Chapter 3
CHAPTER - 3

A CLASS OF SLIPLINE FIELD SOLUTIONS WITH MIXED STRESS AND VELOCITY BOUNDARY CONDITION FOR METAL-MACHINING WITH SLIPPING AND STICKING CONTACT AT THE CHIP/TOOL INTERFACE

3.1 Introduction

During metal machining, the chip undergoes considerable straining at the chip/tool interface due to sliding friction. As a result, the friction stress attains the limiting value of yield stress $k$ in shear over a portion of the plastically stressed contact region nearest to the cutting edge [29]. Experimental studies using a split tool carried out by Childs et al. [74], Barrow et al. [62], Kato et al. [48] and Buryta et al. [82] and photoelastic analysis of the machining process carried out by Chandrasekhar et al. [33], Bagchi et al. [70] and Usui and Takeyami [25] have revealed the existence of both sticking and slipping regions in the zone of plastic contact at the chip/tool interface. These investigators further reported that beyond the region of contact between the chip and the tool over which the chip is plastically stressed, there usually exists an extensive region of elastic contact the forces acting across which contribute significantly to the cutting and thrust forces.

Even though the length of plastic contact consists of both sticking and slipping regions, due consideration to this observation has not been accorded in the theoretical
analysis of the machining process. In the slipline field solutions proposed by Lee and Shaffer[12] and Kudo[34] the interface friction was assumed to obey coulomb's law. But the friction stress over the whole contact region in their analysis was assumed to be either equal to $k$ (full sticking) or everywhere less than $k$ (full slipping). Similar assumptions about the nature of contact was also made in the slipline field analysis carried out by Dewhurst[55], Childs[60] and Shi and Ramalingam[79] though the chip formation in their study was assumed to take place under condition of constant friction stress ($\tau = mk$).

In the present analysis slipline field solutions for the machining process are presented assuming sticking and slipping contact in the plastically stressed region at the chip/tool interface. Coulomb friction is assumed and the chip is assumed to emerge from the deforming zone with an angular velocity (chip curl). With this friction law, however, the relation between the angular range of $\alpha$- and $\beta$- lines bordering the slipping region becomes non-linear. Using the linear approximation to this non-linear relation, the solutions are constructed by the matrix methods developed by Dewhurst[65] and Dewhurst and Collins[49]. It is shown that sticking begins when the coefficient of friction exceeds a certain minimum value. Length of sticking and slipping regions in the interface and the stress distribution are computed as a function of the coefficient of friction for different tool rake angles. Results for cutting and thrust forces, chip curvature and cutting ratio are also presented. The computed values are compared with some experimental results available in literature.
3.2 Slipline field solutions

Two slipline fields which cover the complete range of friction conditions encountered in metal machining are shown in Fig 3.1 and Fig. 3.2 along with their associated hodographs.

Solution 1 (Fig. 3.1) is the modified Dewhurst field[55] when coulomb friction obtains on the interface CE. The solution applies when the friction stress \( \tau \) on the chip/tool boundary nowhere equals the yield stress \( k \) in shear \( (\tau \leq k) \). This condition is satisfied when for any given value of the coefficient of friction \( \mu \), the angular range \( \eta \) of the \( \alpha \)-line ED is less than a certain limiting value \( \eta_L \). The relation for this limiting value is derived as follows:

Equilibrium of an infinitesimal element at point P(Fig.3.2(c)) gives the values of the normal stress \( \sigma_n \) and the shear stress \( \tau \) as,

\[
\sigma_n = p_p + k \sin 2\phi_p
\]

and

\[
\tau = k \cos 2\phi_p
\]

where \( p_p \) and \( \phi_p \) are the hydrostatic pressure and the intersection angle at point P and \( k \) is the yield stress in shear.

The co-efficient of friction is given as:

\[
\mu = k \cos 2\phi_p \div (p_p + k \sin 2\phi_p) = \cos 2\phi_p \div (p_p \div k + \sin 2\phi_p).
\]

When sticking starts at point \( p \), \( \phi_p \) becomes equal to zero.

