Chapter 1

Introduction

1.1 Motivation and research tracks

A supply chain is a network of stages that represent functionalities (including warehouses, manufacturers, suppliers and retailers) which must be provided to convert raw materials into the specified end-products and deliver these end-products to retailers or customers (Simuchi-Levi et al.[102]). Each stage may have one or more options that can satisfy a required function. For example, a function might be the procurement of a raw material, the manufacture of an assembly, or a shipment of a product to a distribution center. Moreover, each stage is a potential location for holding a safety-stock inventory of item processed at this stage to manage the demand variations.

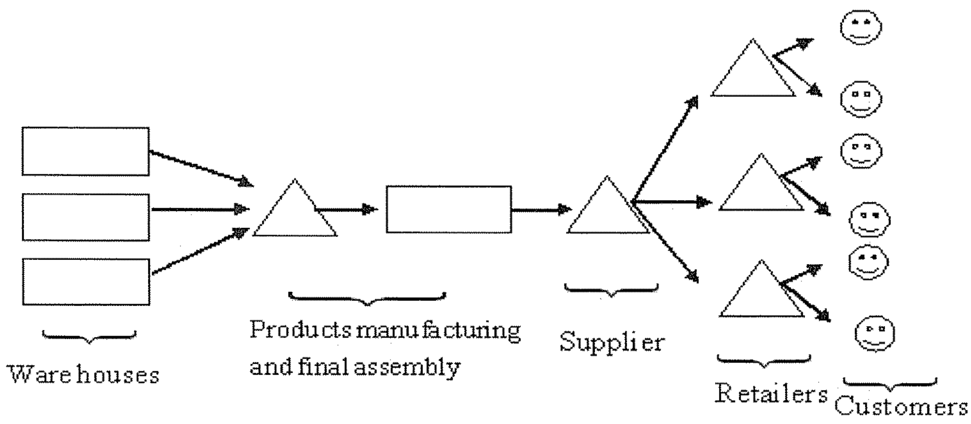

Figure 1: Supply chain (example)

Fig. 1 shows a supply chain which deals with divergent and convergent flows within a complex network resulting from many different customer orders to be handled in parallel.

The term supply chain is also applied to a large company with several sites often located in different countries. Coordinating materials, information and financial flows for a multi-national company can be executed efficiently by utilizing the latest developments in information and communication technology. We can define the inventory as a quantity of goods or materials in the control of an enterprise, and held for a time in a relatively idle or unproductive state, awaiting its intended use or sale. Inventory appears in a supply chain in several forms such as raw materials, components, work-in-process, and finished goods. Efficient and effective management of inventory throughout the supply chain significantly improves the ultimate service provided to the customer. The gains that have been realized when adopting optimal inventory policies in a supply chain are impressive:

- Hewlett-Packard (HP) cut desk jet printer supply costs by 25% with the help of inventory models analyzing the effect of different locations of inventories within its supply chain. This analysis convinced HP to adopt a modular design and postponement for deskjet printers (Lee and Billington [68]). As a result, HP company achieved the savings of $80 million (in net present value).

- Intel Corporation devised a suite of capacity models of production facilities along the semiconductor supply chain in collaboration with its key suppliers. Now, Intel has access to all the supplier's models that may well be used for various planning horizons (next 5 years, next 9 months, or next 8 weeks). Dollar savings of hundreds of millions are estimated for the suppliers and tens of millions for Intel (Shirodkar and Kempf [99]).

These impressive gains have motivated us to build inventory models for the supply chains under some real life situations such as trade credit scenario, stochastic demands, stochastic lead times and fuzzy environment etc. Inventories in supply chains are always the result of inflow and outflow processes (transport, production etc.). The inventory analysis enables us to decompose the average inventory level in a supply chain. In this thesis, we intend to find the ***optimal inventory policies*** in order to minimize the total costs incurred or maximize the profit.

In this thesis, we consider ***two-echelon supply chains*** which consist of a supplier at the higher echelon and many identical retailers (or a single retailer) at the lower echelon. Supplier fulfills the demands from retailers and the retailers in turn fulfill the demands from common customers. The two-echelon supply chains have many real life applications. As one example, the ***Norton Auto Supply company*** (see. Hammond [48]) has one central distribution center (CDC) and 20 regional distribution centers (RDC). The market is partitioned so that the RDCs face similar demand patterns. The RDCs are then divided into five groups according to their geographic proximity. There are four RDCs in each group. Each group follows a weekly order schedule, and the order schedules for different groups are staggered so that only one group of RDCs orders on each day from Monday to Friday. The straggered ordering schedule is reasonable when, say, the CDC has limited capacity. For example, the transportation fleet at the CDC can only deliver to one group of RDCs in a single day. This can be viewed as a two-echelon supply chain system with CDC at the higher echelon and RDCs at the lower echelon.

Large multi-echelon supply chain systems usually consist of hundreds of thousands of Stock Keeping Units (SKUs). In large supply networks like Wal-Mart, and the US-Navy, thousands of SKUs are stocked at different Inventory Holding Points (IHPs). These holding points might be at different echelons where the higher echelons supply the lower echelons. If all of these locations are owned or managed by a centralized management system, a single inventory control system might be implemented. Tremendous improvements are attainable if a centralized inventory management system is considered for the entire supply network. This motivated us to build inventory models that consider the entire supply network and the interactions between their constituent IHPs.

