Chapter - 5

A DIRECT PIECEWISE LINEARISED APPROACH FOR PREDETERMINATION OF TRUE SATURATION CHARACTERISTICS
5.1 Introduction:

For the study of power system transients due to switching operations, analog or digital models of the system are in use. Modelling transformers and reactors form an integral part of studying switching transients. The data for the instantaneous saturation characteristics of these elements is essential for modelling. But one frequent problem in modelling instantaneous characteristics of a transformer is the difficulty in obtaining the curves in first place, even the r.m.s. saturation characteristics are obtainable from the manufacturer with high price tag, and the manufacturer has little enthusiasm for true saturation characteristics. Therefore the instantaneous curve, not easily obtainable from the manufacturer, is to be computed from the available saturation curve in r.m.s.. This computation requires engineering effort and expense. A method has been suggested by Talukdar et.al. [1, 20] to derive the instantaneous saturation curve (I.C.) from the characteristic available in r.m.s. form. This method is based on iterative technique. The suggested algorithm generates successively improving guesses at the desired I.C. The r.m.s. curve corresponding to each guess is computed and then compared to the available r.m.s. curve, the discrepancy between them being used to obtain the next and better guess. This process continues till the computed r.m.s. curve approaches the available one with certain allowable tolerance. The computed time required to derive the I.C. from the r.m.s. curve is quite high. Therefore, it is felt necessary to develop a direct approach to derive I.C. from the given r.m.s. curve so that no trial is necessary to obtain each point on the I.C. and the r.m.s. curve for the points on the I.C. exactly.
matches with those on given r.m.s. curve. In the present chapter, the I.C. developed from segment to segment by computing the appropriate slopes successively for each segment using generalised equation for the slope of IC for the nth segment. The method suggested is based on an analytical technique and as a result there is no need to compute nth point on the r.m.s. curve for the corresponding point on I.C. and compare it with the given r.m.s curve. In addition the method does not involve any trial and error process. The method has also been extended to derive an I.C. with optimum number of segments having some specified accuracy.

5.2 Preliminaries:

The true saturation characteristic of a transformer or an iron cored reactor gives the relationship between peak value of flux linkage(\(\lambda\)) and the magnetising current (\(i\)), when core loss is neglected. This is shown in the fig.(4.2). This symmetrical non-linear characteristic can be represented by polynomial

\[
i = a_1 \lambda + a_3 \lambda^3 + \ldots \ldots \ldots a_n \lambda^n \quad \ldots \ldots \ldots (5.1)
\]

over some finite interval where \(n\) is an odd integer and \(a_1, a_2 \ldots \ldots \ldots a_n\) are constants. In many situations two terms approximation is sufficient [11,34,38] to represent the characteristic for power system transient studies. Therefore the equation for true saturation characteristic of transformer is

\[
i = a_1 \lambda + a_5 \lambda^5 + \ldots \ldots \ldots a_n \lambda^n \quad \text{for } n=7,9,11 \quad \ldots \ldots (5.2)
\]
For analog modelling, usually the true saturation characteristic is approximated by two or three straight line segments [1, 16]. This greatly facilitates modelling of transformers or reactors. In case, more accurate representation of the saturation characteristic is desired, the number of straight line segments can be increased. To get accurate representation of the characteristic, the unsaturated zone and saturated air core region can be represented by two straight lines whereas the region around the knee can be well approximated by few straight line segments.

5.3 Piecewise Linearised Method:

Let \((V_1, I_1), (V_2, I_2), \ldots, (V_n, I_n)\) be the points on the available r.m.s. curve shown in Fig. 5.1 and the corresponding amplitudes of flux linkage and peak currents be \(\tilde{\lambda}_1, \tilde{\lambda}_2, \ldots, \tilde{\lambda}_n\) and \(\tilde{i}_1, \tilde{i}_2, \ldots, \tilde{i}_n\) respectively on the instantaneous curve illustrated in Fig. 5.2(a).