Then

\[
\mu = k \div p_p
\]

Using Hencky’s equation we obtain,
FIG. 3.1(a) SLIPLINE FIELD

FIG. 3.1(b) HODOGRAPH FOR SLIPLINE FIELD IN FIG. 3.1(a)
FIG. 3.2(a) SLIPLINE FIELD

FIG. 3.2(b) HODOGRAPH FOR SLIPLINE FIELD IN FIG. 3.2(a)
FIG. 3.2(c) ANGULAR COORDINATES OF SLIPLINE FIELDS
\[ p_p = p_E + 2k (\alpha + \beta) \]

where, \( p_E \) is the hydrostatic pressure at \( E \) and \( \alpha \) and \( \beta \) are the angular coordinates of the point \( p \) as shown in Fig. 3.2(c).

For sticking condition at \( P \),

\[ \beta = \phi_E + \alpha \]

Hence,

\[ \frac{p_p}{k} = \frac{p_E}{k} + 2(\phi_E + 2\alpha) \]

where, \( \phi_E \) is the angle at which slipline \( ED \) meets the tool face at \( E \).

Thus,

\[ \frac{p_E}{k} + 2\phi_E + 4\alpha = \frac{1}{\mu} \]

The limiting value \( \eta_L \) of \( \alpha \) is therefore given as,

\[ \eta_L = \alpha = \frac{1}{4} \left[ \frac{1}{\mu} - 2\phi_E - \frac{p_E}{k} \right] \quad (3.1) \]

The matrix equations yielding the base curve \( ED \) in solution I (Fig. 3.1(a)) and curve \( BH \) in solution II (Fig. 3.2(a)) are derived as follows:

Referring to Fig. 3.1(a), let the column vector for the base slipline \( ED \) be denoted by \( \sigma_1 \).

Hence,

\[ CD = CL_{\eta_E} \sigma_1 \quad (3.2) \]

where, \( CL \) is the coulomb operator as defined in [65]. Thus,

\[ BD = Q_{\eta_E} CL_{\eta_E} \sigma_1 \quad (3.3) \]

Referring now to the hodograph diagram shown in Fig. 3.1(b), because of the rigid body rotation of the chip, the geometrical similarity of sliplines \( AB, BD, DE \) and the corresponding hodograph curves may be expressed as,
bd = \omega Q \eta_\psi \text{CL}_{\eta_\phi} \sigma_1 \quad (3.4)

and \quad ed = \omega \sigma_1 \quad (3.5)

where, \omega is the angular velocity of chip curl.

The material on entering the deformation zone suffers a velocity discontinuity of magnitude \rho as shown in Fig. 3.1(b). Thus circular arc bc is written as,

\[ bc = \rho \bar{e} \quad (3.6) \]

The curve c'd in the hodograph is calculated from bc and bd (superposition principle) using the relation

\[ c' d = P_{\eta_\psi} \rho \bar{e} + Q_{\eta_\psi} \eta_\psi \text{CL}_{\eta_\phi} \sigma_1 \quad (3.7) \]

Also \[ ed = \text{CL}_{\eta_\phi} cd \quad (3.8) \]

Using equations (3.5), (3.7) and (3.8) the matrix equation determining \sigma_1 is finally written as,

\[ (I - \text{CL}_{\eta_\phi} Q_{\eta_\psi} P_{\eta_\psi} Q_{\eta_\psi} \text{CL}_{\eta_\phi}) \sigma_1 = (\rho / \omega) \text{CL}_{\eta_\phi} P_{\eta_\psi} \bar{e} \quad (3.9) \]

where, I is the unit matrix, \bar{e} is a column vector representing a unit circle and P,Q ,CL are the standard matrix operators and the coulomb operator respectively as defined in chapter -2.

