Chapter - 3

Data Reduction and Analysis

In this chapter we shall attempt to discuss briefly, some of the experimental details, such as the schematic of the electronics set up to acquire the $\gamma - \gamma$ coincidence data. The details of data analysis tools and techniques for example calibration and gain matching would also be presented. The sorting of the coincidence information into the $E_\gamma - E_\gamma$ matrices (symmetric and antisymmetric) and their application in the subsequent data analysis is also outlined. The theoretical aspects underlining the measurements such as angular distribution, polarization and lifetime measurements are also briefly touched upon.

3.1 Electronics for $\gamma - \gamma$ Coincidence

A rapidly rotating residual nucleus is populated following fusion evaporation reaction, which de-excites by emission of a cascade of discrete $\gamma$-ray transitions, that is atleast two or more $\gamma$-ray transitions are in coincidence with in very small interval of time, assuming the absence of any long lived isomeric state ($\tau \sim 100$'s of ns), between them. This de-excitation pattern (decay sequence) encodes the fingerprint(s) for the response of the nucleons (nucleus) to the imposed stimulus of angular momentum and reveals the interaction between the constituent nucleons. Hence, it is crucial that we faithfully reconstruct this decay sequence from the recorded data, an exercise which is analogous to the solution to a conventional cross-word puzzle. Just as in a cross-word puzzle two clues are required to arrive at an unambiguous solution, we require that atleast two $\gamma$-rays are observed in prompt coincidence (within say 200 ns) to reconstruct the decay-sequence. This coincidence condition, referred to as the Master Gate is usually set in the hardware during the implementation of the pulse processing electronics along with the data acquisition system. A signal corresponding to $HPGe\overline{ACS}$ is generated from each Compton suppressed Clover, which indicates that the energy deposited in the Clover detector does not originate from a Compton event.
This signal being a conventional NIM signal has an amplitude of about 50 mV, with a pulse duration of around 200 - 300 ns. These signals from the individual detectors are then used as inputs to an analog adder circuit, whose output is proportional to the number of Compton suppressed Clovers which have non-zero data (this is also referred to as \textit{Clover Multiplicity}). For example if two detectors have recorded data, then the output of this module would be \(\sim 100\) mV, whereas in case of a triple coincidence, the value would be \(\sim 150\) mV. This analog sum is the used as the input to a discriminator, whose threshold can be adjusted to obtain an output when the desired Clover Multiplicity is achieved [37]. For example, if we desire a two-fold coincidence between the Clovers, then the threshold is set at \(\sim 75\) mV, resulting in an output when we have at least two Clovers firing simultaneously. The schematic representation of this pulse processing philosophy using the conventional analog electronics is presented in Fig 17. This output then is used to generate a \textit{Master Gate}, the signal which validates that the given event has satisfied the experimental conditions, and we can record or store these non-zero energies. We also record the individual Clover timings with respect to some reference, which is usually the Master Gate. The data acquisition for one of the experiments was configured using CAMAC systems as the interface standard. The electronics consisted of the indigenously developed Clover module at IUAC, New Delhi Ref.[37].

Thus the aim of the pulse processing methodology, is to extract the information regarding the energy of the detected γ-ray, its arrival time with respect to a reference, to name a few. We would also like to suppress unwanted events, \textit{i.e.} events which do not qualify our experimental requirements such as Compton scattered events, low multiplicity events, such as singles, etc. and then to compress information from valid events in a compact format, which saves both disk space and pre-sorting time, this was achieved using a combination of the conventional NIM analog electronics for pulse processing and CAMAC or VME as the interface standards for the data acquisition.

Recently, digital pulse processing has made it possible to replace the afore detailed philosophy implemented using discrete analog electronics, with an equivalent and efficient processing in the digital domain. This is referred to as Digital Pulse Processing (DPP), which is a signal processing technique wherein the preamplifier pulse (output)
from the detector is directly digitized using fast ADC, which are economically available. This digitized signal, is then subjected to digital filters to achieve all fast (timing) and slow (energy) channel operations, using either algorithms implemented within the programmable hardware, or later using offline processing software. Besides making the system highly compact the DPP has several advantages such as a faster system response time, \textit{i.e.} a significant lower dead-time, which increases the throughput, the ability to upgrade the system software to handle pulses from various detectors, thereby achieving an almost detector independent hardware.

Recently, for the INGA campaign at TIFR, Mumbai, a DSP based pulse processing and DAQ, from M/s XIA, referred to as PIXE-16 was used [38]. One single Pixie - 16 card (which is housed in a VXI crate) has a provision to accept 16 preamplifier pulses and hence could handle 4 Clovers within the same card. Since the energy and timing information is derived from the same signal, we do not require two pre-amplifier output signals from each clover. The preamplifier signal is digitized with a 12 bit 100 MHz
Flash Analog to Digital Converter (FADC). When the digitized data stream enters the signal processing unit where it is shaped accordingly, to obtain the energy and timing information. The signal from the Anti-Compton Shield was input as a veto after a preliminary processing it through conventional NIM electronics. This signal is then converted using a LVDS converter. Time stamped data which satisfied the given user defined criteria was then stored for subsequent analysis. Since the number of Clovers used in the experiment was around 20, the data was acquired using 5 PIXE-16 cards, and hence the event was constructed using the data from the various cards, which was time merged, and then subjected to the conventional analysis using the MARCOS [39] programe.