The point \((\tilde{\lambda}_1, \tilde{i}_1)\) corresponding to \((V_1, I_1)\) point is found out by the following method. For sinusoidal impressed voltage,

\[
\tilde{\lambda}_1 = \frac{\sqrt{2} V_1}{\omega}
\]

(5.3)

\(i_1\) at any point on the first straight line segment joining the origin and \((\tilde{\lambda}_1, \tilde{i}_1)\) point and having a slope \(k_1\) as indicated in Fig. (5.2a) is given by

\[
i_1 = k_1 \tilde{\lambda}_1 \sin \theta
\]

(5.4)

\[
I_1 = \left[ \frac{2}{\pi} \int_0^{\pi/2} k_1^2 \tilde{\lambda}_1^2 \sin^2 \theta \, d\theta \right]^{1/2}
\]
Fig. 5.1 Linearised segments on r.m.s. curve.
Thus the 1st straight line segment on the true saturation curve is determined by its slope \( k_1 \), with its initial and final flux linkage values 0 and \( \lambda_1 \) respectively. To obtain good accuracy, the point \((V_1, I_1)\) should be chosen on the unsaturated linear part of r.m.s. curve. Next \((V_2, I_2)\) point is to be considered. Let the slope of the instantaneous curve for the second segment joining \((\lambda_1, i_1)\) and \((\lambda_2, i_2)\) be \( K_2 \). It can be proved that

\[
I_2^2 = \frac{\pi}{2} \int_0^{\theta_1} \left( K_1 \lambda_2 \sin \theta \right)^2 d\theta + \int_{\theta_1}^{\pi/2} \left\{ K_1 \lambda_2 + K_3 (\lambda_2 \sin \theta - \lambda) \right\}^2 d\theta \quad \ldots \quad (5.6)
\]

where \( \theta_1 = \sin^{-1} \frac{\lambda_1}{\lambda_2} \), or,

\[
K_2^2 \int_{\theta_1}^{\pi/2} \left\{ (\lambda_2 \sin \theta - \lambda_1) \right\}^2 d\theta + K_2 \left[ 2k_1 \lambda_1 \int_0^{\pi/2} \left\{ (\lambda_2 \sin \theta - \lambda_1) \right\} d\theta \right] + \int_{\theta_1}^{\pi/2} k_1^2 \lambda_2 \lambda_1 d\theta + \int_0^{\theta_1} \left( k_1 \lambda_2 \sin \theta \right)^2 d\theta - \frac{\pi I_2^2}{2} = 0
\]

or, \( A_2 K_2^2 + B_2 K_2 + C_2 = 0 \) \quad \ldots \quad (5.7)

In equation (5.7) \( A_2 > 0, B_2 > 0 \) and \( C_2 < 0 \) therefore the only positive value of \( K_2 \) which is admissible can be found from equation (5.8)
Thus, successively $K_3$, $K_4$ .............................. $K_N$ can be evaluated. Now a general equation for finding $K_N$ where $K_1$, $K_2$ .............................. $K_{n-1}$ are known in order.

\[
\frac{\pi}{2} I_N^2 = \int_0^{\theta_1} \left( K_1 \lambda_N \sin \theta \right)^2 d\theta + \int_1^{\theta_2} \left\{ i_{11} + K_2 \left( \lambda_N \sin \theta - \lambda_1 \right) \right\}^2 d\theta + \cdots + \int_{\theta_{n-1}}^{\theta_n} \left\{ K_n \left( \lambda_N \sin \theta - \lambda_{n-1} \right) + i_{n-1} \right\}^2 d\theta + \cdots (5.9)
\]

where

\[
i_n = K_n \left( \lambda_n - \lambda_{n-1} \right) + K_{n-1} \left( \lambda_{n-1} - \lambda_{n-2} \right) + \cdots + K_1 \lambda_1
\]

\[
\theta_n = \sin^{-1} \left( \frac{\lambda_n}{\lambda_N} \right), \quad \lambda_N \text{ is the flux linkage}. \quad \cdots \quad (5.10)
\]

The angle obtained at various flux linkages $\lambda_1$, $\lambda_2$ .......................... $\lambda_N$ are shown in the fig. (5.2b).

Simplifying eqn. (5.9), we have

\[
A_N K_N^2 + B_N K_N + C_N = 0 \quad \cdots \quad (5.11)
\]

where

\[
C_N = d_N + \sum_{n=1}^{N-1} \left( K_n^2 A_n + K_n B_n + d_n \right) - \frac{\pi}{2} I_N^2 \quad \cdots \quad (5.12a)
\]

and for $n \leq N$
Fig. 5.2(a) Amplitudes of flux linkage on an I.C.

