Chapter 10

A two-echelon supply chain under fuzzy environment

10.1 Introduction

Recently, many scholars have studied the supply chain (SC) problems as well as emphasized the need of integration among SC stages to make the supply chain effectively and efficiently for satisfying customer demands. Beside the integration issue, uncertainty has to be dealt in order to define an effective SC inventory policy. In addition to the uncertainty on supply (e.g. lead times) and demand, uncertainty is also associated with the estimates of inventory costs as well as backorder costs. Linguistic expressions, such as the demand rate will be about $\boldsymbol{x}$ units per unit time, the unit inventory (or backorder) cost will be more or less $\boldsymbol{y}$ monetary units per unit time and the lead time is about 'T' time units, depend on possibilities rather than probability (because statistics are unreliable). In fact, as in the case of market demand, managers can find it difficult to estimate costs through crisp numbers because they mostly depend on factors that can be hardly quantified. Fuzzy set theory can be used to model the uncertainty associated with holding cost and backorder cost by simply using linguistic expressions.

Typically, stochastic techniques have been used to cope with these uncertainty SC problems. In such techniques the sources of uncertainty are modeled by probabilistic distributions that are derived from the analysis of past available data. However, past data are not always available or reliable (e.g. due to market turbulence). Moreover, when they can be utilized, the integrated inventory policies determined are usually difficult to be implemented, which makes them unattractive even if efficiently computed. Shorter and shorter product life cycles as well as growing innovation rates make demand extremely variable and the collection of statistics (which are required by stochastic models) less and less reliable. Therefore, the probability theory is not appropriate to assess market demand and inventory related costs. Moreover, due to the ambiguity of the available information and the existence of conflicting evidence (such as market demand and inventory costs), possibility theory should be used to model uncertainty rather than probability theory. Possibility theory in fact deals with the analysis of similarities between an object and some given properties and does not convey information about relative frequencies, as probability does. In this chapter, fuzzy set theory has been adopted because of its potential to deal with the concept of possibility (rather than probability) and linguistic expressions (i.e. management judgments) so as to properly address the uncertainty issue. Progressively shorter product life cycles and growing innovation rates make product demand as well as other SC parameters (e.g., lead-time and cost) even more difficult to predict accurately, because the collection of statistical data becomes increasingly unreliable (Fisher, [36]).

Little research has applied fuzzy set theory in the area of SC management and most research that has been done focused on inventory management and supplier selection. Petrovic et al. [85, 86] modeled the uncertain demand and suppliers reliability (percentage of raw material order delivered) with fuzzy sets and developed a fuzzy isolated inventory model to determine the order-up-to level for each individual stage independently on the serial inventory system control. According to the obtained order-up-to levels, a simulation approach was developed to evaluate the performance of the inventory system. Giannoccaro et al. [41] developed an inventory policy using the periodical review policy based on the concept of fuzzy echelon stock. The market demand and inventory holding cost were represented by fuzzy sets and the order-up-to levels of inventory items were determined to minimize the holding cost.

In fuzzy-programming problem, the constraints and goals are taken as fuzzy sets. It is also assumed that their membership functions (MFs) are known. But it is not always easy for a decision maker (DM) to satisfy these functions precisely. Following

Zacleh [116], the fuzzy numbers describe the imprecise coefficients. These imprecise coefficients may then approximate to crisp set of interval numbers. Grzegorzewski [45] suggested a method to substitute a fuzzy set by a crisp one. Tanaka and Ishibuchi [106] and Inuiguchi and Kume [57] developed a concept for optimization of multi-objective programming problem with interval objective function.

In a fuzzy interactive linear/non-linear multi-objective decision making problem, DM plays an important role. He has every right to choose the suitable membership functions to achieve his optimum goal. In industry, supply chain is continuous process and DM interacts and changes his decision as per situation/environment. So far, Operations Research scientists have ignored this approach. Though the supply chain models in real world are full of uncertainties in non-stochastic sense, none has considered to solve two-echelon supply chain models in a fuzzy environment. This is what we aim in this chapter.