In this thesis, we focus the research in ***three main tracks.***

- Firstly, in chapters 3 to 7, we investigate supply chain models in which the inventory parameters such as demand, lead time, inventory costs etc. are considered to be deterministic. In chapters 3 to 6, we mainly focus on implementing the ***two-echelon trade credit financing*** under various real life environments. In chapter 7, we evaluate supply chain structures using ***delayed product differentiation process.***
- Secondly, in chapters 8 and 9, we develop supply chain models in which the inventory parameters such as demand and lead time are stochastic. In such models, the sources of uncertainty are modeled by ***probabilistic distributions*** that may be derived from the past data. In these chapters, we focus on implementing the continuous review ***(R, Q)*** replenishment policy at all installations.
- Since past data are not always available or reliable (e.g. due to market turbulence), the ***fuzzy set theory*** is appropriate to assess market demand, lead time and inventory related costs. So in chapter 10, thirdly, we construct a supply chain model under fuzzy environment.

1.2 Methodology - Mathematical modeling

In this thesis, the research approach is based upon ***mathematical modeling.*** In short, mathematical modeling is to describe a real world problem with tractable mathematical formulations. These mathematical formulations should not only produce values of performance measures, but also provide insights, and a deeper understanding for the phenomenon studied. In other words, a good mathematical model should tell us why the system behaves the way it does. Building mathematical models is often considered to be an art, rather than a science. The concept of mathematical modeling can in general be separated into three fundamental steps.

- The first step is to formulate a model which captures the essence of the system properties. In our case, we first of all understand the supply chain system, and then identify decision variables, objective functions, and constraints. In this modeling step, we must also identify which properties of the system that are more important and disregard other less important features of the system. Apparently, in order to make the model mathematically tractable, we can not (in general) take all system features into account.

- In the second step, the model is usually solved by some analytical method (and some times by simulation). In connection with inventory models, methods from disciplines like queueing theory, probability theory, fuzzy set theory, optimization theory, etc., are often used when solving a model analytically. After this second step, we form a number of equations to predict the behavior of the model.

- The third and last step is the evaluation of an acquired model. Obviously, we have to check that our model predicts the system behavior adequately through numerical examples. Sometimes, this is done by simulating the system and comparing the simulated results against the analytical results.

1.3 Description of the thesis

1.3.1 A short description of the research contributions

- An EPQ-based model with perfect items is developed **to** investigate partial trade credit financing at the retailer of a supply chain.
- Supply chain models with deteriorating items are developed along with the concepts like two-echelon trade credit financing, two-warehousing, price dependent demand and price discounts under advance payment scheme.
- Incorporating the demand which is a function of both selling price and credit period to the retailer of a supply chain, an EPQ-based model is developed for perishable items under two-echelon trade credit financing.
- An inventory model for non-instantaneous deteriorating items is constructed where the unsatisfied demand at the retailer is partially backlogged with time proportional backlogging rate. System structures are evaluated using a delayed product differentiation process.
- Supply chain models with perfect items are developed under continuous review ***(R, Q)*** replenishment policy along with concepts like stochastic lead times, Poisson distributed demands and shortages with lost sale and partial backordering at the retailers.
- A realistic supply chain model with fuzzy demand rate, fuzzy lead time and fuzzy inventory costs is developed and Pareto-optimum solutions are obtained through interactive fuzzy decision-making procedure.

1.3.2 Summary of chapters

Chapter 1

This chapter is an introductory in nature. A brief description of various concepts used in this thesis is given. Further it contains some of the necessary preliminaries and notations which are used in the thesis.

Chapter 2

Literature survey is presented in this chapter. Various inventory and supply chain models studied by different authors are mentioned here. It includes deterministic, stochastic and fuzzy supply chain or single-echelon inventory models. Literature gaps are also given.

Chapter 3

In this chapter, we consider that the supplier usually is willing to provide the retailer a full trade credit period for payments and the retailer just offers the partial trade credit period to his customers. We develop an EPQ-based model to investigate the retailer's inventory system in a supply chain as a cost minimization problem under partial trade credit option to their customers. Mathematical theorems are proved to determine optimal inventory policy for the retailer and numerical examples are given to illustrate the theorems. We deduce some previously published results of other researchers as particular cases and perform sensitivity analysis for various inventory parameters.

Chapter 4

In this chapter, we consider that the retailer purchases more goods than that can be stored in his own warehouse and these excess quantities are stored in a rented warehouse. Here, the retailer's demand depends on the selling price and the retailer offers the partial trade credit option to his customers under two-echelon trade credit scenario. We develop an EOQ- based model with perishable items and two-storage facility as a profit maximization problem. Mathematical theorems are established to determine optimal inventory policy for the retailer and numerical examples are given to illustrate the theory. We also perform sensitivity analysis for various inventory parameters.

Chapter 5

In this chapter, the dominant, retailer announces price discount offer under advance payment (AP) scheme prior to the selling period. Due to the advancement of internet and on-line money transactions, the AP scheme is common and useful to decrease the estimation error in demand and to increase the market sales. When the items are arrived to the inventory, the priority will be given to the customers who use AP scheme. Here, we consider a supply chain where the supplier provides the dominant retailer a **full** trade credit period for payments whereas the dominant retailer offers the partial trade credit to their customers. We intend to develop an EOQ-based model with perishable items in order to investigate the dominant retailer's inventory system as a cost minimization problem under AP scheme and two-echelon trade credit option. Mathematical theorems are established to determine optimal price discounting and lot-sizing policies. We perform sensitivity analysis for various inventory parameters.

Chapter 6

In this chapter, we consider that the retailer's demand is a function of both the selling price and credit period rather than the constant demand. Incorporating this demand function to the retailer of a supply chain, we develop an EPQ - based model for perishable items under two-echelon trade financing. The purpose of this chapter is to maximize the profit by determining the optimal selling price, credit period and replenishment time. It is shown that the model developed by Jaggi et al. [60] is a particular case of our work. Finally, through numerical examples, sensitivity analysis shows the influence of key model parameters.