The online data which is archived for analysis typically has the following format

1. File Header

2. **Event ID**, Parameter-Id, Energy, Time; Parameter-Id, Energy, Time;...

3. **Event ID**, Parameter-Id, Energy, Time; Parameter-Id, Energy, Time;...

4. ...

5. EOF

where the Parameter-Id encodes the detector identity, followed by the corresponding energy and time information. It is assumed that the data from these detectors satisfy the experimental pre-requisite such as the desired detector multiplicity. If an excited nucleus, which is populated in a typical heavy-ion fusion experiment, were to decay to it’s ground state, by following the sequence, presented in Fig. 18, then, it would emit the following sequence of $\gamma$-rays :

1. $\gamma_8$, $\gamma_6$, $\gamma_5$, $\gamma_2$

2. $\gamma_8$, $\gamma_6$, $\gamma_7$

Hence $\gamma_7$ and $\gamma_5$ or $\gamma_2$ would never be in coincidence. The aim of any in-beam gamma-ray spectroscopic investigation is to develop this level scheme, from the observed coincidence relation between the de-exciting $\gamma$-rays. Due to the limitation on the number
of detectors that can be physically housed in the array, which in turn constrains the solid angle covered by the detectors, it is not possible to detect all these emitted coincidence $\gamma$-rays. If we have about 20 detectors in the array, at a distance of about 25 cm from the target, which approximately corresponds to about a $4 - 5\%$ coverage of the total solid angle, we typically would detect at least two of the coincidence partners. Accordingly, our Master Gate (valid event) is configured to accept at least two detectors which are in prompt coincidence (typically within 100 - 200 ns). We then would have a data set which generally corresponds to the following events,

1. $Event - 1$, $\gamma_8$, $\gamma_6$
2. $Event - 2$, $\gamma_8$, $\gamma_7$
3. $Event - 3$, $\gamma_8$, $\gamma_5$, $\gamma_6$
4. $Event - 4$, $\gamma_8$, $\gamma_2$
5. $Event - 5$, $\gamma_6$, $\gamma_7$
6. $Event - 6$, $\gamma_6$, $\gamma_5$

FIG. 18: A hypothetical decay scheme of a nucleus.
7. Event $-7$, $\gamma_6$, $\gamma_2$

8. Event $-8$, $\gamma_5$, $\gamma_2$

9. ...

It is to be noted that the above does not represent the exact structure of the recorded event by event coincidence data (and is a purely generic representation of the recorded info), which is eventually sorted (stored) into a $E_N^\gamma$ histogram as detailed in the subsequent section. A detailed analysis of this data set, is aimed to extract the observed genetic (sequential) relationships between the de-exciting $\gamma$-transitions and this information is then amalgamated with the intensity arguments. The level scheme, is thus developed following a rigorous and meticulous analysis of the recorded $\gamma-\gamma$ coincidence information.

3.2 Analysis of Gamma Gamma Coincidence Data set

The signal from the detector, following the detailed pulse processing, is used as the input to the ADC (Analog to Digital Converter), which digitizes the information for subsequent storage. The information from the ADC is purely in terms of a channel number, which is related to the energy of the observed $\gamma$-ray. The process of energy calibration establishes this relationship between the recorded ADC channel and the corresponding $E_\gamma$, using radio-active sources ($^{152}\text{Eu}$, $^{133}\text{Ba}$). However, these sources span an energy range from $\sim 80$ keV to $\sim 1408$ keV only. If one were to observe high energy $\gamma$-rays, then we have have to incorporate the beam-off radioactivity data which usually contains a few higher energy ($E_\gamma \geq 2$ MeV at $A \sim 30$ region) transitions as calibration points. The Energy ($E_\gamma$) - Channel ($x$) relation is usually parametrized as

$$E_\gamma = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 \sqrt{x}. \quad (25)$$

Where $x^3$ term is of relevance at high energy ($\geq 1.5$ MeV) and $\sqrt{x}$ term is crucial at low energies ($\leq 200$ keV) [40, 41]. Thus using the constants, known as calibration
constants, we have mapped the ADC channel numbers to their corresponding energies. Once the data has been calibrated, the next process known as \textit{gain matching}, which ensures that the data obtained from each detector has a constant energy dispersion, thereby eliminating the detector dependence on the data. The gain matching process can be mathematically represented using the equation

\[
x_{gm} = \frac{1}{epc} \times (a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 \sqrt{x}).
\]

(26)

FIG. 19: Corresponds to the spectra (a) before gain matching and (b) after gain matching.