Fig. 5.2(b) Angles at different values of flux linkage.
\[ A_n = \frac{\lambda_N}{2} (t_n - S_n) + 2 \lambda_N \lambda_{n-1} g_n + \lambda_{n-1} t_n \] .......................... (5.12b)

\[ B_n = -2 \lambda_{n-1} \left( \lambda_N g_n + \lambda_{n-1} t_n \right) \] .......................... (5.12c)

\[ d_n = \lambda_{n-1} t_n \]

\[ t_n = \theta_n - \theta_{n-1} \]

\[ S_n = \frac{1}{2} (\sin 2\theta_n - \sin 2\theta_{n-1}) \] .......................... (5.12d)

\[ g_n = \cos \theta_n - \cos \theta_{n-1} \]

It can be proved that \( A_N > 0, B_N > 0 \) & \( C_N < 0 \) except for \( N=1 \) when \( A_N > 0, B_N > 0 \) and \( C_N < 0 \). Therefore with the help of eqn. (5.10) and (5.11),

\[ K_N \] can be written

\[ K_N = \frac{-B_N \pm \sqrt{B_N^2 - 4A_N C_N}}{2A_N} \] .......................... (5.13)

5.4 Root Mean Square Error in modelling:

Using eqn.(5.13), the slope of the \( N^{th} \) segment is calculated and then currents of the true saturation curve at different voltages can be computed with help of equation (5.10).
The error $\varepsilon$ in the computed current at applied voltage is given by

$$\varepsilon = (i_{\text{computed}} - i_{\text{experiment}}) \quad \text{(5.14)}$$

In order to estimate the error in the curve derived with the help of piecewise linearised approach suggested in the section (5.3), the definition of error should be such as to account properly the derivation of the curve from actual one over given range with its initial and final flux linkage values $0$ and $\lambda_1$ respectively.

Since a small error in the computed curve in lower range and a large error at higher voltages after the knee portion of saturation curve may lead to erroneous assessment of error in a curve, if the definition of error in curve is based on only the error eqn.(5.14). The small errors resulted in lower range become negligible when they are added to larger errors to obtain the error in a curve. Thus a definition based on the sum of the squares of errors [17,35,58] will further give a wrong calculation regarding the accuracy of the curve and can't be relied on. A smaller error in the lower range may yield large per unit error with respect to the point itself and a large at a higher range of curve may yield very much small per unit error. Hence in order to present correctly the extent of the curve, the definition of error is to be based as per unit error.

Moreover higher per unit error in the derived saturation curve from the actual one in the lower range may lead to very much erroneous conclusions for the prediction of behaviour of the transformer in the linear zone [33] and it is therefore essential to obtain such information of error in a curve with the help of proper
definition for it. It is essential that the definition must be able to reflect the accuracy of the curve correctly over the entire range of the curve. So a new definition of r.m.s. error based on the sum of the squares of the per unit errors at different data points is used for the assessment of accuracy of the model and is given by, equation (5.15).

\[
\text{r.m.s. error} = \sqrt{\frac{1}{n} \sum_{r=1}^{n} \left( \frac{i_r \text{Comp} - i_r \text{Exp}}{i_r \text{Exp}} \right)^2} \quad \ldots \ldots \ (5.15)
\]

where \(i_r \text{Comp}\) = Computed value of the instantaneous current at \(r^{\text{th}}\) data point

\(i_r \text{Exp}\) = Experimental value of the instantaneous current at \(r^{\text{th}}\) data point

\(n\) = number of points considered for the calculation of error in the curve.

5.5 Application of the method and results:

The method suggested to predetermine the true saturation curve from a given r.m.s. saturation curve is not an iterative one, hence it does not involve trial and error process. It is purely an analytical method. The available saturation curve of single phase transformer is shown in Fig.(5.3a). The slope of various linearised segments for I.C. have been computed using equation (5.12) and the corresponding true saturation curve is drawn in the fig.(5.3b). By comparison with the experimental true saturation curve, it is observed that when the number of segments representing the r.m.s. curve has been increased, improved accuracy has been obtained. It is also
Fig. 5.3(a) r.m.s. curve of the transformer.
Fig. 5.3(b) Experimental and computed I.C.'s for different number of segments.
seen from the results provided in Table (5.1) that r.m.s. error in the curve has reduced for an increase in the number of segments. When a large number of points are taken on the available r.m.s. curve, the computed true saturation curve will be very accurate but at the cost of much computation time.

