

6.2.5 Fuzzy Inventory Model with Imprecise Costs and Resources:

In a multi-item inventory system, a manufacturing company may initially have a warehouse of capacity \( w \) units to store the items. But in course of business, to take the advantage of special discount or minimum transportation cost, etc., decision-maker may have to augment the storage area, if the situation demands, i.e., in that case, the warehouse capacity becomes uncertain in non-stochastic sense and storage area can be expressed by fuzzy set. Depending upon different aspects, inventory cost parameters fluctuate. So, holding cost and setup cost are assumed as fuzzy numbers. So, it is assumed that all the parameters \((a_i, b_i, c_i, \alpha_i, \beta_i, H_i)\) and storage spaces \((w_i, w)\) are fuzzy in nature. Then the above crisp inventory model reduces to

\[
\text{Min } TC(D_i, S_i, q_i, r_i) = \sum_{i=1}^{n} \left[ D_i S_i q_i^{-1} r_i^{-1} + \alpha_i D_i^{-1} q_i^{-1} r_i^{-1} + \beta_i q_i^{-1} r_i^{-1} + \gamma_i D_i S_i^{-1} q_i^{-1} r_i^{-1} \right]
\]
\begin{align*}
\sum_{i=1}^{n} & \tilde{\varphi}_i q_i \leq \tilde{\omega} \\
\text{s.t} & \\
D_i, S_i, q_i, r_i > 0 \quad (i = 1, 2, 3, \ldots, n)
\end{align*}

Using approximated value of TFN, \(\hat{a}_i, \hat{b}_i, \hat{c}_i, \hat{\alpha}_i, \hat{\beta}_i, \hat{H}_i, \hat{w}_i\) and \(\hat{w}_i\) are the approximated values of the TFN parameters.

\begin{align*}
\hat{\varphi}_i &= (a_{i1}, a_{i2}, a_{i3}), \\
\hat{\rho}_i &= (b_{i1}, b_{i2}, b_{i3}), \\
\hat{\delta}_i &= (c_{i1}, c_{i2}, c_{i3}), \\
\hat{\alpha}_i &= (\alpha_{i1}, \alpha_{i2}, \alpha_{i3}), \\
\hat{\beta}_i &= (\beta_{i1}, \beta_{i2}, \beta_{i3}), \\
\hat{H}_i &= (H_{i1}, H_{i2}, H_{i3}), \\
\hat{w}_i &= (w_{i1}, w_{i2}, w_{i3})
\end{align*}

The above fuzzy model reduces to

\begin{align*}
\text{Min } & TC^*(D^*, S^*, q^*_i, r^*_i) = n \sqrt{d(\alpha')} = \sum_{i=1}^{n} \left[ D_i S_i q_i^{1/r_i} + \hat{a}_i D_i^{1/\hat{\beta}_i} r_i^{1/\hat{\delta}_i} + H_i q_i + \hat{a}_i D_i^{1/\hat{H}_i} q_i^{1/\hat{r}_i} \right] \\
\text{s.t} & \\
\sum_{i=1}^{n} \hat{w}_i r_i q_i \leq \hat{w} \\
D_i, S_i, q_i, r_i > 0 \quad (i = 1, 2, 3, \ldots, n)
\end{align*}

To obtain the optimal solution of developed model, MGP technique is used. This can be described step by step as follows:

\subsection*{6.2.6 Primal Problem:}

\begin{align*}
\text{Min } & TC^*(D^*, S^*, q^*_i, r^*_i) = n \sqrt{d(\alpha')} = \sum_{i=1}^{n} \left[ D_i S_i q_i^{1/r_i} + \hat{a}_i D_i^{1/\hat{\beta}_i} r_i^{1/\hat{\delta}_i} + H_i q_i + \hat{a}_i D_i^{1/\hat{H}_i} q_i^{1/\hat{r}_i} \right] \\
\text{s.t} & \\
\sum_{i=1}^{n} \hat{w}_i r_i q_i \leq \hat{w} \\
D_i, S_i, q_i, r_i > 0 \quad (i = 1, 2, 3, \ldots, n)
\end{align*}

The coefficient of each term of the objective function is positive. Hence this is the posynomial primal geometric programming problem.

