Energy dependence of the ion-ion interaction potential

1. Introduction

In the previous chapters we have presented the results of our calculations of the real and imaginary parts of the ion-ion interaction potentials for several pairs of closed-shell nuclei in the framework of Brueckner's energy density formalism. It was shown there that the positions and heights of the calculated Coulomb barriers for all the pairs of nuclei considered agree well with those referred to in the literature. The imaginary potential also compares well with the phenomenological potential in the region of strong absorption and beyond it. However, in the above mentioned calculations the effects arising out of the relative motion of the two colliding ions on the interaction potential were not taken into account. Brink and Stancu have shown (fig. 13) in detail the dependence of the real part of the potential for the system $^{16}_0 - ^{16}_0$ on the relative motion by taking proper account of the antisymmetrization effects on the particle densities and kinetic energy densities of the two colliding ions by evaluating them in a two-centre harmonic oscillator potential. However, the numerical difficulties pose a serious problem in extending these calculations to pairs of medium and heavy nuclei.

Recently, Moszkowski has shown how the dependence of the real part of ion-ion interaction potential on the relative energy of the ions can be included in a very simple but approximate way. Using the simple energy density introduced by Skyrme, the effects of relative energy of the two ions were taken into account by adding
correction terms to the energy independent potential. These correction terms arise from (i) the antisymmetrization effects on the kinetic energy and (ii) the effective mass term. In ref. 61, while the first correction was calculated by using a Fermi gas model, the correction arising out of the effective mass term was estimated by considering the energy dependence of the nucleon-nucleus optical potential in an empirical way. Besides, the necessary modification of the effective mass term in the energy density when the nuclei are in relative motion, is not taken into account in the work of Moszkowski. As shown in ref. 57 this modification introduces an extra repulsion in the ion-ion interaction potential at any energy. Further in the generalised folding model, Sinha and Moszkowski have tried to estimate the energy dependence of the real part of ion-ion interaction potential. The energy dependence in this model, is entirely due to knock on exchange, reflecting the weakening of the two-body force with increasing energy. The increase in the interaction potential with an increase in energy (in the low energy region) due to relaxation of Pauli blocking is not included in this model. Therefore, the calculated potential decreases with increasing energy which contradicts the result of Brink and Stancu and Moszkowski.

On the other hand Sinha has shown the energy dependence of the imaginary part of ion-ion interaction potential obtained in the folding model by using forward scattering amplitude approximation. A similar approach has also been made to compute the energy dependent imaginary potential by Saloner and Toeffer, taking proper account of Pauli principle. However, in these works for the incident nucleon, the free two-body matrix t have been used rather than the complex two-body matrix, T (where the propagator has the nuclear many-body Hamiltonian) in
deriving the imaginary potential in the forward scattering amplitude approximation. This approximation is expected to affect the magnitude of the calculated potential.

In this chapter we have extended our earlier calculations, given in chapters II and III, of the real and imaginary parts of heavy-ion interaction potential to include the effects arising out of relative motion, the antisymmetrization on the kinetic energy densities of the colliding ions is taken into account by evaluating the kinetic energy density of the composite system of two ions in a Fermi gas model. However, since we use the sudden approximation of Brueckner in our calculation, the antisymmetrization effects arising out of the modifications on the particle densities of the two ions are completely ignored. This is justified to a good extent because most of the antisymmetrization corrections come from the modifications on the kinetic energy densities. Besides, this approach of calculating the ion-ion interaction potential has the advantage that it can be extended to any pair of nuclei without facing the problem of complicated numerical calculations. Moreover, since the energy density used in our calculations is derived from a simple two-body effective interaction (II.2.1), the necessary modification of the effective mass term when the nuclei are in relative motion, can be easily taken into account and the energy dependence of the ion-ion potential arising from this term, as will be shown in what follows, is included automatically and one need not consider it separately in an empirical way as in ref.61.

In sec.2 of this chapter we outline the calculations of the energy dependent real and imaginary parts of ion-ion interaction potential. Sec.3 deals with the results and discussions.
2. Calculations of the energy dependent real and imaginary parts of ion-ion interaction potential

