Chapter 1

Basic Concepts and Literature Review
1.1. RELIABILITY THEORY

The reliability of product or system is defined as the quality of performing consistently well or of being trustworthy. A measure is said to reliable if it can perform its required mission or function under given condition for a mentioned time period. One can find many definitions for reliability, according to Shelemyahu Zacks (1992), “The term reliability is used generally to express a certain degree of assurance that a device or a system will operate successfully in a specified environment during a certain time period”. According to the definition, the basic elements of reliability are probability, operating conditions, adequate performance and duration of adequate performance.

Reliability can also be defined as the quality over time, which means its significance is always cognate to time and environment unlike quality, where we do not consider the time length and environment of operation but only consider the degree of confirmation. For example, if an automatic teller machine (ATM) becomes unsuccessful in performing transactions, even after following specified instructions, the ATM will be considered as unreliable. There have been cases of machines giving out money without debiting the account, or giving out higher value notes. Under such cases, the machine is said to be unreliable. If these things do not happen before its stated life, it would be considered as reliable.

It is not easy to predict at the start whether a particular ATM machine will be reliable because no two are ever absolutely identical, even though they are of the same make. There are always small manufacturing differences and a few may contain defects. Suppose that out of every 100 cars of a particular type, 99 prove to be trouble free, if used and maintained correctly and one fails to work as intended, then it can be said that the reliability of each car is 99 percent, since reliability is a probability, it is expressed in decimals of 1.00 as given below:
Reliability = 1.00 means certain to work as intended.
Reliability = 0.99 means 99 percent likely to work as intended.
Reliability = 0.50 means 50 percent likely to work as intended.
Reliability = 0.00 means absolutely certain not to work as intended.

Reliability at a time "t" can be defined as:

\[ R(t) = \frac{\text{Number surviving at instant 't'} \text{ Number at start(when 't'= 0)}} {\text{Number at start(when 't'= 0)} } \quad (c.f. \text{ Sinha, 1985}) \]

In the field of statistics, “Reliability is defined as the characteristic of a system or product expressed in terms of the probability that a system or product will perform its particular mission under given condition with respect to some circumstances and time”. It is generally designed by R. From a qualitative point of view, it means the ability of the system or product to continue its function or mission. Quantitatively, it is defined as the probability that operational interruptions will not take place during stated time interval. The estimation and statistical analysis of the reliability of systems and component using different statistical procedures is known as reliability theory.

1.2. MEASUREMENTS OF RELIABILITY

1.2.1. Reliability Function

The reliability function denoted by \( R(t) \) (also called survival function in the field of biological and medical science) is defined as the probability of a system or device not failing prior to time \( t \). Mathematically, it can be defined as

\[ R(t) = P(T > t) = 1 - F(t) \]  \hspace{1cm} (1.1)  

where, \( F(t) \) is the cumulative distribution function (CDF), which is defined as the probability in a random trial that the random variable is not greater than \( t \), i.e.

\[ F(t) = P(T \leq t) = \int_{0}^{t} f(t) dt \]

where, \( f(t) \) is the probability density function (PDF) of a random variable \( t \).
Also, when differentiating equation (1.1), we have

$$f(t) = -\frac{dR(t)}{dt}$$

### 1.2.2. Failure Rate

The probability of failure in a given time interval $[t_1, t_2]$ can be expressed by the given function

$$\int_{t_1}^{t_2} f(t) dt - \int_{t_2}^{\infty} f(t) dt = R(t_1) - R(t_2)$$

The failure rate is defined as the rate at which the failures occur in the interval $[t_1, t_2]$. It is denoted by $\lambda(t)$. Mathematically, it is defined as the ratio of the probability that failures occurs in the interval, given that it has not occurred prior to $t_1$, the start of the interval, divided by interval length (Barlow et al. 1965).