\subsection*{6.2.7 Dual Problem:}
It is a constrained posynomial primal geometric problem. Here number of terms in the $i^{th}$ function is 4 and the numbers of decision variables (namely $D_i$, $q_i$, $S_i$, $r_i$) in that function is 4. Applying modified Geometric Programming (GP) technique according to Abou-El-Ata and Kotb (1997), the dual problem is

$$\text{Max } d(\alpha) = \prod_{i=1}^{n} \left( \frac{1}{\alpha_i^{q_i}} \right) \left( \frac{\hat{a}_i}{\alpha_i^{q_i \gamma_1}} \right) \left( \frac{\hat{H}_i}{\alpha_i^{q_i \gamma_3}} \right) \left( \frac{\hat{a}_i}{\alpha_i^{q_i \gamma_5}} \right) \left( \sum_{j=1}^{n} \alpha_j^{a_j} \right)^{a_j}$$

s.t. 

$$\alpha_{i1} + \alpha_{i2} + \alpha_{i3} + \alpha_{i4} = 1$$

$$\alpha_{i1} + (1-\beta)\alpha_{i2} + \alpha_{i4} = 0$$

$$\alpha_{i1} - \hat{b}_i \alpha_{i4} = 0$$

$$-\alpha_{i1} + \alpha_{i3} - \alpha_{i4} + \alpha_{i5} = 0$$

$$-\alpha_{i1} - \alpha_{i2} + (\hat{c}_i - 1)\alpha_{i4} + \alpha_{i5} = 0$$

$$0 \leq \alpha_j \leq 1 \quad (j=1,2,3,4,5; \quad i=1,2,3,\ldots,n)$$

On solving the above system of equations (6.8), (6.9), (6.10), (6.11), and (6.12), dual values of $\alpha_{i1}^\ast, \alpha_{i2}^\ast, \ldots, \alpha_{i5}^\ast$ can be obtained.

$$\alpha_{i1}^\ast = \hat{b}_i (\hat{\beta}_i - 1)/(\hat{c}_i + 2\hat{b}_i\hat{\beta}_i + 2\hat{\beta}_i - \hat{c}_i\hat{\beta}_i)$$

$$\alpha_{i2}^\ast = (\hat{b}_i - 1)/(\hat{c}_i + 2\hat{b}_i\hat{\beta}_i + 2\hat{\beta}_i - \hat{c}_i\hat{\beta}_i)$$

$$\alpha_{i3}^\ast = (\hat{b}_i\hat{\beta}_i + \hat{\beta}_i - \hat{c}_i\hat{\beta}_i)/(\hat{c}_i + 2\hat{b}_i\hat{\beta}_i + 2\hat{\beta}_i - \hat{c}_i\hat{\beta}_i)$$

$$\alpha_{i4}^\ast = (\hat{\beta}_i - 1)/(\hat{c}_i + 2\hat{b}_i\hat{\beta}_i + 2\hat{\beta}_i - \hat{c}_i\hat{\beta}_i)$$

$$\alpha_{i5}^\ast = -(\hat{b}_i + \hat{c}_i - \hat{c}_i\hat{\beta}_i + 1)/(\hat{c}_i + 2\hat{b}_i\hat{\beta}_i + 2\hat{\beta}_i - \hat{c}_i\hat{\beta}_i)$$

(i=1, 2, 3, \ldots, n)
So the dual objective value is given by

\[
d(\alpha^*) = \prod_{i=1}^{n} \left( \frac{1}{\alpha_{i1}^{*}} \right)^{\alpha_{i1}^{*}} \frac{\hat{a}_{i}}{\alpha_{i2}^{*}}^{\alpha_{i2}^{*}} \frac{\hat{H}_{i}}{\alpha_{i3}^{*}}^{\alpha_{i3}^{*}} \frac{\hat{\alpha}_{i}}{\alpha_{i4}^{*}}^{\alpha_{i4}^{*}} \frac{\hat{w}_{i}}{w^{\alpha_{i5}^{*}}}^{\alpha_{i5}^{*}} \sum_{j=1}^{n} \alpha_{i5}^{*} \right) \quad \ldots (6.13)
\]

Thus, the minimum value of the average cost:

\[
TC^*(D^*, S^*, q^*, r^*) = n \sqrt{d(\alpha^*)} \quad \ldots (6.14)
\]

