The present thesis entitled “Modelling and Analysis of Inventory Control Systems in Fuzzy Environment” explores different inventory models under fuzzy environment by incorporating various market oriented scenarios such as inflation, trade credit, limited storage capacity, deterioration, learning effect, and volume flexibility. This thesis presents the impact of various demand patterns (such as initial stock dependent demand, selling price dependent demand, credit linked dependent demand, and advertisement frequency dependent demand) on inventory modelling. Integrated models with two or three players are developed as now a day inventory management is not a single problem but it is integrated into the whole of logistic of supply.

**In Chapter 3** impact of advertisement on demand has been discussed in imprecise environment. Effect of learning on ordering, holding costs and on percentage of defective items present in each lot has been analyzed. This chapter has been divided into two sections. In section 3.2, a fuzzy model for an inventory problem is devised for single item containing imperfect items with partial backlogging. It is also assumed that due to high complexities in business environment, demand of items is imprecise in nature. Results shows that holding cost, ordering cost and percentage of defective item present in each lot decreases from one shipment to other due to learning effect and hence profit increases. There is little significant change in profit and optimal order quantity due to the change in tolerance level of demand. This is due to incorporating impreciseness in demand.

In section 3.3, the work of previous section is reformulated by highlighting the effect of advertisement on demand. Annual demand was considered as triangular fuzzy number. For fuzzy model, a method of defuzzification, namely the signed distance, is employed to find the estimate of net profit per unit time in crisp sense. Keeping in mind for those inventory practitioners not from mathematics background, global optimal lot size and backordering level is obtained by using arithmetic-geometric mean inequality, which is easier to apply and simple to understand. Results show that partial backlogging is better than the total backordering. From analysis it is observed that there is a correlation between frequency of advertisement and net profit. So, it can be concluded that if the manager properly selects the mode and frequency of advertisement then that company has competitive advantage over the other companies. Hence the manager of every firm must pay attention about the advertisement
policy to capture the untouched part of demand of the market while deciding the inventory policy.

In Chapter 4, a mixed inventory model has been developed by applying fuzzy set theory to deal with the impreciseness of demand. To increase the applicability of proposed model, a combination of backorder and lost sale with backorder price discount is considered. Crashing cost is considered to reduce the lead-time and setup cost. Equivalent crisp expression has been obtained by employing signed distance and centroid method as cost expression is imprecise due to imprecise demand. The backorder price discount, the lead-time and the order quantity were taken as decision variables (whereas the probability distribution of lead-time demand is unknown but its first two moments are known). Chebyshev inequality and Minimax-distribution free procedure are employed to capture the boundary of demand during lead-time. Results show that order quantity is inversely proportional to upper bound of discount ratio. It is observed that retailer can capture the backorders and hence decrease inventory cost by increasing the price discount. Numerically it is shown that proposed model provides better result in comparison to Lin (2008) and Chang et al. (2004). It is also found that model developed by Ben-Daya and Raouf (1994) as special case of this proposed model.

In Chapter 5, an inventory model is developed by incorporating two interesting phenomena viz., inflation and trade credit with different demand pattern. This chapter has been further bifurcated in two sections.

In section 5.2, Economic Order Quantity (EOQ) inventory model is developed by incorporating some more realistic features like impreciseness in costs, deterioration and credit period offered by suppliers to the retailer which can be associated with a number of different types of inventory. In this section demand is considered as function of credit period or selling price. By taking impreciseness in cost parameters, decision-makers absorb all the turbulence in cost due to market fluctuation. Function Principle given Chen (1985) is used for the fuzzy operation and signed distance method is used for defuzzification. It is found that as the inflation rate increases order quantity and profit are gradually decreases. This is due to that as inflation rate increases purchasing power of the retailer decreases. So, inflation permits a proper recognition of the financial implication of the opportunity cost in inventory analysis. The profit of the retailer shows an increasing trend due to increasing nature of demand as the
value of trade credit increases. So in this situation retailer have capital to payoff in less time. This result identifies that trade credit is an effective strategy for inventory policy. Results also show that increase in deterioration rate result in a short cycle length, therefore payment time to settle the account, order quantity and profit of retailer decreases.

Section 5.3, reconsidered the previous section by taking demand rate as the function of initial stock level. Graphically it is shown that profit function is concave with respect to initial inventory level. Numerically it found that demand of the product is fairly sensitive of credit period, selling price and initial inventory level. There is 20% hike in profit when retailer got 20 days as credit period whereas this crop up about 60% when supplier provide credit period of 25 days. So, it is all up to the retailer how they chose credit policy to maximize their profit.

The findings in this chapter are important to the real world. The prices of fashion-based goods, for example clothes or computers, will be marked down gradually with the passage of time. These types of products are usually characterized as having a short shelf life. Promotional activities, such as price discount and trade credit, are commonly employed to speed up the movement of the goods. Our proposed models provide an insight for the decision-maker which is useful and practical. The results are also applicable for other types of perishable goods such vegetables/fruit, baked goods, etc. which are similar in nature to the fashionable goods described above.