Thus,

$$\lambda(t) = \frac{R(t_1) - R(t_2)}{(t_2 - t_1)R(t_1)}$$

Alternatively, it can also be written as

$$\lambda(t) = \frac{R(t) - R(t + \Delta t)}{\Delta t R(t)}, \quad \text{where; } t = t_1, t + \Delta t = t_2$$

### 1.2.3. Hazard Rate

The hazard rate (HR), also called the instantaneous failure rate is defined as the limit of the failure rate as the interval length approaches to zero. It is denoted by $h(t)$.

$$h(t) = \lim_{\Delta t \to 0} \left[ \frac{R(t) - R(t + \Delta t)}{\Delta t R(t)} \right] = -\frac{1}{R(t)} \left[ \frac{dR(t)}{dt} \right] = \frac{1}{R(t)} \left[ -\frac{dR(t)}{dt} \right]$$  \hspace{1cm} (1.2)

Since, $f(t) = -\frac{dR(t)}{dt}$ (previously shown)
Therefore, substituting \( f(t) \) into equation (1.2)

\[
h(t) = \frac{f(t)}{R(t)}
\]

The reliability function \( R(t) \) is always decreasing function of time but \( f(t) \) may be decreasing, constant, increasing, symmetric or skew-symmetric. Therefore the behaviour of \( h(t) \) only depends on \( f(t) \).

Also, based on the concept of hazard function, the integrated hazard function or cumulative hazard function, denoted by \( H(t) \) is given by

\[
H(t) = \int_{0}^{t} h(t) \, dt, \quad t \geq 0
\]

1.2.4. Mean Time to Failure (MTTF)

It means the expected time to failure of a non-repairable products or systems and is defined as follows

\[
MTTF = \int_{0}^{\infty} tf(t) \, dt = \int_{0}^{\infty} \left[ -\frac{dR(t)}{dt} \right] \, dt
\]

Integrating by parts, we get

\[
MTTF = \int_{0}^{\infty} R(t) \, dt
\]

Hence, MTTF depends directly on the reliability function \( R(t) \). For repairable product or system, MTTF is defined as the mean time to first failure (Bourne and Greene, 1972).

1.2.5. Mean Life

The mean life, denoted by \( \theta \), refers to the total population of units or systems considered (Calabro, 1962). Given an initial population of \( n \) units, if all are operated until they fail then the mean life is merely the arithmetic mean time to failure of the total population.
Therefore,

\[ \theta = \frac{\sum_{i=1}^{n} t_i}{n} \]

where, \( t_i \) denotes the failure time of \( i^{th} \) unit in the population, \( n \) denotes the total number of units in the population.

1.2.6. Mean-Time-Between-Failures (MTBF)

The concept of MTBF is related to repairable products or systems. It means the expected time between two failures and is expressed in terms of the expected value of the density function \( f(t) \) as

\[ MTBF = \int_{0}^{\infty} tf(t)dt, \]

where, \( f(t) \) is the probability density function which satisfies;

\[ \int_{0}^{\infty} f(t)dt = 1 \]

1.3. ANALYSIS OF RELIABILITY

Reliability and maintainability are most important components in engineering design of products and systems. Their growth has been motivated by several factors which include the increased complexity and sophistication of systems, insistence on quality of products and public awareness. The reliability analysis is the most important characteristic of system quality as things have to be working satisfactorily. Therefore it has become essential for manufacturers to analyze the product failures, their causes and consequences. Usually, some specific measures are used in reliability analysis in order to check the performance of a product. If the performance is below a certain level, a failure can be said to have occurred.

Accurate prediction and control of reliability play an important role in scheduling preventive maintenance, determining warranty conditions and periods and other associated aspects. Therefore, the manufacturers find the need to develop
asystem that can economically design products meeting the customer’s expectations and have the strong competitive advantage. For that, there are several tests which provide the useful reliability information. Also, each test has different objectives, such as functional or operational tests, environmental stress testing, reliability qualification tests, reliability life testing, safety testing, and reliability growth testing. The primary objective of reliability life testing is to obtain information concerning failures, thus to quantify reliability, determine satisfaction of reliability goals and to improve product reliability.

The traditional reliability experiments consist in analysing failure time data obtained under normal conditions in order to quantify the product’s life characteristics, reliability and other associated parameters. However, in many situations, it is very hard to obtain such failure data because of the high reliability of today’s products, the short span time between product design and release and the challenge of testing systems which are in continuous operation. Considering this difficulty, researchers have developed a class of reliability testing method called accelerated life testing in order to obtain failure information of systems and products in prompt time period. The main prospect of the developed test method is to utilise failure data from accelerated conditions in order to analyse product life under usage conditions.