Where \( E_\gamma \) is the energy corresponding to the ADC channel \( x \), “epc” corresponds to the energy per channel and \( x_{gm} \) represents the gain matched channel. The results of this procedure, for a 1 keV/channel gain matching (eg for \( E_\gamma = 121 \) keV, where the peak centroid would correspond to channel number 121), are presented in Fig. 19. As seen from the figure, following a painstaking, calibration and gain matching, the energies from all detectors are aligned (the recorded energy \( E_\gamma \) in all the detectors are, observed at the same channel number), thereby ensuring that the data is now detector independent.
### 3.2.A. Development of Level Scheme

The primary requisite to develop the level scheme is to ensure that we detect at least two $\gamma$-rays which are in coincidence (within a time window of 100 - 200 ns), during the de-excitation, of the rapidly rotating nucleus to its ground state, thereby preserving the correlation between them. Such correlations can be conveniently, investigated in detail, if one were to store the coincidence information, say a two parameter coincidence information, in a 2D histogram, which stores the number of events corresponding to a particular combination of the two recorded energy values. Hence, in a $E_\gamma - E_\gamma$ matrix each element $M(i, j)$ contains the number of recorded coincidences between $E_\gamma_i$ and $E_\gamma_j$. Ref.[42]. If the two dimensional array is formed without any pre-condition on the detector (such as angle of the detector) where the axes do not correspond to specific detectors then it is termed as “symmetric matrix”. Suppose, the decay sequence is such that we have the following cascade $E_\gamma_i \rightarrow E_\gamma_j$, results in two coincident $\gamma$-rays $E_\gamma_i$ & $E_\gamma_j$ and the detection of this sequence would increment the contents of the location having address $(i, j)$ in the histogram ($M(i, j) = M(i, j) + 1$). However, it is quite possible that the same event is recorded as $E_\gamma_j$ & $E_\gamma_i$, then this event would increment the contents of the $(j, i)$ cell in the 2D histogram. Thus the contents of the $(i, j)$ and $(j, i)$ cells in the matrix correspond to the same decay sequence (event). Hence, such matrices, need to be symmetrized before analysis, i.e, $M(i, j) = M(i, j) + M(j, i)$, thus, in practice we usually store the symmetrized or folded matrices for detailed analysis.

The level sequence presented in Fig 18 would result in the 2D histogram shown in the upper panel of Fig 20. As seen from the figure the cells corresponding to the $\gamma$-rays which are not in coincidence, do not record any data. This histogram is best viewed or analyzed using 1D spectra, i.e, we project this information along one axis say $X$-axis, (shown in the middle panel of Fig 20), which is identical to the $Y$-axis, under the constraints of two limits on the other axis. This exercise, reveals the energy of the gamma-rays which are in coincidence with the gamma-ray specified by the limits. The limits are referred to as “gate”, and the spectrum obtained is termed as “gated spectrum” (shown in the bottom panel of Fig 20). This procedure establishes the genetic relationship between the gamma-rays, and helps us in constructing the level sequence. The exact ordering of the $\gamma$-rays are based on intensity considerations.
FIG. 20: The upper panel corresponds to the schematic representation of the two dimensional matrix (based on the level scheme shown in Fig. 18) and using this matrix we construct the level scheme that we have discussed in the text. The middle panel corresponds to the x projection spectra of the above matrix. The lower panel corresponds to the 200 keV gated spectrum.

When gated on a particular transition all transitions above it, have their intensities in decreasing order, whereas the transitions below it have nearly the same intensity. However, the intensities are calculated after incorporating the efficiency corrections as described in details by R. Chakrabarti et. al [43].
It is often required, to analyze the coincidence information, between a certain set of detectors, for example we may require to look at the coincidences between detectors at $90^\circ$ and $132^\circ$ (where the angles are defined with respect to the beam-direction). These matrices which correspond to non-identical detectors along the individual axes are known as “asymmetric matrices” and are primarily of relevance in establishing the spin-parity and the level lifetime of the level.

The dimension (channels) of the 2D histogram is typically $4096 \times 4096$. If the matrix memory allocation is either 2 or 4 bytes per channel then the memory requirement is $4096 \times 4096 \times 2 \ (or \ 4) = 32$ MBytes or 64 MBytes ([44, 45]). Usually the software gain matching is typically 0.5 keV/channel then using 4096 channels, we shall observe gamma-rays with $E_\gamma \leq 2$ MeV. If we expect to observe, high energy gamma-rays (routinely observed in the light mass nuclei) we have to use an energy dispersion of 1.0 keV/channel so as to cover an energy range upto 4 MeV.

Thus the 2D histograms both symmetric and angle dependent provide an efficient way to represent the coincidence correlations among the recorded quantities, which in-turn help us develop the level schemes.

Each event we record could originate from either a photo-peak interaction ($P_i$), or background ($B_i$). Therefore, when we record a coincidence between two events, we have a combination of $(P_i, B_i) \otimes (P_j, B_j)$, thus the 2D histogram would contain events such as $(P_i, P_j)$, $(P_i, B_j)$, $(B_i, P_j)$, $(B_i, B_j)$, of which only the $(P_i, P_j)$ events are of our interest as they correspond to full-energy coincidences [40, 41].

Several techniques [40, 41] have been devised to estimate the $(P_i, B_j)$, $(B_i, P_j)$, $(B_i, B_j)$, events which contribute to the observed background in the 2D histograms. These background subtraction procedures when applied to the raw $E_\gamma - E_\gamma$ matrix, results in the preservation of mostly the photo-peak photo-peak information. Such a background subtracted data set is then used for the subsequent analysis (shown in Fig. 21) which is free from spurious / contaminant peaks. As seen from the Fig. 21 in the background unsubtracted gated spectrum we find peaks originating from
FIG. 21: The background subtracted gated spectrum (1809 keV ($2^+ \rightarrow 0^+$) transition of $^{26}\text{Mg}$ nucleus) is shown in the above panel and background unsubtracted gated spectrum is shown in the lower panel.

contaminants, whereas in the background subtracted gated spectrum we essentially find peaks pertaining to the nucleus of our interest.

