3 The Means of Exchange Function

3.1 Introduction

The role of money in facilitating transactions was well known to the historical writers on the subject. They contrasted monetary economies with economies based on direct barter. Barter economies suffered from the 'double coincidence of wants' problem—an exchange could happen only when each of the trading parties needed what the other had to offer. The introduction of a generally accepted money solves this problem by allowing goods to be exchanged for money in the confidence that the money could be exchanged for other goods. Thus Jevons (1875) writes about the inefficiency of barter:

Some years since, Mademoiselle Zélie, a singer of the Theatre Lyrique at Paris, made a professional tour round the world, and gave a concert in the Society Islands. In exchange for an air from *Norma* and a few other songs, she was to receive a third part of the receipts. When counted, her share was found to consist of three pigs, twenty-three turkeys, forty-four chickens, five thousand cocoa-nuts, besides considerable quantities of bananas, lemons,
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and oranges. At the Halle in Paris [...] this amount of live stock and vegetables might have brought four thousand francs, which would have been good renumeration for five songs. In the Society Islands, however, pieces of money were very scarce; and as Mademoiselle could not consume any considerable portions of the receipts herself, it became necessary in the mean time to feed the pigs and poultry with the fruit.

This 'double coincidence' problem and the superiority of monetary over barter exchange became a commonplace in textbooks on monetary theory. The necessity of holding money for carrying out transactions was also implicitly recognised by the writers on the quantity theory of money. For the quantity theorists, however, the relationship between the quantity of money held and the volume of transactions, i.e. the velocity of money, was an institutionally given datum. The systematic examination of the transactions demand for money followed only after the Keynesian revolution in macroeconomics and the attempts to provide an account of Keynesian monetary theory in neoclassical terms.

One of the earliest such attempts was the inventory-theoretic model of Baumol (1952). In this model agents must choose between holding interest paying bonds and barren money. The transactions role of money is introduced by assuming that the agents need to make a steady stream of payments which can be done only in terms of money. It is assumed that bonds could be sold to obtain money but that there was a fixed cost of doing so. This led to the well-known result that the average amount of money holding would
be proportional to the volume of spending and inversely proportional to the square-root of the interest rate. A higher rate of interest on bonds would increase the opportunity costs of holding money and thereby lead to more frequent conversion of money into bonds. Baumol's model thus provided a microeconomic story of the velocity of circulation of money, showed contrary to quantity theorists that the velocity would depend on the interest rate and showed that in terms of Keynes' classification of the sources of demand for money, not just speculative demand but also transactions demand would be interest sensitive. However, these results depended not just on the essential role of money in transactions and the cost of converting non-monetary assets into monetary assets. They depended crucially on these costs being fixed costs. If the costs of converting bonds into money had been entirely proportional to the value of transactions then agents would have converted bonds into money at the last possible moment before carrying out a transaction, leading to the transaction demand for money tending to zero. Thus, in retrospect, Baumol's model illustrates the importance of investigating the detailed structure of transactions costs for building a transactions theory of money.

Another early attempt to model the transactions demand for money was made in Don Patinkin's *Money, Interest and Prices* (Patinkin, 1965). Patinkin's book was a magisterial attempt to integrate money into neoclassical general equilibrium theory and to provide an interpretation of Keynes on that basis. The formal method used by Patinkin to carry out this integration was to
include money holdings and prices into agents’ utility functions.\textsuperscript{1} This utility function was seen as an analytical shortcut, an \textit{indirect} utility function arising from an underlying choice made by agents on how much money to hold. In Chapter V of his work, Patinkin discusses how the demand for money may arise from the transactions role of money and a stochastic payments process. The model functions in a discrete time setting, with agents entering into contracts to buy and sell commodities and bonds at the boundaries of Hicksian weeks and the contracts being carried out during the week. Buyers must make payments in terms of money and a buyer who is short of money to make a payment when it falls due suffers “embarrassment” or a monetary cost of converting bonds into money. Mismatches between the timing of purchases and sales give rise to a demand for money. Patinkin assumes that the timing of payments is distributed randomly over the week thus turning the maximum value of this mismatch into a random variable. By arranging to hold a larger quantity of money at the beginning of the week or planning to purchase less an agent could reduce the probability of “embarrassment” but also thereby forego the utility from consumption or the income from the interest payment from bonds. This tradeoff gives rise to a downward sloping demand curve for money in terms of the rate of interest.