1.4. ACCELERATED LIFE TESTING

Due to increase in the competition of business in the current market, the requirements on the improvement of reliability and quality of the products/systems have also increased. Simultaneously, the products/systems are becoming more complex and an increasing number of components have become vital for the function of a product. Therefore, the reliability practitioners find the need to put severe reliability requirements on components and parts of such products/systems. At the same time, it has become necessary for manufacturers producing goods to check the reliability and life characteristics of these products/systems before letting them serve in the market. The traditional life testing method involves analysing failure-time data in order to obtain these life characteristics. But it is almost impossible to obtain such failure-time data for modern products because of their improved efficiency. Given this
difficulty, reliability practitioners have developed a method called accelerated life testing in order to obtain such failure-time data. In this method of testing the products are put at higher stresses in order to quickly obtain information about the life distribution of a material, component or product. The higher stress means that the items are subjected to conditions that are more severe than the normal ones, which yields shorter life but probably does not change the failure mechanisms. Based on these assumptions, the life distribution under normal stress levels can be estimated. Accelerated life testing has mostly been used to estimate or measure reliability with reduced time and cost involved. It helps to identify weak points in a design and also determines when and how a product will fail in its intended environment. This type of information can be used to improve the reliability, see for example Andersson (1991).

The ALT model usually has two components: a parametric distribution describing the failure-time distribution at fixed level of stress and a mathematical model describing the relationship between distributions parameters and levels of stresses. This mathematical model, also called the life-stress relationship is used to extrapolate the lifetime of products at use level of stress. If the levels of stress exceed a highest possible level of stress, the linear relationship may no longer hold. Therefore, one should always avoid using stress above this critical level in practice.

The main aim of ALT is to find the failure data of products and systems by subjecting them to the higher levels of stresses. For analysing ALT efficiently and to obtain performance data, the experimenter needs to determine the testing method, statistical model, form of the life data and a suitable statistical method. When properly analysed, yields reasonable estimates of the product’s life and performance under normal conditions. Figure 1-1 shows different types of ALT, which are described as follows
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1.5. TYPES OF ACCELERATED LIFE TESTING

There are various types of accelerated testing approaches based on the stress type used. Each type is carefully designed to fit the product under consideration. There are two common accelerated life test procedures which are described as follows.

i. Qualitative Accelerated Life Tests

ii. Quantitative Accelerated Life Tests

1.5.1. Qualitative Accelerated Life Tests

Qualitative accelerated life tests do not quantify the reliability of the product under normal use conditions. The primary aim of these tests is to reveal probable failure modes for the product so that product manufacturers can improve the design of the unit or product. Such test can study the behaviour of product in various operational conditions and thus identify design weakness. However, they provide valuable information about the stress types and associated levels to proceed with the quantitative test.
Qualitative tests are performed on small samples of the products subjected to a single severe level of stress, to a number of stresses or to a time-varying stress. If the specimen survives, it passes the test. Otherwise, appropriate actions will be taken to improve the product’s design in order to eliminate the cause(s) of failure. The main objective of these tests is to identify failures and failure modes without attempting to make any predictions as to the product’s life under normal use conditions. Typical qualitative ALT includes **Highly Accelerated Life Tests (HALT)** and **Highly Accelerated Stress Screen (HASS)**. Both are based on techniques that shorten the time required to identify potential causes of failure. The structure of each can be briefly discussed as follows.

**Highly Accelerated Life Tests (HALT):**

HALT is used in the design phase of the product in order to remove design related weaknesses. This is done by testing product at a severe condition which forces failures to occur in significantly less time than under normal conditions. It exposes the products to a step-by-step cycling in environmental variables such as temperature, shock and vibration. The goal of the HALT is to break the product, find the weak components and reinforce or improve the weak spots. Therefore, it is useful to lower product development costs and compress time to failure. It can also expose problems caused by new manufacturing processes when used at the time a product is being introduced into the market.

**Highly Accelerated Stress Screen (HASS):**

HASS is an on-going screening test where products are subjected to high stress frequently well beyond the qualification level but not at the extreme levels as conducted in HALT tests. The main aim is to identify process and vendor problems during the production process. Unlike HALT, it is a screening of the actual products being produced through manufacturing using a less stressful level of stimuli.