5.6 Optimisation of piecewise linearised method:

As explained (Section 5.2 & 5.3) r.m.s. saturation curve is approximated by a number of straight line segment and the slope of corresponding straight line segments of I.C. are calculated with the help of equation (5.12). To get an accurate representation of the r.m.s. curve, the unsaturated and saturated air core region can be represented by two straight lines whereas the region around the knee can be well approximated by many linearised segments as necessary. The more the number of segments used to represent the r.m.s. curve, the more will be accuracy of I.C. but the computation time will also increase. Hence to reduce the computer time at the same time to obtain a true saturation curve with desired accuracy, it is necessary to optimise the number segments for it.
Table - 5.1

Computed I.C. for different number of linearised segments

<table>
<thead>
<tr>
<th>E in volts</th>
<th>λ in wb.turns</th>
<th>Experimental current in amp. (A)</th>
<th>Computed Current</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>A</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>A</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>A</td>
</tr>
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<td>0.4501</td>
<td>0.21</td>
<td>0.1909</td>
</tr>
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<td>0.51</td>
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</tr>
<tr>
<td>200</td>
<td>0.9001</td>
<td>0.95</td>
<td>0.8832</td>
</tr>
<tr>
<td>220</td>
<td>0.9902</td>
<td>1.32</td>
<td>1.2236</td>
</tr>
<tr>
<td>240</td>
<td>1.0801</td>
<td>1.81</td>
<td>1.7598</td>
</tr>
<tr>
<td>250</td>
<td>1.1201</td>
<td>2.16</td>
<td>2.0144</td>
</tr>
<tr>
<td>260</td>
<td>1.1704</td>
<td>2.56</td>
<td>2.5605</td>
</tr>
<tr>
<td>280</td>
<td>1.2605</td>
<td>3.72</td>
<td>-</td>
</tr>
<tr>
<td>290</td>
<td>1.3046</td>
<td>4.43</td>
<td>-</td>
</tr>
<tr>
<td>300</td>
<td>1.3503</td>
<td>5.61</td>
<td>5.6209</td>
</tr>
</tbody>
</table>

No. of segments 04 08 12
R.M.S. (%) error 8.705 8.04 7.46

By the method suggested [14,15,16,17] in section (5.3) for the calculation of the slopes of segments of I.C. the data points on the r.m.s. saturation curve are used and there is no verification for the accuracy of the derived curve at any of the
intermediate points of the segments. The verification of the accuracy of the derived segment may be made at the mid point of segment.

In the next section of this chapter, the method of predetermination of true saturation curve, having desired accuracy and optimal number of straight line segments have been presented. By this method not only the computer time, when compared that of [1, 20] is economised but also the desired accuracy is achieved through out the entire range of each segment, derived from true saturation curve.

5.6.1. Preliminaries:

The slope of a segment is evaluated with the help of equation (5.11) for the optimisation of the number of segments for I.C., it is necessary to verify the accuracy of the slope of the segment. For the purpose of verification, the r.m.s current corresponding to the mid point of the segment is calculated. The linear segment AB in Figure (5.4) represents the R\textsuperscript{th} segment for the r.m.s. curve corresponding to R\textsuperscript{th} segment of I.C. The curvilinear portion ACD shows the part of the experimental r.m.s. curve between the two terminating points (V\textsubscript{R,L}, I\textsubscript{R,L}) and (V\textsubscript{R},I\textsubscript{R}) of the R\textsuperscript{th} segment. It can be seen from the figure (5.4) that the difference between the section ACB and linear segment AC\textsubscript{B} is maximum at mid point of the segment. Hence the computed r.m.s. current has been compared with the r.m.s. current at the mid point of the segment of the given data. If the difference between the two currents is within allowable tolerance then the segment is acceptable and it becomes one of the optimal number of segments for the I.C.. If the segment proves to
Fig. 5.4 Verification of accuracy of an I.C. segment.
be not acceptable, the segment on the r.m.s. curve is bisected and new segment is formed. Then the accuracy of the new segment is verified to satisfy a specified tolerance. This process of bisection and verification of accuracy of the segment is repeated until the derived IC segment satisfies the specified accuracy.