The nuclear part of the ion-ion interaction potential as a function of the separation distance \( R \) between the centres of the two colliding ions can be given by,

\[ V(R) + iW(R) = \int [H_c(P, \tau) - H_c(P_1, \tau_1) - H_c(P_2, \tau_2)] d^3r, \]

\[ (IV.2.1) \]

where \( H_c(P, \tau) \) is the complex energy density of the composite system, \( H_c(P_1, \tau_1) \) and \( H_c(P_2, \tau_2) \) are the same for two separate nuclei \( P \) and \( \tau \) are respectively the particle density and the kinetic energy density. In the framework of the density matrix expansion proposed by Negele and Vautherin\(^84\), the complex energy density of a nucleus derived from a complex two-body effective interaction (III.2.4) can be given by,

\[ H_c(P, \tau) = \frac{\hbar^2}{2m} \tau + (1 + i\gamma) [A(P) + B(P)\tau + P(P)(\nabla P)^2 + Q(P)\nabla^2 P] \]

\[ (IV.2.2) \]

Eqns. (IV.2.2) and (IV.2.1) are same as eqns. (III.3.7) and (III.3.1) respectively. While the first term in eqn. (IV.2.2) is the pure kinetic energy term, the second term in the square bracket comes from the exchange part of the two-body effective interaction and determines the density dependence of the nucleon effective mass. This term gets modified\(^57\) when the nuclei are in relative motion with respect to each other. This modification is done in replacing \( P\tau \) by \( P\tau - \vec{J}^2 \) where \( \vec{J} \) is the momentum density. With this change, the complex energy density becomes
If in the centre of mass system the two nuclei are moving towards each other with momenta $\vec{K}$ and $-\vec{K}$ respectively, it can be shown that this motion can be approximated by a plane wave. Under this approximation the densities $P_{1,2}$ of the two separate nuclei are independent of the relative energy whereas the kinetic energy densities $\zeta_{1,2}$ are linear in $k_{1,2}^2$

$$\zeta_{1,2} = \zeta_{1,2}^s + k_{1,2}^2 P_{1,2},$$

where $k_{1,2}^s = k_{1,2}/A_{1,2}$ and $\zeta_{1,2}^s$ are the static kinetic energy densities. In the same approximation, the momentum densities of the two nuclei can be given by,

$$\vec{J}_1 = k_1 P_1,$$

$$\vec{J}_2 = -k_2 P_2.$$  

The calculation of the interaction potentials $V(R)$ and $W(R)$ in eqn.(IV.2.1) requires the knowledge of the density $P$, the momentum density $\vec{J}$ and the kinetic energy density $\zeta$ of the composite system of the two nuclei. For the sake of simplicity, we use the sudden approximation in our calculation so that $P = P_1 + P_2$. For the momentum density $\vec{J}$ we use the approximation of Brink and Stancu

namely $\vec{J} = \vec{J}_1 + \vec{J}_2$. We have also used the Thomas-Fermi approximation together with the surface corrections for the kinetic energy density so that

$$\zeta_{1,2}^s = \frac{3}{5} k_{1,2}^2 P_{1,2} + \frac{1}{36} \left( \nabla P_{1,2} \right)^2 + \frac{1}{3} \nabla^2 P_{1,2}.$$  

(IV.2.6)
This approximation for $Z_{1,2}^s$ has been examined by Brink and Stancu$^{63}$ and also by us (given in the previous chapters) and it has been observed that this gives a better description of the ion-ion potential in the tail region than the relation obtained by taking Thomas-Fermi approximation only for the kinetic energy density, i.e.

$$Z_{1,2}^s = \frac{3}{5} k_f^2 \rho_{1,2} + \frac{1}{2} \nabla^2 \rho_{1,2} . \quad (IV.2.7)$$

The Thomas-Fermi term in the kinetic energy densities (IV.2.6) takes account of the antisymmetrisation corrections in the ion-ion potential. When the ions are in relative motion, the necessary modification of this term for the composite system of the two ions is done by using a Fermi-gas model$^{61}$. In this model, while the Fermi spheres of the two colliding ions overlap perfectly at zero relative energy, they overlap partially at finite energy and do not overlap at all beyond a critical energy. Defining a reduced energy $\Sigma$ by,

$$\Sigma = E_{cm}/a \quad (IV.2.8)$$

where $E_{cm}$ is the total relative energy in the centre of mass system and $a$ is the reduced mass number $A_1 A_2 / (A_1 + A_2)$, it can be shown$^{61}$ that the Fermi seas of the two ions do not overlap at all if

$$\Sigma \geq 4T_F \quad (IV.2.9)$$

where $T_F$ is the Fermi kinetic energy at the Fermi surface. For $\Sigma < 4T_F$, the Fermi seas of the two ions overlap partially and in that case the Thomas-Fermi term in the kinetic energy density of the composite system will be proportional to

$$F(\rho_1^{5/3} + \rho_2^{5/3}) + (1 - F)(\rho_1^2 + \rho_2^2)^{5/3} \quad (IV.2.10)$$
where $F$ is the fractional volume of each Fermi sphere which does not overlap with the other. We take the following relation of $F$ given by Roszkowski: 