Just as we did in the case of Baumol, we must ask of Patinkin’s model too why agents do not turn bonds into money right at the moment before a purchase. In the formal structure of Patinkin’s model this is ruled out by assumption: bond holdings can be changed only in between weeks whereas

\textsuperscript{1}He assumed that these functions were homogeneous of degree zero with respect to the money holdings and prices taken together, i.e. only the real money holding mattered.
payments become due within a week.\textsuperscript{2} Patinkin justifies this by appealing to “the recurrent theme in the literature that the individual can make more rapid adjustments in the composition of his stock of assets (particularly financial ones) than in his consumption flow of commodities” (Patinkin, 1965, pp. 80). This timing assumption plays the same role as the fixed transaction costs in Baumol’s model in generating a positive demand for money. To the extent that this timing assumption is not explained in terms of more fundamental features of the institutional setting Patinkin’s treatment cannot be considered to have reduced the demand for money to its microeconomic fundamentals, though of course it takes a significant step forward by bringing to focus the role in the timing of payments in giving rise to a transactions demand for money.

Patinkin’s money in the utility function formalism was criticised by Clower (1967) on the ground that it did not take into account the fact the money was different from other commodities in being generally accepted in exchange. As an alternative he proposed the cash-in-advance constraint. This constraint, applicable in discrete-time models, states that at the beginning of the period each agent’s money holdings should be at least as large as their gross purchases. Given our discussion of Patinkin’s stochastic payments model, it is clear that despite Clower’s claims, his approach does not differ fundamentally from Patinkin’s. Indeed, we can think of Clower’s constraint as arising from a deterministic special case of Patinkin’s world where every

\textsuperscript{2} Though in one place Patinkin allows for the possibility of intraweek bond sales, this is not part of his overall framework and is assumed by Patinkin to involve an additional cost.
agent believes that all the payments that they have to make will fall due at the beginning of the week before the payments due to them are received. Lucas and Stokey (1983); Svensson (1985); Lucas and Stokey (1987) have developed models with more flexible cash-in-advance constraints. However, unless the underlying economic mechanisms that induce the cash-in-advance requirements is made explicit, these models do not provide much of an advance over simple introducing real balances in utility functions. Indeed, Feenstra (1986) has shown that a large variety of models with explicit transactions costs, include models which embody Clower's cash-in-advance constraints, have equivalent money-in-the-utility function formulations. To break this impasse we need models that study the process of exchange in detail so as to clearly bring out the social role money plays in the exchange process. A survey of early work on these lines can be found in Ostroy and Starr (1990). However, the study of the transactions role of money has received an increased impetus in recent times due to introduction of methods from search theory and mechanism design. It is a model of think kind that we study in the present chapter.

In section 3.3 we construct a simple model of explicit search and exchange which satisfies these conditions in order to examine how an intrinsically valueless money may still become acceptable in exchange and how the introduction of money affects economic welfare. We conclude by showing how even the primitive environments discussed in this chapter throw up some of the questions that must be answered by any theory of a monetary economy.
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3.2 Necessary conditions for monetary exchange

3.2.1 Decentralization and control

What are the minimum properties that a hypothetical economy must satisfy for it to be a meaningful framework for discussing monetary questions? The first certainly is decentralization and some form of private property. It is only in a world of discrete agents each of whom has a prior claim on a part of the community’s product that we can talk about the problems of exchange and their possible resolution through the institution of money. We believe that both these conditions would be fulfilled by any community provided we look at it at a sufficient degree of detail. The existence of decentralization follows from the discreteness of humanity itself. While our species has evolved remarkable abilities of communication and co-ordination, they are neither so rigid nor so costless that we can ignore the existence of individuals. Once the existence of decentralization is accepted, that of prior claims follows directly from the limited scope of control that can be directly exercised by one individual. Possession, after all, is the most direct form of property. This prior claim may be denied through force and fraud, or might be abdicated by conscious choice. But that such institutions of coercion or consent are required to enforce a particular system of property in itself demonstrates that the the prior individual claims that we have been talking about actually exist.

