3.1 Introduction

Many papers have been published in the past 50 years pertaining to the problem of ferroresonance. Most of the earlier papers are related to series ferroresonance phenomena and the method of analysis therein are either graphical or grapho-analytical. Difficulties in the analysis arise due to the presence of nonlinear element in ferroresonant circuits. In the sixties, many researchers were attracted to the problem and made successful attempts in applying nonlinear control theory to analyse ferroresonance problem. It is found that analysis became simpler with the aid of nonlinear control theory together with deeper physical insight regarding the phenomena.

The ferroresonance problem in power systems was attacked by G.W.Swift in 1969. He suggested a quasi-analytical approach to the problem of ferroresonance in the steady state at fundamental frequency. The nonlinearity is tackled by using incremental describing function method and some generalised conclusions have been made. Later, in 1972 B.S.Ashok Kumar et al made considerable improvement in Swift's method and revealed some new aspects of the ferroresonance problem.
The analytical treatment first proposed by Swift\(^1\) and later modified and improved by B.S. Ashok Kumar et al\(^{42}\) basically relates to single phase parallel ferroresonant circuit and therefore it is worthwhile to review and evaluate the papers published by the authors\(^1,42\).

The present chapter briefly deals with the work done by Swift\(^1\) and B.S. Ashok Kumar et al\(^{42}\).

3.2 Review of the work done by G.W. Swift\(^1\)

G.W. Swift\(^1\) in 1969 applied incremental describing function method to a class of power system problems where ferroresonance overvoltage may occur. The method is restricted to any general configuration having only one nonlinearity and only one input voltage. In addition, balanced operating conditions are assumed and the nonlinearity must be single valued.

Swift describes a method for deriving the condition for the occurrence of ferroresonance and the system input voltage level at which it occurs. He found that conditions necessary for subharmonic response are more critical and associated overvoltages less severe and therefore ferroresonance occurring at fundamental frequency is of prime concern.

The circuit considered by Swift is shown in Fig. 3.1, which represents one phase of an unloaded transformer bank.
at the receiving end of a long transmission line. Ferroresonance being a low frequency phenomena, Pi configuration of the transmission line is justified. The analysis is carried out in steady state and the block diagram representation of the equivalent circuit of Fig.3.1 is shown in Fig.3.2, where

\[
G(s) = \frac{R + sL}{s(s^2LC + SRC + 1)} \quad \quad \quad (3.1)
\]

\[
G_o(s) = \frac{1}{s(s^2LC + SRC + 1)} \quad \quad \quad (3.2)
\]

The magnetic characteristic of the transformer is represented by 'true saturation curve' instead of commonly available RMS- saturation curve. True saturation curve of the transformer is approximated by a quintic nonlinearity given by \( i_L = c_1 \lambda + c_5 \lambda^5 \). A method has been indicated for obtaining true saturation curve experimentally. The coefficients \( c_1 \) and \( c_5 \) are obtained by curve fitting to the experimental results. The author believes that core loss of the transformer has no significant effect on the jump phenomena and therefore in the analysis core loss is not considered.

For the actual transformer, the nonlinearity is taken as \( i_L = \lambda + 4 \lambda^5 \) in the normalised form. For the sake of convenience in analysis, any per unit jump in flux
linkage $\lambda$ is taken to be equal to jump in voltage since transformer voltage is proportional to flux linkages neglecting the transformer winding resistance.

As shown in the block diagram of Fig.3.2, $G(s)$ being a low pass filter, its output is essentially sinusoidal although the input contains harmonics and therefore, since $G_0(s)$ also has sinusoidal output, $\lambda$ will be sinusoidal.

With reference to Fig.3.2, considering the system to be operating under the steady state, an incremental perturbation is given to $\lambda(t)$ such that

$$\lambda(t) = \lambda_m \cos(\omega t + \phi) + \mu \cos \omega t \quad \cdots \quad (3.3)$$

where $\mu \ll \lambda_m$

The incremental perturbation signal $\mu \cos \omega t$ suffers a gain in passing through the nonlinearity which is given by

$$K = 1 + \frac{15}{2} \lambda_m^4 + 5 \lambda_m^4 e^{2i\phi}$$
$$= A + B e^{2i\phi} \quad \cdots \quad (3.4)$$

The stability criterion is given by

$$K(\lambda_m, \phi) \cdot G(j\omega) = -1$$

or

$$G(j\omega) = -\frac{1}{K(\lambda_m, \phi)} \quad \cdots \quad (3.5)$$

It is shown that the locus of $-\frac{1}{K}$ as $\phi$ varies are circles for various $\lambda_m$ with radius $\frac{B^2 - A^2}{B^2 + A^2}$ and centre at $\left(\frac{A}{B^2 - A^2}, 0\right)$. 
The plots of the right and left hand sides of equation (3.5) are shown in Figs. 3.3 and 3.4 respectively.

Swift states that at a particular frequency if the point on the Nyquist locus lies on the intersection points of two \( \lambda_m \) circles, jump occurs. The amplitude of \( \lambda_m \) after the jump would be such that the operating point is brought back to a state of nonencirclement of \(( -1, 0)\) point on the \( G(j\omega) \) plane. It is also claimed that jump occurs from lower lambda circle to the upper lambda circle.

It is shown that for a specified value of circuit parameters, \( R = 0.002 \) p.u., \( L = 0.025 \) p.u and \( C = 50 \) p.u., jump is possible for a particular input voltage provided the peak flux linkage reaches the value of 0.96 p.u. But when the value of \( L \) is taken to be equal to 0.021 p.u., jump does not occur. Fig. 3.5 shows \( G(jL) \) plot for various \( L \) and it is found that ferroresonance occurs for any value of \( L \) above 0.0223 p.u.