To find the optimal values of \(D^*_i, S^*_i, q^*_i, r^*_i\), we have

\[
\frac{D_i S_i q_i r_i}{\alpha_{11}^{*}} = \frac{\hat{r}_i^{*1-\hat{\beta}_1}}{\alpha_{21}^{*}} \frac{\hat{q}_i^{*1-\hat{\beta}_2}}{\alpha_{31}^{*}} \frac{\hat{D}_i}{\alpha_{41}^{*}} = \frac{S_i}{\alpha_{12}^{*}} \frac{\hat{q}_i^{*1-\hat{\beta}_3}}{\alpha_{22}^{*}} \frac{\hat{r}_i^{*1-\hat{\beta}_4}}{\alpha_{32}^{*}} \frac{\hat{D}_i}{\alpha_{42}^{*}}
\]

\[
= \frac{D_n S_n q_n r_n}{\alpha_{1n}^{*}} = \frac{\hat{r}_n^{*1-\hat{\beta}_n}}{\alpha_{2n}^{*}} \frac{\hat{q}_n^{*1-\hat{\beta}_n}}{\alpha_{3n}^{*}} \frac{\hat{D}_n}{\alpha_{4n}^{*}}
\]

\[
= \frac{\sum_{i=1}^{n} \left[ D_i S_i q_i r_i \hat{a}_i D_i^{\hat{\beta}_1} r_i^{\hat{\beta}_1} + H_i q_i r_i + \hat{a}_i D_i S_i q_i r_i^{\hat{\beta}_1} \right]}{\sum_{i=1}^{n} (\alpha_{i1}^{*} + \alpha_{i2}^{*} + \alpha_{i3}^{*} + \alpha_{i4}^{*})}
\]

\[
= \frac{TC^*(D^*, S^*, q^*, r^*)}{n} = \frac{n \sqrt{d(\alpha^*)}}{n} = \sqrt{d(\alpha^*)}
\]

\[
So, \hat{H}_i q_i r_i = \alpha_{i3}^{*} \sqrt{d(\alpha)}, \hat{a}_i D_i S_i q_i r_i^{\hat{\beta}_1} = \alpha_{i4}^{*} \sqrt{d(\alpha)}
\]

\[
D_i S_i q_i r_i^{\hat{\beta}_1} = \alpha_{i1}^{*} \sqrt{d(\alpha)}, \hat{a}_i D_i^{\hat{\beta}_1} r_i^{\hat{\beta}_1} = \alpha_{i2}^{*} \sqrt{d(\alpha)}
\]
Solving the above four non-linear equation for i = 1, 2, 3... n. We get optimal values of
\( D_i^* \), \( S_i^* \), \( q_i^* \), \( r_i^* \) (i=1, 2, 3... n) as follows:

\[
D_i^* = \left\{ \frac{(b\hat{\beta}_i - b_i^*)}{(\hat{\alpha}_i + b_i^*)} \right\} \left[ \frac{(b\hat{\beta}_i - b_i^*)}{(\hat{\alpha}_i + b_i^*)} \right]^{(2b_i^* + c_i^*)} \left( \frac{n \sqrt{d(\alpha^*)}}{(c_i + 2b\hat{\beta}_i + 2\hat{\beta}_i \hat{\beta}_i)^{\frac{1}{2}}} \right)^{\frac{1}{(1 + b_i^* - c_i^* \hat{\beta}_i)}}
\]

\[
S_i^* = \left( \frac{(b\hat{\beta}_i - b_i^*)}{(\hat{\alpha}_i + b_i^*)} \right)^2 \left( \frac{n \sqrt{d(\alpha^*)}}{(c_i + 2b\hat{\beta}_i + 2\hat{\beta}_i \hat{\beta}_i)^{\frac{1}{2}}} \right)^{\frac{1}{(1 + b_i^* - c_i^* \hat{\beta}_i)}}
\]

\[
q_i^* = \left( \frac{(b\hat{\beta}_i - b_i^*)}{(\hat{\alpha}_i + b_i^*)} \right)^2 \left( \frac{n \sqrt{d(\alpha^*)}}{(c_i + 2b\hat{\beta}_i + 2\hat{\beta}_i \hat{\beta}_i)^{\frac{1}{2}}} \right)^{\frac{1}{(1 + b_i^* - c_i^* \hat{\beta}_i)}}
\]

\[
r_i^* = \left( \frac{(b\hat{\beta}_i - b_i^*)}{(\hat{\alpha}_i + b_i^*)} \right)^2 \left( \frac{n \sqrt{d(\alpha^*)}}{(c_i + 2b\hat{\beta}_i + 2\hat{\beta}_i \hat{\beta}_i)^{\frac{1}{2}}} \right)^{\frac{1}{(1 + b_i^* - c_i^* \hat{\beta}_i)}} \quad (i = 1, 2, 3, \ldots, n)
\]

6.3 Volume Flexible Production-Policy with Repairable Defective Product:

In this case the production cost is considered as a function of production rate.

6.3.1 Assumptions:

The mathematical model in this section is developed on the basis of the following assumptions:

1. Q is the production rate and considered as a decision variable.
2. Production process is imperfect.
3. Defective product repaired at incurring some cost.
4. The production cost per unit item is a function of the production rate.
5. Shortages are not allowed.