Our claim that every community can ultimately be decomposed into individuals with private claims should not be taken to mean that such atomistic
analysis is always the most fruitful. We may choose to ignore the questions of allocation and coordination within certain groups and consider only the interactions between these groups in our analysis. This does not imply a fundamental change in method since we are still working in terms of individuals and their spheres of control—only it is the case that the individual agents of our analysis do not correspond to individual humans. Of course, this leaves us with the task of ascribing valid behavioural properties to these collective agents without having to resort to the atomistic analysis after all. On the other hand, empirical evidence may allow us to ascribe behaviour to collective agents even when we do not know or do not care how this behaviour emerges from the behaviour of the constituent individuals.

3.2.2 Specialisation

Even with decentralisation and private property, trade and money are not necessary unless there is heterogeneity among agents. This heterogeneity may come from various sources— inhereted endowments, intrinsic abilities, geographical distribution of resources, position in the life cycle. Even among identical agents there might be heterogeneity of outcomes. Thus we may imagine a community of identical agents living over a number of periods whose endowment in each period is a random quantities of some perishable good. If these agents are risk-averse then they will desire the opportunity for smoothing their income which is offered by intertemporal exchange, i.e., borrowing and lending.

More interesting that these exogenous sources of heterogeneity is endoge-
nous heterogeniety which itself depends on the opportunities for exchange. This follows Adam Smith's original intuition about gains in productivity made possible by specialisation and the division of labour (Smith, 1904; Young, 1928). In this case, if the introduction of money reduces the costs of exchange then the result is not just a realisation of the productive potential that exists at the time money is introduced but also the initiation of a cumulative process of increase in productivity.

3.2.3 Information and utility constraints

Traditionally, the existence of decentralisation, private property and specialisation in an economy has been considered sufficient to produce an essential role for money. The argument works by contrasting monetary exchange with a system of direct barter where agents accept only those commodities that they wish to consume and offer only those commodities that they have themselves produced. Thus we have, in a classical statement by Jevons (1875):

Some years since, Mademoiselle Zélie, a singer of the Theatre Lyrique at Paris, made a professional tour round the world, and gave a concert in the Society Islands. In exchange for an air from Norma and a few other songs, she was to receive a third part of the receipts. When counted, her share was found to consist of three pigs, twenty-three turkeys, forty-four chickens, five thousand cocoa-nuts, besides considerable quantities of bananas, lemons and oranges. At the Halle in Paris [...] this amount of live stock
and vegetables might have brought four thousand francs, which would have been good renumeration for five songs. In the Society Islands, however, pieces of money were very scarce; and as Mademoiselle could not consume any considerable portions of the receipts herself, it became necessary in the mean time to feed the pigs and poultry with the fruit.

Such anecdotes, which are fairly commonplace in textbooks of monetary economics, are in one sense misleading. For while they certainly are effective in showing that direct barter is an inefficient exchange mechanism, it does not necessarily imply that monetary exchange is the only alternative. There may well be other mechanisms which can also support an efficient allocation.

The simplest such mechanism is that of indirect exchange where at least some agents accept commodities in trade which they do not intend to consume. Once we add differential storage, transport and quality-inspection costs to our model, it is likely that only a few commodities will participate in these indirect exchanges. This we might take as an explanation of the emergence of commodity money and then see fiat money as a further refinement of commodity money.