Chapter 7

In this chapter, we consider a supply chain in which the supplier supplies multiple items to the retailer. The retailer orders different products in response to the demands of the customers. The items are non-instantaneous deteriorating over the cycle time. The unsatisfied demand at a retailer is partially backlogged with a time-proportional backlogging rate. We formulate models for a postponement system and an independent system to minimize the total average cost function per unit time for ordering and keeping ‘non-instantaneous deteriorating items’. An algorithm is given to derive the optimal solutions of the proposed models. The impact of the deterioration rate on the inventory replenishment policies is studied with the help of both theoretical and numerical results.

Chapter 8

In general, the control parameters (order quantity ***(Q)*** and reorder point (i?,)) depend on both the demand process and the replenishment lead time. Although many studies have treated lead time as constant, focusing solely on demand, a continuous review ***(R, Q)*** model with stochastic lead time could be a building block in supply chain management. Variability in lead time between successive stages is often what disturbs supply chain coordination. In a two-stage system consisting of one supplier and many identical retailers, we concentrate on lead time at the supplier as a random variable. We assume Poisson demands with constant transportation times for the retailers. Unsatisfied demands assumed to be lost in the retailers and unsatisfied retailer orders are backordered at the supplier. We have developed numerical examples when the lead time for the supplier’s orders is assumed to be gamma distributed and the related accuracy is assessed through simulation.

Chapter 9

In this chapter, we consider a supply chain with a number of identical, independent ‘retailers’ at the lower echelon and a single supplier at the upper echelon controlled by continuous review inventory policy (***R,Q***). Each retailer experiences Poisson demand with constant transportation times. We consider constant lead time for replenishing supplier orders from an external warehouse to the supplier and unsatisfied retailer orders are backordered in the supplier. The unsatisfied demand is partially backordered in the retailers. The partially backordering policy is implemented in the retailers using an explicit control parameter ‘6’ which limits the maximum number of backorders allowed to be accumulated during the lead time. We develop an approximate cost function to find optimal reorder points for given batch sizes in all installations, the optimal value of ***b*** in the retailers and the related accuracy is assessed through simulation.

Chapter 10

In this chapter, a realistic supply chain model with fuzzy demand, fuzzy lead-time and fuzzy inventory costs is formulated and an optimal inventory policy is found to minimize the total cost of the supply chain. We consider a two-level supply chain with single supplier at the upper echelon and many identical retailers at the lower-echelon. The unsatisfied customer demands at the retailers are partially backordered using the control parameter '6'. The replenishment policy at all installations is continuous review policy. The imprecise parameters of lead-time, inventory costs and demand are expressed through linear/non-linear membership functions. These are represented by different types of membership functions, linear or quadratic, depending upon the prevailing supply condition and marketing environment. The imprecise parameters are transformed into corresponding interval numbers and then following the interval analysis, the objective function for total average cost is changed into respective multi-objective functions. These functions are minimized and solved for a Pareto-optimum solution by interactive fuzzy decision-making procedure. The model is illustrated numerically and the results are presented in tabular forms.

Chapter 11

This chapter sums up the thesis with the intention of further research.

1.4 Basic mathematical theory

Definition 1. A set X is said to be a ***metric space*** if with any two points p and q of X there is associated a real number $d(p,q)$, called distance from p and q, such that

(a) $d(p,q) > 0$ if $p \neq q$; $d(p,p) = 0$

(b) $d(p,q) = d(q,p)$

(c) $d(p,q) \leq d(p,r) + d(r,q)$, for any $r \in X$

Definition 2. By an ***open cover*** of a set E in a metric space X we mean a collection $\{G_\alpha\}$ of open subsets of X such that $E \subset \bigcup_\alpha G_\alpha$.

Definition 3. A subset K of a metric space X is said to be ***compact*** if every open cover of K contains a finite subcover.

Since closed subsets of compact sets are compact, any closed interval in real number system R is compact. The cartesian product of any two closed intervals in R is also a compact set.

Definition 4. Let X and Y be metric spaces with metrics d_X and d_Y respectively. Suppose $E \subset X, p \in E$, and f maps E into Y. Then f is said to be ***continuous*** at p if for every $\epsilon > 0$ there exists a $\delta > 0$ such that

$$d_Y(f(x), f(p)) < \epsilon$$

for all points $x \in E$ for which $d_X(x,p) < \delta$.

Definition 5. Let f be defined (and real-valued) on $[a,b]$. For any $x \in [a,b]$, the derivative of f at x is

$$f'(x) = \lim_{t \to x} \frac{f(t) - f(x)}{t - x}$$

provided this limit exists. If f' is defined at a point x, we say that f is ***differentiable*** at x. If f' is defined at every point of a set $[a,b]$, we say that f is differentiable on $[a,b]$.

Definition 6. A differentiable mapping f of an open set $E \subset R^n$ into R^m is said to be ***continuously differentiable*** in E if f' is a continuous function.

Definition 7. A function f defined on an open interval (a, b) is said to be ***concave*** if for $x, y \in (a, b)$ and each $\lambda, 0 \leq \lambda \leq 1$, we have

$$f(\lambda x + (1-\lambda)y) \geq \lambda f(x) + (1-\lambda) f(y) \tag{1}$$

If f is concave then $(-f)$ is ***convex***.

Positive definite matrix. An $n \times n$ real symmetric matrix M is positive definite if $z^T M z > 0$ for all non-zero vector z with real entries ($z \in R^n$) where z^T denotes the transpose of z. For example, the matrix $M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is positive definite, since for any $z = [z_1, z_2]$, $z^T M z = z_1^2 + z_2^2 > 0$.