5.6.2 Proposed method for optimisation of number of segments:

The r.m.s. saturation curve is approximated by piece wise linearised zones. Considering an N\textsuperscript{th} zone of r.m.s. curve, its data is used for computation of the slopes of linearised segments in the corresponding zones of I.C.. Each of the three slopes are verified to satisfy the specified accuracy, hence they are the optimum number of segments representing the required I.C. for the zone. Initially whole range of N\textsuperscript{th} zone is considered as a single R\textsuperscript{th} segment and its slope is computed using equation (5.13) and then accuracy of the segment is verified. If it does not satisfy the specified accuracy, a bisection of R\textsuperscript{th} segment is made and new R\textsuperscript{th} segment is formed up to the point of bisection. The slope of R\textsuperscript{th} segment is computed and its accuracy is verified. This process is repeated till a specified accuracy is obtained. The procedure of optimisation is described in details as follows.

**Step - 1 Optimisation in the first Zone:**

The linear part of the r.m.s. curve joining the origin to a point \((V_1, I_1)\) on the curve is considered first zone \((N=1)\) and the same is taken as first segment. The point \((V_1, I_1)\) should be chosen on the unsaturated linear part of the r.m.s. curve for obtaining good accuracy. The slope \(K_1\) of the segment \(K_1 = \frac{2I_1}{\bar{\lambda}_1}\) where \(\bar{\lambda}_1\) is the
amplitude of flux linkage corresponding to $V_1$. The equation for $K_1$ shows one to one correspondence between the r.m.s. current and instantaneous current for the segment. Hence there is no necessity of verification of accuracy of the segment. Thus the first zone yields only one segment ($R=1$) in the process of optimisation.

Step - 2 Computation in the second Zone and extension for the generalised zone:

The whole range of second zone ($N=2$) is at first considered as one segment ($R=2$) and it is a linearised segment between the points ($V_1, I_1$) and ($V_2, I_2$) of the r.m.s. curve. The slope $K_2$ of the segment joining the corresponding points ($\bar{\lambda}_1, \bar{i}_1$) and ($\bar{\lambda}_2, \bar{i}_2$) respectively of the instantaneous curve is evaluated from equation (5.7). The value of $K_2$ is given by (Reference equation 5.8).

$$K_2 = \frac{-B_2 + \sqrt{B_2^2 - 4A_2C_2}}{2A_2}$$

where

$$A_2 = \frac{1}{2} \bar{\lambda}_2^2 (T_2 - Q_2) + 2 \bar{\lambda}_2 \bar{\lambda}_1 G_1 + \bar{\lambda}_1^2 T_2$$

$$B_2 = -J_1 \bar{\lambda}_2 . G_2 + \bar{\lambda}_1 . T_2$$

$$C_2 = C'_2 + d_2 - \frac{\pi}{2} I_2^2$$

$$T_2 = \frac{\pi}{2} - \theta_1$$

$$Q_2 = -\frac{1}{2} \sin 2\theta_1$$

$$G_2 = -\cos \theta_1$$
\[
d_2 = J_1^2 \cdot T_2 \quad \quad \theta_1 = \sin^{-1} \frac{\lambda_1}{\lambda_2}
\]

\[
J_1 = K_1 \cdot \lambda_1
\]

\[
C_2 = K_1^2 \cdot A_1, \quad A_1 = \frac{1}{2} \lambda_2^2 (T_1 - Q_1) + 2 \lambda_2 \lambda_1 \cdot G_1
\]

\[
T_1 = \theta_1
\]

\[
Q_1 = \frac{1}{2} \sin 2\theta_1
\]

\[
G_1 = \cos \theta_1 - 1
\]

and hence the following boundary values are true for the above calculation

\[
\lambda_0 = 0, \quad \theta = 0, \quad J_0 = 0, \quad B_1 = 0 \quad \text{and} \quad d_1 = 0
\]

Then the r.m.s. current at the mid point of the segment is calculated. The computed r.m.s. current at the voltage of the mid point is compared with the r.m.s. current at the voltage for the given data to verify the accuracy of slope of the segment. Thus the voltage of the mid point of the 2nd segment is given by

\[
V_{2a} = V_1 + \frac{1}{2} (V_2 - V_1) \quad ............. \quad (5.16)
\]