But another possibility studied formally in the papers of Kocherlakota (1998) and Kocherlakota and Wallace (1998) shows that this may not be the most illuminating way to see the role of money in exchange. These papers study allocation mechanisms in an environment that encompasses overlapping generations and random matching models of money as well as the
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turnpike model proposed by Townsend (1980). In these environment they look at trading mechanisms based on what they call 'memory'—the possibility of there being a public record of the trading history of the agents who make up a community. They are able to show that a community with 'memory' can not only achieve all those allocations which a monetary economy can achieve, but it can achieve even more. The way 'memory' can substitute for money is easy to see. Assume that in our community 'memory' is implemented as a huge scoreboard. Each time one agent delivers a commodity to another agent, the giving agent's score goes up and the receiving agents score goes down by the amount of money that would otherwise have changed hands. Exchanges are constrained by the fact that scores can never be negative. Then this system tracks the monetary system exactly. Indeed it is analogous to our system of payments through cheques drawn on banks—the only difference being that unlike Kocherlakota's 'memory', the record of all commodity transfers are not publicly visible in a bank-based payments systems and therefore we need some additional reason to trust the bank.

But if we can actually implement something like 'memory' we need not constrain ourselves to quid pro quo either. With all transactions being public, agents may follow the policy of offering their commodity to anyone who needs it without demanding anything in return. Since givers in some pairings will be receivers in others, an optimal allocation can be achieved in this way. The question then arises of what prevents someone from receiving but refusing to give. It is here that the assumption of a publicly visible 'memory' becomes necessary. In (Kocherlakota, 1998) a deviation by anyone from
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a strategy triggers the punishment of everyone refusing to participate in any trades from that point onwards. This threat is sufficient to ensure that no one actually deviates.

3.2.4 Conclusion

Thus we see that for money to be necessary for an economy, the economy must must satisfy the assumption of decentralization, private property and heterogeneity. But even in such an economy, it is not the physical properties of the money object which are important. Rather, we can turn the result of (Kocherlakota, 1998) on its head to say that money is a way to economise on memory. Many allocations which would otherwise have required a costless centralised repository of trading records can now be achieved by passing useless tokens in a decentralized manner.

3.3 Valueless money and explicit search

While the previous section discussed situations in which money might be necessary, in this section we construct a model to show how in a simple world where money is necessary it can also be sufficient in making possible hitherto impossible exchanges. The model of this section is that of bilateral exchange between randomly-matched agents. There is an extensive literature on this class of models, starting with the work of Kiyotaki and Wright (1989, 1993). In the present study we shall limit ourselves to the simplest case for the sake of tractability. In particular, while the search-theoretic liter-
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ature has also explored the emergence of commodity money and the coexistence of commodity and valueless money, in our model we shall exclude the direct exchange of commodities in order to restrict our attention to the process through which intrinsically valueless money circulates.

3.3.1 The model

Consider an island economy where the only commodity is coconuts. Coconuts are indivisible and come in three varieties—red, green and blue. The island is inhabited by three tribes of people—the Red, Green and Blue tribes. There is a continuum of agents on the island, with the population of each tribe being equal to 1. The assumption of a continuum, with the random matching assumption we make below, means that the probability of two agents meeting repeatedly is zero and hence systems of exchange based on reputation and credit are ruled out.

Time is discrete and infinite and agents discount future payoffs at the rate $\delta$.

As a proxy for intrinsically useless money we assume that a proportion $M$ of agents are endowed with an unit each of an indivisible and useless object which we shall call 'money'.

An agent can be in two states. She can be carrying an object—coconut or money—or she can be trying to search for a coconut. We assume that the island is so fertile that search is instantaneous. So, an agent who has been able to consume a coconut in one period gains utility $U$ at the end of that period and enters at the beginning of the next period carrying a fresh
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<table>
<thead>
<tr>
<th>Tribe</th>
<th>Colour collected</th>
<th>Colour consumed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>Red</td>
<td>Blue</td>
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<tr>
<td>Green</td>
<td>Green</td>
<td>Red</td>
</tr>
<tr>
<td>Blue</td>
<td>Blue</td>
<td>Green</td>
</tr>
</tbody>
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Table 3.1: Colour of coconuts collected and consumed by different tribes

We introduce a double coincidence of wants problem with the following assumption. First, there is specialisation—red coconuts can be collected only by the people of the Red tribe, green coconuts by the Green tribe and blue coconuts by the Blue tribe. Second, there is a diversity of preferences—the people of the Red tribe consume only blue coconuts, people of the Green tribe consume only red coconuts and people of the Blue tribe consume only green coconuts. With this pattern of specialisation and preferences, depicted in Table 3.1, there is no possible pairing in which there is a double coincidence of wants, i.e. a pairing in which both participants have something which the other participant wants.