**1.5.2. Quantitative Accelerated Life Tests**

Quantitative ALT is designed to quantify the life characteristics of the system, component or product under normal use conditions and thereby provide reliability information. The main aim is to produce the data required for accelerated life data
analysis and induce information about life characteristics of products that include the prediction of mean life, reliability of the product under normal use conditions, projected returns and warranty costs. Units are tested at higher than usual levels of stress (e.g. temperature, voltage, pressure, vibration, and cycling rate) to induce early failure. Data obtained from Quantitative ALT are then analysed based on models that relate the lifetime to stress. Finally, the results are extrapolated to estimate the life distribution at the normal use condition.

There are two methods of acceleration in quantitative accelerated life testing that includes usage rate acceleration and overstress acceleration. Both are devised to obtain times-to-failure data of products and systems at an accelerated pace.

**Usage Rate Acceleration:**

This type of acceleration is related to the products which do not operate continuously under normal conditions, e.g. microwave oven, washing machine, etc. The procedure is to operate such test units continuously to encounter early failures. For example, a microwave oven operates for small periods of time every day. One can accelerate a test on microwave ovens by operating them more frequently until failure. The same could be said for washers. The main aim is to reduce the test time by operating these products continuously.

**Overstress Acceleration:**

This type of acceleration is applied on products which have very high or continuous usage such as computer servers and peripherals. In such cases, engineers exceed the stress on test units that they will encounter under normal use conditions. The failure information obtained under these conditions are then used to extrapolate to use conditions. The overstress acceleration can be in the form of high or low temperature, humidity, voltage, pressure, vibration, etc. Special care must be taken to choose the stress levels so that they accelerate the failure modes under considerations but do not introduce failure modes that will never occur under use conditions.
1.6. APPLICATION AREA OF ACCELERATED LIFE TESTING

Accelerated life testing is applied in different areas and serves various purposes, for example:

- To predict reliability
- To develop the relationship between reliability and operating conditions.
- To identify design failures and manufacturing defects in order to eliminate or reduce them.
- To identify dominating failure causes.
- To verify that product reliability surpasses customer specification.
- To study the result of design and manufacturing changes.
- To determine burn-in time and conditions in order to eliminate early failures.
- To improve reliability.
- To test materials.
- To compare different designs, components, etc., in order to identify the best one
- To measure reliability and to estimate warranty and service costs by giving information about the lifetime distribution.
- To evaluate alternative manufacturing processes.

Accelerated life testing may have one or several of these purposes. The increasing demand for reliability improvement puts new requirements on planning and use of accelerated life testing experiments. It is important to use the obtained information to develop the design and improve the reliability.

1.7. STRESSES AND THEIR CLASSIFICATIONS

Let us consider a non-negative random variable $T(x)$ which represents the time of failure of a component depending on a vector of covariates $x$. In reliability theory, the vector $x$ is called a stress vector. The probability of failure of an item is then a function of stress given by

$$F_x(t) = P[T(x) > t], t \geq 0$$

Assume that $F_x(t)$ is differentiable and decreasing on $(0, \infty)$ as a function of $t$ for every $x \in S$, where $S$ denotes a set of possible stress values.
Stress is classified into following categories constant stress, step stress, progressive stress, cyclic stress and random stress. We now define and describe these stress classifications (see Nelson (1990).

1.7.1. Constant stress

A constant stress ALT may employ two or more stress levels and each stress level remains constant during the test. All test units are allocated into several groups according to the number of stress levels to be employed. Each group is tested under a certain stress level. For some materials and products, accelerated test models for constant stress are better developed and experimentally established. Examples of constant stress are temperature, voltage and current.

1.7.2. Step-stress

In a step stress ALT, the test can be divided into several stages and each stage employs one stress level. In this, a specimen is subjected to successively higher levels of stress. At first, all test units are subjected to a specified constant stress for a specified length of time, then the non-failed units are further tested i.e. subjected to a next higher stress level for a specified time. The stress is increased step by step until all units fail. Usually, all specimens go through the same specific pattern of stress levels and test times. But sometimes different patterns are applied to different specimens. The increasing stress levels ensure that the failures occur quickly resulting in data, appropriate for statistical purposes.
1.7.3. Progressive stress

The stress levels in a progressive stress ALT are set to be a consecutive function of time, and all units are tested at these continually changing stress levels until the end.