The amplitude of flux linkage at the mid point of segment is to be calculated for the computation of r.m.s. current at the voltage is given by

\[
\tilde{\lambda}_{2a} = \frac{\sqrt{2} V_{2a}}{\alpha} \quad ............. \quad (5.17)
\]

The r.m.s current computed for the voltage of the mid point is given by

\[
I_{2a} = \sqrt{2} \left( K_1^2 A_1 + K_2^2 A_2 + K_2 B_2 + d_2 \right) \quad ............. \quad (5.18)
\]
where

\[ A_2 = \frac{1}{2} \lambda_{2a} T_2' - Q_2' + 2 \lambda_{2a} \lambda_1 \cdot G_1 + \lambda_1 \cdot T_2' \]

\[ B_2 = -2J_1 \lambda_{2a} \cdot G_2' + \lambda_1 \cdot T_2' \]

\[ T_2' = \frac{\pi}{2} - \theta_1' \]

\[ Q_2' = -\frac{1}{2} \sin 2\theta_1', \quad G_2' = -\cos \theta_1', \quad d_2' = J_1^2 \cdot T_2' \]

\[ \theta_1' = \sin^{-1} \frac{\lambda_1}{\lambda_{2a}} \]

\[ J_1 = K_1 \lambda_1, \quad A_1' = \frac{1}{2} \lambda_{2a} \left( T_1' - Q_1' \right) + 2 \lambda_{2a} \lambda_1 \cdot G_1' \]

\[ T_1' = \theta_1' \]

\[ G_1' = \cos \theta_1' - 1, \quad Q_1' = \frac{1}{2} \sin 2\theta_1' \]

The above method can be extended for the computation of slope of any general segment say that the slope of R\textsuperscript{th} segment in \( N \)\textsuperscript{th} zone is to be computed. It may be noted that there is knowledge of slopes of the previous (R-1) segment and R\textsuperscript{th} segment is formed by joining \((X_{r-1}, I_{r-1})\) and \((X_r, I_r)\) of I.C. . The point corresponds to \((V_{3,1}, I_{r-1})\) and \((V_r, I_r)\) of the r.m.s. saturation curve.

From equation (5.7), the quadratic equation for the R\textsuperscript{th} segment having slope, \( K_r \), can be written as \[ A_r K_r^2 + B_r K_r + C_r = 0 \] ........................ (5.20)

From equation (5.20) slope of the R\textsuperscript{th} segment is given by

\[ K_r = \frac{-B_r \pm \sqrt{B_r^2 - 4A_r C_r}}{2A_r} \] ........................ (5.21)
where the parameters of the above equation are given by

\[ A_R = \frac{1}{2} \lambda_R T_R - Q_R + 2 \lambda_2 \lambda_{R-1} \cdot G_R + \lambda_{R-1}^2 \cdot T_R \]

\[ B_R = -2 J_{R-1} \lambda_R G_R + \lambda_{R-1} \cdot T_R \]

\[ C_R = C'_R + d_R - \frac{\pi}{2} I_R \]

\[ T_R = \frac{\pi}{2} - \theta_{R-1} \]

\[ \theta_R = -\frac{1}{2} \sin 2\theta_{R-1}, \quad G_R = -\cos \theta_{R-1} \]

\[ d_R = J_{R-1}^2 \cdot T_R \]

\[ C'_R = \sum_{x=1}^{R-1} (K_x^2 A_x + K_x B_x + d_x) \]  \[ \ldots \ldots (5.22) \]

where the parameters of the equation for \( C'_R \) are defined for \( x = 1 \) to \( R-1 \) as

\[ A_x = \frac{1}{2} \lambda_R T_x - Q_x + 2 \lambda_2 \lambda_{x-1} \cdot G_x + \lambda_{x-1}^2 \cdot T_x \]

\[ B_x = -2 J_{x-1} \lambda_R G_x + \lambda_{x-1} \cdot T_x \]

\[ d_x = J_{x-1}^2 \cdot T_x \]

\[ \theta_x = \sin^{-1} \frac{\lambda_x}{\lambda_R} \]

\[ T_x = \theta_x - \theta_{x-1}, \quad Q_x = \frac{1}{2} (\sin 2\theta_x - \sin 2\theta_{x-1}) \]

\[ G_x = \cos \theta_x - \cos \theta_{x-1} \quad \text{and} \]
Thus the slope of $K_R$ is computed and then the r.m.s. current $I_{Ra}$ at the mid point of the $R^{th}$ segment is computed. For the purpose, the voltage at the mid point of the $R^{th}$ segment is calculated from