When the angular range of the \alpha-line ED attains the limiting value \eta_L as given by equation(3.1), both sticking and slipping regions may be present in the chip/tool contact length. The slipline field that satisfies this requirement is shown in Fig 3.2( solution II). EDC and CGF in the above figure define the slipping and sticking zones respectively with \tau \leq k on EC and \tau = k on CF, while FGI is the singular field constructed about
point F. Referring to the corresponding hodograph shown in Fig. 3.2(b), it is demonstrated that all velocity boundary conditions are also satisfied.

Let the sliplines ED, DH and BH in Fig. 3.2(a) be denoted by the column vectors \( \sigma_1 \), \( \sigma_2 \) and \( \sigma_3 \) respectively. Hence,

\[
\begin{align*}
CD &= CL_\eta \otimes e_\sigma 1 \\
and \quad DC &= R_\eta CL_\eta \otimes e_\sigma 1 
\end{align*}
\]

Curves GH and CG can be defined from curves DC and DH as,

\[
\begin{align*}
GH &= P_\eta \delta DC + Q_\delta \eta \sigma_2 \\
and \quad CG &= P^* e_\eta \sigma_2 + Q^* e_\eta DC
\end{align*}
\]

Since friction stress \( \tau \) is constant on CF, FG is calculated from CG using the rough boundary operator. This relation is written as,

\[
FG = G_{\delta \xi} CG
\]

FGI is a singular field. Hence,

\[
GI = Q^* e_\delta FG
\]

BH ( \( \sigma_3 \) ) is finally derived from slipline curves GH and GI using the relation,

\[
\sigma_3 = P_\eta \eta_1 GI + Q_\eta \eta \eta GH
\]

Substituting equations (3.10), (3.11), (3.12) and (3.13) in equation (3.14), \( \sigma_3 \) is finally expressed in terms of \( \sigma_1 \) and \( \sigma_2 \). This is written as,

\[
\sigma_3 = U \sigma_2 + V \sigma_1
\]

where, \( U \) and \( V \) are resultant matrix operators defined as,

\[
\begin{align*}
U &= P_\psi \eta_1 Q^* \delta G_{\delta \xi} P_\eta_1 + Q_\eta_1 \eta \eta Q_\delta \eta_1 \\
and \quad V &= P_\psi \eta_1 Q^* \delta G_{\delta \xi} Q^* \eta_1 R_\eta \eta CL_\eta \otimes e_\sigma + Q_\eta_1 \eta P_\eta_1 \delta R_\eta_1 CL_\eta \otimes e_\eta
\end{align*}
\]

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Referring now to Fig. 3.2(b), rigid body rotation of the chip requires that the hodograph curves $bh$, $dh$ and $ed$ are geometrically similar to their slipline images. Thus,

$$ed = \omega \sigma_1$$
$$dh = \omega \sigma_2$$

and

$$bh = \omega \sigma_3 \quad (3.17)$$

Further, $bf_1$ is a circular arc of radius $\rho$, the velocity discontinuity across the primary shear line FIBA. Therefore, $bf_1$ is written as,

$$bf_1 = \rho \bar{c} \quad (3.18)$$

Curve $f_2h$ can be defined from curves $bh$ and $bf_1$ as,

$$f_2h = P_{\xi} \rho c + Q_{\psi} \omega \sigma_3 \quad (3.19)$$

where, $\xi = \eta + \delta$.

Also $gh$ and $gc$ are calculated from $f_2h$. These relations are written as,

$$gh = S_{\delta} f_2 h$$

and

$$gc = R_{\delta} G_{\delta} f_2 h \quad (3.20)$$

Curves $cd$ and $dh$ can be defined from curves $gc$ and $gh$ as,

$$dh = \omega \sigma_2 = P_{\delta} \eta c + Q_{\eta \delta} gh$$

$$cd = Q^*_{\delta} gc + P^*_{\delta} gh \quad (3.21)$$

Further, $ed$ is calculated from $cd$ using the coulomb operator. Thus,

$$ed = \omega \sigma_1 = CL_{\eta_k} cd \quad (3.22)$$

Substituting equations (3.18), (3.19), (3.20) in equations (3.21) and (3.22), $\sigma_1$ and $\sigma_2$ are finally expressed in terms of $\sigma_3$. These relations are written as,

$$\sigma_1 = X\sigma_3 + \rho / \omega Wc$$
\[ \sigma_2 = Z\sigma_3 + \rho/\omega Y \bar{\epsilon} \]  