### 3.2.B. Multipolarity Determination

The mere placement of the transitions in the level sequence does not give us the complete information on the level sequence. The spin parity assignments of the levels are essential to have a comprehensive understanding of the level structure. The state with angular momentum and parity ($J_i$ and $\pi_i$) decays to a final state characterized by ($J_f$ and $\pi_f$). The angular momentum and parity selection rules for the $l$ th order
multipole have to be followed for the $J_i \rightarrow J_f$ transition, *viz.*

$$|J_f - J_i| \leq l \leq (J_f + J_i) \text{ and}$$

$$\Delta \pi = \text{no : even electric, odd magnetic}$$

$$\Delta \pi = \text{yes : odd electric, even magnetic.}$$

For $J_i = J_f = 0$, the selection rule demands a $l = 0$ transition, hence such a radiative transition will be forbidden as the photon cannot be emitted with zero angular momentum [46]. The states will decay via emission of orbital electron and it is accompanied with the emission of X-rays. A stretched transition is one in which photon carries the algebraic difference between the angular momentum of the initial and final state. The lowest permitted multipoles are usually the most preferred modes of emission. Hence, if we can determine the change in angular momentum $\Delta l$ associated with a particular transition, then from the knowledge of $J_i$, the $J_f$ can be assigned (or vice-versa). The $\gamma$-ray emitted during the $J_i \rightarrow J_f$ de-excitation, would have would have an angular dependence, which is governed by the associated change in angular momentum, $\Delta l$. Hence, the transition would have an angle dependent intensity distribution. This information is used to arrive at the multipolarity assignment for the given transition.

Now let us consider the $1^+ \rightarrow 0^+$ transition. The initial angular momentum state $J_i = 1^+$ has three $m$ sub-states ($m_i = +1, 0, -1$) and the final state has only one such sub-state ($m_f = 0$). The selection rules allow for transitions connecting all the initial $m$ sub-states $m_i = +1, 0, -1$, to the final $m_f = 0$ state. As all the $m$ sub states are degenerate (under the present circumstances), thus the resulting $\gamma$ transition would have an identical energy. Although the transitions have same energy but they possess different characteristics distribution ($F_j^m(\theta)$) whose value depends on the angle ($\theta$) between the emitted $\gamma$-ray and the Z-axis (whose choice is arbitrary). This distribution function is essentially obtained by calculating the Poynting vector (energy flow) as a function of $\theta$, such that

$$F_j^m(\theta) = F_j^{-m}(\theta)$$

$$\int F_j^m(\theta) = 8\pi$$
In case of dipole transition [47] we have:

\[ F_1^0(\theta) = 3\sin^2\theta \]  
\[ F_1^{\pm 1}(\theta) = \frac{3}{2}(1 + \cos^2\theta) \]  

Since all the above three components are equally probable, hence the resultant angular distribution is given by the equation,

\[ W(\theta) = \frac{1}{3}(F_1^1(\theta)) + \frac{1}{3}(F_1^0(\theta)) + \frac{1}{3}(F_1^{-1}(\theta)) \]  

\[ W(\theta) = \frac{1}{3} \times 3/2(1 + \cos^2\theta) + \frac{1}{3} \times 3\sin^2\theta + \frac{1}{3} \times 3/2(1 + \cos^2\theta) \implies W(\theta) = 2 \]  

From the above calculations we find that \( W(\theta) \) is constant, and hence the intensity would be isotropic. Similarly for quadrupole transition we have,

\[ F_2^0(\theta) = \frac{5}{2}(6\cos^2\theta - 6\cos^4\theta) \]  
\[ F_2^{\pm 1}(\theta) = \frac{5}{2}(1 - 3\cos^2\theta + 4\cos^4\theta) \]  
\[ F_2^{\pm 2}(\theta) = \frac{5}{2}(1 - \cos^4\theta) \]  

\[ W(\theta) = \frac{1}{5}(F_2^0(\theta)) + \frac{1}{5}(F_2^{+1}(\theta)) + \frac{1}{5}(F_2^{-1}(\theta)) + \frac{1}{5}(F_2^{+2}(\theta)) + \frac{1}{5}(F_2^{-2}(\theta)) \]  

We find that, as in the case of a dipole transition, \( W(\theta) \) is also constant for quadrupole transition hence the intensity has no angle dependence. However, if we were to preferentially populate one of the the \( m_i \) sub-states, say for example the \( m_i = 0 \) state, for the \( J_i = 1^+ \rightarrow J_f = 0^+ \) transition, [47] then

\[ W(\theta) = 0 \times (F_1^1(\theta)) + 1 \times (F_1^0(\theta)) + 0 \times (F_1^{-1}(\theta)) \]  
\[ W(\theta) = 1 \times (F_1^0(\theta)) \implies W(\theta) \propto \sin^2\theta \]  