$$V_{Ra} = V_{R-1} + \frac{1}{2}(V_R - V_{R-1}) \quad \text{(5.24)}$$

The amplitude of flux linkage at the voltage of the mid point $V_{Ra}$ is

$$\lambda_{Ra} = \frac{\sqrt{2} V_{Ra}}{O} \quad \text{(5.25)}$$

Then the computed r.m.s. current at the voltage of the mid point of the $R^{th}$ segment is given by

$$I_{Ra} = \sqrt{\frac{2}{\pi} \sum_{x=1}^{R} K_x^2 A'_{xl} + K_x B'_{xl} + dx'_1} \quad \text{(5.26)}$$

where the parameters of the equation for $I_{Ra}$ are defined for $x = 1$ to $R$ as

$$A'_{xl} = \frac{1}{2} \lambda_{Ra} T_x - \theta'_x + 2 \lambda_{Ra} \lambda_{x-1} G_x + \lambda_{x-1} T'$$

$$B'_{xl} = -2J_{k-1} \lambda_{Ra} G_x + \lambda_{x-1} T'$$

$$dx'_1 = J_{k-1}^2 \quad \text{T'}$$

$$T_x' = \theta'_x - \theta'_{x-1}$$
\[ \theta_x' = \sin^{-1}\frac{\lambda_x}{\lambda_{Ra}}, \quad Q_x' = \frac{1}{2} \left( \sin 2\theta_x' - \sin 2\theta_{x-1}' \right) \]

\[ G_x' = \cos \theta_x' - \cos \theta_{x-1}' \]

\[ J_x = K_x \left( \lambda_x - \lambda_{x-1} \right) + J_{x-1} \] \hspace{1cm} \ldots \ldots \ (5.27)

Step - III Verification of accuracy of the R\textsuperscript{th} segment:

For verification of accuracy the value of the computed r.m.s. current, \( I_{Ra} \) at the mid point of R\textsuperscript{th} segment is to be compared with the measured r.m.s. current, \( I_{Ra} \) from the data at the voltage. For the purpose of the r.m.s. saturation characteristic is given by

\[ S_a = \frac{I_N - I_{N-1}}{V_N - V_{N-1}} \] \hspace{1cm} \ldots \ldots \ (5.28)

Then the measured r.m.s. current at the mid point of the R\textsuperscript{th} segment is given by

\[ R_{Ra} = S_N (V_{Ra} - V_{R-1}) + I_{R-1} \] \hspace{1cm} \ldots \ldots \ (5.29)

where \( I_{R-1} \) is the measured current for the voltage corresponding to the end of (R-1)\textsuperscript{th} segment and is already known current.

The per unit error in the computed r.m.s. current at the voltage of the mid point of the R\textsuperscript{th} segment is given by

\[ \varepsilon_{cu} = \frac{I_{Ra} - I_{Ra}}{I_{Ra}} \] \hspace{1cm} \ldots \ldots \ (5.30)
Step - IV  Acceptance of $R^{{th}}$ segment:

The per unit error $e_M^*$ and specified error $e_\sigma^*$ in the r.m.s. current are compared and if $e_M^* \leq e_\sigma^*$, the $R^{{th}}$ segment can be accepted and the computations for $(N+1)^{th}$ zone are taken up.

Step - V  First bisection of the $N^{{th}}$ zone and a new $R^{{th}}$ segment:

If $e_M^* > e_\sigma^*$, then the voltage $E_{Ra}$ is assumed to be the higher voltage of the new $R^{{th}}$ segment. In other words, the old $R^{{th}}$ segment has been bisected for the 1st time and the 1st part of bisection is considered as the new $R^{{th}}$ segment.