6.3.2 Notations:

Following notations are used for the development of the proposed model.
\( \hat{p} \) = Fuzzy opportunity cost percentage \((i_1, i_2, i_3, i_4)\)

\( \tilde{S} \) = Fuzzy setup cost \((S_1, S_2, S_3, S_4)\)

\( \hat{E} \) = Fuzzy cost incurred by repairing a defecting item \((e_1, e_2, e_3, e_4)\)

\( \tilde{H} \) = Fuzzy daily holding cost per unit \((h_1, h_2, h_3, h_4)\)

\( p \) = The probability that the production process can go ‘out-of-control’

\( a_p \) = The investment is required to reduce the ‘out-of-control’ probability \( P \)

D = Total demand over the Planning time period [0,T]

R = Daily demand

B = Daily production

Q = Production quantity

\( C(Q) = \) The unit production cost \( (= r + g/Q + \beta Q) \)

Where \( r, g, \beta \) are all positive constants. This cost is based on the following factors:

1. The material cost \( r \) per unit item is fixed.

2. As the production rate increases, some costs like labour and energy costs are equally distributed over a large number of units. Hence the production cost per unit \( (g/Q) \) decreases as the production rate \( Q \) increases.

3. The third term \( (\beta Q) \), associated with tool/die, is proportional to the production rate.

6.3. Mathematical Formulation of Inventory Model:

The production cycle begins with zero inventory level. The production starts with the rate \( Q \) at time \( t=0 \) and continues up to time \( t_{G_t} \). During time span \([0, t_{G_t}]\) inventory piles up, adjusting demand D in the market.
Furthermore,

\[ t_{G_t} = \frac{Q}{B} \] is the length of the product run in days

\[ R_{G_t} = R \frac{Q}{B} \] is the sale quantity for the product run

\[ M = Q - R \frac{Q}{B} = Q \left( 1 - \frac{R}{B} \right) \] is the inventory quantity at the end of the product run, where R should be less than B and average inventory is

\[ M_s = \frac{M}{2} = \frac{1}{2} Q \left( 1 - \frac{R}{B} \right) \]

Production cost = \( C(Q)D \)

Investment cost required for fixed process = \( l a_p T \)

Setup cost = \( S \left( \frac{D}{Q} \right) \)

Holding cost = \((IC(Q)+H)(1-R/B)TQ/2\)

Repairing cost for defective product = \( D p E \)

**Inventory Level**

\[ \text{Fig. 6.1 Inventory level of Production Model} \]
The total cost \( F \) per cycle time \( T \) with imperfect production = Production Cost + Fuzzy investment cost required for fixed process + Fuzzy setup cost + Fuzzy holding cost + Fuzzy repairing cost for defectives

\[
F = C(Q) D + I_a T + S + (IC(Q) + H) \left( Q \left( 1 - \frac{R}{B^j} \right)^j \right) T + D p E
\]

In order to find the optimal production rate, all the costs are represented by trapezoidal fuzzy numbers. The trapezoidal fuzzy average annual cost is calculated using arithmetic operations based on the Function Principle. The Function Principle defines an efficient way to perform the basic fuzzy arithmetic operations on fuzzy numbers. Now, let us consider that input costs, \( S, E, H, I \) are imprecise and expressed by trapezoidal fuzzy numbers \( b_i, b_i, b_i, b_i \) respectively. A normal trapezoidal fuzzy number, \( \tilde{A} = [k_1, k_2, k_3, k_4] \), satisfies the condition \( 0 \leq k_1 \leq k_2 \leq k_3 \leq k_4 \) and has the following membership function:

\[
\mu_{\tilde{A}}(x) = \begin{cases} 
0 & x < k_1, x > k_4 \\
\frac{x - k_1}{k_2 - k_1} & k_1 \leq x < k_2 \\
1 & k_2 \leq x < k_3 \\
\frac{k_4 - x}{k_4 - k_3} & k_3 \leq x \leq k_4 
\end{cases}
\]

In this study, Chen’s (1985) Function Principle that simplifies the calculation of the fuzzy total cost function is used. According to the Function Principle, the fuzzy total annual cost \( \hat{F} (F_1, F_2, F_3, F_4) \), becomes a trapezoidal fuzzy number where

\[
F_l = D \left( r + \frac{g}{Q} + \beta Q \right) + e_i p_i + a_p T_l + \frac{DS_l}{Q} + \frac{T}{2} \left( 1 - \frac{R}{B^j} \right) \left( r + \frac{g}{Q} + \beta Q \right) + h_k Q
\]

