CHAPTER 6

AN INVENTORY MODEL WITH NON-INSTANTANEOUS DETERIORATION, PARTIAL BACKLOGGING AND INFLATION OVER A FINITE TIME HORIZON

6.1 Introduction

Maintaining and controlling inventories of deteriorating items is very important. In general, deterioration is defined as the damage, spoilage, dryness, breakage, vaporization, etc., that result in the degradation of the original condition of the product. As pointed out in Chapter 2, number of researchers developed inventory models for deteriorating items. In all the literature, it was assumed that the deterioration occurs as soon as the retailer receives the commodities. However, in real life, most of the goods would have a span of maintaining the quality or the original condition, for certain period. That is, during that period there was no deterioration occurring. This phenomenon is termed as “non-instantaneous deterioration”. Thus we assume that deterioration of the goods is non-instantaneous in this chapter.

Inflation has a major effect on the demand of the goods, especially for fashionable goods for middle and higher income groups. The concept of the inflation should be considered especially for long-term investment and forecasting. Most of the inventory models did not take into account the effects of inflation and time value of money. During the last few years, the
economic situation of most countries have been changed such that it has not been possible to ignore the effects of inflation and time value of money further.

When shortages occur, it is either completely backlogged or completely lost. But practically some customers are willing to wait for backorders and others would turn to buy from other sellers.

To the author’s best of knowledge, there is no inventory model with non-instantaneous deterioration, partial backlogging and inflation over a finite time horizon. Therefore, in the present chapter, a deterministic inventory model for non-instantaneous deteriorating items with price and advertisement dependent demand pattern over a finite horizon is proposed in which the inflation and time value of money are considered. Shortages are allowed and partially backlogged in this model. We have shown the effect due to changes in various parameters by taking a suitable numerical examples and sensitivity analysis.

The rest of the chapter is organized as follows: In section 6.2, the problem description of this chapter is given and followed by notations and assumptions used throughout this chapter. In section 6.4, the mathematical model to minimize the total inventory cost is established. Section 6.5 characterizes the optimal solutions and provides a simple algorithm. Several numerical examples are provided in section 6.6 to illustrate the theory and the solution procedure. This is followed by managerial implications in section 6.7 and conclusion in section 6.8.
6.2 Problem description

This chapter deals with an EOQ model for non-instantaneous deteriorating items with price and advertisement dependent demand pattern under the effect of inflation and time value of money over a finite planning horizon. In this model, shortages are allowed and partially backlogged. The backlogging rate is dependent on the waiting time for the next replenishment. The main aim of this chapter is to minimize the total inventory cost of the retailer by finding the optimal interval and the optimal order quantity. An algorithm is designed to find the optimum solution of the proposed model. Numerical examples are given. The effect of changes in the different parameters on the optimal total cost is graphically presented and the implications are discussed in detail.

6.3 Notations and assumptions

6.3.1 Notations

We additionally used the following notations to develop the mathematical model of the chapter.

- $T$ The length of the order cycle.
- $q_1(t)$ The inventory level at time $t$, $0 \leq t \leq \mu$.
- $q_2(t)$ The inventory level at time $t$, $\mu \leq t \leq t_1$.
- $q_3(t)$ The inventory level at time $t$, $t_1 \leq t \leq T$.
- $C_1$ The holding cost per item per unit time.
- $Q$ The 2nd, 3rd, ..., mth order size.
- $TC_F$ The total cost for first replenishment cycle.
- $TC$ The total cost of the system over a finite planning horizon $H$.
6.3.2 Assumptions

To develop the mathematical model, the following assumptions are being made.

