CANONICAL TRANSFORMATIONS AND LOOP FORMULATION OF
SU(N) LATTICE GAUGE THEORY.

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Abstract

In this thesis, we construct canonical transformations to reformulate pure SU(N) Kogut-Susskind lattice
gauge theory in terms of a set of fundamental loop & string flux operators along with their canonically
conjugate loop & string electric fields. The canonical relations between the initial SU(N) link operators
and the final SU(N) loop & string operators, consistent with SU(N) gauge transformations, are explicitly
constructed over the entire lattice. We show that as a consequence of SU(N) Gauss laws, all SU(N) string
degrees of freedom are frozen and decouple from the physical Hilbert space \( \mathcal{H} \). The Kogut-Susskind
Hamiltonian rewritten in terms of the fundamental physical loop operators has global SU(N) invariance.
There are no gauge fields. We show that the SU(N) loop Hamiltonian has only nearest neighbour interactions
upto \( O(g^2) \). Since the canonical transformations are \( 1 - 1 \) transformations, this loop formulation is
free of Mandelstam and Bianchi identity constraints. Specializing to the SU(2) case, we also show that the
physical loop Hilbert space of SU(2) lattice gauge theory is isomorphic to that of a collection of Wigner
coupled hydrogen atoms, one per plaquette, with no net angular momentum. We also analyze the special
role of the hydrogen atom dynamical symmetry group SO(4,2) in the pure SU(2) lattice gauge theory loop
dynamics.

Further, we exploit above canonical transformation techniques to obtain old and well known Kramers-
Wannier duality in \((1 + 1)\) dimensions and Wegner’s \( Z_2 \) gauge-\( Z_2 \) spin duality in \((2 + 1)\) dimensions. In fact,
it is shown that the SU(N) loop formulation constructed above is a non-abelian (SU(N)) generalization
of the simple Wegner’s \( Z_2 \) gauge-\( Z_2 \) spin duality in \((2 + 1)\) dimensions. More precisely, the SU(N) loop
formulation in \( 2 + 1 \) dimensions is in terms of the fundamental magnetic fields & their conjugate scalar
electric potentials without any local Gauss law constraints. This is dual to the standard Kogut-Susskind
link formulation of SU(N) lattice gauge theory which is in terms of fundamental electric fields & their
conjugate magnetic vector potentials along with SU(N) Gauss law constraints at each lattice site.