$$F = \frac{3}{2} \left( \frac{e}{4T_F} \right)^{1/2} - \frac{1}{2} \left( \frac{e}{4T_F} \right)^{3/2}$$  \hspace{1cm} (IV.2.11)

and use for the Fermi kinetic energy $T_F$ at the Fermi surface, the value corresponding to nuclear matter at normal density. With these modifications in the Thomas-Fermi term, we approximate the kinetic energy density for the composite system as,

$$\mathcal{C} = \frac{2}{5} \left( \frac{\rho_T^2}{2} \right)^{2/3} F \left( \frac{5}{3} P_1 + \frac{5}{3} P_2 \right) + \left( 1 - F \right) \left( P_1 + P_2 \right)^{5/3}\right)$$

$$\mathcal{C} = \frac{1}{36} \sqrt{\rho}^2 + \frac{1}{3} \sqrt{\rho}^2 + k^2 \rho_1 + k^2 \rho_2$$  \hspace{1cm} (IV.2.12)

where $\rho = \rho_1 + \rho_2$. From eqns. (IV.2.10) and (IV.2.11) it is seen that for $\varepsilon = 0$, there is a perfect overlap of the Fermi levels whereas this overlap decreases to zero as $\varepsilon$ approaches $4T_F$.

3. Results and Discussions

(a) Energy dependence of the real part of ion-ion interaction potential

The real part of the ion-ion interaction potential can be obtained from the real part of the complex energy density $\mathcal{H}_r(\rho, \varepsilon)$ given in eqn. (IV.2.3)

$$\mathcal{H}_r(\rho, \varepsilon) = \frac{1}{2m} \varepsilon^2 + A(\rho) + B(\rho) \varepsilon + C(\rho) (\nabla \rho)^2$$  \hspace{1cm} (IV.3.1)

where the terms $P(\rho)(\nabla \rho)^2 + Q(\rho)\nabla^2 \rho$ in eqn. (IV.2.3) have been replaced by $C(\rho) (\nabla \rho)^2$ obtained by using integration by parts as given in eqn. (III.3.10). Using the parametrized versions of the
density functionals $A(P)$, $B(P)$ and $C(P)$ as given in the previous chapters and the formalism of sec. 2 of this chapter, we have calculated the real part of the ion-ion interaction potential $V(R)$ for the system $^{16}\text{O} - ^{16}\text{O}$ at different relative energy $E_{\text{cm}}$. This result is shown in fig. 36, where the interaction potential has been plotted as a function of the separation distance $R$. It is seen from this figure that the potential becomes more and more attractive when $E_{\text{cm}}$ varies from 0 to 250 MeV. Beyond this range of $E_{\text{cm}}$ the potential curve starts to move in the opposite direction and for large values of energy, for example $E_{\text{cm}} = 1000$ MeV, rises above the curve for zero energy. This behaviour of the ion-ion potential $V(R)$ with energy agrees with the findings of Brink and Stancu at least qualitatively.

From eqn. (IV.2.12) it is readily seen that the antisymmetrization effects on the kinetic energy density decreases as $E_{\text{cm}}$ increases. This effect which causes the potential to become more and more attractive for low values of $E_{\text{cm}}$ is shown in fig. 37, where we have plotted $V_A(R)$ as a function of $R$ for different energies. The quantity $V_A(R)$ represents the antisymmetrization correction due to the modifications made on the Thomas-Fermi term in the kinetic energy density of the composite system of both the ions through eqn. (IV.2.10) and can be given by,

$$V_A(R) = \int \left[ (\frac{\hbar^2}{2m} + B(P)) \zeta_{TF}(C) - (\frac{\hbar^2}{2m} + B(P_1)) \zeta_{TF}(1) 
- (\frac{\hbar^2}{2m} + B(P_2)) \zeta_{TF}(2) \right] d^3r. \tag{IV.3.2}$$

Here $\zeta_{TF}(C)$ is the Thomas-Fermi term of the kinetic energy density of the composite system and $\zeta_{TF}(1)$ and $\zeta_{TF}(2)$ are the same for the two separate ions. The major contribution to $V_A(R)$ comes from the pure
Fig. 36

$^{16}_0 - ^{16}_0$

$V(R)$ (MeV)

$R$ (fm)

(1) $E_{cm}=0$ MeV
(2) $E_{cm}=50$ MeV
(3) $E_{cm}=100$ MeV
(4) $E_{cm}=250$ MeV
(5) $E_{cm}=500$ MeV
(6) $E_{cm}=1000$ MeV
(1) $E_{cm} = 0$ MeV  
(2) $E_{cm} = 50$ MeV  
(3) $E_{cm} = 100$ MeV  
(4) $E_{cm} = 250$ MeV  
(5) $E_{cm} = 500$ MeV  
(6) $E_{cm} = 1000$ MeV  

Fig. 37
The kinetic energy term and this contribution vanishes for energies $E \geq 4T_f$. However, the contribution from the other term associated with the density functional $B$ (the so-called effective mass term) does not vanish at this critical energy because $B(P)$ is always greater than $B(P_1)$ as well as $B(P_2)$.