1. A single item is considered over the prescribed period of planning horizon.
2. There is no replacement or repair of deteriorated items takes place in a given cycle.
3. The lead time is zero.
4. During the fixed period $\mu$, the product has no deterioration. After that, it will deteriorate with constant rate $\theta$, $0 < \theta < 1$.
5. The replenishment takes place at an infinite rate.
6. The effects of inflation and time value of money are considered.
7. The demand rate $D$ is a deterministic function of selling price, $s$ and advertisement cost, $A_C$ per unit item; i.e., $D(A_C, s) = A_C^n s^{-b}$, $a > 0, b > 1, 0 \leq \eta < 1$, $a$ is the scaling factor, $b$ is the index of price elasticity and $\eta$ is the shape parameter.
8. Shortages are allowed and partially backlogged. During stock out period, the backlogging rate is variable and is dependent on the length of the waiting time for the next replenishment. So the backlogging rate for negative inventory is, $B(t) = \frac{1}{1 + \delta(T - t)}$, where $\delta$ is backlogging parameter $0 \leq \delta \leq 1$ and $(T - t)$ is waiting time $(t_j \leq t \leq T)$, $(j = 1, 2, \ldots, m)$. The remaining fraction $(1 - B(t))$ is lost.
6.4 Formulation of the model

Suppose that the planning horizon $H$ is divided into $m$ equal parts of length $T = H/m$. Hence the reorder times over the planning horizon $H$ are $T_j = jT$ ($j=0, 1, 2, \ldots, m$). When the inventory is positive, demand rate is dependent on selling price and advertisement, whereas for negative inventory, the demand is partially backlogged. The period for which there is no-shortage in each interval $[jT, (j+1)T]$ is a fraction of the scheduling period $T$ and is equal to $kT$ ($0 < k < 1$). Shortages occur at time $t_j = (k+j-1)T$, ($j = 1, 2, \ldots, m$) and are build up until time $t = jT$ ($j = 1, 2, \ldots, m$) before they are backordered. This model is demonstrated in Figure 6.1.

![Figure 6.1: Representation of the inventory system by graph](image)

The first replenishment lot size of $S$ is replenished at $T_0 = 0$. During the time interval $[0, \mu]$, the inventory level is decreasing only owing to demand rate. The inventory level is dropping to zero due to demand and deterioration during the time interval $[\mu, t_1]$. During the interval $[t_1, T]$,
shortages occur and are accumulated until \( t = T_i \) before they are partially backlogged.

Based on the above description, during the time interval \([0, \mu]\), the inventory level reduces owing to demand only. Hence, the differential equation representing the inventory status is given by

\[
\frac{dq_1(t)}{dt} = -D, \quad 0 \leq t \leq \mu \tag{101}
\]

With the condition \( q_1(0) = S \), the solution of Equation (101) is

\[
q_1(t) = S - Dt \quad 0 \leq t \leq \mu \tag{102}
\]

In the second interval \([\mu, t_1]\), the inventory level decreases due to demand and deterioration. Thus, the differential equation below represents the inventory status:

\[
\frac{dq_2(t)}{dt} + \theta q_2(t) = -D, \quad \mu \leq t \leq t_1 \tag{103}
\]

With the condition \( q_2(t_1) = 0 \), we get the solution of Equation (103) is

\[
q_2(t) = \frac{D}{\theta} \left[ e^{\theta(t_1-t)} - 1 \right], \quad \mu \leq t \leq t_1 \tag{104}
\]

Put \( t = \mu \) in Equations (102) and (104) we find the value of \( S \) as

\[
S = D \left[ \mu + \frac{e^{\theta(t_1-\mu)} - 1}{\theta} \right], \tag{105}
\]

Substituting Equation (105) in Equation (102) we get

\[
q_1(t) = D \left[ \mu - t + \frac{e^{\theta(t_1-\mu)} - 1}{\theta} \right], \quad 0 \leq t \leq \mu \tag{106}
\]
During the third interval \([t_1, T]\), shortage occurred and the demand is partially backlogged. That is, the Inventory level at time \(t\) is governed by the following differential equation:

\[
\frac{dq_3(t)}{dt} = \frac{-D}{1 + \delta(T - t)}, \quad t_1 \leq t \leq T
\]  

(107)

With the condition \(q_3(t_1) = 0\), the solution of Equation (107) is

\[
q_3(t) = D(t_1 - t) \left[ 1 - \delta t + \frac{\delta}{2} (t + t_1) \right], \quad t_1 \leq t \leq T
\]

(108)

Therefore, the maximum inventory level and maximum amount of shortage demand to be backlogged during the first replenishment cycle are

\[
S = D \left[ \mu + \frac{e^{\theta(kH/m - \mu)} - 1}{\theta} \right]
\]

(109)

\[
BL = \frac{DH(1 - k)}{2m^2} \left[ 2m - \delta H(1 - k) \right]
\]

(110)

respectively.