Graphical Representation of Progressive Stress Loading

All three types of ALTs have their advantages and disadvantages, as discussed in detail in Nelson (1990). More literature on ALT plans can be found in Nelson (2005).

1.8. COMPLETE AND CENSORED DATA

In the field of life testing, the failure can be classified into the two categories.

i. Complete data (all the failure data are available)

ii. Censored data (some of the failure data are missing)
Data is said to be complete if it contains the failure time information of each and every unit of the test. The failure information consists of the exact failure time of test units, which means that the failure time of each test unit is observed or known. There are many cases in which the failure data of all the units in the sample is not known. This type of data is called incomplete or censored data. Such data is obtained when the experimenter terminate the experiment before all the units run to failure. The censored data can be broadly classified into two types, type-I censored data and type-II censored data, briefly discussed below.

**Type-I censoring:** Type-I censored (or time censored) data is usually obtained when censoring time is fixed but there is no restriction on the number of failures obtained during this time period. Hence the number of failure in that fixed time is a random variable.

**Type-II censoring:** Type-II censored (or failure censored) data is usually obtained when the test is terminated after a pre-specified number of failures but there is no restriction on the time of the test. Hence time to obtain these fixed number failure is a random variable.

The mixture of these two censoring schemes is called hybrid censoring scheme. Figure 2-1 shows different censoring schemes in ALTg, which are described as follows:

![Censoring Diagram](image-url)
For a detailed description of these censoring schemes, one can refer to *Hybrid censoring: Models, inferential results and applications*, Balakrishnan and Kundu, (2015). Apart from these censoring schemes, there are some advanced censoring schemes named as adaptive censoring schemes and also some generalised censoring schemes. Also, there is an important censoring scheme called progressive censoring scheme which is extensively used in accelerated life testing along with abovedefined censoring schemes.

1.9. LIFE-STRESS RELATIONSHIPS

The mathematical model describing the relation between life and stress of the units is called the life stress relation. It describes a characteristic point or a life characteristic of the distribution from one stress level to another stress level. This life characteristic is expressed as a function of stress. One can usually use life-stress relationship statistical analysis when dealing with Step-Stress Accelerated Life Testing (SSALT).

Nelson (1990) and Zhao and Elsayed (2005) summarised several life-stress relationship models in a log-linear format. Bai et al. (1989), Bai and Chun (1993), Khamis and Higgins (1998), Xiong (1998), and Yeo and Tang (1990) developed inference models for SSALT assuming that the mean life of a test unit is a log-linear function of stress. Moreover, Khamis and Higgins (1996) and Elsayed and Zhang (2007) considered a log-quadratic model for a 3-step stress plan and 2-step test plan, respectively. Depending on assumed underlying life distribution, different life characteristics can be considered. Some of the life-stress relations in the accelerated life testing analysis are:

i. Arrhenius life-stress relationship/Arrhenius model.
ii. Eyring life stress relationship/Eyring model.
iii. Log-Linear Relationship/Log-Linear model.
iv. The Temperature-Humidity Relationship.
v. The exponential relationship.
vi. Inverse power law model (IPL).

The detailed description of these life-stress relationships is given in Nelson (1990). Here, only inverse power law model is discussed in detail.
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Inverse Power Law (IPL)

The IPL is commonly used for non-thermal accelerated stress. The inverse power relationship between nominal life $\tau$ of a product and a stress variable $V$ is given by

$$\tau(V) = \frac{A}{V^{\gamma_1}}$$

where $A$ and $\gamma_1$ are parameters to be estimated. Equivalent forms are

$$\tau(V) = \left(\frac{A^*}{V}\right)^{\gamma_1} \quad \text{and} \quad \tau(V) = A^\ast \left(\frac{V_0}{V}\right)^{\gamma_1}$$

where $V_0$ is a specified (standard) level of stress.

The acceleration factor $AF_i$ of the inverse power law is

$$AF_i = \frac{\tau(V_o)}{\tau(V_s)} = \left(\frac{V_s}{V_o}\right)^{\gamma_1}$$

1.10. GENERAL STEPS FOR DESIGNING AN ALT

The ALT model usually has two components: a parametric distribution describing the failure-time distribution at fixed level of stress and a mathematical model describing the relationship between distributions parameters and levels of stresses. A proper test plan should be used to decide the appropriate distribution of failure times, life-stress relation, appropriate stress levels and also a number of the test units that need to be allocated to the different stress levels. But it is sometimes an expensive and difficult endeavour. Therefore, it is advisable to have a plan that accurately estimates reliability at operating conditions for minimising test time and costs. Different acceleration models and life distributions can be combined to build up a suitable model for analysing the data from ALT. Following are some common test plans for ALTs.