Negative definite matrix. An $n \times n$ real symmetric matrix M is negative definite if $z^T M z < 0$ for all non-zero vector z with real entries ($z \in R^n$) where z^T denotes the transpose of z. For example, the matrix $M = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ is negative definite, since for any $z = [z_1, z_2]$, $z^T M z = -z_1^2 - z_2^2 < 0$.

Optimization problems

An ***optimization problem*** is the problem of finding the best solution from all feasible solutions. In the simplest case, it is nothing but solving problems in which one seeks to minimize or maximize a real valued function by systematically choosing the values of real or integer variables from an allowed set.

Let f be a real valued function of single variable x. If $\frac{d^2 f}{dx^2} < 0 (> 0)$ then f is strictly concave (convex) function. Suppose f is a real valued function of multiple variables i.e., $f(x_1, x_2, ..., x_n)$, then we follow the following procedure to check convexity/ concavity of f. If all second order partial derivatives of f exist, then ***Hessian matrix*** of f is the matrix

$$H(f) = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_n} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \cdots & \frac{\partial^2 f}{\partial x_2 \partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 f}{\partial x_n \partial x_1} & \frac{\partial^2 f}{\partial x_n \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_n^2} \end{bmatrix}$$

If $H(f)$ is negative definite (positive definite) then f is a concave (convex) function.

Stochastic programming is a framework for modeling optimization problems that involve uncertainty. Stochastic programming models are similar in style but take advantage of the fact that probability distributions governing the data are known or can be estimated. The aim here is to find some policy that is feasible for all (or almost all) the possible data instances and maximizes the expectation of some function of the decisions and the random variables.

Multi-objective programming is the process of simultaneously optimizing one or more objectives subject to certain constraints. For example, the multi-objective minimization problem can be written as

$$\begin{aligned} \text{Minimize} \quad & [\mu_1(x), \mu_2(x), \cdots, \mu_n(x)]^T \\ \text{subject to} \quad & \\ g(x) &\leq 0 \\ h(x) &= 0 \\ x_l &\leq x \leq x_u. \end{aligned}$$

where μ_i is the i-th objective function, g and h are inequality and equality constraints respectively, and x is the vector of optimization or decision variables. x_l and x_u are the respective lower and upper bounds for x .

If a multi-objective problem is well performed, there should not be a single solution that simultaneously optimizes each objective to its fullest. The solution to the above problem is a set of Pareto-points. ***Pareto solutions*** are for which improvement in one objective can only occur with the worsening of at least one other objective. A solution $\mu^* = \{\mu_1^*, \mu_2^*, ..., \mu_n^*\}$ in the feasible region is termed Pareto optimal (for the above problem) if there does not exist another feasible solution $\mu = \{\mu_1, \mu_2, ..., \mu_n\}$ such that $\mu_i \leq \mu_i^*$ for all i, $i \in \{1, 2, \cdots, n\}$ and $\mu_j < \mu_j^*$ for at least one index of j, $j \in \{1, 2, \cdots, n\}$.

Theorem 2.1. (Rudin [90]). Suppose f is a continuous real function on a compact metric space X, and

$$M = \sup f(p), \; m = \inf f(p) \text{ for } p \in X.$$

Then there exist points $p, q \in X$ such that $f(p) = M$ and $f(q) = m$.

Theorem 2.2. (Intermediate value theorem [90]) . Suppose f is a real differentiable function on $[a, b]$ and suppose $f'(a) < \lambda < f'(b)$. Then there is a point $x \in (a, b)$ such that $f'(x) = \lambda$.

From the above theorem, we have that if $f'(a)f'(b) < 0$, then there exists a number $d \in (a, b)$ such that $f'(d) = 0$.

Theorem 2.3. (Implicit function theorem [90]). Let f be a continuously differentiable function from an open set $E \subset R^{n+m}$ into R^n, such that $f(a, b) = 0$ for some point $(a, b) \in E$, $a \in R^n$ and $b \in R^m$. Put $A = f'(a, b)$ and assume that the linear transformation A_x is invertible. Then there exist open sets $U \subset R^{n+m}$ and $W \subset R^m$, with $(a, b) \in U$ and $b \in W$, having the following property:

Every $y \in W$ corresponds a unique x such that

$$(x, y) \in U \text{ and } f(x, y) = 0.$$

If this x is defined to be $g(y)$, then g is a continuously differentiable mapping of W into R^n, $g(b) = a$,

$$f(g(y), y) = 0, \ (y \in W), \text{ and } g'(b) = -(A_x)^{-1}A_y$$

The equation $f(x, y) = 0$ can be written as a system of n equations in $n + m$ variables:

$$f_1(x_1, \ldots, x_n, y_1, \ldots, y_m) = 0$$

$$\vdots$$

$$f_n(x_1, \ldots, x_n, y_1, \ldots, y_m) = 0$$

The assumption that A_x is invertible means that the n by n matrix

$$\begin{bmatrix} D_1 f_1 & \ldots & D_n f_1 \\ \ldots & \ldots & \ldots \\ D_1 f_n & \ldots & D_n f_n \end{bmatrix}$$

evaluated at the point (a, b) defines an invertible linear operator in R^n. Then the implicit function theorem states that the solutions $x_1, x_2, .., x_n$ (in terms of $y_1, y_2, .., y_n$) are continuously differentiable functions.

Poisson density function.