Besides the decreasing effect of antisymmetrization, there is an additional energy dependence of $V(R)$ at any energy which comes from the linear dependence of $\zeta$ and $j^2$ on $k^2$. From the relation of $\zeta_{1,2}$ and $\tilde{j}_{1,2}$ in eqns. (IV.2.4) and (IV.2.5) we see that

$$ (P_{1,2} \zeta_{1,2} - j_{1,2}^2) = P_{1,2} \frac{8}{\zeta_{1,2}} , \quad \text{(IV.3.3)} $$

and the contribution $V_e(R)$ from this additional energy dependence to the potential $V(R)$ can be obtained as,

$$ V_e(R) = (k_1 + k_2)^2 \int \frac{B(P)}{\rho_1 \rho_2} \rho_1 \rho_2 d^3r. \quad \text{(IV.3.4)} $$

This is linear in $E_{cm}$ and is determined completely by the density functional $B$. In fig.38 we have shown this energy dependence of the potential, where $V_e(R)$ has been plotted as a function of $R$ at different energy.

Thus we see that the complete energy dependence of the real part of ion-ion interaction potential $V(R)$ comes from two distinct effects, namely (i) the decreasing effect of antisymmetrization on the kinetic energy densities which causes the potential to become more and more attractive with increasing energy and (ii) the linear dependence of $\zeta$ and $j^2$ on $k^2$ which tends to decrease the strength of the potential as energy increases. While both the kinetic term and the effective mass term contribute to the first effect, the contribution to the second effect comes only from the effective mass term. The
Fig. 38

$V_E(R)(\text{MeV})$

$R \, (\text{fm})$

(1) $E_{CM} = 50 \, \text{MeV}$
(2) $E_{CM} = 100 \, \text{MeV}$
(3) $E_{CM} = 250 \, \text{MeV}$
(4) $E_{CM} = 500 \, \text{MeV}$
(5) $E_{CM} = 1000 \, \text{MeV}$

$^{16}O - ^{16}O$
Increasing attraction brought into the potential due to the first effect through $V_{\text{A}}(R)$ dominates the repulsive effect $V_{\text{E}}(R)$ coming from the effective mass term and this causes the total potential $V(R)$ to be more and more attractive at low energies. However, at relatively high energies, $V_{\text{E}}(R)$ completely dominates over $V_{\text{A}}(R)$; thereby making the ion-ion potential $V(R)$ less and less attractive. These findings are in conformity with those of Moszkowski$^{61}$. However, as mentioned earlier, the contribution from the effective mass term which has two opposite effects, comes automatically in our calculation through the density functional $\beta$ whereas these effects are treated phenomenologically in ref.$^{61}$. Moreover, the extra repulsion brought by the $j^2$ term in the energy density is not taken into account in the work of ref.$^{61}$.

(b) Energy dependence of the imaginary part of ion-ion interaction potential

The imaginary part of the ion-ion interaction potential can be obtained from the imaginary part of the complex energy density given in eqn.(IV.2.3).

$$H_i(P, \tau) = \left[ A(P) + B(P) \tau + P(P)(\nabla P)^2 + Q(P) \nabla^2 P \right] \tau$$

(IV.3.5)

Here the terms $P(P)(\nabla P)^2 + Q(P) \nabla^2 P$ are not replaced by $C(P)(\nabla P)^2$ as done in the previous case, because such a replacement will require the differentiation of $\tau$ with respect to $P$ which is very much complicated. Using the parametrized versions of $A(P)$ and $B(P)$ given in eqn.(II.2.24) and $P(P)$ and $Q(P)$ given in eqn.(III.3.11) and the formalism of the section 2 of this chapter, we have calculated
the energy dependent imaginary part of ion-ion interaction potential $U(R)$ for the system $^{16}O-^{40}Ca$ at different relative energy $E_{\text{cm}}$. It may be noted that $\gamma$ has been calculated in this work by the help of eqn.(III.2.7). The wave number $k$ appearing in this equation is evaluated by eqn.(III.2.6). The terms appearing in these equations have the same meaning as in chapter III. However, the real part of nucleon-nucleus optical potential $U$ in eqn.(III.2.8) is modified to incorporate the effect of relative motion of the ions through eqns.(IV.2.4) and (III.2.8) as,