Chapter 6 has been devoted to address the volume flexibility and reliability of production system in fuzzy environment. Most of the researchers considered different cost parameters associated to inventory as crisp variables and production rate as pre-determined parameter. On focusing this, two different production inventory models are being addressed here.

In section 6.2, a multi-item Economic Production Quantity (EPQ) model with deterioration is considered, where unit production cost, holding cost are assumed to be imprecise in nature and the production process is assumed not to be 100% perfect. In the proposed model here, the storage area is considered to be limited due to high rent and scarcity of space in market place. Due to the general nature of the proposed relationship between production setup cost, process reliability and flexibility, optimization by calculus often leads to a system of non-linear equations which, in general, are very hard to be solved explicitly and
numerical methods are always needed to obtain approximate solutions. Consequently, closed-form optimal solutions to this EPQ problem are not easily available through calculus-based optimization technique. A Modified Geometric Programming (MGP) approach is employed here to solve the proposed multi-item EPQ model due to posynomial nature of objective function (as MGP is one of the most efficient optimization techniques to deal with posynomials). Results show that profit is fairly sensitive with respect to total cost of interest.

In section 6.3, a Fuzzy Production Inventory Model (FPIM) with volume flexible production rate has been considered. Different inventory associated costs are taken as trapezoidal fuzzy number. During the production-run-time, the manufacturing process may shift to an ‘out-of-control’ state. In ‘out-of-control’ state, a part of produced items are defective. The defective items are reworked immediately at a cost. The inventory cost function both in the crisp sense and fuzzy sense are derived. The fuzzy model is defuzzified by using Graded Mean Integration Representation (GMIR) method. Cost function is convex and it is minimized by considering production rate as decision variables. It is common belief that more production gives less inventory cost and inventory cost is least when unit production cost is least. Through the present analysis, these perceptions are proved to be incorrect. This analysis provided the direction to the decision-maker to take care of setup cost, demand of customer and production cost more in comparison to the other parameters. As production rate dips when demand and setup cost decreases whereas it rises as production cost decreases.

Chapter 7 discusses a supply chain model for newly launched product under inflationary environment in an imprecise planning horizon, i.e., lifetime of the product is fuzzy in nature. Vendor offered price discount to buyer to boost the demand. Demand depends on time and selling price during the price discount period. After withdrawal of price discount, demand depends on selling price only. Learning effect is incorporated on production and setup cost for vendor and on ordering cost for buyer. Supply chain model is formulated for both the crisp and fuzzy inventory parameters. Form the numerical analysis; it is observed that \( Q_i \) for the \( i^{th} \) cycle increases in every cycle. It happens because company gives some price discount in every cycle to increase the demand potential. It is observed that due to discount, demand of the item at the beginning of each cycle \( D_0 \) is relatively larger than the demand at the end of the
cycle, \( D_r \). Demand increases during price discount period and attains its maximum value, \( D_{t_1} \), at \( t = t_1 \).

**Chapter 8** devoted to derives the optimal numbers of deliveries with single-producer, single-distributor and single-retailer when the integrated joint total cost is globally minimum in fuzzy environment. In the proposed model, producer offers a single product to the distributor by considering that the production system produces some defective units during production process. Distributor in turn delivers the items in fixed quantities to his retailer. Demand and production rate are uncertain in both stochastic and fuzzy sense i.e., demands and production rate are simultaneously random and imprecise. The expression for the average inventory cost of Supply Chain (SC) is fuzzy random variable due to demand and production rate. By using graded mean integration method, equivalent deterministic expression is obtained for the total cost of the SC. Graphically it is found the total cost function is convex in nature. Results show that as deterioration rate increases from 0.02 to 0.2, \( TC_p, TC_d, TC_r \) and TC also increases. The reason is that the deterioration quantity of finished products increases with deterioration rate, which results in an increase in ordering cost and deterioration cost. For higher values of \( \delta \), the imperfect items are produced more that result in more production to adjust the demand for perfect quality products and in turn the total cost of producer, distributor, and retailer and hence supply chain is increased. From sensitivity analysis it is observed that as the setup/ordering cost increases from -20% to +20% at all the level of supply chain, it produces very little effect on replenishment quantity and number of replenishment as a result there is not so much variation in inventory cost of different player.

**Scope for Future Research:**

Our study in particular provides ample scope for future research and exploration. Although many different kinds of rates have been experimented with, even then, there a number of different functions available which can be approximate the demand for any kind of inventory. These function along with newer ideas and ideology can be combined to form different fields of study. In the future study, multiple producers, multiple distributors and multiple retailers can be considered by considering different incentive schemes to attract the different players to form the supply chain for long term.