An interesting density function which has many physical applications is Poisson density. For all non-negative x, it is defined as

$$p(x; \lambda) = \frac{\lambda^x e^{-\lambda}}{x!}, \quad x = 0, 1, 2, ... \tag{2}$$

where λ is constant. The complementary cumulative Poisson distribution is

$$P(x; \lambda) = \sum_{j=x}^{\infty} \frac{\lambda^j e^{-\lambda}}{j!} \tag{3}$$

Negative binomial distribution.

The density function for the negative binomial distribution of order n is

$$p(x) = b_N(x; n, \rho) = \begin{pmatrix} x + n - 1 \\ n - 1 \end{pmatrix} \rho^n (1 - \rho)^x, \quad x = 0, 1, 2, ... \tag{4}$$

where $0 < \rho < 1$ and n is a positive integer.

Gamma distribution.

The gamma density function is defined by

$$\gamma(x; \alpha, \beta) = \begin{cases} \frac{(\beta x)^\alpha \beta e^{-\beta x}}{\alpha!} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases} \tag{5}$$

where α is a non-negative integer and β is any positive number.

Definition 8. An ordered pair of brackets defines an interval $A = [a_L, a_R] = \{a : a_L \leq a \leq a_R, a \in R^+\}$ where a_L and a_R are, respectively, left and right limits of A. R^+ denotes the set of all positive real numbers. Let $* \in \{+, -, \cdot, /\}$ be a binary operation on the set of positive real numbers. If A and B are closed intervals then $A * B = \{a * b : a \in A, b \in B\}$ defines a binary operation on the set of closed intervals. In the case of division, it is assumed that $0 \notin B$. The operations on intervals may be explicitly calculated as below.

$$\begin{aligned}
\frac{A}{B} &= \frac{[a_L, a_R]}{[b_L, b_R]} = \left[\frac{a_L}{b_R}, \frac{a_R}{b_L}\right] \text{ where } 0 \notin B, 0 \le a_L \le a_R \text{ and } 0 < b_L \le b_R \\
A + B &= [a_L, a_R] + [b_L, b_R] = [a_L + b_L, a_R + b_R] \\
kA &= \begin{cases} [ka_L, ka_R] & \text{for } k \ge 0 \\ (ka_R, ka_L) & \text{for } k < 0 \end{cases}
\end{aligned}$$

The lower case letters denote the real number and upper case letters denote closed intervals.

Order relations between intervals

The order relation $\le_{LR}$ between $A = [a_L, a_R]$ and $B = [b_L, b_R]$ is defined as

$$A \le_{LR} B \text{ iff } a_L \le b_L \text{ and } a_R \le b_R$$

$$A <_{LR} B \text{ iff } A \le_{LR} B \text{ and } a_R \ne b_R.$$

The order relation $\le_{LR}$ represents the Decision Maker's performance for the alternative with minimum cost, that is, if $A \le_{LR} B$, then A is preferred to B.

Basics in fuzzy set theory

Definition 9. A ***fuzzy set*** $\widetilde{A}$ in a universe of discourse X is defined as the following set of pairs

$$\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x)) : x \in X\}$$

where $\mu_{\widetilde{A}} : X \to [0, 1]$ is a mapping called the membership function of the fuzzy set $\widetilde{A}$. The larger $\mu_{\widetilde{A}}(x)$ is the stronger grade of membership form in $\widetilde{A}$.

Definition 10. A fuzzy set $\widetilde{A}$ of the universe of discourse X is ***convex*** if and only if $\forall x_1, x_2 \in X$,

$$\mu_{\widetilde{A}}(\lambda x_1 + (1 - \lambda)x_2) \ge \min(\mu_{\widetilde{A}}(x_1), \mu_{\widetilde{A}}(x_2)) \text{ where } 0 \le \lambda \le 1$$

Definition 11. A fuzzy set $\widetilde{A}$ in the universe of discourse X is called a ***normal fuzzy set*** if there exists at least one $x \in X$ such that $\mu_{\widetilde{A}}(x) = 1$. A fuzzy number is a special case of fuzzy set. Different definitions and properties of fuzzy numbers are encountered in the literature but they all agree that a fuzzy number represent the conception of a set of real numbers close to 'a' where 'a' is the number being a fuzzy field.

Definition 12. A fuzzy set which is both convex and normal is called as a ***fuzzy number*** in the universe of discourse X. Fuzzy numbers are often represented by two types of membership functions:

(a) Linear membership function, e.g., Triangular fuzzy number, Trapezoidal fuzzy number etc.

(b) Non - linear membership function, e.g., Parabolic fuzzy number, Exponential fuzzy number etc.

Definition 13. A $\alpha-$ ***cut of a fuzzy number*** $\widetilde{A}$ is defined as crisp set

$$A_\alpha = \{x : \mu_{\widetilde{A}}(x) \geq \alpha, x \in X\} \text{ where } \alpha \in [0, 1]$$

A_α is a non-empty bounded closed interval contained in X and it can be denoted by $[A_L(\alpha), A_R(\alpha)]$, $A_L(\alpha)$ and $A_R(\alpha)$ are the lower and upper bounds of the closed interval respectively.

1.5 Inventory analysis in a supply chain

Inventories in supply chains are always the result of inflow and outflow processes (transport, production etc.) of commodities. The inventory analysis enables us to decompose the average inventory level in a supply chain. In this section some basic concepts in inventory theory is presented. The purpose is to give the reader a background to better understand the contents of this thesis. To fully understand the assumptions and the results presented in the chapters of this thesis, some basic mathematical theory is also needed.