Hence $V_R = V_{Ra}$ for the new $R^{{th}}$ segment, where $V_{Ra}$ is the mid point voltage of old $R^{{th}}$ segment. Then the procedures suggested in step II and III are followed. After calculating the slope of the present $R^{{th}}$ segment, the computed r.m.s. current $I_{Rb}$ at the voltage of the mid point of the $R^{{th}}$ segment is evaluated and compared with the measured current $I_{Rb}$ at the voltage. The mid point voltage of the segment is given by

$$V_{Rb} = V_{R-1} + \frac{1}{2}(V_R - V_{R-1}) \quad \cdots \quad (5.31)$$

Where $V_R = V_{Ra}$ as given above. The computed r.m.s. current is given by

$$I_{Rb} = \sqrt{\frac{2}{R} \sum_{r=1}^{R} (K_r^2 A_r + K_r B_r + d_r)} \quad \cdots \quad (5.32)$$

where all the parameters required for equation (5.27) are calculated by substituting $\lambda_{Rb}$ in the place of $\lambda_{Ra}$. The measured r.m.s current at the voltage $E_{Rb}$ is

$$I_{Rb} = S_n(V_{Rb} - V_{R-1}) + I_{R-1} \quad \cdots \quad (5.33)$$
The per unit error, $\varepsilon_{eb}$ at the voltage is computed from

$$\varepsilon_{eb} = \frac{I_{eb} - I_{Rb}}{I_{Rb}} \quad \ldots \ldots \quad (5.34)$$

**Step VI** Acceptance of the first bisected zone and formation of the next $R^{th}$ segment:

The per unit computed error $\varepsilon_{eb}$ is compared with the specified tolerance $\varepsilon$, for the curve.

If $\varepsilon_{eb} \leq \varepsilon$, then the $R^{th}$ segment satisfying the specified accuracy is up to the mid point of the linearised segment of $N^{th}$ zone of r.m.s. curve.

Hence the segment joining the point $(V_{R1}, I_{R1})$ and the end point $(V_{N}, I_{N})$ of the zone forms the new $R^{th}$ segment of r.m.s. curve and the slope of the corresponding segment for I.C. is computed.

**Step VII** Second bisection of the $N^{th}$ zone and repetition of the process:

If $\varepsilon_{eb} > \varepsilon$, the first part of the bisection of the linearised segment for $N^{th}$ zone is again bisected. Then the $R^{th}$ segment is the line joining the point $(V_{R11}, I_{R11})$ and the point $(V_{Rb}, I_{Rb})$. Then the step II, III and IV are repeated to verify the acceptance of the linearised segment obtained by second bisection. Thus the process of bisection is repeated until the derived IC segment satisfies the specified accuracy. Then Step VI is followed to form the next linearised segment. It is to be remembered that the bisection of a linear segment is made on the basis of voltage of the segment. This
process is repeated till the linearised segments for the I.C. of the desired zone satisfies the specified tolerance. Then the I.C. in the zone has optimum number of linearised segments.

5.6.3 Application of the suggested method and the results:

The data of the voltages and the corresponding currents for the various linearised zones of the r.m.s. curve for laboratory type single phase transformer rated 3KVA, 220/230 V, 50 Hz has been used for the computation of optimised number of segments for the corresponding I.C. having specified accuracy. The tolerance values for the error in I.C. of the transformer has been specified as 1%, 0.75%, and 0.5%. The results obtained for the instantaneous curve having the specified tolerances have been presented in Table - (5.2). It is observed from the computed results that specification of higher accuracy for the true saturation curve more number of segments for the desired range of I.C. Moreover each of the optimised segments of the I.C. has the required accuracy through out the range of the segment.

The flow chart for the computer programme to derive an I.C. having a specified accuracy and optimum number of segments is shown in the fig.(5.5).
Fig. 5.5
Flow chart of computer programme
### Table - 5.2

**Computed I.C. for different tolerance**

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<th>( E ) in volts</th>
<th>( \lambda ) in wb.turns</th>
<th>Experimental current in amp. (A)</th>
<th>Computed Current A</th>
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<th>No. of segments for optimisation</th>
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<td>Tolerance value %</td>
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5.7 Conclusion:

The piecewise linearised approach suggested in section (5.3) is useful to derive analytically transformer from the available r.m.s. saturation data. As it suggests, the representation of I.C. by piecewise linear segments, it does not require any mathematical expression to describe the instantaneous curve. The suggested method is accurate over any extended range of the I.C. including very high voltages in the saturated air core region where as modelling by mathematical expression usually have limited accuracy. Moreover the method of computation of I.C. does not involve any trial and error procedure where as the method suggested by Talukdar et.al. [1, 20] involves cut and try process.

The method has been improved by optimising the number of segments by which the I.C. is represented. Thus the optimisation method is more accurate as it is verified through out the desired range.