Chapter- 5

FRACTIONALLY SPACED NEURO FUZZY EQUALIZER AND ANN BASED DEMODULATOR

5.1. Introduction:

Modern digital data transmission systems commonly use an adaptive equaliser as a key part of the receiver. The design of this equaliser is important since it determines the maximum quality attainable from the system and represents a high fraction of the computation used to implement the demodulator. One fundamental advantage of signal processing technique over traditional analog method is the easy implementation of a constant delay that is the signal samples are simply stored in a buffer memory for the given time. The technique works perfectly as long as the desired delay is a multiple of the used sample interval. When a delay of a fraction of the sampled interval is needed or if it is desired to control the delay value continuously, more advanced methods must be used. A fractional delay filter is a device for band limited interpolation between samples. It finds application in numerous fields of Signal Processing including Communication, Array Processing and Music Technology.

Quadrature Amplitude Modulation (QAM) was developed to increase the bit-per-hertz ratio for transmission where fractional delay is used. It is first introduced for the voice
band modems, the technology was then applied to microwave radio relay system. Its application has led to great interest in its use for other communication situations, where economic or regulatory consideration limits the available transmission bandwidth.

5.2. Development of a Fractionally Spaced Equaliser

In the digital communication system the signal is first applied to the modulator and then the signal is sent through the transmitter. The transmitter converts the data sequence to a band limited wave form and its frequency is translated into the frequency band which is appropriate for transmission. The signal while propagated through the channel it is delayed, attenuated and in many practical cases it is distorted in a frequency dependent manner. The receiver accepts the delayed, noisy and attenuated input data sequence. In a QAM system, bits are modulated before transmission. The constellation for a 16 QAM has been shown in Fig.5.1.

![Fig.5.1. Constellation for 16 QAM](image-url)
The conventional QAM receiver employs an adaptive fractionally spaced equaliser scheme followed by a demodulator. These equalisers are fractionally spaced, because the equaliser taps are closer together in time than the symbol interval $T$. This means that the input to the equaliser is sampled faster than the symbol rate, $f_b$. The output rate is still at the symbol rate, making the FSE a decimating or a resampling filter. It is shown in Fig. 5.2. that the equaliser neutralises the effect of channel coloration and reduces the additive noise of the channel. In other words the equaliser is used to reconstruct the transmitted message in its undistorted form. The demodulator takes the equaliser output and produces the desired base band message.

A fractionally spaced equaliser (FSE) is based on sampling the incoming signal at least as fast as Nyquist rate. If the transmitted signal consists of pulses having a raised cosine spectrum with roll-off factor $\beta$, its spectrum extends to $F_{\text{max}} = (1 + \beta)/2T$, where $T$ is the symbol interval. The signal can be sampled at the receiver end,
\[ 2F_{\text{max}} = \frac{(1 + \beta)}{T} \] (5.1)

and then passed through an equaliser with tap spacing of \( \frac{T}{1 + \beta} \).

In general a digitally implemented fractionally spaced equaliser has tap spacing \( \frac{MT}{N} \) where \( M \) and \( N \) are integers and \( N > M \). Usually, a \( T/2 \) spaced equaliser is used in many applications.

5.3. Fractionally Spaced Neuro Fuzzy Equaliser:

In this chapter a Neuro Fuzzy equaliser has been used for the equalisation purpose and an ANN based demodulator is used to recover the desired binary bits. The proposed scheme is shown in Fig.5.3.

![Proposed Scheme For Decoding QAM Signals into Bits](image-url)
The equaliser samples the channel output at a rate $T/L$ secs., where $T$ is the time period in sec. allocated to each transmitted bit. Then the samples passes through the Gaussian membership function and a set of $M$ fuzzy rules are constructed using the following Gaussian membership function,

$$
\mu_{\mu_i}(x_i) = \exp \left[ -\left( \frac{1}{2} \right) \left( \frac{x_i - c_{im}}{\sigma_{im}} \right)^2 \right]
$$

(5.2)

where $m = 1, 2, ..., M$, $i = 1, 2, ..., n$, $c_{mi}$ and $\sigma_{mi}$ are the free parameters which are to be updated during the learning procedure and $n$ is the time index.

