CHAPTER 2

LITERATURE REVIEW

Structural members subjected to axial or in-plane periodic loads exhibit flutter instability for the critical values of the load. Prediction of the instability regions is important to avoid the catastrophic failure due to the flutter instability. All the earlier studies used classical methods, variational/energy methods and the versatile FE method. However, the dynamic stability solution in all these studies is obtained through rigorous higher mathematical analysis.

An attempt is made in the present thesis to develop simple formulations to predict the dynamic stability behavior of the structural members using approximate analytical methods, thus avoiding the complex mathematical formulations used by many researchers working on this topic. Though, many dynamic stability studies in the literature, pertaining to the research in the area of structural mechanics, is available, a critical review of the relevant research findings is presented here to give an overview of the research carried out in this area.
2.1 STABILITY AND FREQUENCY ANALYSIS OF STRUCTURAL MEMBERS

The stability (buckling) and the frequency analysis of the structural members has been investigated by many researchers using the exact or the approximate continuum and numerical methods.

The elastic stability of the structural members such as columns, plates and shells is well discussed by Timoshenko and Gere [1]. A wide range of practical cases on the stability of bars has been represented in Ref.[2]. The stability of structural members subjected to axial/in-plane loads is studied in detail in Refs. [3,4]. Shastry and Rao [5], used the finite element (FE) method to study the stability analysis of columns subjected to intermediate concentrated load. In this study, stability parameters are evaluated for fixed-fixed, hinged-hinged, fixed-hinged and fixed-free boundary conditions. Subramanian and Rao [6] studied the stability behavior of columns, with the commonly used boundary conditions, subjected to intermediate load with one end restrained to move axially using the FE method. In Ref.[7], the stability of columns subjected to intermediate load with ends are restrained to move axially is studied using the FE method.

The stability behavior of columns, subjected to uniformly distributed compressive loads with the one end restrained to move axially
studied by Rao and Subramanian [8] by using the FE method and also evaluated the stability parameters for various values of the axial spring stiffness parameter representing the axial elastic restraint for four types of boundary conditions. The stability and vibration behavior of short beams studied by Raju and Rao [9] using the famous Rayleigh-Ritz (RR) method. In this study, the fundamental frequency parameters are evaluated in terms of the axial load parameters for the simply supported and clamped beam boundary conditions and also discussed the effect of the shear deformation and rotatory inertia on the frequency and the buckling load parameters. In [10], the first five natural frequencies of the short beams are evaluated for guided-hinged, clamped-free, pinned-pinned, fixed-pinned and fixed-guided conditions for various slenderness ratios, using the FE formulation. The stability and frequency analysis of short cantilever columns subjected to an axial compressive load is studied in Ref. [11] using the FE formulation. In this study, the stability and the fundamental frequency parameters for columns subjected to an axial compressive load for different values of slenderness ratios are evaluated. Accurate closed form solutions are presented for the simply supported, clamped and cantilever boundary conditions of the short uniform columns subjected to distributed axial compressive loads by Raju and Rao [13] using the RR two term solutions. Bokain [14], the frequency of an initially loaded beam under axial concentrated compressive loads is evaluated. The development a simple design formula
to predict the fundamental frequencies of an initially loaded tapered beam with the end elastic rotational restraints in Ref.[15] and initially stressed tapered rectangular beam in Ref. [16] by Rao et al.

### 2.2 Dynamic Stability Behavior of Beams

The Dynamic stability analysis of the beams is briefly discussed by Timoshenko and Gere [1]. Dynamic Stability of prismatic bars subjected to pulsating axial loads studied by Beliave [12] and evaluated the critical loads of the bar. In the classic work of Bolotin [17], the dynamic stability behavior of the structural members including the beams using analytical methods is exhaustively discussed. Various other studies on the dynamic stability of the beams are presented by the other researchers, based on the work, of Brown et al. [18], by using the FE formulation. It is also reported in Ref.[18] that if the mode shapes of the buckling and the free vibration are similar, the dynamic stability regions of columns subjected to pulsating load are more or less remain the same by a proper non-dimensionalization of the basic physical quantities involved in the problem. The end conditions of the bar are assumed to be pinned-pinned, pinned-fixed, fixed-fixed and fixed-free. The evaluation of the buckling and the frequency parameters for columns subjected to uniformly distributed concentrated axial and varying distributed axial compressive loads and the dynamic stability behavior of the short
cantilever columns is studied by Shastry and Rao [19, 20] based on Brown et al. [18] using the FE method for various slenderness ratios.