There are \(m\) cycles during the planning horizon. Since, inventory is assumed to start and end at zero, an extra replenishment at \(T_m = H\) is required to satisfy the backorders of the last cycle in the planning horizon. Therefore, there are \(m + 1\) replenishments in the entire planning horizon \(H\).

The first replenishment lot size is \(S\).

The 2\(^{nd}\), 3\(^{rd}\), ..., \(m\(^{th}\) replenishment order size is

\[
Q = S + BL
\]

(111)

The last or \((m+1)^{th}\) replenishment lot size is \(BL\).

Since replenishment in each cycle is done at the start of each cycle, the present value of ordering cost during the first cycle is
\[ OC = A. \] (112)

The holding cost \( HC \) during the first replenishment cycle is

\[
HC = \int_0^{\frac{t}{\mu}} C_1 q_1(t) e^{-R' t} dt + \int_{\mu}^{\frac{t}{\mu}} C_1 q_2(t) e^{-R' t} dt
\]

\[
= \frac{C_1 D}{R} \left\{ e^{-R k H / m} \theta e^{\theta (k H / m - \mu)} e^{-R' \mu} \frac{e^{-R' \mu} - 1}{R + \theta} + e^{-R' \mu} \frac{1}{R} + \mu + \frac{e^{\theta (k H / m - \mu)}}{\theta} - 1 \right\} 
\]

(113)

The deteriorating cost \( DC \) during the first replenishment cycle is

\[
DC = C_2 \int_{\mu}^{\frac{t}{\mu}} \theta q_2(t) e^{-R' t} dt
\]

\[
= \frac{C_2 D}{R (R + \theta)} \left\{ \theta e^{-R k H / m} - e^{-R' \mu} \left[ \theta + R \left( 1 - e^{\theta (k H / m - \mu)} \right) \right] \right\}
\]

(114)

Total shortage cost \( SC \) during the first replenishment cycle is given by

\[
SC = -C_3 \int_0^{\frac{T}{T}} q_3(t) e^{-R' t} dt
\]

\[
= -\frac{C_3 D}{R} \left\{ e^{-R H / m} \left[ \frac{k H^2}{m^2} \left( 1 - \frac{k}{2} \frac{m}{\delta H} \right) + \frac{H}{m} \left( 1 - \frac{\delta H}{2m} \right) + \frac{1}{R} \left( 1 + \frac{\delta}{R} \right) \right]
\]

\[
+ e^{-R k H / m} \left[ \frac{\delta H}{m} \left( 1 - k - \frac{m}{HR} \right) - 1 \right] \right\}
\]

(115)

The lost sales cost \( LC \) during the first replenishment cycle is

\[
LC = C_4 \int_0^{\frac{T}{T}} \left( 1 - \frac{1}{1 + \delta (T - t)} \right) D e^{-R' t} dt
\]

\[
= -\frac{C_4 D \delta}{R^2} \left[ e^{-R H / m} + e^{-R k H / m} \left( \frac{RH}{m} (1 - k) - 1 \right) \right]
\]

(116)
Replenishment is done at \( t = 0 \) and \( T \). The present value of purchasing cost \( PC \) during the first replenishment cycle is

\[
PC = pS + pe^{-RT} (BI) = pD \left\{ \mu + \frac{e^{\theta(kH/m-\mu)}}{\theta} - 1 + e^{-RH/m} \frac{H}{m}(1-k) \left[ 1 + \frac{\delta H}{2m}(k-1) \right] \right\} \tag{117}
\]

Hence, Total cost = ordering cost + inventory holding cost + deterioration cost + shortage cost + lost sales cost + purchasing cost.