\( l = 1, 2, 3, 4. \)
\( P(F) \) satisfies the conditions 0 ≤ \( F_1 \) ≤ \( F_2 \) ≤ \( F_3 \) ≤ \( F_4 \) and its membership is given by

\[
\mu_{F}(x) = \begin{cases} 
0 & x < F_i, x > F_i \\
\frac{x - F_1}{F_2 - F_1} & F_1 \leq x < F_2 \\
1 & F_2 \leq x < F_3 \\
\frac{F_4 - x}{F_4 - F_3} & F_3 \leq x \leq F_4 
\end{cases}
\]

In order to minimize the fuzzy total cost, the fuzzy total inventory cost is defuzzified by using graded mean integration representation method.

\[
P(\tilde{F}) = \frac{1}{6} \left[ D \left( 2 \left( r + \frac{g}{Q} + \beta Q \right) + \left( e_1 + e_4 \right) \frac{D}{Q} \left( S_2 + S_4 \right) + \frac{R}{B} \right) + \left( r + \frac{g}{Q} + \beta Q \right) \left( i_2 + i_4 \right) + \left( h_2 + h_4 \right) + \frac{T}{2} \left( 1 - \frac{R}{B} \right) \right] \]

Optimal production quantity \( Q^* \) can be obtained by calculus method when \( P(\tilde{F}) \) is minimum.

\[
\frac{1}{6} \left[ 3D \left( -\frac{2g}{Q} + 2\beta \right) - \frac{D}{Q} \left( S_1 + 2S_2 + 2S_3 + S_4 \right) + \frac{T}{2} \left( 1 - \frac{R}{B} \right) \left( r + 2\beta Q \left( i_1 + 2i_2 + 2i_3 + i_4 \right) \right) + \left( h_1 + 2h_2 + 2h_3 + h_4 \right) \right] = 0
\]

On solving the above expression one can get the optimal value of \( Q \).

**Theorem-1**: There exists a global minimum value of \( P(\tilde{F}) \).
Proof: \[
\frac{d^2 P(\beta)}{dQ^2} = \frac{1}{6} \left[ 3D \left( \frac{4g}{Q^2} \right) + \frac{2D}{Q^3} \left( K_1 + 2K_2 + 2K_3 + K_4 \right) + \right.

\frac{T}{2} \left( 1 - \frac{R}{B} \right) 2\beta \left( i_1 + 2i_2 + 2i_3 + i_4 \right) \left] > 0 \quad \ldots (6.19) \right.
\]

From equation (6.19) it is clear that total cost function is convex with respect to production rate.

6.4 Numerical Examples: The feasibility and applicability of developed models has been illustrated in the following sections through numerical examples:

Example-1: This example illustrates the model developed in first section. Suppose a manufacturing company produces two types of machines. The demand rate of each machine is uniform over time and can assume to be deterministic. The relevant data for the machines are given in Table 6.1. The imprecise shape parameters \( \alpha_i, \beta_i, a_i, b_i, \) and \( c_i \) are given in Table 6.2. Now we have to determine the demand rates \( (D_1, D_2) \), setup cost \( (S_1, S_2) \), and production quantity \( (q_1, q_2) \) and production reliability \( (r_1, r_2) \) of each machines and also optimal average cost \( (TC) \) of the production system. Table 6.3 presents the optimal values of decision variables.

Table-6.1: Input Data:

<table>
<thead>
<tr>
<th>Types of Machines (i)</th>
<th>Production Cost ( (D_i) )</th>
<th>Interest and depreciation cost ( (a_iS_i^{1-h}r_i^c) )</th>
<th>Holding cost, ( H_i($) )</th>
<th>Storage space area per unit item ( (w_i \text{ sq.ft}) )</th>
<th>Total storage space area ( (w \text{ sq.ft}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( D_1^{b_1} )</td>
<td>( a_1S_1^{b_1}r_1^c )</td>
<td>(8.5,10,12)</td>
<td>(8,10,12)</td>
<td>(1500,2000,3500)</td>
</tr>
<tr>
<td>2</td>
<td>( D_2^{b_2} )</td>
<td>( a_2S_2^{b_2}r_2^c )</td>
<td>(11,12,9,13)</td>
<td>(9,11,13)</td>
<td>(1500,2000,3500)</td>
</tr>
</tbody>
</table>

Table-6.2: Input Imprecise Data for Shape Parameters:

<table>
<thead>
<tr>
<th>Types of Machines( i )</th>
<th>( \alpha_i )</th>
<th>( \beta_i )</th>
<th>( a_i )</th>
<th>( b_i )</th>
<th>( c_i )</th>
</tr>
</thead>
</table>

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Table-6.3: Optimal Solution:

<table>
<thead>
<tr>
<th>Methods</th>
<th>$D_1^*$</th>
<th>$S_1^*($)$</th>
<th>$q_1^*$</th>
<th>$r_1^*$</th>
<th>$D_2^*$</th>
<th>$S_2^*($)$</th>
<th>$q_2^*$</th>
<th>$r_2^*$</th>
<th>TC* ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MGP</td>
<td>2660.3</td>
<td>4.90</td>
<td>140</td>
<td>0.77</td>
<td>1560.8</td>
<td>4.90</td>
<td>88.0</td>
<td>0.88</td>
<td>680.9</td>
</tr>
</tbody>
</table>

Observations:

1. **From Table 6.4** it is clear that demand is less sensitive with respect to ‘a’ whereas setup cost is more sensitive with respect to ‘a’. Effect of ‘a’ on demand is reverse i.e., as ‘a’ increases demand decreases whereas setup cost has directly related to ‘a’ i.e., as ‘a’ increases setup cost also increases and finally profit of production system increases as ‘a’ increases.

2. **From Table 6.4** it is clear that as the value of ‘b’ increases, demand and profit increases whereas setup cost decreases. Demand is more sensitive then setup cost and profit is very less sensitive with respect to ‘b’.

3. **From Table 6.4** it is clear that as parameter ‘c’ increases demand as well as profit decreases whereas setup cost increases. Profit is much sensitive with respect to ‘c’ and changes very rapidly with change of ‘c’.

4. Profit is much more sensitive with respect to parameter ‘c’ whereas less sensitive with respect to ‘b’. Demand is highly sensitive with respect to ‘b’ and ‘c’ and very less sensitive with respect to ‘a’. Setup cost is very sensitive with respect to ‘a’.

Table-6.4: Effect of Variation in Parameters a, b, and c on Demand, Setup Cost and Total Profit:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>(%) Variation</th>
<th>% Variation in demand</th>
<th>% Variation in setup cost</th>
<th>%</th>
</tr>
</thead>
</table>
### Table-6.5: Effect of Variation in Parameters $\alpha$ and $\beta$ on Demand and Total Profit:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variation (%)</th>
<th>Variation in demand</th>
<th>Variation in Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td><strong>$D_1$</strong></td>
<td><strong>$D_2$</strong></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-20</td>
<td>1.27</td>
<td>2.46</td>
</tr>
<tr>
<td></td>
<td>-10</td>
<td>0.60</td>
<td>1.15</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>-0.54</td>
<td>-1.03</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>-1.03</td>
<td>-1.96</td>
</tr>
<tr>
<td>$\beta$</td>
<td>-20</td>
<td>-6.22</td>
<td>-4.98</td>
</tr>
<tr>
<td></td>
<td>-10</td>
<td>-2.98</td>
<td>-2.38</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>2.78</td>
<td>2.20</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>5.38</td>
<td>4.26</td>
</tr>
<tr>
<td>$c$</td>
<td>-20</td>
<td>6.45</td>
<td>5.99</td>
</tr>
<tr>
<td></td>
<td>-10</td>
<td>3.36</td>
<td>3.98</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>-2.98</td>
<td>-2.78</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>-7.89</td>
<td>-6.08</td>
</tr>
</tbody>
</table>

**Table-6.5: Effect of Variation in Parameters $\alpha$ and $\beta$ on Demand and Total Profit:**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variation (%)</th>
<th>Variation in demand</th>
<th>Variation in Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td><strong>$D_1$</strong></td>
<td><strong>$D_2$</strong></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-20</td>
<td>17.24</td>
<td>18.56</td>
</tr>
<tr>
<td></td>
<td>-10</td>
<td>13.45</td>
<td>12.67</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>-14.56</td>
<td>-11.08</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>-20.56</td>
<td>-19.78</td>
</tr>
<tr>
<td>$\beta$</td>
<td>-20</td>
<td>-9.89</td>
<td>-10.90</td>
</tr>
<tr>
<td></td>
<td>-10</td>
<td>-7.41</td>
<td>-8.98</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>13.83</td>
<td>14.50</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>12.67</td>
<td>13.67</td>
</tr>
</tbody>
</table>
Observations:

1. From Table 6.5 it is clear that demand and profit are very sensitive with respect to ‘α’. As ‘α’ increases, demand decreases whereas profit increases. Both demand and profit changes rapidly with respect to ‘α’.