Thus, the resultant intensity depends on the value of \( \sin^2\theta \), it implies that the intensity has an angular dependence. Similarly, for quadrupole transition if we were preferentially populate the \( m_i = 0 \) state, then
FIG. 22: The intensity distribution for dipole and quadrupole transitions.

\[ W(\theta) = 1 \times (F_2^0(\theta)) + 0 \times (F_2^{+1}(\theta)) + 0 \times (F_2^{-1}(\theta)) + 0 \times (F_2^{+2}(\theta)) + 0 \times (F_2^{-2}(\theta)) \]

\[ W(\theta) = \frac{5}{2} \times (6\cos^2\theta - 6\cos^4\theta) = \frac{5}{2} \times 6\cos^2\theta(1 - \cos^2\theta) \Rightarrow W(\theta) \propto \cos^2\theta\sin^2\theta \]

(41)

Thus, the resultant intensity of a quadrupole transition depends on the value of \( \cos^2\theta\sin^2\theta \). In the above calculations, we have ignored the normalization constant \( 1/8\pi \). Such states, wherein we have a preferential population of \( m \) sub-states, are referred to as oriented states. The gamma-ray emission from such states, would have an angular anisotropy for the observed intensity, which is dependent on the multipolarity \( (l) \) of the gamma-ray. Hence, dipole and quadrupole transitions originating from oriented sources, are expected to have a distinct angular anisotropy for the intensity, \( I_\gamma(\theta) \). The angular distribution for a dipole radiation \( (m = 0) \) is illustrated in the upper panel of Fig 22, whereas a quadrupole radiation for \( (m = 0) \), is shown in the bottom panel of Fig 22. As seen from the figures, the intensity of the two radiation quanta have a distinct angular dependence. Such measurements in turn, help
us identify the multipolarity of the radiation.

One of the technique to produce such oriented states, is to use heavy-ion induced fusion evaporation reactions. These reactions preferentially populate $m_i = 0$ states, since the angular momentum is constrained within a plane perpendicular to the reaction plane.

The angular distribution (variation of intensity as a function of the angle) \[47\] for the $\gamma - \gamma$ cascade for $J_i \rightarrow J \rightarrow J_f$, in which both the $\gamma$-rays are of multipole order $L_1$ and $L_2$ respectively and are pure, is conventionally represented by the following equation,

$$ W(\theta) = 1 + A_{22}P_2(\cos \theta) + A_{44}P_4(\cos \theta) \quad (42) $$

Where $W(\theta)$ is the $\gamma$-ray intensity is measured at angle $\theta$ to the beam direction, $A_{22}$ and $A_{44}$ are the angular distribution coefficients and $P_2(\cos \theta)$ and $P_4(\cos \theta)$ are the Legendre polynomials. The values of the angular distribution co-efficients, for pure multipoles can be written as

$$ A_{22} = (F_2(L_1 L_1 J_i J_i) \ast F_2(L_2 L_2 J_f J_f)), A_{44} = (F_4(L_1 L_1 J_i J_i) \ast F_4(L_2 L_2 J_f J_f)) \quad (43) $$

Now, if the transitions are mixed \( ie., \) two multipole components $l$ and $(l + 1)$ contribute to each of the $\gamma$-transitions, then the contribution of dominant individual component is quantified by the mixing ratio $\delta$,

$$ \delta^2(E(l + 1)/M(l)) = \frac{W(E(l + 1))}{W(M(l))} $$

$$ \delta^2(M(l + 1)/E(l)) = \frac{W(M(l + 1))}{W(E(l))} \quad (44) $$

Where $\delta$, $W(\lambda l)$ are the mixing ratio and the transition probability respectively and $F$ co-efficients $(F_{2,4}(L_i L_i J_i, J_j))$ depend on the angular momenta involved in the transition and have been tabulated in Refs.[48]. The co-efficients for mixed transitions are expressed as

$$ A_{22} = \frac{1}{1 + \delta^2} (F_2(L_1 L_1 J_i J_i) + 2\delta F_2(L_1 L'_1 J_i J_i) + 2\delta^2 F_2(L'_1 L'_1 J_i J_i)) $$

$$ A_{44} = \frac{1}{1 + \delta^2} (F_4(L_1 L_1 J_i J_i) + 2\delta F_4(L_1 L'_1 J_i J_i) + 2\delta^2 F_4(L'_1 L'_1 J_i J_i)) \quad (45) $$

Since the nuclei are partially aligned after their formation, to include this effect, we use
a multiplicative factor $\alpha_j$, known as the “attenuation co-efficients”, where

$$A_{22}(cal) = A_{22}\alpha_2$$

$$A_{44}(cal) = A_{44}\alpha_4$$

(46)

The values of $\alpha_{2,4}$ coefficients are listed in the Ref.[49]. Efficiency corrected angle dependent intensities are used to obtain angular distribution coefficient. The experimental mixing ratio can be obtained from the $\chi^2$ minimization procedure [50], where the value of $\chi^2$ is defined by the equation,

$$\chi^2 = \frac{[A_{22}(expt) - A_{22}(cal)]^2}{3[\Delta A_{22}(expt)]^2} + \frac{[A_{44}(expt) - A_{44}(cal)]^2}{3[\Delta A_{44}(expt)]^2}$$

(47)

Where $\Delta A_{22,44}(expt)$ are the uncertainties in the experimentally observed angular distribution coefficients and $A_{22,44}(expt)$ are defined as

$$A_{22}(expt) = \frac{A_{22}}{A_{00}}$$

$$A_{44}(expt) = \frac{A_{44}}{A_{00}}$$

(48)

Angular distribution measurements have been used in the present thesis to deduce the information on the multipolarity (and the mixing ratio) of the $\gamma$-ray, and are presented in the subsequent chapters. However, the angular distribution measurement, is a singles measurement, $^1$ and therefore, such measurement has its inherent limitation. In a heavy-ion induced fusion evaporation reaction a large number of nuclei are populated, in addition to a reaction specific characteristic continuum background, making it difficult to identify the weak transitions of interest in the singles spectrum. As a consequence, it is very difficult to perform the angular distribution measurement for these transitions.