1.5.1 Key performance indicators

(a) Demand structures

An important feature of an inventory system is the demand structure. To find suitable demand pattern, forecasting methods can be used. In general, there are two types of approaches that may be of interest: ***extrapolation of historical data*** and ***forecasts based on other factors.***

The simplest structure is that the demand is known. Many products can be represented well by this ***deterministic demand,*** especially products that are in a mature stage of a product life cycle and are used regularly. Examples are consumable products like tooth paste, many standard tools and various spare parts. In chapters 3 to 7, we consider the following types of demand function in deterministic case.

- Demand is known and uniform
- Selling price dependent demand
- Credit period dependent demand.

The assumption of deterministic demand may be unrealistic in most cases. However, in cases when company delivers according to a long term contract, the assumption may well be adequate. In most contexts, demand is ***uncertain,*** i.e., in the best case, only the probabilistic distribution of customer demand is known. Consequently, when the demand is uncertain the exact values of the demand can not be predicted. In chapters 8 and 9, we shall make the assumption that the customer demand follows the

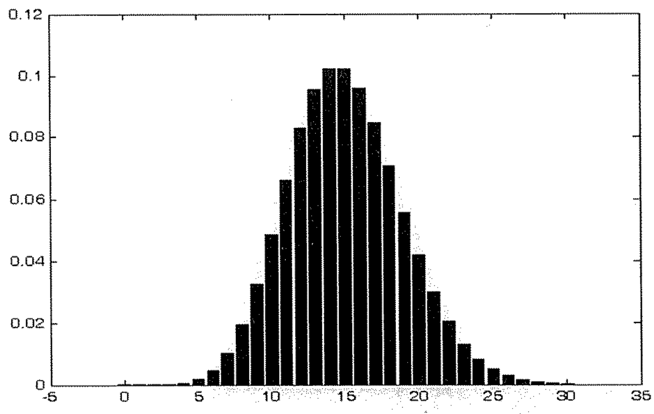

Figure 2: Probability function for Poisson demand with mean and variance equal to 15

so called ***Poisson process***. In inventory models, it is common to assume that customer arrivals occur randomly in time. In many cases, consecutive customer arrivals are completely independent. A Poisson process capture both of these properties. A Poisson process $\{X(t),\ t \geq 0\}$ (with intensity λ) has independent, stationary, Poisson distributed increments with $X(0) = 0$. This means that the probability for the arrival of k customers within a time space of T time units becomes,

$$P(X(T) = k) = \frac{(\lambda T)^k}{k!} e^{-\lambda T},\ k = 0, 1, 2, ...,$$

with mean and variance equal to λT. The intensity λ can be viewed as the average number of customer arrivals per unit of time.

Another important and interesting result regarding the Poisson is that the superposition of many independent renewal processes is asymptotically a Poisson process. In other words, if demand originates from a large number of independent sources, it is appropriate to assume that the demand follows a Poisson process. Fig.2 shows the graph for the Poisson distribution with mean is equal to 15.

Whenever statistical data is unreliable or even unavailable, stochastic models may not be the best choice. Fuzzy set theory (Klir and Yuan, [65]) may provide an alternative approach for dealing with the uncertainty in demand. In real life, the demand rate is normally vague, imprecise and flexible in nature and so a ***fuzzy parameter***. For example, demand rate is about λ units.

(b) Time horizon

The time period over which the inventory level will be controlled is called the time horizon. This horizon may be finite or infinite depending upon the nature of the demand for the commodity.

(c) Planning cycle time

Responsiveness describes the ability of the complete supply chain to react according to the changes in the market place. The main indicator in this area is the planning cycle time which is simply defined as the time between the beginning of two subsequent planning cycles. Long planning cycle times prevent the plan from taking into account the short-term changes in the real world. The appropriate planning cycle time has to be determined with respect to the aggregation level of the planning process, the planning horizon and planning effect.

(d) Lot-size or order quantity

By the 'order quantity' we mean the quantity produced or procured during one cycle. The cycle stock (ordering quantity) is used to cover the demand between two consecutive production runs of the same product. The role of cycle stock is to reduce the costs for setting up and cleaning the production facility (setup or change over costs). The actual replacement of stock may occur instantaneously or uniformly. Instantaneous replenishment occurs in case the stock is purchased from outside sources whereas the uniform replenishment may occur when the product is manufactured by the company. Economic Order Quantity (EOQ) is that size of order which minimizes total annual costs of the inventory. If the production or supply of commodity is instantaneous then the inventory model is treated as an EOQ model. If the production or replenishment rate of the commodity is finite and uniform then the inventory model is treated as a Economic Production Quantity (EPQ) model. The use of EOQ and EPQ models for inventory analysis can be found in Osteryoung [81].

(e) Lead time

The time gap between placing of an order and its actual arrival in the inventory is known as lead time. Lead time has two components, namely the administrative lead time - from initiation of procurement action until placing an order and the delivery lead time - from placing of an order until the delivery of the ordered material. If the *lead time is constant*, it does not affect the system and we can therefore assume that the lead time (L) is equal to zero. This is what we consider in chapters 3 to 7. The only difference in case of positive lead time is that we need to order L time units earlier.