In each period agents are randomly matched. They have to decide whether to exchange the coconut or money that they are carrying with the coconut or money carried by the other agent in the pairing. We assume that when an exchange happens it imposes a transaction cost of $\epsilon > 0$ in terms of utility on both the parties to the exchange. We can informally think of this cost as the disutility of effort involved in communication and in ascertaining the genuineness of money or quality of coconut offered by the other agent. If on the other hand, the members of a pairing choose not to exchange they do not incur any cost and carry over their holdings to the next period. Successful
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consumption of a coconut of the preferred colour provides an utility of $U$. As mentioned above, consumption and search for new coconuts happens in between periods.

3.3.2 The optimization problem

For each matched pair the agents must decide whether or not to enter into an exchange. If the object offered for exchange is a coconut of the colour the agent can consume, then the agent should certainly accept the offer provided $e$ is small enough. On the other hand if the object offered is money or a coconut of another colour then the decision of the agent will depend on the degree to which he believes that he will be able to exchange the object offered now for a coconut of his choice in the future. This in turn will depend on beliefs and actions of future agents. This is the central problem in the acceptability of intrinsically valueless money. We approach this problem in our environment by using the tools of game theory.

We limit our attention to situations which are symmetric across colours of coconuts and tribes and stationary over time.

We argue that coconut-coconut exchanges cannot occur in a symmetric equilibrium. Because of our assumptions ruling out the double coincidence of wants, at least one party to a putative coconut-coconut exchange would be exchanging a coconut he cannot consume for another coconut that he cannot consume. In a symmetric equilibrium the coconut offered would be no more acceptable in exchange in the future than the coconut the agent currently holds. Thus this agent would be in exactly the same economic con-
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dition after the exchange as he was in before the exchange. He would therefore not want to incur the disutility cost of the exchange. For this reason coconut-coconut exchanges would never happen even if they could have benefited one of the parties to the exchange.

The strategies of the agents can therefore be completely summarised by the probability \( \pi_0 \) of an agent offering a coconut in exchange for money and the probability \( \pi_1 \) of an agent offering money in exchange for a coconut. \( \pi = \pi_0 \pi_1 \) is the probability that monetary exchange happens in situations where it is possible.

Consider a coconut-carrying agent. With probability \( \frac{M}{3} \) she meets a money-carrying agent who desires coconuts of a colour that our agent carries. In this case the agent either sells her coconut with probability \( \pi \). In all other cases she remains a coconut-carrying agent. If we take \( V_c \) to be the lifetime expected utility of a coconut-carrying agent, then the Bellman equation is:

\[
(1 + \delta)V_c = \max_{\pi_0} \left\{ \frac{\pi_0 \pi_1 M}{3} (V_m - \epsilon) + \left( 1 - \frac{\pi_0 \pi_1 M}{3} \right) V_c \right\}
\]

\[
= \max_{\pi_0} \left\{ V_c + \frac{\pi_0 \pi_1 M}{3} (V_m - V_c - \epsilon) \right\} \tag{3.1}
\]

\[
\delta V_c = \max_{\pi_0} \left\{ \frac{\pi_0 \pi_1 M}{3} (V_m - V_c - \epsilon) \right\}
\]