So, the total cost for first replenishment cycle is formulated as

\[
TC_{F} = OC + HC + DC + SC + LC + PC \tag{118}
\]

So, the present value of total cost of the system over a finite planning horizon \( H \) is

\[
TC(m,k) = \sum_{j=0}^{m-1} TC_{F} \ e^{-Rj/T} + Ae^{-RH} = TC_{F} \left\{ \frac{1-e^{-RH}}{1-e^{-RH/m}} \right\} + Ae^{-RH} \tag{119}
\]

where \( T = H/m \) and \( TC_{F} \) derived by substituting Equations (112) to (117) in Equation (118).

On simplification we get

\[
TC(m,k) = G \left\{ A + \frac{C_{2}D}{R} \left\{ \frac{e^{-R(kH/m)} - e^{\theta(kH/m-\mu)}}{R + \theta} \right\} + \frac{e^{-R\mu}}{R} - 1 + \mu \right. \\
+ \left. \frac{e^{\theta(kH/m-\mu)}}{\theta} - 1 \right\} + \frac{C_{2}D}{R(\theta + R)} \left\{ \theta e^{-R(kH/m)} - e^{-R\mu} \left[ \theta + R(1 - e^{\theta(kH/m-\mu)}) \right] \right\} \\
- \frac{C_{3}D}{R} \left\{ e^{-RKH/m} \left[ \frac{kH^{2}D}{m^{2}} \left( 1 - \frac{k}{2} - \frac{m}{\delta H} \right) + \frac{H}{m} \left( 1 - \frac{\delta H}{2m} \right) + \frac{1}{R} \left( 1 + \frac{\delta}{R} \right) \right] \\
+ \frac{e^{-RkH/m}}{R} \left[ \frac{\delta H}{m} \left( 1 - k - \frac{m}{HR} \right) - 1 \right] \right\} \right)
\]
\[
\frac{-C_4 D \delta}{R^2} \left[ e^{-R H/m} + e^{-R k H/m} \left( \frac{R H}{m} (1 - k) - 1 \right) \right] \\
+ pD \left\{ \mu + \frac{e^{\theta (k H/m - \mu)}}{\theta} - \frac{1}{1 + e^{-R H/m} \left( (1 - k) \left( 1 + \frac{\delta H}{2m} (k - 1) \right) \right)} \right\} + \Lambda e^{-R H} 
\]

where
\[
G = \left( \frac{1 - e^{-R H}}{1 - e^{-R H/m}} \right) 
\]

6.5 Solution procedure

The present value of total cost \(TC(m,k)\) is a function of two variables \(m\) and \(k\), where \(m\) is a discrete variable and \(k\) is a continuous variable. For a given value of \(m\), the necessary condition for \(TC(m,k)\) to be minimized is \(dTC(m,k)/dk = 0\) which gives