Since, the fusion reaction preferentially populates $m = 0$ sub-states, we would observe an anisotropy in the angle dependent intensity. Hence, if we were to sort the data observed at two angles, then all transitions which have the same multipolarity would have a similar angular dependence of their respective intensities.

For example, if we set a gate on a quadrupole ($\Delta J = 2$) transition, say $E_\gamma = 2167$ keV ($2^+ \rightarrow 0^+$) [23] transition in $^{38}$Ar, and we observe that, the intensity of $E_\gamma = 777$

$^1$ A singles measurement corresponds, to recording the data in all the $N$ number of detectors, when anyone of the detectors has a non-zero data, hence this data set corresponds to an uncorrelated set of events.
keV is nearly identical at both 90° and 32° (shown in Fig 23), which indicates that this transition also involves a $\Delta J = 2$, change in angular momentum. Indeed, this transition is a quadrupole transition, $(5^- \rightarrow 3^-)$. However, the intensity of $E_\gamma = 1643$ keV transition, differs by almost a factor of two (from Fig 23), at the two angles. This implies that this transition is a dipole in nature, and indeed it is so $(3^- \rightarrow 2^+)$.

Thus this procedure merely identifies the change in angular momentum, $\Delta J$ and is not sensitive to the electromagnetic nature of the transition. Thus $M1$ and $E1$ would exhibit to a similar angular intensity pattern.

This angular intensity anisotropy which is referred to as $R_{DCO}$ (directional correlation from oriented nuclei) is extracted from the angle dependent $\gamma - \gamma$ matrices, where one axis contains the energy recorded in detectors placed at an angle $\theta_1$ (with respect to beam axis) and the other axis corresponds to the coincident energy recorded in the
detectors placed at an angle \((\theta_2)\). Then the \(R_{DCO}\) is defined as:

\[
R_{DCO} = \frac{I_{\gamma_2}^{\theta_2}[Gate_{\gamma_1}^{\theta_1}]}{I_{\gamma_2}^{\theta_2}[Gate_{\gamma_1}^{\theta_1}]} \tag{49}
\]

Where, for example, \(I_{\gamma_2}^{\theta_2}[Gate_{\gamma_1}^{\theta_1}]\) represents the intensity of transition \(\gamma_2\) as recorded in detectors at the angle \(\theta_2\), when energy gate is set on \(\gamma_1\) in detectors at \(\theta_1\), and vice-versa for \(I_{\gamma_1}^{\theta_1}[Gate_{\gamma_2}^{\theta_2}]\). Assuming, the commonly encountered stretched transitions \(^2\), and if we know the multipolarity of the gated transition then:

1. If the value of \(R_{DCO} \sim 1\) then the observed transition has a similar multipolarity as the gated transition.

2. If the value of \(R_{DCO}\) is \(\sim 0.5\) or \(\sim 2\) then observed and gating transition have opposite multipolarity.

The above described technique, demands that the gate to be set on a detector at an angle (forward or backward \(w.r.t\) the beam direction). This technique has one limitation, if the observed transition exhibits a Doppler shape (which originates when the stopping time of the recoils is \(\sim\) or greater than the level lifetime, due to which the emission occurs in flight). It is not possible to properly set gate on \(\gamma\)-rays with shapes at forward and or backward angle, since the gates would then be quite wide, which in turn would introduce considerable contaminants in the gated spectrum. Besides, it is not easy to precisely determine the gating limits for such transitions. In view of these difficulties, it is not possible to set gates on the transitions at angles other than \(90^\circ\), to extract the \(R_{DCO}\) value. To circumvent this problem we define a parameter known as Anisotropy Ratio \([9]\),

\[
R_{anisotropy} = \frac{I_{\gamma_1} (at \ \theta_1 \ gated \ by \ \gamma_2 \ at \ 90^\circ)}{I_{\gamma_1} (at \ \theta_2 \ gated \ by \ \gamma_2 \ at \ 90^\circ)} \tag{50}
\]

The advantage of this procedure is that the gates are always set on transitions at detectors placed at \(90^\circ\), thus avoiding the Doppler shapes. The intensity of the coincident \(\gamma\)-rays are noted at two different angles other than \(90^\circ\). This prescription was followed to obtain the information on the dominant multipolarity of the de-exciting transition and is detailed in the subsequent chapters.

\(^2\) stretched transitions involve the algebraic difference of the initial and final spins
Thus the coincidence angular anisotropy helps us deduce information on the multipolarity of the de-exciting transition.