Once the critical values of \( \lambda_m \) are determined, ordinary phasor calculations are employed to know the input voltage at which jump occurs. Thus for the circuit considered, jump occurs for \( E_o = 0.19 \) p.u. It is also probable that a shock of some sort may force the system to jump even at a input voltage lesser than 0.19 p.u. An analogue computer was programmed for a check of the result.
Thus Swift concluded that a circuit need not be of the simple $R - L - C$ series or parallel form in order that ferroresonance may occur. The ferroresonant behaviour of the circuit can be predicted as long as there is only one nonlinearity and one input voltage by applying incremental describing function techniques.

Disadvantages:

(i) The approach is essentially graphical as it necessitates first the plotting of constant $\lambda_m$ circles in the left half of the $G(j\omega)$ plane and then the plot of Nyquist locus. To determine the critical points, two intersecting $\lambda_m$ circles are to be found out by trial and error such that the Nyquist locus passes through the same intersecting point at the frequency under consideration. Thus the method involves much labour and suffers from the disadvantage of inherent graphical inaccuracies.

(ii) The conception that jump occurs from lower $\lambda_m$-circle to higher $\lambda_m$-circle is found to be erroneous. This has been proved later by B.S.Ashok Kumar et al.\textsuperscript{42}

(iii) In the paper, nothing is mentioned about the critical values of $R$ and $C$ for the jump to occur.

(iv) Lastly, the effect of core loss of the transformer has not been considered in the analysis. This may lead one to have pessimistic results in so far as the behaviour of ferroresonant circuit is concerned.
3.3 Review of work done by B.S. Ashok Kumar et al

In 1972, B.S. Ashok Kumar et al. made considerable improvements of Swift's method. The authors considered the circuit used by Swift and revealed many new aspects to the ferroresonance problem. The approach to the problem is similar to that of Swift in respect of the choice of base quantities and development of block diagram.

The error made in the analysis by Swift was pointed out by the authors. Swift's claim that two intersecting \( \lambda_m \) circles represent the jump-from and jump-to points was found to be erroneous. In fact, the two intersecting \( \lambda_m \) circles correspond to the upper jump-from point and lower jump-from point. A mathematical proof to this effect is given by them.

The authors derived the nonlinear envelope equations for different types of nonlinearities. This envelope is a plot of incremental describing function in the complex plane and defines the zone of ferroresonance. A general \( n \)th degree nonlinearity of the type \( i_L = a_1 \lambda + a_n \lambda^n \), for \( n = 3, 5, 7, ... \) etc. has been considered for the transformer and it is found that envelope equations for different types of nonlinearities are part of circles. It is further shown that the susceptibility of a system to ferroresonance is dependent on the order of nonlinearity e.g., a seventh-order nonlinearity being more susceptible than a fifth order or a cubic type of nonlinearity.
For the general polynomial type nonlinearity, no analytical expression for the nonlinear envelope is developed; instead a graphical plotting is suggested. Fig.3.6 shows envelopes for different types of nonlinearities.

A new term 'percentage unstable zone'(p.u.z.) has been defined by the authors to estimate the jump severity for different types of nonlinearities. It is defined as

$$p.u.z. = \frac{\lambda_{m2} - \lambda_{m1}}{\lambda_{m1}} \times 100$$

where $\lambda_{m1}$ and $\lambda_{m2}$ are flux linkages corresponding to upper and lower jump-from points respectively. For the circuit of Fig.3.1, the p.u.z. is found to be of the order of 70% for the cubic, 50% for quintic and 37% for septimic nonlinearities. The jump occurs at higher $\lambda_m$ for cubic when compared with quintic and septimic nonlinearities. Therefore the severity of jump is found to be maximum for cubic type and it decreases with increase in the order of nonlinearity.

A direct analytical method is developed to compute the two jump-from points for different types of nonlinearities. But as regards the computation of jump-to points, a direct method is given for only cubic type of nonlinearity. For higher order nonlinearities, difficulties are experienced and hence an indirect method has been suggested for the computation of jump-to points.
The possibility of occurrence of ferroresonance in long e.h.v. lines has been investigated by the authors. The effects of series and shunt compensation, frequency and load on the susceptibility of the system to ferroresonance are also studied.

An envelope equation in the L - C parameter plane has been derived for fixed R and \( \omega \). The equation to the envelope is fourth degree in L. This envelope equation facilitates the study of variation of series inductance or shunt capacitance on the susceptibility of the system to ferroresonance.

The authors concluded that the susceptibility of a system to ferroresonance jump is dependent on the order of nonlinearity, higher the degree of nonlinearity, more susceptible it becomes to ferroresonance. On the other hand, the jump severity decreases with increase in the order of nonlinearity. A direct analytical method to compute all the four critical points of ferroresonance has been developed for cubic type of nonlinearity. In long e.h.v. lines, with usual compensations, the possibility of ferroresonance is minimum under normal conditions.

3.4 Conclusions

The authors\(^1\)\(^,\)\(^42\) have opened new avenues for analysing ferroresonance problem and contributed a good
deal of valuable information on the subject. The method developed by Swift provides a basic idea to tackle the problem of ferroresonance, but the method fails to convey adequate informations with regard to parallel ferroresonant circuit behaviour. Swift's method has been extended by B.S. Ashok Kumar et al. to make it more sophisticated and some new aspects have been presented.

In the present thesis, the behaviour of parallel ferroresonant circuit in the steady state has been thoroughly investigated. Since parallel ferroresonant circuit studied in the thesis is similar to that used by the authors, an humble attempt has been made to review their works. In subsequent chapters, analysis of parallel ferroresonant circuit is carried out step by step using the techniques developed by the present author. It is believed that there may be broad range of applications of the techniques developed in several fields of control and power systems.