The dynamic instability behavior of columns, with two symmetrically placed intermediate supports, is studied by Shastry and Rao [22] using the FE method. In this study, it is also evaluated the critical frequencies of a clamped column subjected to intermediate and concentrated periodic load. It is reported that, by increasing the constant part of the axial load, decreases the critical frequency parameter with increases in a/L (ratio of load acting from the fixed end to the length of the column) and then it remains constant for ratio a/L beyond 0.625. In [24], the stability and the frequency parameters are evaluated for a cantilever column, and also the stability boundaries for the cantilever column subjected to an intermediate periodic concentrated axial load is analyzed using the FE method. The stability of the columns and stringers subjected to periodically varying loads is studied by Stroker and Lubkin [25], and evaluated the critical loads.
2.3 DYNAMIC STABILITY OF BEAMS RESTING ON ELASTIC FOUNDATION

When a beam is resting on a flexible medium it is treated as elastic foundation. The first study on the dynamic stability behavior of beams resting on a one elastic foundation is by Smith and Herrmann [26]. It is observed in this study that the critical flutter loads are independent of the foundation (Winkler) stiffness for uniform beams. The dynamic stability analysis of tapered, fixed-free beam resting on a Winkler-type elastic foundation is investigated by Lee [27]. It is reported that the critical flutter loads are insensitive to variation of in the modulus of the elastic foundation in the presence or absence of damping.

Abbas and Thomas [28] studied the dynamic stability of beams resting on an elastic foundation. In this study, a FE model is developed for the dynamic stability analysis for beams resting on an elastic foundation. It is also reported that when the elastic foundation parameter increases, the width of the regions of dynamic stability decreases, and the dynamic stability regions shifts away from the vertical axis and thus making the beam, less sensitive to periodic loads. It is reported in Ref. [18] that the mode shape of the buckling of bars resting on an elastic foundation depends on the value of the foundation stiffness parameter. There exists a transition foundation stiffness value and beyond this transition foundation stiffness value, mode shape changes
for the stability problem. The transition foundation parameter for the buckling problem for the elastic foundation is discussed by Timoshenko and Gere [1], for a simply supported column the value of this first transition foundation parameter is found to be 4. The frequency parameter for the cantilever beams on an elastic foundation for various foundation parameters are evaluated by Shastry and Rao [29, 30], by using the FE method.

Shastry and Rao [31] studied the cantilever beam resting on an elastic foundation and evaluated the stability and the frequency parameters for different values of slenderness ratios of the beam and also studied the dynamic stability boundaries for different values of the foundation modulus. The dynamic stability behavior of the simply supported columns resting on a Winkler foundation is studied by Rao and Shastry [32] by considering the effects of shear deformation and rotatory inertia using the FE method. It is observed that the shear deformation and rotatory inertia effect leads to the dynamic stability boundaries shifts away from the vertical axis.

Raju and Rao [33, 34] studied the effect of the elastic foundation, on the mode shapes in the stability and the vibration problems for tapered beams. They have reported that for a uniform simply supported column the first transition foundation value is 4 as given in [33].
[34], the value of transition foundation decreases with decreasing slenderness ratio. However, as the value of transition foundation is so large, that the short simply supported columns vibrate with one–half wave only for all practical values of foundation parameter. In Ref. [35], the effect of elastic foundation on mode shapes in the stability and the vibration problems of simply supported rectangular plate are studied. The phenomenon of changing mode shapes for the stability and the vibration are discussed. In their study, it is reported that for the vibration problem the transition value does not exist, for a simply supported condition of rectangular plate.