### 3.2.C. Linear Polarization Measurements

The use of Clover detectors uniquely facilitates the measurement of linear polarization of the observed $\gamma$-ray transitions which helps in determining the electro-magnetic nature of the transition. Each crystal of the Clover detector acts as a scatterer and the adjacent two crystals are the absorbers, along the perpendicular and parallel direction, with respect to the reaction plane. The distinction between the electric and magnetic transition can be obtained from the observed asymmetry between the number of perpendicular and parallel Compton scattered events for a given $\gamma$ transition. For example, the 1596 keV $(7/2^- \rightarrow 5/2^+)$, transition in $^{29}$Si, is predominantly an electric transition with $\delta = 0.14 \pm 0.04$ [28]. Hence, it should have a preferential scattering along the perpendicular direction (with respect to the reaction plane), which indeed is observed by us as presented in the Fig 24. Similarly, an magnetic transition is expected to have a preferential scattering along the parallel direction, which is observed (in the lower panel in Fig 24) for the 670 keV $(4^- \rightarrow 3^-)$ transition in $^{38}$Ar nucleus.

The asymmetry between the perpendicular and the parallel scattering is quantified as,

$$\Delta_{Pol} = \frac{aN_\perp - N_\parallel}{aN_\perp + N_\parallel}, \quad (51)$$

where $N_\perp$ and $N_\parallel$ are the number of photons scattered in the perpendicular and the parallel direction respectively. The parameter “$a$” denotes a correction factor due to the asymmetry in the geometrical response of the Clover segments in the array. It is measured by using unpolarized $\gamma$-ray emission, and the following equation as detailed in Ref.[51].

$$a = \frac{N_\parallel}{N_\perp}. \quad (52)$$

The value of the asymmetry factor is energy dependent ($a(E_\gamma) = a_0 + a_1 E_\gamma$) and the parameters $a_0$ and $a_1$ are determined using radioactive sources and beam-off radioactivity under identical experimental conditions (source position etc.) as the in-beam
FIG. 24: The intensity variation for electric and magnetic transitions, in the perpendicular and parallel scattered events. The spectra are displaced with respect to each other for clarity.

measurements. This asymmetry is anticipated to be nearly independent of the γ-ray energy and hence the expected values of the constant are $a_0 \sim 1$ and $a_1 \sim 1 \times 10^{-6}$. For the present experiment, the typical value of the fitting parameters are, $a_0 = 1.003 \pm 0.002$ and $a_1 = 1.0 \pm 4.0 \times 10^{-6} \,(keV)^{-1}$. Owing to the very small value of the $a_1$ coefficient ($\sim 10^{-6}$) it was not considered in the present work. The value of the $\Delta_{Pol}$ is extracted from the two asymmetric γ - γ matrices where the Y-axis contains the parallel (perpendicular) scattered events in detectors at 90° and X-axis contains the coincident events in all other detectors. Gates are applied on X-axis and counts corresponds to the parallel ($N_\parallel$) and the perpendicular ($N_\perp$) scattering are obtained from the resulting spectra. The value of $\Delta_{Pol}$ is indicative of the electro-magnetic nature of the γ-transition. For a pure electric transition $\Delta_{Pol}$ is positive while a negative value of $\Delta_{Pol}$ implies a magnetic transition. A near zero value of $\Delta_{Pol}$ is suggestive of the mixed nature for the transition. The polarization asymmetry is related to the degree of
linear polarization \((P)\) as
\[
P = \frac{\Delta P_{\text{pol}}}{Q(E_{\gamma})},
\]
where \(Q(E_{\gamma})\) \([51]\) is the energy dependent polarization sensitivity,
\[
Q(E_{\gamma}) = (C E_{\gamma} + D)Q_0(E_{\gamma})
\]
where,
\[
Q_0(E_{\gamma}) = \frac{(\alpha + 1)}{(\alpha^2 + \alpha + 1)},
\]
with,
\[
\alpha = \frac{E_{\gamma}(\text{MeV})}{0.511}.
\]
The theoretical value of polarization \((P(\theta = 90^\circ))\) \([52]\) can be expressed as
\[
P(\theta = 90^\circ) = \pm \frac{3A_{22}(\text{cal})H_2 - 7.5A_{44}(\text{cal})H_4}{2A_{22}(\text{cal}) + 0.75A_{44}(\text{cal})}
\]
Where \(\theta\) is the angle of emission of the \(\gamma\)-rays from an oriented source with respect to the orientation axis, and \(H_{2,4}\) are functions that depend on the initial and final spins and the mixing ratio. The \(A_{22,44}(\text{cal})\) are the angular distribution coefficients. The \(+\) (-) sign applies for a transition without (with) change in the parity. The value of polarization attains a maximum value at \(\theta = 90^\circ\). The value of \(P = \pm 1\) for completely polarized \(\gamma\)-rays and is 0 for unpolarized \(\gamma\)-rays. For pure transition (M1 or E1) the value of \(H_{2,4}\) are \(1, -1/6\) respectively. For a mixed dipole and quadrupole transition the values of \(H_{2,4}\) are given by
\[
H_2(1, 2) = \frac{F_2(1, 1) - (2/3)\delta F_2(1, 2) + \delta^2 F_2(2, 2)}{F_2(1, 1) + 2\delta F_2(1, 2) + \delta^2 F_2(2, 2)}
\]
and
\[
H_4(1, 2) = -1/6.
\]
Similarly for an admixture of \(\Delta J = 2 \& 3\) the values are
\[
H_2(2, 3) = \frac{-F_2(2, 2) - \delta F_2(2, 3) + (2/3)\delta^2 F_2(3, 3)}{F_2(2, 2) + 2\delta F_2(2, 3) + \delta^2 F_2(3, 3)}
\]
and
\[
H_4(2, 3) = \frac{5F_4(2, 2) - 2\delta F_4(2, 3) + 20\delta^2 F_4(3, 3)}{30(F_4(2, 2) + 2\delta F_4(2, 3) + \delta^2 F_4(3, 3))}.
\]
As a part of the present thesis, the linear polarization measurements were performed for several transitions in $^{26}\text{Mg}$ and $^{28}\text{Si}$ to uniquely assign their electro-magnetic nature. The polarization asymmetry $\Delta_{\text{pol}}$, were experimentally obtained, and this allowed us to determine the experimental polarization, which was successfully compared with the predicted theoretical polarization values, and a reasonable agreement was noted.