1.10.1. Plan for Step-Stress ALT

Accelerated life tests at constant stress can take too long to induce failure and also there is usually a great scatter in failure times. Therefore, Step-stress testing is a more suitable choice to use for modern products. These tests are intended to reduce
test time and to assure that failures occur quickly. A step-stress test runs through a pattern of specified stresses, for a specified time until the specimen fails or the test is stopped when a certain censoring scheme has been used.

**Assumptions and Test Procedure**

i. Stresses are employed in ascending order. If there are two stress levels \( x_1 \) and \( x_2 \), then \( x_2 > x_1 \).

ii. A random sample of \( n \) identical products is placed on the test under initial stress level \( x_1 \) then at the time \( \tau \), the stress is changed to \( x_2 \) and the test is continued until all products fail or a certain censoring time has been reached.

iii. The failure times of the test units at each stress level are independent and identically distributed according to some probability distribution.

iv. The mathematical model relating to life and stress of test units is known or assumed.

v. A Cumulative Exposure (CE) model holds, i.e. the remaining life of test items depends only on the current cumulative fraction failed and current stress regardless of how the fraction accumulated. Moreover, if held at the current stress, items will fail according to the Cumulative Distribution Function (CDF) of stress, but starting at the previously accumulated fraction failed; see for more detail Nelson (1990). According to CE model, the CDF in SSALT is given by

\[
F(t) = \begin{cases} 
F_1(t) & 0 \leq t < \tau_1 \\
F_2(t - \tau_1 + s_1) & \tau_1 \leq t < \infty 
\end{cases}
\]

where, the equivalent starting time \( s_1 \) is a solution of \( F_1(\tau) = F_2(s_1) \).

The general CE model given by Nelson (1980) is given by

\[
F(t) = \begin{cases} 
F_1(t) & 0 \leq t < \tau_1 \\
F_2(t - \tau_1 + s_1) & \tau_1 \leq t < \tau_2 \\
\vdots & \vdots \\
F_i(t - \tau_{i-1} + s_{i-1}) & \tau_{i-1} \leq t < \tau_i \\
\vdots & \vdots 
\end{cases}
\]
Where \( i \) is a stress level, \( \tau_{i-1} \) is a switch point between stress level \( i - 1 \) and \( i \) and \( s_i \) is an equivalent starting point under stress level \( i \) to an ending point under stress level \( i - 1 \). \( s_i(i > 0) \) is the solution of \( F_{i+1}(s_i) = F_i(\tau_i - \tau_{i-1}) \) for \( i = 1, 2, \ldots, k - 1 \).

It was first of all Nelson (1980), who proposed CE model for planning SSALT. After that many studies regarding SSALT planning based on the CE model have been performed. Miller and Nelson (1983) presented the optimum simple SSALT plan. Then Bai et al. (1989) and Bai and Chun (1991) extended this model to the case where a prescribed censoring time is involved. Khamis and Higgins (1998) proposed a new model for SSALT as an alternative to the CE model, which is based on a time transformation of the exponential CE model.

1.10.2. Partially Accelerated life tests

In ALT, the major assumption is that the mathematical model which specifies the relationship between lifetime and stress of units is known. However, there are some situations in which such models are not known or cannot be assumed. In such cases accelerated life testing is not a suitable choice to use. Therefore we need a better criterion called partially accelerated life testing (PALTs) in order to find the failure times of products. In this method of testing the products are tested at both normal as well as accelerated conditions. It consists of a variety of test methods for shortening the life of certain products or hastening the degradation of their performance. The commonly used stress methods are constant-stress partially accelerated life test (CSPALT) and step-stress partially accelerated life test (SSPALT).