The effect of variable elastic foundation on the mode shapes for the stability and the vibration of a uniform simply supported beam is studied by Raju and Rao [36] using RR method. It is observed that the first transition value and the buckling load parameter decreases with an increase in the value of variable elastic foundation.

In Ref.[37], the effect of variable nonlinear elastic foundation on mode shapes for the stability and the vibration problems of simply supported columns/beams is studied using the RR method and it is reported that no mode shape change occurs irrespective of the variation of the foundation parameter.
The dynamic stability analysis of an axially loaded beam resting on an elastic foundation with damping is studied by Engel [38]. It is observed that an increase in the foundation stiffness, it increases the critical dynamic load and shifts the regions of instability to a higher applied frequency. Rao and Naidu [40] studied the stability and the free vibration behavior of a uniform beams and columns with elastic and rotational restraints using the FE formulation. In this work, the frequency parameters of a beam with rigid translation and stability parameters of a column with end axial compressive loads are evaluated. The stability and frequency parameters for simply supported, clamped and fixed-hinged conditions of tapered columns subjected to nonlinear elastic and rotational restraints are evaluated by Naidu and Rao [41] using the FE method. In [42], the dynamic stability behavior of layered composite plates resting on an elastic (Winkler) foundation is studied. Naidu et al. [43] studied the stability behavior of tapered columns with nonlinear elastic and rotational restraints using the FE formulation. In this work, the stability parameters for various values of the linear depth taper and the rotational spring stiffness parameters are evaluated. The stability and fundamental frequency parameters of simply supported and clamped beams with two symmetrically place intermediate supports are evaluated by Shastry and Rao [44], using the FE formulation. Rao and Neetha [45], developed a simple design formula for evaluation of the fundamental frequency parameter of initially stressed beam resting on an
Winkler type elastic foundation. In Ref. [46], the existence of a transition foundation stiffness parameter in the buckling problem of uniform axially loaded simply supported beam resting on whether uniform or variable Winkler foundation is evaluated. The vibration frequencies of initially stressed simply supported beams and plates below and above the transition foundation value are studied by Raju and Rao [47]. This study focused to evaluate the critical load parameter and the frequency parameters for a simply supported beam. It is also reported that the transition foundation parameter does not exist for the vibration problem for the simply supported beam. Rao and Raju [48] studied the stability behavior of tapered cantilever columns resting on an elastic foundation subjected to a follower force at the free end. The critical load and coalescence frequency of a tapered column for different taper ratios are investigated in this study. The stability behavior of Timoshenko beam resting on a Winkler foundation is studied by Lee et al [50]. Kien[51], studied the prestress Timoshenko beams resting on one parameter elastic foundation and evaluated the free vibration frequencies of the beam.

Rao and Neetha [52] developed a simple design formula, to evaluate the first transition foundation parameter for columns resting on Winkler foundation. In this study, simply supported and clamped boundary conditions are considered. It is observed that for simply
supported column on the Winkler foundation, the first foundation transition value (i.e. 4) is found to be the same as the exact solution by Timoshenko and Gere [1]. It is also reported that except for the simply supported boundary condition, analytical expressions for the transition foundation parameters cannot be easily calculated as the transition phenomenon is complex. Free vibration analysis of the beams is investigated by Tambiratnam and Zhuge [53] by using the FE method and evaluated the free vibrations for stepped beams and beams on stepped elastic foundation (Winkler type).

