Preface

This doctoral thesis entitled *Approximation by some positive linear operators* embodies the research work carried out by me under the supervision of **Prof. M. Mursaleen, Chairman**, Department of Mathematics, Aligarh Muslim University, Aligarh, India and under the co-supervision of **Prof. Jan Lang**, Department of Mathematics, Ohio State University, Ohio, USA. The present thesis contains some newly constructed operators and their approximating properties. Results studied here are more general than the corresponding previously known results. Also, some new results have been obtained for $s$-numbers of the Volterra operator.

*Approximation theory* is the branch of Mathematics in which we study approximation of functions. The theorem given by the German mathematician K. Weierstrass in the year of 1885 is considered as the beginning of this interesting branch of Mathematics. This theorem has played a key role in the development of *Approximation Theory*. In this exposition we majorly study the approximation of functions by positive linear operators which possesses key relevance in study of numerical analysis, computer aided geometric design and solutions of differential equations.

The application of $q$-calculus has galvanised the research in the field of *Approximation Theory* since the last quarter of the last century. It has appeared as a connection between Mathematics and Physics.

The $q$-calculus is the generalisation as well as the modification of the usual calculus. Recently, a generalisation of the $q$-calculus, known as the $(p,q)$-calculus, has also been applied to study the approximation of operators.

Present thesis is structured into seven chapters. Each chapter is divided into sections and subsections wherever required. The definitions, propositions, theorems, examples and remarks have been specified by numerals in which the first digit denotes the chapter, the second digit stands for the section and the third digit (wherever it occurs) represents the subsection within that section. Elementary concepts like function, sequence, convergence etc. have been presumed and shall not be discussed here.

In Chapter 1 we give a brief history of the development of the *Approximation Theory*
and present some well known definitions and results which are both inspired by and used throughout the thesis. We mention some important tools which measure the quality of approximation by the positive linear operators, representatively, the modulus of continuity $\omega$, the moduli of smoothness $\omega_2$ and the Peetre’s $K$-functional. We give a brief account of the Korovkin’s type theorems. The basics of the $q$-and the $(p, q)$-calculii have been presented. Also, we delve into the theory of Volterra operators in brief.

Chapter 2 presents approximating results for the Stancu type Jakimovski-Leviatan-Durrmeyer operators and the generalized Dunkl analogue of Szász operators. We study the convergence of these operators in the weighted space through the Korovkin’s theorem. The accuracy of convergence is estimated by means of the modulus of continuity and Lipschitz type functions. Some direct results are also studied.

Chapter 3 is dedicated to the study of Stancu type Bernstein-Schurer-Kantorovich operators constructed via $q$-integers. We calculate the central moments for these operators and investigate the uniform convergence on the compact interval $[0, 1]$. We construct the bivariate analogue of these operators and study their approximation behaviour using the Lipschitz class of functions.

Chapter 4 deals with the approximation of the Dunkle generalisation of $q$-analogue of the Kantorovich-Szász-Mirkjan operators. Convergence properties of these operators are studied via the Korovkin’s theorem. Rate of approximation is estimated in terms of the modulus of continuity, the Lipschitz functions and in the weighted class of functions.

Chapter 5 aims at presenting the two parametric Stancu-Beta operators constructed with the help of the $(p, q)$-integers. Peetre’s $K$-functionals and the modulus of continuity are used to study some approximation results. The weighted convergence of these operators is studied. We present the Voronovskaya type theorem for these operators. With the help of the Lipschitz-type maximal functions the pointwise estimate investigated.

In the penultimate chapter, the $q$-analogue of the Bernstein-Schurer-Kantorovich operators and the $(p, q)$-Stancu-Beta operators have been studied. Korovkin’s type theorem is verified for these operators. We investigate into the rates of convergence using first and the second modulus of continuity. The rate of smoothness is also obtained in terms of the Lipschitz functions. The convergence rate is also estimated using modulus of continuity of the derivative of function.

The last chapter is devoted to the study of Volterra operator and $s$-numbers. We
prove some lemmas for this operator and establish equality among certain \( s \)-numbers. The Volterra operator under study is not compact but we show that it is nicely behaving in terms of \( s \)-numbers which are powerful tools to study the asymptotic behaviour of an operator.

Finally, at the end, a bibliography is given which is extensive but by no means exhaustive. It hosts only the books and research papers which have been referred to in this thesis.