In the analysis of the costs, when one variable is studied, it is studied in isolation without taking into account the other variables effect on this variable. To get the combined effect of all the related variables as the ultimate result, the gross production multiple regression equation analysis has been in use (Earl O. Heady and John L. Dillon. 1961). This short of analysis answers some of the problem like (a) the contribution of each of the inputs to the total output and the extent, (b) the significant of otherwise contribution of each of the inputs to total production.

5.1: Cobb Douglas Function:

After a careful review of several production functions for the present study (Waheeduddin Khan and Tripathy R. N. 1972), the Cobb – Douglas type has been finally selected in the form: (Table- 5.1)

\[ Y = AX_1^{b_1} X_2^{b_2} \ldots X_n^{b_n} \]

Where \( A = \) constant, \( b_1 = \) regression coefficients (production elasticities)

The sum of the regression coefficients gives the nature of returns to scale. By adopting the least square method \( A, b_1 \) are estimated.

\( Y = \) Gross value of total output (Rs.)
\( X_1 = \) Land (Hectares)
\( X_2 = \) Human labour (man days) HL
\( X_3 = \) Animal labour (paid days) AL
\( X_4 = \) Fixed cost (Rs.) FC
\( X_5 = \) Variable cost (Rs.) VC
All the variables taken for analysis are estimated carefully, and simple correlation (r) values between independent variables are found to be less than the multiple correlation coefficient (r) value. Hence the multi-collinearity problem does not arise to mar the results and hence the results are not spurious (Lawrance, R. K. Lein. 1965, Introduction to Econometrics). The results are presented for each village separately for the farm business as a whole.

Table- 5.1
Cobb – Douglas Analysis Irrigated Villages
Y= 1, 03,700

<table>
<thead>
<tr>
<th></th>
<th>G.M.</th>
<th>bi</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area X₁</td>
<td>127</td>
<td>-5.13</td>
<td>-1.16</td>
</tr>
<tr>
<td>HL X₂</td>
<td>5081</td>
<td>-0.33</td>
<td>-0.07</td>
</tr>
<tr>
<td>AL X₃</td>
<td>1076</td>
<td>5.67</td>
<td>0.88</td>
</tr>
<tr>
<td>FC X₄</td>
<td>10250</td>
<td>0.84</td>
<td>0.71</td>
</tr>
<tr>
<td>VC X₅</td>
<td>26950</td>
<td>-0.05</td>
<td>-0.03</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>0.99</td>
<td></td>
</tr>
</tbody>
</table>

n = 1105
A = 7.58
G.M. = Geometric Mean
R² = 0.99
R² = 0.99 Ave.
Rel. error = 4.99
SEE of the eqt. = 0.13
Durbin Wastsen Statistics 24,899
Table- 5.2
Cobb – Douglas Analysis Un-irrigated Villages

\[ Y = 25,640 \]

<table>
<thead>
<tr>
<th>( \text{Area } X_1 )</th>
<th>( \text{G.M.} )</th>
<th>( bi )</th>
<th>( t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( )</td>
<td>63.46</td>
<td>-0.21</td>
<td>-0.56</td>
</tr>
<tr>
<td>( \text{HL } X_2 )</td>
<td>1210</td>
<td>0.21</td>
<td>0.24</td>
</tr>
<tr>
<td>( \text{AL } X_3 )</td>
<td>361</td>
<td>0.05</td>
<td>0.08</td>
</tr>
<tr>
<td>( \text{FC } X_4 )</td>
<td>2257</td>
<td>-1.23</td>
<td>-1.02</td>
</tr>
<tr>
<td>( \text{VC } X_5 )</td>
<td>5118</td>
<td>1.06</td>
<td>2.45</td>
</tr>
<tr>
<td>( \text{Total} )</td>
<td>( 1.00 )</td>
<td>( )</td>
<td>( )</td>
</tr>
</tbody>
</table>

n= 175
A = 9.60
G.M. = Geometric Mean
\( R^2 = 0.97 \)
\( R^2 = 0.83 \) Ave.
Rel. error = 3.25
SEE of the eqt. = 0.09
Durbin Watsen Statistics 1.60

The five input variables, considered together have explained the variations in the total production to the extent of 97% in Dry villages, while it is 99% in wet villages. The geometric mean values, presented for each of the variables in both villages, showed that irrigated villages values are higher than that for un-irrigated villages. This can be explained as due to extensive cultivation (due to irrigation) in irrigated villages over un-irrigated villages over un-irrigated villages. More area is cultivated in irrigated villages, which demands more of HL and AL. More of fixed cost means depreciation on the agricultural implements, draught
power, interest on the (own or borrowed) working capital and land value (cultivated). More of variable cost indicates the usage of manure, fertilizer, PPM and other hire charges (Table- 5.2).