\[
\frac{dTC(m,k)}{dk} = \frac{GDH}{R m} \left\{ C_1 \left\{ e^{\theta (k H/m - \mu)} - \frac{e^{-R k H/m}}{R + \theta} - \frac{e^{-R \mu} e^{\theta (k H/m - \mu)}}{R + \theta} \right\} \\
+ \frac{C_2 \theta R}{(R + \theta)} \left\{ e^{-R \mu} e^{\theta (k H/m - \mu)} - e^{-R k H/m} \right\} \\
+ C_3 \delta \left\{ e^{-R k H/m} \left[ \frac{H}{m} (1 - k) - \frac{1}{\delta} \right] - e^{-R H/m} \frac{H}{m} \left[ 1 - k - \frac{m}{\delta H} \right] \right\} \\
+ C_4 \delta R \left\{ e^{-R k H/m} \left[ \frac{H}{m} (1 - k) \right] \right\} \\
+ pR \left\{ e^{\theta (k H/m - \mu)} + e^{-R H/m} \left[ 1 - (1 - k) \frac{\delta H}{m} \right] \right\} = 0 
\]
Now,
\[
\frac{d^2 TC(m, k)}{dk^2} = \frac{GDH^2}{Rm^2} \left\{ C_1 \left\{ \theta e^{\theta(kH/m - \mu)} + \text{Re} \frac{e^{RkH/m} - \theta^2 e^{R\mu} e^{\theta(kH/m - \mu)}}{R + \theta} \right\} \\
+ \frac{C_2 \theta R}{(R + \theta)} \left\{ e^{R\mu} e^{\theta(kH/m - \mu)} + \text{Re} e^{RkH/m} \right\} \\
+ C_3 \delta e^{-RH/m} + e^{-RkH/m} \left[ R \left( \frac{H}{m} (k-1) + \frac{1}{\delta} \right) - 1 \right] \right\} \\
+ C_4 \delta R \left\{ e^{-RkH/m} \left[ \frac{H}{m} (k-1) - 1 \right] \right\} \\
+ pR \left\{ \theta e^{\theta(kH/m - \mu)} + \delta e^{-RH/m} \right\} \right\} > 0
\] (122)

Furthermore, the Equation (122) shows that $TC(m, k)$ is convex with respect to $k$. So, for a given positive integer $m$, the optimal value of $k$ can be obtained from (121).

**Algorithm 6.1**

**Step 1:** Start with $m = 1$.

**Step 2:** Using (121) solve for $k$. Then substitute the solution obtained for (121) into (120) to compute the total inventory cost.

**Step 3:** Increase $m$ by 1 and repeat step 2.

**Step 4:** Repeat step 2 and step 3 until $TC(m, k)$ increases. The value of $m$, which corresponds to the increase of $TC$ for the first time, is taken as the optimal value of $m$ (denoted by $m^*$) and the corresponding $k$ (denoted by $k^*$) is the optimal value for $k$. 

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Using the optimal solution procedure described above, we can find the optimal order quantity and maximum inventory level to be

\[ S^* = D \left[ \mu + e^{\theta(k^*H/m^* - \mu)} - 1 \right] \]

\[ Q^* = D \left[ \mu + e^{\theta(k^*H/m^* - \mu)} - 1 \right] + \frac{DH(1 - k^*)}{2m^*} \left[ 2m^* \delta H(1 - k^*) \right] \]

6.6 Numerical examples

**Example 6.1**

Consider an inventory system with the following data: \( p = $12; \ s = $20; \ A = $100; \ C_1 = $2; \ C_2 = $1; \ C_3 = $6; \ C_4 = $8; \ \theta = 0.2; \ \delta = 0.1; \ \mu = 0.4 \text{ year}; \ A_c = $150; \ \eta = 0.4; \ a = 40000; \ b = 2.5; \ R = 0.2; \ H = 8 \text{ years}. \)

Using the solution procedure described above, the results are presented in Table 6.1. From this table we see that when the number of replenishments \( m = 8 \), the total cost \( TC \) becomes minimum. Hence, the optimal values of \( m \) and \( k \) are \( m^* = 8 \), \( k^* = 0.3657 \), respectively, and the minimum total cost \( TC(m^*, k^*) = $8,788.0230 \). We then have, \( T^* = H/m^* = 8/8 = 1 \text{ year}, \ t_{1}^* = k^*H/m^* = 0.3657 \text{ year}, \ Q^* = 162.6105 \text{ units}. \)