2. From Table 6.5 it is clear that demand increases whereas profit decreases with the increasing value of ‘β’. Both the value of demand and profit changes with respect to ‘β’.

3. Profit is much sensitive with respect to ‘α’ in comparison to ‘β’.

Example-2: As illustration of model in second section, consider the data from a manufacturing company. A manufacturing company produces commercial refrigerator units in batches. The firm estimated: \( H \% \) is about $1 per unit, \( G \) is about $90000, \( P \) is greater or less than $1000, \( P \) is about 0.2 \%, \( p \) is 0.02, \( a_p \) is $100000, \( R \) is 25 units, \( B \) is 30 units, \( D = 365R \), \( r = 900 \), \( g = 9000 \), \( \beta = 0.01 \). Now the problem is to find the optimal number of refrigerators to be produced in each batch in order so that total inventory cost remains minimum.

To solve the problem, a general rule is used to transfer the linguistic data, “greater or less than” and “about”, into trapezoidal fuzzy number. “greater or less than” = (0.9, 0.95, 1.05, 1.1) and “about” = (0.95, 1, 1, 1.05).

By the above rule, the fuzzy parameters in this example can be transferred as follows:

\( H = (0.95, 1, 1, 1.05) \), \( G = (85500, 90000, 90000, 94500) \), \( P = (900, 950, 1050, 1100) \), \( P = (0.0019, 0.002, 0.002, 0.0021) \).

By using the above data, the optimal value of production rate (\( Q^* \)) is 2247 units.

From Fig.6.2, Theorem-1 is numerically verified and the total cost is convex with respect to production rate.
Most of the producer perception is that the total cost decreases with the increase of production. This is not true in real life as can be seen from Fig.6.2. From Fig.6.2, it is observed that as the production rate increases, the total cost decreases up to 2247 units first; and afterwards it increases with the increase of production rate. This is due to wear and tear, holding cost, repairing cost of defective items, etc., become predominant over the other costs.

One of other wrong perceptions of producers is that total cost is minimum with the lowest unit production cost. From Fig.6.3, it was observed that lowest unit production cost is $919 corresponding to the production rate 948.68 units but for these units total cost is $9550600 which is quite high than the lowest cost. Fig.6.3 reveals that at the lowest total cost, the unit production cost is about $928.23 which is much higher than the minimum unit cost $919. Hence, through this analysis two wrong perceptions of the producers are proved to be incorrect in reality.

![Fig.6.2 Convexity of Total Cost Function with Respect to Production Rate](image)
Sensitivity Analysis:

In order to analyze how the parameters affect the optimal production rates, sensitivity analysis was conducted with respect to the parameters D, r, g, β, h, i, S. Sensitivity analysis was performed by using the data of above numerical example. The value of Q* is obtained, when one of the parameters increases or decreases by 5%, 10%, 15% and 20% while all other parameters remain unchanged. From Table 6.6, the following facts are observed:

1. Because D affects the production system directly so that production rate is fairly sensitive to D. From Fig.6.4, production rate increases from -5.55% to 4.31% as the demand rate increases from -20% to 20%. Producer adjusts production rate according to the customer demand and avoids overstocking to reduce the total inventory cost.

2. From Fig.6.5, effect of holding cost, opportunity cost and setup cost was observed on total production cost. As holding cost increases from -20% to 20% the production rate decreases from 1.73% to -1.63%. It is reasonable that when holding cost is higher, production rate decreases in order to reduce the inventory cost. As opportunity cost increases -20% to 20% then the production rate decreases from 3.35 to -3.03%.
3. **From Fig.6.6**, it was tried to analyze the effect of different components of production cost on production rate. It was observed that when the raw material cost increases -20% to 20% the total production quantity decreases from 3.18% to -2.89%. This means that as the material cost decreases, producer can produce more products in same cost. As the labour and energy cost increases from -20% to 20% then the production rate increases from -0.90% to 0.90%. Production rate decreases 5.66% to -4.82% as tool/die cost increases from -20% to 20%.

4. Here, D, g, S influence $Q^*$ in the same direction whereas the influence of $\beta$, h, i, on $Q^*$ is in the opposite direction.

5. The value of $Q^*$ is more sensitive to the parameter S. The lower value of K results lower value of $Q^*$ whereas higher value of S results higher value of $Q^*$.