The production elasticity’s (regression coefficients) indicate that the total output is increasing with an increase in any one of the inputs, while others are kept constant, at their geometric mean levels. Though in magnitude the b value is higher for irrigated villages (−5.13) than that of un-irrigated villages (−0.22) for the cultivated area, both the values are found to be statistically not significant (lower ‘t’). This indicates that the extent of magnitude of area cultivated has no significant impact on higher yields. The human labour and animal labour contribution also seems to be not effecting the total yield in both the villages (with non significant ‘t’ values). Relatively speaking (HL) $b_2^{0.22}$ is more than (AL) $b_3^{0.05}$ in un-irrigated villages while it is the other way in irrigated villages. Fixed cost is also not significantly contributing to the total yield in both villages. While variable cost is significant in un-irrigated in irrigated villages the variable cost also could not bring in a change (influence) on the total output.

At this point it would be of interest to note how the importance of each of the X- variables can be graded or ranked (Relative importance of different X- variables-398pp. Snedecor and Cochran: in Statistical Methods). The standard can be graded or ranked. The standard partial regression coefficient $\beta_i \sqrt{\frac{\partial^2 y}{\partial^2 x_i}}$ are calculated for each of the inputs and their magnitudes are ranked (irrespective of the sign). In un-irrigated villages the order is: variable cost, fixed cost and agricultural labour, area and fixed cost. The coefficients estimate the change in Y as a fraction of Y produced by one SD change in $X_1$.

Variable cost is the main stake in un-irrigated villages i.e. more of manure and fertilizer usage of resulting in better yields. This is supported
by the usage of working capital, animal and implements (which are the components in fixed cost); animal labour is ranked as the 3rd important input. But in irrigated villages fixed cost and animal labour are identified as the important variables. Also area under crops has come as an important variable. The productivity is no doubt a function of area as the overhead costs would come down with more area under the same crop, for the same investment.

In un-irrigated villages, the elasticity of output to variable cost (VC) is high and significant i.e., a unit increase in VC with other factors held at their GM levels, adds more to gross output, than an increase in other input factors does. The elasticity for FC is negative, but not significant. The response of output to an increase in land input is negative but not significant.

On irrigated village’s farms, the production elasticities for individual input factors are more variant. The elasticity of output to animal labour input is positive and high but not significant. So also for FC As in the case of un-irrigated villages, here also the elasticity of output to land input is negative and not significant. The other inputs have also contributed to the output, but not significantly.

The sums of elasticity ($\sum bi$) are tested for deviation from unit to decide the nature of returns to scale. For both the villages, the test revealed increasing returns to scale ($\sum bi=1.00$) un-irrigated villages and ($\sum bi=0.99$) irrigated villages.

5.2: Standard Normal From:

It is customary or conventional to fit a “log” function to the variables in the production function. In this study another attempt is made to transform the variables in to their standard normal (SNV) from ($x-x^{-}\hat{x}$) with zero mean and SD as unity. With this transformation
again, the multiple regression equations are fitted and the results are given below, for both the villages separately.

**Table- 5.3**

**Multiple Regression Equations**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Un- irrigated villages</th>
<th>Irrigated villages</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bi</td>
<td>t</td>
</tr>
<tr>
<td>Area</td>
<td>0.05</td>
<td>0.08</td>
</tr>
<tr>
<td>HL</td>
<td>0.72</td>
<td>0.53</td>
</tr>
<tr>
<td>AL</td>
<td>0.38</td>
<td>0.25</td>
</tr>
<tr>
<td>FC</td>
<td>-2.20</td>
<td>-0.93</td>
</tr>
<tr>
<td>VC</td>
<td>1.82</td>
<td>1.53</td>
</tr>
<tr>
<td>bi</td>
<td>0.78</td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>-0.00</td>
<td>-0.09</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.95</td>
<td>0.99</td>
</tr>
<tr>
<td>$R^{-2}$</td>
<td>0.65</td>
<td>0.99</td>
</tr>
<tr>
<td>SEE of the eqt.</td>
<td>0.58</td>
<td>0.07</td>
</tr>
</tbody>
</table>

In Case of un-irrigated villages, it is observed that none of the elasticities (regression coefficients) are statistically significant and their sum is 0.70 which is significantly different from unity, indicating constant returns to scale. But in case of irrigated villages, the elasticities for cultivated area and fixed cost are statistically significant. The sums of elasticities are found to be 1.00 and are not significantly different from unity indicating increasing returns to scale. The SEE of the equations is almost nil in case of irrigated villages, whereas it is (relatively) high for un-irrigated villages (Table 5.3).
Thus in addition to the normal procedure of log transformation, the SNV transformation has yielded some more realistic, meaningful and interesting results.