In step stress PALT, the test unit is first run at use condition, and if the unit does not fail by the end of the specified time \( \tau \), the test is switched to a higher level and continued until the unit fails or the test is censored. Thus, the total lifetime \( Y \) of the unit in step PALT is given as follows

\[
Y = \begin{cases} 
X & \text{for } X \leq \tau \\
\tau + \beta^{-1}(X - \tau) & \text{for } X < \tau 
\end{cases}
\]

where, \( X \) is the lifetime of an item at use condition, \( \tau \) is the stress change time and \( \beta > 1 \) is the AF. Because the switching to the higher stress level can be regarded as tampering with the ordinary life test, \( Y \) is called a tampered random variable, \( \tau \) is called the
tampering point and $\beta^{-1}$ is called the tampering coefficient. Observed value of $Y$ is called non-tampered observation if $Y < \tau$ otherwise, it is called a tampered observation. The above model is referred to as a tampered random variable (TRV) model.

1.11. LITERATURE REVIEW

There is a lot of literature available on design and analysis of ALT procedures. For example; Nelson (1990) has indicated that the stress can be applied in various ways, commonly used methods are step-stress and constant stress. Viertl (1998) provides a briefer and more technical overview of the available statistical methods for ALTs, with more focus than Nelson (1990) on a large class of statistical methods that, for a variety of practical reasons, seem not to have been used widely in practice. These methods include nonparametric, semi-parametric and Bayesian methods for ALT planning and analysis. Mann et al. (1974) overview all available ALT methods at the time that book was written. See for more details; Cox and Oakes (1984), Davis (1952), Kalbfleisch and Prentice (1980), Lawless (1982), Kececioglu (1993), Meeker and Escobar (1998), Bagdonavicius and Nikulin (2002), Meeker and Hahn (1985) and Nelson (1982).

Practically, most devices such as lamps, semiconductors, and microelectronics are run at a constant stress. Therefore many authors have studied inference in CSALT where stress is kept at a constant throughout the life of the test until the occurrence of failure or the observation is censored. For example, see Bugaighis (1990), Lawless (1982), Abdel-Ghaly et al. (1998), McCool (1980), Watkins (1991), Watkins and John (2008) and Fan and Yu (2012). Since modern age is the age of competition, therefore is essential for the manufacturers to make decisions regarding the optimum method to estimate the reliability of the products or the services before letting them serve in the market. Moreover, a test plan needs to be developed to obtain appropriate and sufficient information in order to accurately estimate the reliability performance at operating conditions, significantly reduce test times and costs, and achieve other objectives. The criteria for choosing an appropriate test plan depend on the purpose of the experiment, see Meeker et al. (1998). Assuming that the lifetimes of the units follow different statistical distributions, Optimum CSALT plans were studied using different censoring scheme; for example, Nelson and Kieplinski (1976) studied optimum ALT plans for normal and lognormal life distributions. Yang (1994) proposed an optimal design of 4-level CSALT plans considering different censoring

Studies are also available on statistical inference model for SSALT which is based on CE model using MLE technique; see Zhao and Elsayed (2005), Xiong (1998) and Balakrishnan et al. (2007). Their studies contained new assumptions and assumptions to obtain MLE and CI of parameters for the models under consideration. Tang et al. (1996) suggested the use of linear CE model in order to analyse data from SSALT using 3-parameter Weibull distribution with failure-free life which is characterised by a location parameter. Since CE model was proposed by Nelson (1980), many studies regarding SSALT plan based on the CE model have been performed. Bai et al. (1989) and Bai and Chun (1991) extended the optimum simple
SSALT presented by Miller and Nelson (1983) to a case where prescribed censoring
time is involved. In addition, many authors have dealt with the optimal SSALT and
ALT planning using CE model to extend to special cases for real industrial
applications and present new optimality criterion; see Meeker and Nelson (1975),
the fiducial inference for ALT with Weibull distribution using progressive type-II
censoring.

Khamis and Higgin (1998) proposed a new model for SSALT as an
alternative to CE model, which is based on a time transformation of the exponential
CE model. The proposed KH model is mathematically simpler than the CE model
without sacrificing flexibility for fitting data. A few studies for KH model have been
performed in life testing models while most researchers are related to CE model for
SSALT. Alhadeed and Yang (2002) provided the optimal plan for SSALT using
KH model when the shape parameter is unknown. Elsayed and Zhang (2007)
presented the optimal plan for a simple SSALT without any assumption of failure
time distribution for realistic cases.