**Table-6.6: Sensitivity with Respect to Different Parameters:**

<table>
<thead>
<tr>
<th>Change in %</th>
<th>Parameters</th>
<th>-20</th>
<th>-15</th>
<th>-10</th>
<th>-5</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>-5.55</td>
<td>-4.01</td>
<td>-2.58</td>
<td>-1.24</td>
<td>1.18</td>
<td>2.28</td>
<td>3.33</td>
<td>4.31</td>
<td></td>
</tr>
<tr>
<td>g</td>
<td>-0.90</td>
<td>-0.67</td>
<td>-0.44</td>
<td>-0.20</td>
<td>0.23</td>
<td>0.45</td>
<td>0.67</td>
<td>0.90</td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>5.66</td>
<td>4.15</td>
<td>2.71</td>
<td>1.33</td>
<td>-1.27</td>
<td>-2.49</td>
<td>-3.68</td>
<td>-4.82</td>
<td></td>
</tr>
<tr>
<td>r</td>
<td>3.18</td>
<td>2.36</td>
<td>1.55</td>
<td>0.77</td>
<td>-0.74</td>
<td>-1.47</td>
<td>-2.19</td>
<td>-2.89</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>1.73</td>
<td>1.29</td>
<td>0.85</td>
<td>0.42</td>
<td>-0.41</td>
<td>-0.82</td>
<td>-1.23</td>
<td>-1.63</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>-9.46</td>
<td>-7.00</td>
<td>-4.60</td>
<td>-2.27</td>
<td>2.23</td>
<td>4.41</td>
<td>6.57</td>
<td>8.63</td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>3.35</td>
<td>2.46</td>
<td>1.63</td>
<td>0.81</td>
<td>-0.78</td>
<td>-1.54</td>
<td>-2.29</td>
<td>-3.03</td>
<td></td>
</tr>
</tbody>
</table>
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6.5 Summary and Concluding Remarks:

Inventory cost is an inherent fuzzy notion which can be measured by the synthesis of its components. The uncertainty expressions such as “about” or “greater or less” which used frequently in the cost of inventory control are imprecise. This is due to expression of the human-like way of thinking for the evaluation of the cost of inventory. In reality, when a production system started, it is difficult to predict all the cost parameters of inventory precisely. So at the time of start, decision-maker makes a rough estimate of all the costs. Thus, the decision-maker while modelling uses linguistic expressions. In this chapter two different inventory models have been developed in fuzzy environment.

In first section, an inventory model for multi-item with investment costs to reduce production cost and increase systems reliability and with deterioration under total cost minimization is developed. This model involves one storage space constraint where storage space is imprecise in nature. Process reliability and flexibility considered in this model as the product produced during production are not of perfect quality (which directly gets affect by the reliability of the production process employed to manufacturing the products). The above described model was solved by Modified Geometric Programming (MGP). Its advantage lies in its computational efficiency and in the primal dual relation. For the total cost minimization model, the inventory problem is formulated as a posynomial problem. Optimal solutions can be derived by using dual geometric programming. The proposed model can assist the modeler in accurately determining the optimal demand, setup cost, production quantity per batch, and production reliability and also assists the decision maker to decide the effect of different inventory parameters on total cost. The results in this section not only provide valuable reference for decision-makers in planning the production and controlling the inventory but also present a useful model for many organizations which are dealing in food items, photographic film, electronic components, radioactive materials, and fashionable commodities.

Until now, many production inventory models have been considered in the literature. Some of them assumed that the rate of production is inflexible and others took...
as decision variable. It was observed that production rate depends on many factors such as
cost of raw materials, labour, power, fuel and many more. In general, it is assumed that the
production rate is higher than demand to avoid the stock-out situations. This leads to rapid
accumulation of inventories resulting in higher holding cost and other inventory related
problems. So to cope up this type of problem, in second section a production model has
been developed taking production rate as decision variable to support the real phenomena
that the production increases to adjust the increasing demand and it helps to decline the
unit production cost. It is common belief that more production gives less inventory cost
and inventory cost is least when unit production cost is least. Through the present analysis,
these perceptions are proved to be incorrect.

The sensitive analysis for the model gives us the optimal solution without re-
solving the problem given the change of data. This analysis provided the direction to the
decision-maker to take care of setup cost, demand of customer and production cost more
in comparison to the other parameters. As production rate dips when demand and setup
cost decreases whereas it rises as production cost decreases.

In future this model can make more realistic by incorporated budget constraints,
taking selling price dependent demand and time dependent deterioration rate.

References:

   holding cost under two restrictions: A geometric programming Approach. Production Planning and
   Control, 8(5), 608-611.

   demand and flexibility and reliability consideration. Computers & Industrial Engineering, 56, 411-
   416.