**Table 6.1: Optimal total cost with respect to \( m \).**

<table>
<thead>
<tr>
<th>( m )</th>
<th>( k(m) )</th>
<th>( Q )</th>
<th>( T )</th>
<th>( TC(m, k) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.3759</td>
<td>1255.9595</td>
<td>8.0000</td>
<td>12552.3713</td>
</tr>
<tr>
<td>2</td>
<td>0.4665</td>
<td>665.3547</td>
<td>4.0000</td>
<td>10788.6462</td>
</tr>
<tr>
<td>3</td>
<td>0.4648</td>
<td>437.9606</td>
<td>2.6667</td>
<td>9717.9141</td>
</tr>
<tr>
<td>4</td>
<td>0.4503</td>
<td>326.1313</td>
<td>2.0000</td>
<td>9236.1511</td>
</tr>
<tr>
<td></td>
<td>0.4314</td>
<td>260.0445</td>
<td>1.6000</td>
<td>8993.1223</td>
</tr>
<tr>
<td>---</td>
<td>--------</td>
<td>----------</td>
<td>--------</td>
<td>-----------</td>
</tr>
<tr>
<td>6</td>
<td>0.4104</td>
<td>216.4756</td>
<td>1.3333</td>
<td>8867.3695</td>
</tr>
<tr>
<td>7</td>
<td>0.3884</td>
<td>185.6119</td>
<td>1.1428</td>
<td>8807.4370</td>
</tr>
<tr>
<td>8</td>
<td><strong>0.3657</strong></td>
<td><strong>162.6105</strong></td>
<td><strong>1.0000</strong></td>
<td><strong>8788.0230</strong></td>
</tr>
<tr>
<td>9</td>
<td>0.3426</td>
<td>144.8094</td>
<td>0.8889</td>
<td>8795.2868</td>
</tr>
<tr>
<td>10</td>
<td>0.3191</td>
<td>130.6256</td>
<td>0.8000</td>
<td>8821.0342</td>
</tr>
</tbody>
</table>

From Figure 6.2, it is observed that the total cost decreases with the number of replenishment $m$ and it attains the minimum value 8788.0230 at $m = 8$. If the number of replenishment crosses 8, the total cost then increases. Therefore, if the retailer replenishes the quantities 8 times during the finite horizon he will attain the minimum cost.

Moreover, if $\mu = 0$, this model becomes the instantaneous deteriorating item case, and the optimal values of $m$ and $k$ are $m^* = 9$, $k^* = 0.2583$, respectively, $Q^* = 144.7756$ units and the minimum total cost $TC(m^*, k^*) = 8,814.9407$. The results are illustrated in Table 6.2.

| Table 6.2: Optimal total cost with respect to $m$ when $\mu = 0$. |
|---|---|---|---|
|   | $k(m)$ | $Q$ | $T$ |
| 1 | 0.3600 | 1278.2936 | 8.0000 |
| 2 | 0.4459 | 682.6551 | 4.0000 |
| 3 | 0.4356 | 447.9116 | 2.6667 |
| 4 | 0.4120 | 332.2979 | 2.0000 |
| 5 | 0.3839 | 263.9537 | 1.6000 |
| 6 | 0.3538 | 218.8935 | 1.3333 |
| 7 | 0.3226 | 186.9734 | 1.1428 |
From Figure 6.3, it is observed that when $\mu = 0$ the total cost decreases with the number of replenishment $m$ and it attains the minimum value 8814.9407 at $m = 9$. As the number of replenishment crosses 9, then the total cost increases. Therefore, if the retailer replenishes the quantities 9 times during the finite horizon he will attain the minimum cost.

**Example 6.2**

Consider another inventory system with the following data: $p = $12; $s = $25; $A = $200; $C_1 = $2; $C_2 = $3; $C_3 = $7; $C_4 = $9; $\theta = 0.2; $\delta = 0.1; $\mu = 0.4$ year; $A_c = $250; $\eta = 0.6; $a = 20000; $b = 2.4; $R = 0.2; $H = 5$ years.

Using the solution procedure described above, the results are presented in Table 6.3. From this table we see that when the number of replenishments $m = 5$, the total cost $TC$ becomes minimum. Hence, the optimal values of $m$ and $k$ are $m^* = 5$, $k^* = 0.4017$, respectively, and the minimum total cost $TC (m^*, k^*) = 10,549.0467$. We then have, $T^* = H/m^* = 5/5 = 1$ year, $t_1^* = k^*H/m^* = 0.4017$ year, $Q^* = 238.1753$ units.