In a progressive stress ALT, the stress applied to a test unit is continuously
increasing in time. An ALT with linearly increasing stress is a ramp test. In particular,
a ramp test with two different linearly increasing stresses is called a simple ramp test
and studied by many authors for example; Prot (1952), Solomon et al. (1976), Starr
and Endicott (1962). See for more details; Yin and Sheng (1987), Bai et al. (1989),
Bai and Chun (1991), and Srivastava and Shukla (2009) have all studied simple ramp-
stress ALT tests where stress in the ramp test increases indefinitely in time. This
approach is an impractical situation as too high a stress may cause failure modes other
than that under consideration and the test unit might not be able to provide such a high
stress. Hong et al. (2010) proposed a new optimum ramp-stress test plan by using a
new approach for computing approximate largesample variances of ML estimates of a
quantile of a general log-location-scale distribution with censoring, and time-varying
stress. Bai et al. (1992) proposed an optimum time censored single objective ramp test
generalised the TFR model from the SSALT setting to the progressive stress ALT and
ML estimation is investigated for the parametric setting where the scale parameter
satisfying the equation of the inverse power law. Abdel-Hamid and AL-Hussaini (2007) considered progressive stress ALTs when the lifetime of a product under use condition follows a finite mixture of distributions. Srivastava and Jain (2011) discuss optimum ramp ALT of m identical repairable systems using non-homogeneous power law process under failure truncated case. Most of the previous work on planning ALT has focused on a sole estimating objective, such as some specified 100 quantile lifetime, the reliability of a product over some specified period of time, and AF. It is impractical to estimate only one objective parameter after conducting such costly tests. Srivastava and Mittal (2012) deals with the formulation of optimum multi-objective ramp-stress accelerated life test plans with stress upper bounds for the Burr type-XII distribution under type-I censoring. Srivastava and Mittal (2012) considers optimal design for ramp-stress accelerated life test with multiple stresses and multiple estimating objectives using Burr type-XII life distribution and type-I censoring.

For designing SSPALT, a lot of amount of literature is available. For example, Goel (1971) considered the estimation problem of the AF using both ML and Bayesian methods assuming that the lifetimes of the units follow uniform and exponential distributions. DeGroot and Goel (1979) used Bayesian approach to estimate the parameters of the exponential distribution and AF in SSPALT, with different loss functions. Bai and Chung (1992) estimated the scale parameter and AF for exponential distribution under type-I censored sample using ML method. Bhattacharyya and Soejoeti (1989) estimated the parameters of the Weibull distribution and acceleration factor using ML method. Attia et al. (1996) considered the ML method for estimating the AF and the parameters of SSPALT model under time constraint assuming that the lifetimes of the units follow Weibull distribution. Addel-Hamid (2016) obtained the estimations of exponential lifetime model in SSPALT under progressive type-I censoring and general entropy loss function. Abdel-Ghani (1998) applied ML and Bayesian methods to estimate the parameters of Weibull distribution and the AF for both SSPALT and CSPALT under type-I and type-II censored data. Abel-Ghaly et al. (1997) used Bayesian approach for estimating the parameters of Weibull distribution parameters with known shape parameter under SSPALT with both type-I and type-II censored data. Abdel-Ghani (2004) considered the estimation problem of log-logistic distribution parameters under SSPALT. Abdel-Ghaly et al. (2002) studied the estimation problem of the AF and the parameters of Weibull distribution in SSPALT using ML method with type-I and type-II censoring. Ismail (2015) used ML and Bayesian methods for estimating the AF and the parameters of the Pareto distribution of the second kind. Abdel-Ghaly et al. [2002, 2003] studied both the estimation and optimal design problems for the Pareto distribution under SSPALT with type-I and type-II censoring. Again, Ismail (2006) studied the estimation and optimal design problems for the Gompertz distribution in SSPALT with type-I censored data. Sun et al. (2012) presented the inference for Burr-XII distribution in SSPALT using progressive type-II censoring with random removals. Abd-Elfattah et al. (2015) studied the estimation problem of the AF and the parameters of Burr type-XII distribution in SSPALT using ML method under type-I censoring. Recently, Lone et al. (2016) investigated the parameters of Mukherjee-Islam Distribution under SSPALT using time constraint. Rahman et al. (2016) extended the same work assuming that the failure data is obtained using failure censoring criteria. Ismail (2012) considered SSALT model for the estimation of