The membership functions can be expressed as,

$$
b^m(n-1) = \prod_{i=1}^{M} \left[ \exp \left( -\left( \frac{1}{2} \right) \left( \frac{x_i(n) - c_{mi}(n-1)}{\sigma_{mi}(n-1)} \right)^2 \right) \right]
$$

(5.3)

$$
b(n-1) = \sum_{m=1}^{M} b^m(n-1)
$$

(5.4)

At each time point $n = 1, 2, ..., n$, the input to the equaliser is given by,

$$
a^m(n-1) = \frac{b^m(n-1)}{b(n-1)}
$$

(5.5)
Then these fuzzy membership functions are allowed to pass through the multi-layer neural network having three layers and one neuron in each layer. Then the output of the equaliser is

\[ s_n = g_i \left[ w_{0k}^{(3)} g_k \left( \sum_{m=1}^{M} w_{mj}^{(1)} a_m(n) + \theta_j^{(1)} \right) + \theta_k^{(2)} \right] + \theta_i^{(3)} \] (5.6)

where \( w_{mj}^{(1)} \) and \( \theta_j^{(1)} \) are the connecting weights and bias respectively of the first layer. \( w_{0k}^{(2)} \) and \( \theta_k^{(2)} \) are the connecting weights and bias respectively of the second layer. \( w_{0i}^{(3)} \) and \( \theta_i^{(3)} \) are the connecting weights and bias respectively of the third layer and \( g_i, g_k \) and \( g_j \) are the output sigmoids at the final, second and first layers respectively.

Then this is compared with the actual QAM signal and the difference between these two is the error signal \( e(n) \) which is then used to update the weights, threshold and the free parameters of the Gaussian membership function. The update equations are,

\[ \Delta w^t(n) = \lambda \left[ e^t(n) f^{t-1} \right] \] (5.7)

in this equation \( f \) is equal to \( f_{jk} \) when \( t = 2 \) and is equal to \( s(n) = f_{3l}^3 \) for \( t = 3 \). Similarly \( w^t \) is equal to \( w_{jk}^2 \) when \( t = 2 \) and is equal to \( w_{3l}^3 \) for \( t = 3 \). The threshold update equation can be written as

\[ \Delta \theta^t(n) = \lambda \, e^t(n) \] (5.8)
where $\Delta \theta^1 = \theta^1_k$, $\Delta \theta^2 = \theta^2_k$ and $\Delta \theta^3 = \theta_k$. The free parameters used in the Gaussian membership function can be updated using the following equations:

$$
\Delta c_i^m (n) = \lambda e^{(1)}(n) q(n) \left[ a^m (n-1) \right] \left[ w_{ij} (n-1) - f_j^{(1)}(n) \right] 
$$

(5.9)

$$
\Delta \sigma_i^m (n) = \lambda e^{(1)}(n) q(n) \left[ \frac{x_i (n) - c_i^m (n-1)}{\sigma_i^m (n-1)} \right] \left[ a^m (n-1) \right] \left[ w_{ij} (n-1) - f_j^{(1)}(n) \right] 
$$

(5.10)

where $q(n) = \frac{(x_i (n) - c_i^m (n-1))}{[\sigma_i^m (n-1)]^2}$, and $\lambda$ is the small positive step size.

### 5.4. The Proposed Demodulator

The proposed ANN based demodulator scheme is shown in Fig. 5.4. If we are using an N bit system, the proposed demodulator will have N number of ANN based three layered networks which would estimate the individual transmitted bits at their respective outputs.
Mathematically, the output of \( n \)th network yielding the desired \( n \)th transmitted bit is given by:

\[
B_n' = g_i \left( \sum_{k=1}^{p} w_{ik} g_k \left( \sum_{j=1}^{p'} w_{kj} g_j \left( \sum_{i=1}^{L} w_{ji}s_i + \theta_j^{in} \right) + \theta_k^{in} \right) + \theta_i^{in} \right)
\]

(5.11)
where $w_{ji}^n$, $w_{jk}^n$ and $w_{kl}^n$ are the connecting weights of first, second and third layers respectively, $\theta_j^n$ and $\theta_k^n$ are the bias weights of the first, second and third layers respectively for the $n$th network, $p_2^n$ and $p_3^n$ are the number of neurons in the first and second layers respectively.