Figure 6.4 shows the total cost function versus the number of replenishment.
Table 6.3: Optimum total cost with respect to $m$

<table>
<thead>
<tr>
<th>$m$</th>
<th>$k(m)$</th>
<th>$Q$</th>
<th>$T$</th>
<th>$TC(m, k)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.4754</td>
<td>1237.8105</td>
<td>5.000</td>
<td>14300.6126</td>
</tr>
<tr>
<td>2</td>
<td>0.4846</td>
<td>603.0406</td>
<td>2.500</td>
<td>11595.2251</td>
</tr>
<tr>
<td>3</td>
<td>0.4619</td>
<td>397.8391</td>
<td>1.667</td>
<td>10853.4134</td>
</tr>
<tr>
<td>4</td>
<td>0.4329</td>
<td>297.5398</td>
<td>1.250</td>
<td>10605.6931</td>
</tr>
<tr>
<td>5</td>
<td>0.4017</td>
<td>238.1753</td>
<td><strong>1.000</strong></td>
<td><strong>10549.0467</strong></td>
</tr>
<tr>
<td>6</td>
<td>0.3695</td>
<td>198.9525</td>
<td>0.8333</td>
<td>10585.6852</td>
</tr>
<tr>
<td>7</td>
<td>0.3366</td>
<td>171.1131</td>
<td>0.7142</td>
<td>10674.7364</td>
</tr>
<tr>
<td>8</td>
<td>0.3034</td>
<td>150.3316</td>
<td>0.6250</td>
<td>10796.1393</td>
</tr>
</tbody>
</table>

If $\mu = 0$, the optimal values of $m$ and $k$ are $m^* = 6$, $k^* = 0.2747$ respectively, $Q^* = 198.9567$ units and the minimum total cost $TC(m^*, k^*) = 10,646.8395$. The results are illustrated in Table 6.4 and Figure 6.5 shows the convexity of the total cost function with respect to $m$ when $\mu = 0$.

Table 6.4: Optimum total cost with respect to $m$ when $\mu = 0$

<table>
<thead>
<tr>
<th>$m$</th>
<th>$k(m)$</th>
<th>$Q$</th>
<th>$T$</th>
<th>$TC(m, k)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.4561</td>
<td>1270.6663</td>
<td>5.000</td>
<td>14711.4756</td>
</tr>
<tr>
<td>2</td>
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<td>616.8149</td>
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Figure 6.2: Example 6.1: Total cost with respect to number of replenishment.

Figure 6.4: Example 6.2: Total cost with respect to number of replenishment.

Figure 6.3: Example 6.1: Total cost with respect to number of replenishment when $\mu = 0$.

Figure 6.5: Example 6.2: Total cost with respect to number of replenishment when $\mu = 0$. 
It can be seen that there is a decrease in total cost from the non-instantaneous deteriorating item model. This implies that if the retailer can convert the instantaneously deteriorating items to non-instantaneous deteriorating items by improving stock control, then the total cost per unit time will decrease.

6.7 Sensitivity analysis

We now study the effects of changes in the values of the system parameters $\theta$, $R$, $\mu$, $\delta$, $s$ and $A_c$ on the optimal replenishment policy of the Example 6.1. We change one parameter at a time keeping the other parameters unchanged. The results are summarized in Table 6.5.