If the demand is uncertain, we can not disregard the lead time. In a supply chain, the lead time for the retailer's orders is equivalent to the service time for the supplier to meet demand plus transportation time (fixed). We know from queueing theory that service time follows random process. When treating lead times are independent random variables as well as orders do not cross, it is often convenient to consider Poisson distributed demand and Gamma distributed lead time (Hadley and Whitin [49]). In practice, the lead time distribution, when known, can be fairly well fitted by the Gamma distribution. Let the probability for x units demanded for the period of length t time units be

$$p(x;\lambda t) = \frac{(\lambda t)^x}{x!}e^{-\lambda t}, x = 0, 1, 2, \ldots$$

where λ is the mean rate of the demand. Let the probability for lead time t be

$$\gamma(t;\alpha,\beta) = \frac{\beta(\beta t)^\alpha e^{-\beta t}}{\alpha!}$$

The lead time demand is the marginal distribution $p(x)$ which is

$$\begin{aligned}
p(x) &= \int_0^\infty \frac{\beta(\lambda t)^x(\beta t)^\alpha}{x!\alpha!}e^{-(\beta+\alpha)t}\,dt \\
&= \frac{\beta^{\alpha+1}\lambda^x}{x!\alpha!}\int_0^\infty t^{x+\alpha}e^{-(\beta+\lambda)t}\,dt \\
&= \frac{\beta^{\alpha+1}\lambda^x(\alpha+x)!}{x!\alpha!(\beta+\lambda)^{\alpha+x+1}}\int_0^\infty \frac{(\beta+\lambda)^{\alpha+x+1}t^{x+\alpha}}{(\alpha+x)!}e^{-(\beta+\lambda)t}\,dt \\
&= \frac{(\alpha+x)!}{x!\alpha!}\left(\frac{\beta}{\beta+\lambda}\right)^{\alpha+1}\left(\frac{\lambda}{\beta+\lambda}\right)^x \\
&= b_N\left(x;\alpha+1,\frac{\beta}{\beta+\lambda}\right),\ \alpha > -1 \text{ and } \beta,\lambda > 0.
\end{aligned}$$

Thus, if α is a positive integer, the marginal distribution of lead time demand $p(x)$ has the negative binomial distribution.

(f) Control policies

In principle, the following three fundamental questions arise when constructing an inventory control policy:

1. How often should we inspect the inventory status?
2. When should we place an order?
3. How large should the order be?

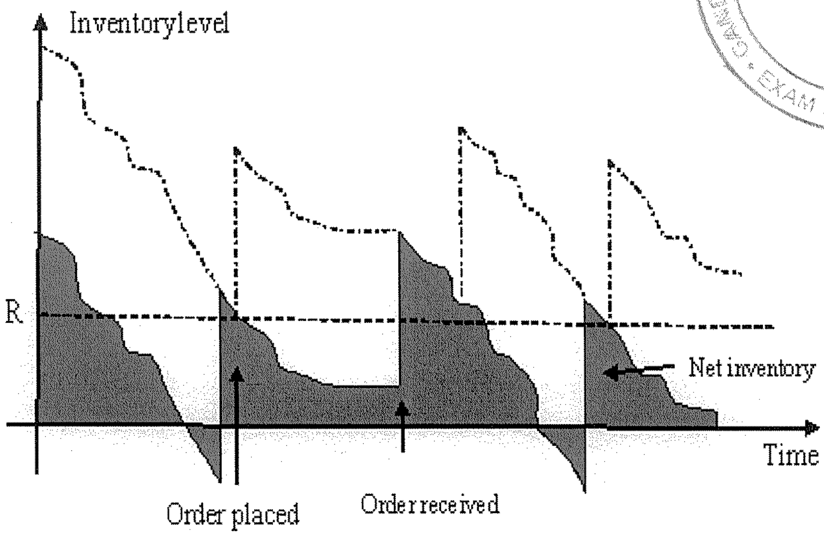

Figure 3: continuous review (R,Q) replenishment policy

In case of deterministic demand, the answers to three questions are more or less trivial. However, when demand is stochastic, many different answers are possible depending on the properties of the inventory system. Let us continue by focusing on first issue, that is, the frequency of inspections of the inventory status. Assume that the time between reviews is T. Then, if $T > 0$ we say that the inventory status is reviewed periodically. The alternative to periodic is continuous review, which (of course) means that the inventory status is reviewed on a continuous basis, i.e. $\boldsymbol{T} = 0$. Periodic review systems are often used when the demand is high, and in cases when coordination of replenishments may be beneficial. Continuous review systems are used when demand is low, since the cost of applying continuous review is essentially same as applying periodic review in this case. An advantage of continuous review (compared with periodic review) is that safety stock can be reduced with retained customer service.

The second and third questions stated above are often answered simultaneously. The most popular and commonly used control policy (replenishment rule) is the so called (***R,Q***) policy (see Fig.3). When applying this policy, a batch quantity of size ***Q*** is ordered whenever the inventory position (stock on hand + outstanding orders - backorders) declines to or below the reorder point ***R.*** The replenishment order arrives after a lead time ***L*** units of time. Some times it is necessary to order a multiple of ***Q*** such that the inventory position becomes larger than ***R.***

(g) Possible stockouts

Having demands when the system is out of stock are called stock outs. There are three types of possible stockouts:

- ***Backorders*** : The demands during stockout period are backlogged and fulfilled upon the arrival of next order quantity. This type of shortages happen when the customer agrees to wait or the demanded items are not available in the market.
- ***Lost sales :*** Here, customers do not wait upto the arrival of next order quantity and hence the sales are lost during stockout period.
- ***Partial backlogging :*** In this type, during stockout period, some of the customers will wait and others do not wait upto the next replenishment.

(h) Delivery performance

As customer satisfaction is a key component of a supply chain, delivery performance is an essential measure for total supply chain. Regarding different aspects of delivery performance, various indicators called service level and order lead time are distinguished in inventory management literature. An exact definition for service level is

$$\text{Service level} = 1 - \frac{\text{Mean backlog at the end of the period}}{\text{Mean demand per period}}$$

(i) Classification of inventory items

Depending upon the age of the items residing in the stock, in this thesis, we classify them into three categories.