Rao [54] studied the uniform initially stressed simply supported beams, resting on a Winkler foundation. In this study, evaluated the transition foundation parameter for an initially stressed simply supported uniform beam resting on an uniform Winkler foundation, there exist a transition foundation parameter, for the vibration problem, where the mode shape of the fundamental frequency changes from first symmetric mode to the first anti symmetric mode. In Ref. [57], the stability of a beams resting on an elastic foundation subjected to conservative and non-conservative forces are evaluated. The dynamic stability behavior of Timoshenko beams resting on an elastic foundation is studied by Yokayama [58].
2.3.1 Two Parameter Elastic Foundations:

Filonenko - Borodich [59], studied some of the approximate theories on the elastic foundation. Two parameter elastic foundation solutions are presented in the study. Pasternak [60] developed some formulations adopting new method for analysis of elastic foundation by two foundation constants. Several types of elastic foundation models are discussed by Kerr [61] and Zhaohua and Cook [62]. Though, different formulations to deal with two parameter elastic foundations are presented in [59, 60 and 62], the Pasternak foundation is widely used for its analytical simplicity. In Ref. [55], an equivalent Winkler foundation stiffness is derived to represent the various two parameter elastic foundations. Rao and Raju [56] studied the simply supported beam resting on a variable two parameter elastic foundation. It is reported that in the case of a uniform two parameters foundation, the first transition foundation stiffness parameter is independent of the second foundation stiffness parameter.

Naidu and Rao [63], studied the stability behavior of columns resting on a two parameter elastic foundation. In this study, evaluated the stability parameters for different values of foundation parameter for the simply supported, cantilever and clamped beam boundary conditions. In [64], it is investigated the initially loaded uniform beams on a two parameter elastic foundation. In this study, the frequency
parameters are evaluated for the simply supported and clamped conditions of the beam resting on a two parameter elastic foundation.

In Ref.[65], the first transition stiffness parameters for uniform and variable Pasternak foundations of simply support beams are evaluated using the energy method. It is observed that for the simply supported beams the value of the first transition, for the buckling problem, is dependent on the first foundation parameter only [1], and such a transition foundation parameter does not exist for the mode shape changes in the vibration problems. Franciosi and Masi [66], studied the beams resting on two parameter elastic soil and investigated the free vibration characteristics.

In [67], the transition foundation modulus parameters for a uniform simply supported beams resting on a variable Winkler or uniform Pasternak foundation are evaluated by using the RR method. In this study, the transition foundation modulus for four types of variable Winkler foundations and a uniform Pasternak foundation is investigated. It is reported that the transition foundation modulus is independent of the shear layer for the uniform Pasternak foundation. Accurate closed form solutions are developed to predict the vibration and stability behavior of uniform simply supported beams resting on a variable Winkler or uniform Pasternak foundation by Rao and Raju [68] using RR
method. In this study, the buckling load and frequency parameters are evaluated for simply supported, cantilever and clamped boundary conditions. In [69], an equivalent Winkler foundation is used to represent the two parameter elastic foundations. In this study, a simple one term standard trigonometric function is assumed to evaluate the buckling load and frequency parameters for simply supported, cantilever and clamped condition of beams. Rao and Raju [70], studied the concept of the uniform Winkler foundation to represent the variable two parameter foundation. Rao [71], the first transition foundation stiffness parameters for a uniform initially loaded simply supported beam resting on the Pasternak foundation are evaluated by using the unified energy formulation. It is also observed that the first transition foundation stiffness parameter is independent of the second foundation stiffness parameter.

2.4 STABILITY AND VIBRATION ANALYSIS OF INITIALLY STRESSED STRUCTURAL MEMBERS

Shastry and Rao [44], the stability and fundamental frequency parameters of the simply supported and clamped beams with two symmetrically placed intermediate supports are evaluated by using the FE formulation. Based on the matrix equilibrium equation, a formula is developed to evaluate the frequency parameter of initially loaded beams resting on an elastic foundation by Rao and Neetha [45]. Simply
supported beam and rectangular plate subjected to biaxial periodic compressive load on a uniform elastic foundation is studied by Raju and Rao [47]. In this study, the buckling load parameter and the fundamental frequency parameter for the simply supported initially stressed beam are evaluated by using the RR method. Naidu and Rao [72] studied the simply supported and clamped tapered beams to evaluate the fundamental frequencies of the initially loaded beams using the FE method. Naidu et al. [73] evaluated the natural frequencies of the initially loaded uniform beams. In Ref.[74], the fundamental frequencies of the initially loaded structural members is discussed. In this study, the initially loaded tapered beam of rectangular cross-section with simply supported and clamped boundary conditions, and cantilever tapered beam of circular cross section are considered. The uniform beam with end rotational restraints, square plate under uniaxial compression and the uniform circular plate subjected to a uniform edge radial compressive load are also considered and the fundamental frequencies of the initially stressed structural members are evaluated.