(3.23)

where, \( W, X, Y \) and \( Z \) are operators given by the relations,

\[
W = CL \eta_{\psi} Q^\ast R G P \xi_p + CL \eta_{\psi} P^\ast S \delta P \xi_p
\]

\[
X = CL \eta_{\psi} G^\ast R G \xi_p + CL \eta_{\psi} P^\ast S \delta Q \psi \xi
\]

\[
Y = P \delta \eta R G \xi_p P \xi_p + Q \eta \delta S \delta P \xi_p
\]

\[
Z = P \delta \eta R G \xi_p Q \psi \xi + Q \eta \delta S \delta Q \psi \xi
\]  

(3.24)

Substituting equation (3.24) in equation (3.16) the matrix equation for \( \sigma_3 \) is finally written as

\[
(1 - (UZ + VX)) \sigma_3 = \rho/\omega \left( UY + V W \right) \bar{\epsilon}
\]  

(3.25)

It may be noticed that both these fields have four degrees of freedom given by the field angles \( \theta, \psi, \eta/\delta \) and the hydrostatic pressure \( p_\delta \) at E. Since there are only three stress boundary conditions to be satisfied for a force free chip, both these fields are non-unique in nature.

It may be seen that the angular coordinates of any point \( P \) on the slipping region EC (Fig. 3.2(c)) are governed by the equation:

\[
\mu \left( p_\delta/k + 2(\alpha + \beta) + \sin 2(\phi + \alpha - \beta) + \cos 2(\phi + \alpha - \beta) \right) = 0
\]  

(3.26)

Following Dewhurst[65] the above non-linear relation between \( \alpha \) and \( \beta \) was approximated by a linear relation given by:

\[
\beta_A = B_0 \alpha
\]  

(3.27)

It may be mentioned that \( \beta_A \) is an approximate value for \( \beta \) calculated from the linear equation (3.27). For low values of \( \mu \), an approximate expression for \( B_0 \) was
presented by Dewhurst[65] based on small angle approximation. However, as reported by Murakami[71] this gives rise to large error when $\mu$ exceeds 0.3.

In the present study, $B_0$ was calculated using the method of linear regression analysis. Let $\alpha$ and $\beta$ be the angular co-ordinates of any point $P$ on the slipping region $EC$ as shown in Fig.3.2(c). For any value of $\alpha_i$, the true value of $\beta_i$ is calculated by solution to equation(3.26).

Thus, the error $e_i$ in the true and approximate values is expressed as,

$$e_i = \beta_i - \beta_0$$

When the calculation is carried out over $j$ number of points on $EC$, the sum of the square of errors is given by

$$\sum_{i=1}^{j} e_i^2 = \sum_{i=1}^{j} (\beta_i - B_0 \alpha_i)^2 \quad (3.28)$$

For the best linear fit,

$$d \sum_{i=1}^{j} e_i^2 / d B_0 = 0$$

Hence,

$$B_0 = \frac{\sum_{i=1}^{j} \alpha_i}{\sum_{i=1}^{j} \beta_i} \alpha_i \quad (3.29)$$

In the present analysis, $B_0$ was determined using equation(3.29) by choosing 10 points ($j = 10$) with known $\alpha$ values on $ED$ and by computing the corresponding $\beta$ values with the help of equation(3.26).

3.3. Computation of slipline fields

The slipline fields shown in Fig.3.1 and Fig.3.2 are of indirect type and these were analysed by solutions to the matrix equations (3.9) and (3.25). A FORTRAN program developed for the purpose required input of friction coefficient $\mu$, hydrostatic
pressure $p_B$ at $E$ and an initial guess for the three field angles $\theta$, $\psi$, and $\eta/\delta$.