<p>| Table 6.5: Sensitivity analysis for various major parameters |
|---------------------------------------------|---|---|---|---|
| Parameter | Parameter value | $m$ | $k$ | $Q$ | $TC(m, k)$ |
| $\theta$ | 0.1 | 10 | 0.8481 | 133.2703 | 9430.6332 |
| | 0.2 | 8 | 0.3657 | 162.6105 | 8788.0230 |
| | 0.3 | 6 | 0.2107 | 212.3985 | 8728.7074 |
| $\mu$ | 0.2 | 9 | 0.3001 | 144.3553 | 8773.3654 |
| | 0.4 | 8 | 0.3657 | 162.6105 | 8788.0230 |
| | 0.6 | 7 | 0.4222 | 186.2425 | 8849.1868 |
| $\delta$ | 0.05 | 8 | 0.3693 | 164.2942 | 8928.9701 |
| | 0.1 | 8 | 0.3657 | 162.6105 | 8788.0230 |
| | 0.15 | 8 | 0.3619 | 160.8868 | 8643.5586 |
| $R$ | 0.18 | 10 | 0.6346 | 132.2280 | 9601.4806 |
| | 0.2 | 8 | 0.3657 | 162.6105 | 8788.0230 |
| | 0.22 | 6 | 0.2604 | 213.2174 | 8194.0680 |</p>
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Based on the numerical results, we obtain the following managerial phenomena:

1. When the deterioration rate $\theta$ is increasing, the optimal cost is decreasing and the order quantity is increasing. Figure 6.6 shows that when the deterioration rate increase, the number of replenishment decreases. So, the ordering cost will decrease. Therefore, the total cost of the retailer will decrease. Moreover, minimum deterioration rate of the products will minimize the deterioration cost of the items for the retailer.

2. When the length of fresh product time $\mu$ is decreasing, the total cost and the order quantity are decreasing. Since the order quantity is reduced, automatically holding cost of the items will also reduced. So, the total cost of the retailer will be minimum. Figure 6.8 shows the convexity of the total cost function with respect to the changes in the parameter $\mu$. Furthermore, if the retailers manage the non-instantaneous deteriorating items instead of instantaneous deteriorating items, then their total cost will be minimum.
3. If backlogging parameter $\delta$ is increased then the total cost and the order quantity will be decreased. In Figure 6.7, if the backlogging parameter increases then the ordering quantity will decrease. Therefore, the total cost of the retailer also decreases. That is, in order to minimize the cost, the retailers should increase the backlogging parameter.

4. When the net discount rate of inflation $R$ is increasing, the optimal cost is decreasing and the order quantity is increasing. Figure 6.9 shows that when the inflation rate increases, the number of replenishment decreases. So, the ordering cost will be decreased. In turn the total cost of the retailer will be decreased. That is, for higher values of the net discount rate of inflation ($R$), the total cost of the retailer will be minimum.

5. When the selling price $s$ is increasing, the total optimal cost and the order quantity are highly decreasing. But the increasing of selling price will decrease the demand as well as decrease the order quantity. So, the total cost of the retailer will decrease, which is illustrated in Figure 6.10.

6. When the advertisement cost $A_c$ is increasing, the total cost and the order quantity are highly increasing. Figure 6.11 shows that the minimum advertisement cost will minimize the total cost of the retailer but more advertisement cost implies more demand as well as more ordering quantity.

7. During shortages, if the retailers reduce the shortage cost as well as lost sales cost while they backlog the items partially, then the total cost of the retailer will be reduced.
Figure 6.6: Total cost vs. different values of $A$

Figure 6.8: Total cost vs. different values of $\mu$.

Figure 6.7: Total cost vs. different values of $\delta$.

Figure 6.9: Total cost vs. different values of $R$. 

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6.8 Conclusion

In this chapter, an EOQ model for non-instantaneous deteriorating items under inflation and time value of money is developed over the finite planning horizon. This chapter considered the demand as increasing function of advertising parameter with decreasing value of selling price. Also, shortage is allowed and it can be partially backlogged, where the backlogging rate is dependent on the time of waiting for the next replenishment. By constructing an efficient computational algorithm, we illustrated through couple of numerical examples that how the optimal order quantity and the optimal total cost can be derived. The results of the proposed model show that there is a decrease in total cost from the non-instantaneously deteriorating items compared with instantaneously deteriorating items. Also, when the net discount rate of inflation and the backlogging rate increase, the optimal total cost will decrease. Furthermore, sensitivity analysis is carried out with respect to the key parameters and useful managerial insights are obtained. The graphical illustrations are also given to analyse the efficiency of the model clearly.