The situation for the money-carrying agent is almost symmetrical. With probability \( 1 - M \) she meets a coconut-carrying agent. With further probability \( 1/3 \) the coconut is of a colour of her choice. In this case she buys the co-
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conut with a probability \( \pi \), consumes it to gain an utility of \( U \) and, given our assumption of instantaneous coconut search, emerges as a coconut-carrying agent in the next period. In all other situation she remains a money-carrying agent. If \( V_m \) is the lifetime expected utility of this type of agent then the corresponding Bellman equation is:

\[
(1 + \delta)V_m = \max_{\pi_1} \left\{ \frac{\pi_0 \pi_1 (1 - M)}{3} (U + V_c - \epsilon) + \left( 1 - \frac{\pi_0 \pi_1 (1 - M)}{3} \right) V_m \right\} \\
= \max_{\pi_1} \left\{ V_M + \frac{\pi_0 \pi_1 (1 - M)}{3} (U + V_c - V_m - \epsilon) \right\} \\
\delta V_m = \max_{\pi_1} \left\{ \frac{\pi_0 \pi_1 (1 - M)}{3} (U + V_c - V_m - \epsilon) \right\}
\]

(3.2)

3.3.3 Non-monetary equilibrium

**Proposition 1.** There is an equilibrium with \( \pi = 0 \).

**Proof.** It is immediately clear that \( \pi_0 = 0 \) is an equilibrium. If \( \pi_0 = 0 \) then from eq. (3.2) \( V_m = 0 \), which then in turn is consistent with \( \pi_0 = 0 \) being an optimal solution for eq. (3.1). \( \square \)

If money is not generally accepted, then it is not in the interest of any agent to accept money. This is a feature that we shall repeatedly meet in the models of intrinsically valueless money. In our opinion, just the existence of a non-monetary equilibrium is not enough to invalidate a model of an monetary economy. Rather, we take it as a confirmation that we have correctly modelled as a money as intrinsically valueless, so that its value depends only on its acceptability. Instead, we shall evaluate different models on the
basis of the monetary equilibria that they imply.

3.3.4 Monetary equilibrium

Proposition 2. There is an equilibrium with $\pi > 0$ provided $\epsilon$ is sufficiently small.

Proof. Let $\hat{V}_c$ and $\hat{V}_m$ be the values of $V_c$ and $V_m$ for some arbitrary value of $\pi$ rather than optimal values. By referring to eq. (3.1) and eq. (3.2) we have,

$$\delta \hat{V}_c = \frac{\pi M}{3} (\hat{V}_m - \hat{V}_c - \epsilon) \quad (3.3)$$

and

$$\delta \hat{V}_m = \frac{\pi (1 - M)}{3} (U + \hat{V}_c - \hat{V}_m - \epsilon) \quad (3.4)$$

By solving (3.3) and (3.4) simultaneously we have,

$$\hat{V}_m - \hat{V}_c - \epsilon = \frac{\pi (1 - M) U - [3\delta + 2\pi(1 - M)]\epsilon}{3\delta + \pi} \quad (3.5)$$

and,

$$U + \hat{V}_c - \hat{V}_m - \epsilon = \frac{(3\delta + M\pi) U - (3\delta + 2\pi M)\epsilon}{3\delta + \pi} \quad (3.6)$$

Since both $[3\delta + 2\pi(1 - M)]$ and $(3\delta + 2\pi M)$ are positive, for any $U > 0$, there exists $E > 0$ such that for $\epsilon < E$,

$$\hat{V}_m - \hat{V}_c - \epsilon > 0 \quad (3.7)$$

and

$$U + \hat{V}_c - \hat{V}_m - \epsilon > 0 \quad (3.8)$$
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By combining inequality (3.8) with eq. (3.2) and inequality (3.7) with eq. (3.1) we see that $\pi_1 = 1$ is the solution of the money-carrier's optimisation problem whenever $\pi_0 > 0$ and $\pi_0 = 1$ is the solution of the coconut-carrier's optimisation problem whenever $\pi_1 > 0$. Taken together this means that $\pi_0 = \pi_1 = 1$ is an equilibrium of our economy. Money buys goods and goods buy money.

3.3.5 Welfare

Non-monetary equilibrium

Proposition 3. In the equilibrium with $\pi = 0$, we have $V_c = V_m = 0$.

Proof. This immediately follows from eq (3.2) and eq. (3.1).