The program first evaluated $\phi_E$ by solution to the equation:

$$\mu \left( \frac{p_E}{k + \sin 2\phi_E} \right) - \cos 2\phi_E = 0$$  \hspace{1cm} (3.30)

and then determined the linear coefficient $B_0$ (equation 3.29). These data were used to generate coulomb and basic matrix operators and determine the column vector for the base sliplines. Other slipline and hodograph curves were now calculated and the forces $F_1$, $F_2$ and moment $M$ on the chip boundary ABDE (Fig 3.1(a)) or ABHDE (Fig 3.2(a)) were evaluated. The field angles were calculated from the requirement of force free condition of the chip. An algorithm developed by Powell[51] for solution to non-linear algebraic equations was employed for the purpose and free chip condition was assumed to be satisfied when the values of the field angles $\theta$, $\psi$, $\eta/\delta$ satisfied the inequality

$$(F_1/k_{t_0})^2 + (F_2/k_{t_0})^2 + (M/k_{t_0})^2 \leq 10^{-10}$$  \hspace{1cm} (3.31)

In all calculations, the scale factor $\rho/\omega$ was set equal to 1. The programme also incorporated the following checks to test the accuracy of calculations:

i) Flatness check: Point $C$ on slipline curve DC (Fig. 3.1(a)) and points $C$, $F$ on slipline curves DC, GF (Fig 3.2(a)) respectively must lie on the tool face. Similarly point $e$ on hodograph curve de must lie on line $oc$ (Fig. 3.1(b)) or line $oc$ (Fig. 3.2(b)).

ii) Mass flux check: The mass entering into the deformation region should be equal to the mass leaving the same. This is written as,

$$t_1(c_e + o_a)/2 = t_0\omega$$  \hspace{1cm} (3.32)
iii) Traction check:- The horizontal and vertical forces calculated from the primary shear line CBA (Fig. 3.1(a)) or FBA (Fig. 3.2(a)) should be equal to those calculated from the sliplines bordering the plastically stressed region (EB and CB for solution I and ED, CD and CG, FG for solution II).

In all calculations the above three requirements were found to be satisfied to 5 significant figures.

In the present study it was assumed that $\tau = k$ on CF ($\phi_E + \eta - \eta_1 = 0$, Fig. 3.2(a)). Due to the linear approximation, $\beta = B_0 \alpha$, however it was not found to be so and $\tau$ differed marginally from $k$. For $\mu = 0.6$, $\tau$ was equal to $0.995k$ and for $\mu = 0.8$, $\tau$ was equal to $0.998k$. Attempt was also made to analyse the fields by the method proposed by Murakami[71] by fitting a cubic polynomial between the angular range of $\alpha$- and $\beta$- lines given by the equation:

$$\beta = B_1 \alpha + B_2 \alpha^2 + B_3 \alpha^3$$

(3.33)

where, $B_1$, $B_2$, and $B_3$ were calculated by Langrange interpolation. But with this procedure the solution did not converge for a value of $\mu$ as low as 0.4 even with matrices of size $20 \times 20$. This was because coefficients in equation (3.33) generally diverge and $B_2$, $B_3$ become very large as $\mu$ increases. This method of analysis was therefore not pursued further. It is also likely that no significant improvement will result when $\tau$ has a value exactly equal to $k$ on CF (Fig. 3.2(a)).
3.4 Range of validity

Not all solutions given by the present slipline fields are necessarily valid. For this to be so, the plastic stress field must be extended to the chip and the work-piece to demonstrate that the yield criterion is not violated in the rigid regions. This requires that the hydrostatic pressure \( p_A \) at A should be such that the rigid vertices at A are not overstressed. Following Hill[16] the range of permissible values of \( p_A \) for which valid solutions are obtained is written as:

\[
p_A / k \leq 1 - 2 \cos (\alpha_1 - \pi / 4), \quad \alpha_1 \leq 3 \pi / 4
\]

\[
p_A / k \leq 1 + 2 (\alpha_1 - 3 \pi / 4), \quad \alpha_1 \geq 3 \pi / 4
\]