- **Perfect items**: Products which have long life such as bricks, stones etc.
- **Perishable items**: Products which deteriorate as time goes. Deterioration is defined as damage, spoilage, decay, obsolescence, evaporation, pilferage etc. that result in decrease of usefulness of the original one. For example, food items. The traditional EOQ models for perishable items can be found in the literature. Ghare and Schrader [40] who derived a revised form of the EOQ model assuming exponential decay. Covert and Philip [32] developed an EOQ model with Weibull distribution deterioration. Misra [77] considered an economic production quantity (EPQ) model for deteriorating items with both a varying and a constant rate of deterioration. Balkhi and Benkherouf [13] proposed a method for obtaining an optimal production cycle time of deteriorating items in a model where demand and production rates are functions of time. Sarma [95] has developed a deterministic inventory model with deteriorating items and two-warehouse facility. Pakkala and Achary [83] have developed an EPQ model with deteriorating items and two-warehouse facility. Goyal and Giri [47] have illustrated recent trends in modeling of deteriorating inventory. Raafat [87] surveyed the literature on continuously deteriorating inventory models. Many recent articles can be found in the literature related to deteriorating items.
- **Non-instantaneous deteriorating items**: In real life, the goods are non-instantaneous deteriorating (e.g., vegetables, fruits, fishes and so on). These items would have a span of maintaining quality or the original condition, namely, during that period, there was no deterioration occurs. But deterioration starts after that period. We call this phenomenon as 'non-instantaneous deteriorating' in the literature.

(j) Costs associated with inventories

Various costs associated with inventory control are often classified as follows:

(1) **Set-up cost.** This is the cost associated with the setting up of machinery before starting production. Set-up cost is generally assumed to be independent of the quantity ordered or produced.

(2) **Ordering cost.** This is a cost associated with ordering of raw material for production purposes. Advertisements, consumption of stationary and postage, telephone charges, telegrams, rent for space used by the purchasing department, traveling expenditures incurred, etc., constitute the ordering cost.

(3) **Purchase cost.** The cost of purchasing a unit of an **item is** known as purchase cost.

(4) **Holding cost.** The holding cost is associated with carrying inventory. This cost generally includes the costs such as rent for space used for storage, insurance of stored equipment, production, taxes, etc.

(5) **Deterioration cost.** The cost incurred due to the deterioration of items in inventory is referred to as the deterioration cost.

(6) **Shortage cost.** The penalty cost for running out of stock (i.e., when an item can not be supplied on the customer's demand) is known as shortage cost. This cost includes the loss of potential profit through sales of items and loss of goodwill, in terms of permanent loss of customers and its associated loss profit in future sales.

(7) **Selling price.** When demand for certain commodity is affected by the quantity stocked, decision problem is based on a profit maximization criterion **that** includes **the** revenue from selling. Salvage value may be combined **with** the cost of storage and hence is generally neglected.

1.5.2 Trade credit financing

In today's business transactions, it is more and more common to see that retailers are allowed a fixed time period before they settle their account to the supplier. We term this period as ***trade credit period.*** Before the end of the trade credit period, the retailer can sell the goods, accumulates revenue and earns interest. A higher interest is charged if the payment is not settled at the end of the trade credit period. In the real world, the supplier would allow a specified credit period (say, 30 days) to the retailer for payment without penalty to stimulate the demand of consumable products. This credit term in financial management is denoted as 'net 30'. Teng [107] illustrated two more ***benefits of trade credit policy:*** (1) it attracts new customers who consider trade credit policy to be a type of price reduction; and (2) it should cause a reduction in sales outstanding, since some established customer will pay more promptly in order to take advantage of trade credit more frequently.

One level trade credit financing refers that the supplier would offer the retailer trade credit but the retailer would not offer the trade credit to his/her customers. That is, the retailer could sell the goods, accumulates revenue and earns interest within the trade credit period; but the customer would pay for the items as soon as the items are received from the retailer. To date, several interesting and relevant papers related to one level trade credit financing exist in the literature. But, in most business transactions, this assumption is unrealistic and usually the supplier offers a credit period to the retailer and the retailer, in turn, passes on this credit period to his customers. For example, in India, the TATA Company can delay the amount of purchasing cost until the end of the delay period offered by his supplier. The TATA Company also offers permissible delay payment period to his dealership. **In chapters 3 to 7, we develop inventory models for the supply chain systems under the two-echelon (or two-level) trade credit financing.**

1.6. Notations

n	number of retailers in a supply chain
λ	demand rate per year
P	production rate per year
ρ	$1 - \frac{\lambda}{P} \geq 0$
h	stock-holding cost per unit per unit time excluding interest charges
A	ordering cost per order
h_1	retailer's holding cost per unit per unit time in the own warehouse W_1 excluding interest charges
$h_2(> h_1)$	retailer's holding cost per unit per unit time in the rented warehouse W_2 excluding interest charges
Z	storage capacity of the own warehouse
c	unit purchasing price
s	unit selling price
π_r	penalty cost per unit of lost sale at a retailer
θ	deterioration rate of an item
I_e	interest earned per \$ per year
I_k	interest charges payable per \$ per year to the supplier
α	customer's fraction of the total amount owed payable at the time of placing an order offered by the retailer, $0 \leq \alpha \leq 1$
M	retailer's trade credit period offered by the supplier in years
N	customer's trade credit period offered by the retailer in years
T	cycle time in years
$I(t)$	the inventory level at time t where $0 \leq t \leq T$
L_r	transportation time for deliveries from the supplier to a retailer
Q_0	Ordering quantity (batch size) of the supplier
Q_r	batch size at a retailer
R_0	reorder point of the supplier (integer value, multiple of Q_r)
R_r	reorder point of a retailer (integer value)
TC	annual total cost