Hence, the angular intensity anisotropy and the asymmetry between the parallel and perpendicular scattering (with respect to the reaction plane) helped us assign unambiguously the spin and parity of the levels de-exciting with the corresponding $\gamma$ transition. Thus we were able to obtain a detailed information of the level scheme of the nucleus under study. However, a more complete and unique information on the underlying configuration of the observed levels is obtained from the measurement of the level lifetimes. The measurement techniques are dependent on range of the expected time scale for the level lifetimes.

### 3.2.D. Lifetime Measurements

The fast moving excited residual nuclei are slowed down and eventually stopped in the backing medium, and the stopping process typically requires about few tens of fs to few ps. If the residual nucleus emits a $\gamma$-ray while in motion, which are detected either in the forward or backward direction (with respect to the beam direction) before it stops, the energy of the detected $\gamma$-ray exhibits Doppler effect (shape or shift). The measurement of this Doppler shape or shift coupled with the slowing down history of the recoils, with an appropriate inclusion of the reaction kinematics, can used to extract the lifetime of the level de-excited by the emitted $\gamma$-ray transition. This is the principle of the Doppler Shift Attenuation Method (DSAM) for determining the level lifetimes in the range of few tens of fs to few ps. The transition strengths deduced from these measurements, are connected with the initial and final wave-functions of the involved levels, and provide us with a stringent test for the validity of nuclear model, which elucidate the observed level structure.
FIG. 25: The schematic diagram for observed energy of $\gamma$-ray as viewed by the (forward) detector when the residue emit $\gamma$-rays in flight.

DSAM exploits the observed Doppler effects during the slowing down of the recoiling nuclei, in an elemental high-Z backing, while de-exciting with the emission of $\gamma$-ray transitions. If the lifetime of the level is less than or comparable with the stopping time then the $\gamma$-ray emitted from such an excited nucleus with recoiling velocity $\beta (= v/c)$ will undergo a Doppler effect (non relativistic) given by,

$$E_{\text{obs}} = E_0 [1 + \beta \cos \theta]$$  \hspace{1cm} (62)

where $E_{\text{obs}}$, is the energy of the observed $\gamma$-ray, $E_0$ is the energy of $\gamma$-ray emitted by a nucleus decaying at rest, and $\theta$ is the angle between the direction of recoil and that of the emitted $\gamma$-ray. In case of very short-lived states (lifetime $\ll$ stopping time) and detector at angles $\neq 90^\circ$, the $\gamma$-ray transition peak in the spectrum is shifted, compared to the actual energy, to higher (lower) energies when observed at forward (backward) angles with respect to the direction of the recoiling nuclei. For levels with lifetime $\sim$ stopping time, the peak has a stopped component at the actual energy and an accompanying shape, extending to higher (lower) energies for detectors at forward (backward) angles.
The LINESHAPE [53] code is widely used to extract the level lifetimes from the observed Doppler shapes and shifts using the Doppler Shift Attenuation Method. This method requires a thin target backed with a high $Z$ elemental backing. The thin target ensures that the residue do not lose appreciable energy while traversing the target, and the elemental backing ensures that the uncertainties in the simulation of the slowing down process is minimized. However, in the present measurements we had a thick molecular $Ta_2O_5$ target, which in contradistinction to the conventional scenario. Hence, the LINESHAPE code was extensively modified to cater to our specific experimental requirements which are circumstantiated in Ref.[7]. Least squares fitting of the observed shapes to the calculated ones was carried out simultaneously at different angles for determination of the level lifetimes. The parameters for the fitting procedure included the level lifetime, the side-feeding time, the peak height, the contaminant peak(s), and the background parameters etc. The results are elaborated in the subsequent chapters.

Following this conscientious analysis, we develop the level scheme of the residual nucleus of interest, along with details such as the level excitation energy, spin, parity, and the level lifetime. All these then help us to corroborate the model predictions, and or rarefy the same. Thus this exercise of a detailed description of the de-exciting process culminates finally, in undertaking the appropriate model calculations, relevant to the nucleus under investigations.