Intuitively, this shows that money is 'essential' in our economy in the sense of Hahn, which in turn is a consequence of the structure of preferences where agents desire commodities other than those they themselves produce and the fact that exchange is expensive.

Monetary equilibrium

The exact expressions for expected utility in this case are cluttered because of terms involving $\varepsilon$. To form an idea of the maximum potential benefits from monetary exchange, we look at the limit where $\varepsilon$ tends to 0. In this limit, with $\pi = 1$ we have,

$$V_c = \frac{M}{3\delta} \left[ \frac{(1 - M)}{3\delta + 1} \right] U > 0$$

(3.9)
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and,

\[ V_m = \frac{1 - M}{3\delta} \left[ \frac{3\delta + M}{3\delta + 1} \right] U > 0 \] \hspace{1cm} (3.10)

More interestingly, since a proportion \( M \) of agents carry money and a proportion \( 1 - M \) of them carry commodities, the expected social welfare\(^3\) of an agent is,

\[ W = MV_m + (1 - M)V_c \]
\[ = \frac{U(1 - M)M}{3\delta} \] \hspace{1cm} (3.11)

Thus we see that welfare is quadratic in the quantity of money, increasing till \( M = 1/2 \) and decreasing thereafter. This is a result of two contradictory tendencies. Beginning from \( M = 0 \), introduction of money increases welfare since it allows trade to take place where it had been impossible before. But since in our economy carrying money and carrying coconuts is mutually exclusive, more agents carrying money also means less agents carrying coconuts and hence reduces the possibility of finding something to consume. For \( M > 1/2 \) is is the latter effect which predominates.

While the details of this results depend a great deal on the specificities of this model, it can be taken as pointing to a contradiction between the role of money in providing liquidity and the transaction costs associated with running an efficient payments system.

\(^3\)The expected social welfare may be interpreted as the expected utility of an agent who knows he will be born into our island community but who doesn't know whether he will be cast into the role of a money-carrier or commodity-carrier initially. Hence it can be used as a metric of the desirability of our social arrangement.
3.4 Conclusion

While the search-theoretic model of this chapter could demonstrate how an intrinsically valueless object could become acceptable in exchange, it did so under extremely restrictive assumptions. For our purposes, the most restrictive of these was that of stationarity, which prevented us from studying the questions at the heart of monetary economics—why the value of money changes, how do those changes affect other economic variables and welfare, how might the changes be controlled? Indeed, the assumption of indivisibility means that there is no meaningful way to talk about the value of money within the context of this model at all.

Another restrictive assumption of this model is that of random matching. In really existing economies exchange is not carried out through random search but rather in organised markets and with the participation of specialist traders. In the environment studied in the model of this chapter the double coincidence of wants problem could be solved without money by introducing a trader who keeps stocks of all the three kinds of coconuts on hand and is willing to exchange coconuts of a colour that a customer produces with one of the colour that the customer consumes. We can relax the constraint of indivisibility to introduce a payoff for this specialised trader.

But this scenario of a mega-mart that is willing to exchange any commodity for any other goes to the other extreme of unrealism. In reality most specialised traders and most marketplaces restrict themselves to only a few goods at a time. As soon as we recognise this fact, we are once again faced with a double coincidence problem: what I produce might be sold
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in a market different from the market which sells what I want to consume. Once again there would be a role for money to bridge this double coincidence problem but if the money is intrinsically valueless then this usefulness would once again be contingent on the belief in the acceptability of the money. Thus while our model of individual traders being randomly matched is unrealistic when taken literally, it does capture important aspects of the problems associated with intrinsically valueless money even in a world with specialised traders and markets.

There have been attempts to remove these restrictions while remaining within the framework of game theory and explicit search (see for example Lagos and Wright (2005)). However, gains in generality come at a high cost in tractability using this approach. In this study, we take the alternative path of ignoring the actual mechanism of exchange—assuming that it functions perfectly efficiently—and then seeing how changes in the value of money interact with the rest of the economy. With this goal in mind, we turn to the study of the polar opposite case of a perfectly cashless economy in the next chapter.