5.3: Marginal Value Products\(^1\) (at Geometric Mean Levels):

By the production function analysis, it is possible now to evaluate the efficiency of factor proportions in production on farms in both villages (Table 5.4).

\[ \text{MVP} = \frac{dy}{dx} = b_i \frac{X y}{x_1} \]

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
Item & Un-irrigated villages M.V.P. & Irrigated villages M.V.P. \\
\hline
Area & -87.47 & -41.95 \\
HL & 4.65 & 6.86 \\
AL & 3.91 & 5.46 \\
FC & -14.66 & 8.58 \\
VC & 5.35 & -0.20 \\
\hline
\end{tabular}
\caption{Marginal Value Products\(^1\) (at Geometric Mean Levels)}
\end{table}

\[
\text{MVP} = \frac{dv}{dx} = \frac{y}{x} = \frac{b_i}{x_1}
\]

The level of MVP of individual input factors in un-irrigated villages shows that between human and bullock labour, the MVP is almost the same; but within the two, MVP is higher for HL than AL. Working capital has large MVP. Area and FC have shown (−) MVP, showing the over capitalisation of farms. For irrigated villages, bullock
labour and FC cost have shown (+) MVP of which bullock labour giving very high MVP. The impact of VC is (−) ve and insignificant. There seems to be wastage in the utilization of human labour. In both the villages it is commonly observed that land’s MVP is (−) ve and very high. This may be interpreted as that in bigger LHS, the productivity is not in bigger LHS, the productivity is not commensurate with LHS but smaller farms are observed to be producing more.

5.4: The Discriminate Function Analysis- Hotellings $T^2$ Analysis:

So far the analyses were the comparison of the sample villages, with respect to one variable at a time. But it may be a fact that each variable may behave in a different way in isolation than when it is taken in a group, i.e. the group behaviour could be different from individual behaviour when comparison between the two villages, is made (Radhakrishna D. 1969).

There is a multivariate technique to study the extent to which different populations overlap one another. As a multivariate generalization of the t-test: given a number of related measurements made on each of the two groups, the investigator may want a single test of null hypothesis; that the two population have the same means, with respect to all the measurements (Snedecor G. W., Cochran W. G. 1968). Historically the Discriminant function was developed independently by fisher (Fisher R. A. 1936), whose primary interest was in classification, by Mahalanobis in connection with a large study of the relation between Indian castes and Tribes and by Hotelling who produced the multivariate t-test (Mahalanobis P. C. 1930).

Let X be a normal variate with known means $\mu_1$ and $\mu_2$ in the two populations and knows S.D. ($p$), assumed to be same in both the population. The value of x is measured for new specimen that belongs to
one of the 2 populations. Our task is to classify the specimen into the correct population.
If \( \mu_1 < \mu_2 \) when \( X > \mu_1 + \mu_2 /2 \) specimen goes to I group.

\[ X > \mu_1 + \mu_2 /2 \] specimen goes to II group.

The mean of the 2 populations serve as boundary point percentage misclassification; if the specimen actually comes from I group, our verdict is wrong whenever

\[
\frac{\mu_1}{\bar{\delta}} < \frac{\mu_2 - \mu}{\bar{\delta}} \quad \text{i.e. whenever} \quad x - \frac{\mu_1 + \mu_2}{2} = \frac{D}{2\bar{\delta}} \quad \text{where} \quad \Delta \quad \text{is the distance}
\]

Between two means, Since \( X - \mu/\bar{\delta} \) are a SNV, the probability of misclassification in the area of the normal tail forms \( \Delta/2\bar{\delta} \) to \( \infty \). For a high degree of accuracy in the classification \( \Delta/2\bar{\delta} \) most exceed 3.

**Variables Selected and the Analysis:**

There are many variables that can judge the performance or levels of living of the cultivators in sample villages. For the present study, only six important variables are taken, as the data is available readily on them, from the field study. Though the list is not exhaustive, for the purpose of assessing the overall performance of the distance between the two groups, the six variables are felt to be necessary and sufficient.