$$U = \left[ \frac{d\phi(p)}{dp} + \frac{\partial\phi(p)}{\partial p} \right] e^{-kR} - \frac{d\phi(p)}{dp} (\phi(p))^2 - 2\zeta(p)\phi(p)^2$$

$$+ \frac{2m}{h^2} (E + \varepsilon_{\text{FT}})(\phi(p) + \frac{d\phi(p)}{dp}p) \left[ 1 + \frac{2m}{h^2} (\phi(p) + \frac{d\phi(p)}{dp}p) \right]^{-1}.$$

(IV.3.6)

The Fermi momenta appearing in the expression of $\gamma$ obtained as mentioned above are evaluated in the Fermi gas model as given in section 2 of this chapter.

The calculated energy dependent imaginary part of ion-ion interaction potential $W(R)$ for the system $^{16}O-^{40}Ca$ for different relative energies $E_{\text{cm}}$ has been shown as a function of the separation distance $R$ in fig.39. It is seen from this figure that the potential $W(R)$ increases with increasing $E_{\text{cm}}$ unlike the potential $V(R)$ as shown in fig.36. This behaviour of $W(R)$ with energy agrees qualitatively with the results of Sinha$^{33}$ which has been shown in fig.19 and Saloner and Toepffer$^{76}$. However, it is worth to mention here that the energy dependence of $W(R)$ comes through $\gamma$, over and above the kinetic energy density as shown for $V(R)$. Therefore, it is not possible to distinguish the repulsive and attractive contributions,
Fig. 39

$W(R)$ (MeV)

$R$ (fm)

-40 -30 -20 -10 0 2 4 6 8 10

$W(R)$ (MeV)

(1) $E_{cm} = 50$ MeV
(2) $E_{cm} = 100$ MeV
(3) $E_{cm} = 250$ MeV
(4) $E_{cm} = 500$ MeV
(5) $E_{cm} = 1000$ MeV

Fig. 39
$W(R) \text{(MeV)}$

Fig. 41

$^{16}_O - ^{16}_O$

$E_{Lab} = 40 \text{ MeV}$
Fig. 42

\( {^{16}}O - {^{16}}O \)

\( E_{\text{cm}} = 15.5 \text{ MeV} \)

\( R (\text{fm}) \)

\( V(R)(\text{MeV}) \)

(1) \( V^{HF} \)
(2) \( V \)
(3) \( V^{PH} \)

Fig. 42
with increasing energy to $\Psi(R)$, made by the energy density functionals, unlike $V(R)$.

We have compared the results of our energy dependent potentials $V(R)$ and $\Psi(R)$ for the system $^{16}_0 - ^{40}_Ca$ at $E_{\text{Lab}} = 40$ MeV, with the potentials\(^8\) extracted from the elastic data, in figs. 40 and 41. In the same figures we have also plotted the microscopic potentials derived by Brink and Stancu\(^62\) which fits the same data. It is seen that our potential curves lie in between the phenomenological and that of ref.\(^62\). The numerical values of our calculated potentials are more close to the phenomenological potentials than the values obtained in ref.\(^62\). It is therefore expected that our potentials will fit the data in a better way. Further, it may be mentioned that as discussed in chapter I, the ion-ion interaction potential is only relevant in the strong absorption region and onwards to describe heavy-ion elastic scattering as shown in fig.4. In this region, the imaginary part of the ion-ion interaction potential for the lighter systems such as $^{16}_0 - ^{16}_0$ at low energy is very weak. In such cases, the real part of the heavy-ion interaction potential determines the scattering data as shown in fig.5. In view of this we have compared the real part of the potential $V^{\text{PH}}$ deduced from experimental elastic data of $^{16}_0 - ^{16}_0$ at $E_{\text{cm}} = 15.5$ MeV, with our energy dependent potential $V$ in fig.42, where the surface region of these potentials have been plotted. We have also shown the Hartree-Fock potential\(^67\) $V^{\text{HF}}$ in the same figure. Surprisingly we find that our potential even matches better than the HF potential (shown in fig.17) with the phenomenological one $V^{\text{PH}}$ in the surface region. This indicates that our potential can fit the elastic scattering data of $^{16}_0 - ^{16}_0$ at $E_{\text{cm}} = 15.5$ MeV as good as the HF potential which reproduces the elastic scattering data, as shown in fig.18, for the same system.