Simple and accurate master design formula is derived by Amba-Rao [75] to evaluate the fundamental frequencies of initially loaded structural members, which are very attractive for designers. Based on the works of Amba-Rao [75], Galef [76] proposed an intuitive formula and is shown through energy methods that the formula is exact as for as the
buckling and the vibration modes are similar. Bokaian [14] made an elaborate and detailed study on the vibration frequencies of compressed, uniform single span beam with various boundary conditions. It is also observed that Galef’s [76] formula is valid for certain cases of boundary conditions of beams, and for other boundary conditions, the formula is either approximate or not valid. Gorman [77] evaluated the vibration frequencies of the initially loaded plates with rotational edge support. The values of the fundamental frequency parameters and buckling load parameters for many structural members with complicating effects are evaluated in Ref. [80]. The frequency parameters of the circular plate with the edge radial load are obtained in the works of [81, 82 and 83]. Rao [84] developed a simple design formula to evaluate the fundamental frequencies of initially loaded square plates. Rao and Neetha [85] obtained the fundamental frequency of initially loaded moderately thick circular plates including the effect of the shear deformation and rotatory inertia. Lurie[81], Singa Rao and Amba-Rao [82, 83] obtained the frequency parameter of circular plates with the edge radial loads with or without elastic edge rotational restraints. Rao [84] evaluated the fundamental frequency of initially loaded square plates using the simple design formula for various boundary conditions.
2.5 **DYNAMIC STABILITY OF STRUCTURAL MEMBERS LIKE PLATES AND SHELLS.**

Stability analysis of circular plates is studied by Rao and Raju [86] using the FE method. Various effects such as geometry and material property variation, elastic restraints and elastic foundation and shear deformation on the buckling characteristics of circular plate are studied. In [89], the dynamic instability regions of laminated composite plates using the FE method are studied by Chen and Yang.

The dynamic instability behavior of rectangular plate on a nonhomogeneous Winkler foundation can be seen in the works of Saha *et al* [90]. Post critical behavior of bi-axially compressed square plates studied by Jayachandran and Vaidhyanadhan [91], using the FE method. It is observed that with increase in foundation stiffness it reduces postbuckling strength and also for particular foundation stiffness; clamped plates resting on an elastic foundation have a higher postbuckling strength compared to simply supported plates and plates with two opposite edges simply supported and the other two clamped. The dynamic stability behavior of layered square plates exhaustively studied by Dey and Singha [92] using the FE method and evaluated the dynamic instability boundaries. It is also reported that, by increasing the both the static and periodic loads, the width of the regions of instability increases with the dynamic load. Recently, Ramachandra and Sarat Kumar [93] investigated the dynamic instability of plates subjected to
non-uniform in-plane loads using the Galerkin method. The stability regions are evaluated for uniform, linear and parabolic dynamic in-plane loads. It is also observed that the width of the regions of instability increases by increasing the static load and dynamic load for the uniform and parabolic in-plane loading with the periodic load. The instability regions of thin elastic shells can be seen in the works of Fung and Sechler [94]. Jinhua and Yiming [95] studied a layered cylindrical shell subjected to uniform end periodic load. It is seen from the results that the width of the regions of instability increases with the increase of static load parameter and with periodic load. Liew et al. [96] studied the dynamic instability regions of rotating cylindrical shells subjected to periodic loads using the Ritz method.

2.6 **NONLINEAR DYNAMIC STABILITY BEHAVIOR OF STRUCTURAL MEMBERS**

The first classic study on the large amplitude free vibration behaviour of beam is due to Woinowsky-Kreiger [103]. In this study, the effect of axial force on the vibration behavior of hinged bars is studied.