The outputs of the networks, $B_n$ are real valued which are then passed through a hard limiter $H_l$ to give discrete 1 or -1 as output. The hard limiter $H_l$ is expressed as:

$$H_l(B_n) = \begin{cases} +1 & \text{for } B_n > 0 \\ -1 & \text{for } B_n < 0 \end{cases}$$

(5.22)

Training:

The demodulator consists of $N$ number of ANN based networks and hence, training for each network is carried out independently. Initially, the weights of the networks are randomised and output bit values, $[B_1, B_2, ..., B_N]$ are computed from the sampled signal $s_i$. Initially the network has not been trained, and therefore its output bits $B_n$ do not match with the desired bits $B_n$ and an error $e$ exists between them. The error $e$ is expressed as $e_n = (B_n - B_n^d)$. These $N$ error values are used to update the weights of all the $N$ demodulator networks, that is, $e_1, e_2, ..., e_N$ update the weights of 1st, 2nd, ..., $N$th networks respectively. The weights are trained using standard Back Propagation algorithms.
5.5. Simulation And Results

For simulation, random nibbles were generated and then converted to 16-QAM signal which was then passed through a British Telecommunication channel-2 having an Eigen Value Ratio (EVR) of 11.6. Then -15dB white Gaussian noise was added to it. This corrupted signal was sampled at the rate of T/L taking the value of L to be eight and then fed to the Neuro Fuzzy equaliser. Then the equaliser output is compared with the actual QAM signal and the error signal is used to update the free parameters of the Gaussian membership function and the weights and thresholds of the equaliser. Similarly British Telecommunication channel-3 having an (EVR) of 66.8 was also used for simulation and the results of the simulation study has been shown in the following graphs.

![MSE Characteristic](image)

Fig. 5.5. MSE Characteristic for British Telecom Channel-2 at 30 dB SNR
In case of 16 QAM signal each constellation represents 4 bits. Hence the proposed ANN based demodulator has 4 different neural networks corresponding to each bit. Training of the four networks were carried out using Back Propagation algorithm. Each of the 4 networks are separately trained and the corresponding learning characteristics obtained from simulation are shown in Fig.5. Each of these characteristics indicates the estimation potentiality of its associated bits. It is observed that the proposed demodulator is potential enough to correctly reconstruct the transmitted bit pattern.

For reconstructing bits of first and second positions a three layered network having one neuron in each layer are sufficient as shown in Fig.5.4. But estimation of other two bits require more complex networks. For the third bit, a network with 5, 2 and 1 neurons in its first, second and third layers respectively was used. Similarly for the fourth bit, a network with 6, 3 and 1 neurons in its first, second and third layers respectively was employed.
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[Graphs showing error sequences for Channel-2 and Channel-3 for Bit-1, Bit-2, and Bit-3.]
Testing: After the ANN based demodulator model is developed, its performance is tested. Random nibble patterns are generated and converted to 16-QAM signal. This signal was then passed through the channel having EVR of 68.6 and white Gaussian Noise of -20dB strength was added to its output. Then the received output is fed to the Neurofuzzy equaliser the equaliser output is fed to the proposed ANN based demodulator. The estimated output of the demodulator model and the actually transmitted nibbles are tabulated in Table.1. It is observed that the transmitted bits are correctly recovered from the noisy channel output. The same observation is made for other simulated channel.

Table.5.1. Comparison between desired and actual outputs

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Input</th>
<th>Output</th>
<th>Sl. No.</th>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>111-1</td>
<td>111-1</td>
<td>4</td>
<td>1111</td>
<td>1111</td>
</tr>
<tr>
<td>2</td>
<td>11-1-1</td>
<td>11-1-1</td>
<td>5</td>
<td>11-11</td>
<td>11-11</td>
</tr>
<tr>
<td>3</td>
<td>-1-1-1-1</td>
<td>-1-1-1-1</td>
<td>6</td>
<td>-1111</td>
<td>-1111</td>
</tr>
</tbody>
</table>

Fig.5.7. Convergence Characteristics for Reconstruction of Different Bits.
5.6. Conclusion

A novel scheme for equalisation and demodulation of QAM signals using ANN technique has been proposed. This demodulator is capable of correctly estimating the desired transmitted bits from the corrupted channel output. The combined approach reduces the hardware complexity of the QAM receiver. Simulation studies are carried out to demonstrate the efficacy of the proposed scheme under different noise conditions and for different channels.