(3.34)

and

\[
1 + 2 (\alpha_2 - \pi / 4) \geq p_A / k \geq -1 + 2 \cos (\alpha_2 - \pi / 4)
\]

\[
\alpha_2 \geq \pi / 4
\]

In some solutions, point \( f_2 \) in the hodograph diagram (Fig 3.2(b)) was found to be below the line \( ox \). This occurred for large values of \( \mu (\geq 0.6) \) and for tool rake angles \( \gamma \) less than 5 degrees. The cutting ratio and the cutting forces for these solutions were also found to be large. It is likely that formation of built up edge takes place under these conditions.

3.5 Results and discussion

For any given value of \( \mu \) and hydrostatic pressure \( p_B \), as the angular range \( \eta \) of slipline ED increases, field angles \( \theta \) and \( \psi \) also increase (Fig3.1(a)). At low values of \( \mu (\leq 0.55) \) the rigid vertices at A are overstressed before \( \eta \) attains the limiting value of \( \eta_L \) (equation 3.1). Under these conditions the machining
behavior is governed by solution 1 only. The range of validity for $\mu = 0.25$ is given in Fig. 3.3(a), defined in terms of the possible range of $\psi$, the angle of the center fan. Referring to the above figure, it may be seen that the upper limit on $\psi$ for this case is given by the curve UL, where the field angles produce overstressing of vertex angle $\alpha_2$ (Fig.3.1(a)). For $\gamma$ greater than 12 degrees, the lower limit on $\psi$ is given by the curve LL which refers to the overstressing of vertex angle $\alpha_1$. It is also seen that for $\gamma$ less than 12 degrees, Lee and Shaffer's solution[12] defines the lower limit. Thus it appears that for this value of $\mu$, chip will always leave the deformation zone with a curvature when $\gamma$ exceeds 12 degrees. For still lower values of $\mu$, curve LL shifts to the left and eventually meets the abscissa at $\gamma$ equal to zero degree for $\mu = 0.0$. For higher values of $\mu$ the above curve shifts to the right. It was also observed that for $\mu > 0.4$, vertex angle $\alpha_1$ is never overstressed for the range of rake angles examined.

Solutions with slipping and sticking zones in the chip/tool contact length are predicted only when $\mu \geq 0.55$. The permissible range of values of $\psi$ for such a case for $\mu = 0.6$ is shown in Fig. 3.3(b). Referring to the above figure, it may be seen that the lower limit for all rake angles in this case is provided by Lee and Shaffer's solution[12] for which $\psi = 0$. The upper limit on $\psi$ for slipping contact only is indicated by the curve SL in the above figure for which $\eta$ has a value equal to $\eta_L$ in solution 1 (Fig.3.1(a)). This curve is nearly flat as for any given value of $\mu$ the field angles are independent of the tool rake angle. This observation was also reported earlier by Dewhurst[55]. When the contact length involves both slipping and sticking zones.
FIG. 3.3 (b) RANGE OF VALIDITY OF SLIPLINE FIELD IN FIG. 3.2 (a)
the maximum permissible value of $\psi$ is restricted by overstressing of vertex angle $\alpha_2$ in solution 11 (Fig. 3.2 (a)). This is indicated by the curve UL in Fig.3.3 (b). It may also be seen that the curves UL and SL meet at point Q corresponding to $\gamma$ equal to 2 degrees. Thus for machining with tools having rake angle less than the above value, the interface will be governed by slipping friction only.