Various studies on the nonlinear vibrations of beams have been reported subsequent to the first classic study of Woinowsky-Kreiger [103]. The experimental studies were carried out in [106] to investigate the nonlinear vibration behavior of beams with pinned ends. The nonlinear frequencies of a beam for simply supported, clamped-clamped
and clamped – hinged end conditions are evaluated by Ray and Bert [106]. Pratap and Vardhan [108] have been studied the nonlinear vibrations of beams subjected to axial forces. The large amplitude vibrations of slender, uniform beams resting on an elastic foundation and Pasternak foundation are studied by Rao [111 and 112] using direct numerical integration technique and evaluated the nonlinear frequencies of uniform beams with edges immovable axially. Free vibration behavior of tapered beams including the non-linear elastic end rotational restraints is studied in the works of Naidu et al [113]. In this study, the fundamental frequencies of the tapered beams with nonlinear end elastic restraints are evaluated using the FE method. Rao [114] studied the large amplitude free vibrations of beams using the principle of the conservation of energy. In this study the ratios of nonlinear frequency to linear frequency are evaluated for different values of amplitude ratios ($a/r$) for both the hinged and clamped beams with immovable ends. Rao and Reddy [115] the ratios of nonlinear to linear frequency frequencies of beams are evaluated. Simple closed form solutions to investigate the large amplitude vibration analysis of composite beams are presented in Ref.[118]. In Ref.[120], the vibrations of circular plates are evaluated and the effect of geometric non linearity, shear deformation and rotary inertia on the vibration are discussed. In Ref.[121], the effect of geometric non linearity on the flexural vibrations of moderately thick rectangular plates are studied.
Rao et al [133] developed a novel formula to evaluate the nonlinear radian frequency of uniform Timoshenko beams for hinged-hinged, clamped and clamped-hinged boundary conditions. In this study, the ratio of nonlinear frequency to linear frequency parameters is presented. Rao and Raju [132] studied postbuckling analysis of columns using the RR method and simple intuitive formulation where admissible displacement functions for postbuckling analysis of columns are suitably assumed for each displacement field variable involved. A closed form solution is developed for the postbuckling deformation of composite beams by Emam et al [78]. Gupta et al [117], developed a simple closed form formulations for post-buckling behavior of composite beams using the RR method. In this study, fixed-fixed, fixed-hinged and hinged-hinged boundary conditions are considered to evaluate the nonlinear frequencies. Rao and Raju [134] studied the vibration and postbuckling behavior of square plates and developed a simple design formula applicable to beams and plates using the FE method.

The influence of the geometric nonlinearity (large amplitudes) on the dynamic stability of bars is investigated by Riess and Mutkowsky [104 and Rubenfeld [105]. The dynamic instability of laminated beams subjected to large pulsating thermal loads is investigated by Suresh et al [110] using the FE method. It is reported that the stability region decreases with increase in amplitude (the abstract nondimensional
parameter when $\mu$ is used) and this reduction is more for the beams with the hinged ends. Tetsuya et al [116] investigated the nonlinear dynamic stability of steel columns under axial loads using the FE method. Some of the studies on the nonlinear dynamic stability of plates using the FE method are seen in the works [119,124,125 and 127]. Some of the studies on nonlinear buckling analysis are analyzed in the works [128 and 129].

In all the earlier studies mentioned in this chapter, the researchers evaluated the buckling load parameters and frequency parameters for commonly used boundary conditions of beams, plates and shells using RR and FE methods. These values are used as reference values to non-dimensionalize the load and applied radian frequency to study the dynamic stability behavior of structural members. Some works on the dynamic stability of beams, plates and shells by using the FE method are available. The generality of the simple formula to study the dynamic stability behavior of any structural member is not recognized by the earlier researchers. Recognition of the simple dynamic stability formula and validation of the same for different structural members subjected to periodic loads with secondary effects is the main contribution in the present work.