1. No. of earners in each household (number)
2. Assets per each household (Rs.)
3. Household income (Rs.)
4. Net returns from all sources per household (Rs.)
5. Intensity of cropping - farm - farm level (%) and
6. Per consumption unit expenditure on food items (Rs.)
For the two sets of villages, the normal eqts obtained are as follows:

<table>
<thead>
<tr>
<th>Equation</th>
<th>L1</th>
<th>Δ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.34 $L_1$</td>
<td>$\Delta$</td>
<td>-0.21</td>
</tr>
<tr>
<td>0.36 $L_1 + 12.15 L_2$</td>
<td></td>
<td>29.38</td>
</tr>
<tr>
<td>1.25$L_1 + 4.00L_2 + 26.72L_3$</td>
<td></td>
<td>-1.29</td>
</tr>
<tr>
<td>-0.03$L_1 + -0.03L_2 + 0.67L_3 + 0.16L_4$</td>
<td></td>
<td>0.03</td>
</tr>
<tr>
<td>7.64$L_1 - 40.59L_2+1384L_3 + 1.30L_4 + 1306.34L_5$</td>
<td></td>
<td>10.03</td>
</tr>
<tr>
<td>49.03$L_1+3.46L_2-188.31L_3-3.06L_4-842.87L_5+47910.15L_6$</td>
<td></td>
<td>178.97</td>
</tr>
</tbody>
</table>

(Lower triangle is given here)

The $L_1$ is computed by the Doolittle method (Rao, C. R.1952) of inversion of the matrix. The diagonal element $L_1$ is given as under:

<table>
<thead>
<tr>
<th>No. of Earners</th>
<th>$L_1$ = 0.8970</th>
<th>$di$ = -0.21</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assets</td>
<td>$L_2$ = 0.1000</td>
<td>29.38</td>
</tr>
<tr>
<td>HH income</td>
<td>$L_3$ = 0.0500</td>
<td>-1.29</td>
</tr>
<tr>
<td>Returns</td>
<td>$L_4$ = 7.2640</td>
<td>0.03</td>
</tr>
<tr>
<td>IOC</td>
<td>$L_5$ = 0.0006</td>
<td>19.03</td>
</tr>
<tr>
<td>PC Expd.</td>
<td>$L_6$ = 0.0004</td>
<td>178.97</td>
</tr>
</tbody>
</table>

Here $\Sigma L_1 \; di = 0.3322$. The value of $\Delta /s$ for the Discriminant is given by

$$\sqrt{\left(n_1+n_2-2\right)} \; \Sigma L_1 \; di = \sqrt{(209+98-2)} \; (0.3322) = 10.06,$$

indicates that the percentage misclassification is negligible or very low.
Analysis of variance of the Discriminant function.

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>S.S.</th>
<th>M.S.S.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Between villages</td>
<td>2</td>
<td>( n_1 n_2 (\sum Ld)^2 / n_1 + n_2 )</td>
<td>3.6327</td>
</tr>
<tr>
<td>2. Within villages</td>
<td>304</td>
<td>((\sum LD))</td>
<td>0.00108</td>
</tr>
</tbody>
</table>

\[
F = \frac{(1)}{(2)} = 3363.6
\]

For (1) d.f. = 2, for (2) d.f. = \( n_1 + n_2 - K - 1 = 304 \).

The value of F is very large for the d.f. (2,304) as it must be, if the discriminate is to be effective in classification. Hotellings T^2 Statistis (Anderson, T. W. 1972).

The multivariate – “t” tests takes into account the combination group of six variables at one time and the distance between the sample villages is tested for statistical significance.

\[
T^2 = \frac{n_1 n_2}{n_1 + n_2} \sum L1di = 22.16
\]

\[
F = \frac{n_1 + n_2 - K + 1}{K (n_1 + n_2 - 2)} x T^2
\]

\[
= (6,302)
\]

\[
= 3.65
\]

Table values for

\[F6. \infty = 2.80 \text{ at } 1 \text{ % L.S.}\]

\[= 3.74 \text{ at } 0.1\% \text{ L.S.}\]

\[= \text{Significant at } 1 \text{ % L.S.}\]
Thus it has been statistically proved, that the irrigated villages has significantly higher (group) values than un-irrigated villages, for the six important variables taken at one time and treated here, the Discriminant function has discriminated significantly the sample villages. Hotellings $T^2$ also has proved statistically the difference between irrigated and un-irrigated villages to be significant. Thus with irrigation as the only limiting factor, irrigated villages has been proved to be superior over un-irrigated villages, either for single variable and or when 6 variables are taken at a time. It is obvious and logically correct to support the conclusion, that when the important, necessary and sufficient variables like HH income, expenditure, returns etc., are considered, the irrigated villages has been observed to be at a higher level than the dry villages and the distance in also statistically significant.