The variation of machining parameters with rake angle as obtained from the present analysis are shown in Fig. 3.4 - 3.8 for $\mu$ values equal to 0.0, 0.4, 0.6 and 0.8. For each value of the rake angle, the range of possible solutions lie within the limits as discussed with reference to Fig. 3.3. Referring to the above figures, it may be seen that the rake angle and the interface friction are the two most important variables in metal machining that influence the cutting forces, cutting ratio and chip curvature. The cutting and thrust forces and cutting ratio are found to decrease with increase in $\gamma$ and with decrease in the value of $\mu$ which is in agreement with experimental observations. It may also be seen that chip curvature ($t_0 / R_m$) decreases as $\mu$ increases indicating that the chip will have a tendency to stream rather than curl as rake friction increases (Fig. 3.6). The computed values of the cutting force, the thrust force and the cutting ratio are compared with the experimental results of Eggleston et al.[23] and Ponkshe[35] in Fig. 3.4, 3.5 and 3.7 respectively. Referring to this figure, it may be seen that there is excellent agreement between theory and experiment for all rake angles with the experimental points mostly lying within the solution range for $\mu = 0.4$ and $\mu = 0.8$.

The variation of non-dimensional contact length $l_t / t_o$, the non-dimensional sticking length $l_s / t_o$ and sticking ratio $l_s / l_t$ with rake angle is shown in Fig. 3.8 where, it is also compared with the experimental results reported in references [23], [35], [33].
FIG. 3.4 VARIATION OF NON-DIMENSIONALIZED CUTTING FORCE WITH $V_b$ AND $J_A$

$\mu=0.8$

- OVERSTRESSING OF $d_2$
- SLIPPING LIMIT
- LEE-SHAFFER LIMIT
- OVER STRESSING OF $d_1$

$\mu=0.6$

$\mu=0.4$

$\mu=0.0$

FIG. 3.4 VARIATION OF NON-DIMENSIONALIZED CUTTING FORCE WITH $\gamma_0$ AND $\mu$
FIG. 3.5 VARIATION OF NON-DIMENSIONALIZED THRUST FORCE WITH $\gamma_0$ AND $\mu$
FIG. 3.6 VARIATION OF CURVATURE OF THE MACHINED CHIP WITH $\gamma_0$ AND $\mu$

OVERSTRESSING OF $d_{2}$
OVERSTRESSING OF $d_{1}$
SLIPPING LIMIT
LEE-SHAFER LIMIT
FIG. 3.7 VARIATION OF THE CUTTING RATIO WITH $\gamma_0$ AND $\mu$
FIG. 3.8 VARIATION OF $l_t/l_0$, $l_s/l_0$ AND $l_s/l_t$ WITH $\gamma_0$ AND $\mu$
For $\gamma = 5, 10$ and $15$ degrees, most of the experimental points are found to lie within the solution range for $\mu = 0.4$ and $0.8$. At higher rake angles, however, the agreement between theory and experiment is not found to be so good. Better agreement may be obtained when a higher value of $\mu$ is assumed for these cases. It may also be seen with reference to Fig. 3.8 that though $l_1 / t_0$ and $l_s / t_0$ vary, the sticking ratio $l_s / l_k$ is virtually unaffected by variation in rake angle.

It is observed that the predicted variations in the machining parameters depend critically upon the choice of the tool rake angle (Figs 3.4 -3.8). The range of possible solutions decrease as the rake angle increases.

The variation of normal and shear stress in the tool/chip contact length is given in Fig. 3.9 for values of $\mu$ varying between $\mu = 0.4$ to $0.8$ for rake angle equal to $30$ degrees. Referring to the above figure it may be seen that for $\mu = 0.4$ the normal pressure increases monotonically in the length of plastic contact. The same trend is also observed for $\mu = 0.6$ and 0.8, though the rise in pressure is not as steep in the zone of slipping contact as it is in the sticking contact area. It may also be seen that the extent of sticking contact is very much influenced by the value of $\mu$. At $\mu = 0.6$, the length of sticking contact forms only $25\%$ of the length of total plastic contact, which increases to $70\%$ for $\mu = 0.8$. Even within the slipping contact zone $\tau$ does not differ much from $k$, the least value being $0.94k$ for $\mu = 0.8$. This may be the reason why the distribution of $\tau$ in the plastic contact area from experimental measurements appears to be nearly flat.
FIG. 3.9 VARIATION OF $\sigma_N$ AND $\varepsilon$ AT THE CHIP-TOOL INTERFACE
3.6 Conclusion:

1. In the present analysis slipline field solutions for the orthogonal machining process are presented assuming sticking and slipping contact in the plastically stressed region at the chip/tool interface. Interface friction is assumed to obey Coulomb's law and the solutions are constructed by linear approximation to this non-linear boundary value problem as suggested by Dewhurst. Limits of validity of these solutions have been examined by applying Hill's overstressing criterion.

2. It is seen that for a given value of friction coefficient $\mu \leq 0.55$ the interface friction is limited by slipping contact only. When $\mu$ exceeds the above value, sticking and slipping zones are predicted in the length of plastic contact. Solutions with slipping contact can be obtained by analysis of the modified Dewhurst field shown in Fig. 3.1. The slipline field shown in Fig. 3.2 applies when the plastically stressed region in the interface consists of both slipping and sticking zones.

3. Computed values of machining parameters such as cutting force, thrust force, cutting ratio, chip curvature and contact length are presented for rake angle values between -5 to 30 degrees and for $\mu$ values between 0.0 to 0.8. For any given value of rake angle and friction coefficient $\mu$, the solutions lie within the limits imposed by overstressing of assumed rigid regions in the chip and the workpiece. Tool rake angle and interface friction are seen to have most significant influence on machining parameters. The predicted variations in the machining parameters depend critically upon the choice of the tool rake angle. The range of possible solutions decrease as rake angle increases.
4. Computed values of machining parameters are found to show excellent agreement with experimental results reported in literature.

3.6 Plotting of some slipline fields

In this section, the slipline field network with associated hodographs for some rake angles are presented. A programme was developed for calculation of coordinates for the slipline curves. These co-ordinates were subsequently plotted on a graph sheet and the network was constructed by joining the consecutive points by a smooth curve. These slipline fields with associated hodographs are shown in Fig.3.10 to 3.13 and in all cases the field angles to which the solution applies are also mentioned.
\[ \gamma = 20^\circ \quad \mu = 0.15 \quad \eta = 50^\circ \quad \eta_s = 6.8^\circ \quad \psi = 5.9^\circ \]

**FIG. 3.10(a) SLIPLINE FIELD SOLUTION-I (FIG.3.1(a))**

**FIG. 3.10(b) HODOGRAPH FOR SLIPLINE FIELD IN FIG.3.10(a)**
FIG. 3.11(a) SLIPLINE FIELD SOLUTION - I (FIG. 3.1(a))

\[ \mu = 0.15 \quad \chi = 30^\circ \quad \eta = 5^\circ \quad \eta_\theta = 6.8^\circ \quad \psi = 15.9^\circ \]
FIG. 3.11(b) HODOGRAPH FOR SLIPLINE FIELD IN FIG. 3.11(a)
FIG. 3.12(a) SLIPLINE FIELD SOLUTION - II (FIG. 3.2(a))

\[ \mu = 0.6 \quad \delta = 3.0 \quad \gamma = 20^\circ \]

\[ \theta = 6.1^\circ \quad \varphi = 17.9^\circ \quad \eta = 3.2^\circ \quad \eta_1 = 16.5^\circ \]
FIG. 3.12(b) HODOGRAPH FOR SLIPLINE FIELD (FIG. 3.12(a))
FIG. 3.13(a) SLIPLINE FIELD SOLUTION - II (FIG. 3.2(a))

\[ \gamma_0 = 30^\circ \quad \delta = 4^\circ \quad \psi = 19.8^\circ \quad \mu = 0.6 \]

\[ \eta = 3.4^\circ \quad \eta_1 = 16.3^\circ \quad \theta = 7.2^\circ \]
FIG. 3.13(b) HODOGRAPH FOR SLIPLINE FIELD (FIG. 3.13(a))