To the best of our knowledge, for the first time, we introduce an interactive fuzzy approach for a supply chain system which consists of a single supplier and a number of retailers under an uncertain parametric conditions. All installations follow the continuous review $(\boldsymbol{R},\ \boldsymbol{Q})$ replenishment policy. The SC parameters are expressed by fuzzy numbers, which are then converted to appropriate interval numbers following Grzegorzewski [45]. Using the concept of interval arithmetic, we have constructed an equivalent multi-objective model, corresponding to the original problem, with interval coefficients. This equivalent problem has been solved using interactive fuzzy decision making procedure. Optimum and Pareto-optimum solutions are derived for the transformed problem using a gradient non-linear optimization technique - Generalized Reduced Gradient (GRG) method. Finally, some numerical examples are presented to illustrate the model.

This chapter is organised as follows. Sub-sections 10.1.1 and 10.1.2 provide some preliminary results which will be used later. In section 10.2, we model the two-echelon supply chain system with partial backorders under fuzzy environment. Section 10.3 illustrates the theory by numerical examples. Finally, section 10.4 concludes the chapter.

10.1.1 Transfer of the interval valued objective function to multi-objective function

This section will be used to transfer the interval valued objective function (total cost) of a supply chain into a multi-objective optimization problem. We define a non-linear objective function with co-efficients of the decision variables as interval valued numbers:

$$\text{Minimize} \quad TC(z) = \frac{\sum_{i=1}^{m}[x_{l_i}, x_{r_i}] \prod_{j=1}^{k} z_j^{a_j}}{\sum_{i=1}^{n}[y_{l_i}, y_{r_i}] \prod_{j=1}^{k} z_j^{b_j}} \tag{246}$$

subject to $z_j > 0, j = 1, 2, ..., k$ and $z \in F \subset R^n$, where F is the feasible region of z, $0 < x_{l_i} < x_{r_i}, 0 < y_{l_i} < y_{r_i}$ and a_j, b_j are positive constants. Eq.(246) can be rewritten as

$$TC(z) = \frac{\sum_{i=1}^{m}[x_{c_i} - x_{d_i}, x_{c_i} + x_{d_i}] \prod_{j=1}^{k} z_j^{a_j}}{\sum_{i=1}^{n}[y_{c_i} - y_{d_i}, y_{c_i} + y_{d_i}] \prod_{j=1}^{k} z_j^{b_j}}$$

where

$x_{c_i} = \frac{x_{l_i}+x_{r_i}}{2}$, $x_{d_i} = \frac{x_{r_i}-x_{l_i}}{2}$, $y_{c_i} = \frac{y_{l_i}+y_{r_i}}{2}$ and $y_{d_i} = \frac{y_{r_i}-y_{l_i}}{2}$

After some significant simplification using interval arithmetic, we have

$$\text{Minimize } TC(z) = \left[\frac{\sum_{i=1}^{m} x_{l_i} \prod_{j=1}^{k} z_j^{a_j}}{\sum_{i=1}^{n} y_{r_i} \prod_{j=1}^{k} z_j^{b_j}}, \frac{\sum_{i=1}^{m} x_{r_i} \prod_{j=1}^{k} z_j^{a_j}}{\sum_{i=1}^{n} y_{l_i} \prod_{j=1}^{k} z_j^{b_j}}\right] \tag{247}$$

The above interval optimization problem can be converted as the multi-objective optimization problem following Inuiguchi and Kume [57],

$$\text{Minimize} \{TC_l, TC_r, TC_c\} \tag{248}$$

subject to the non-negativity constraints of Eq.(246) where

$$\begin{aligned} TC_l &= \frac{\sum_{i=1}^{m} x_{l_i} \prod_{j=1}^{k} z_j^{a_j}}{\sum_{i=1}^{n} y_{r_i} \prod_{j=1}^{k} z_j^{b_j}} \\ TC_r &= \frac{\sum_{i=1}^{m} x_{r_i} \prod_{j=1}^{k} z_j^{a_j}}{\sum_{i=1}^{n} y_{l_i} \prod_{j=1}^{k} z_j^{b_j}} \\ TC_c &= \frac{1}{2}[TC_l + TC_r] \end{aligned}$$

10.1.2 The nearest interval approximation of a fuzzy number

Here, we want to approximate a fuzzy number by an interval. Suppose $\tilde{X}$ and $\tilde{Y}$ are two fuzzy numbers with α - cuts $[x_l, x_r]$ and $[y_l, y_r]$ respectively. Then the distance between $\tilde{X}$ and $\tilde{Y}$ with respect to the metric m is

$$m(\tilde{X}, \tilde{Y}) = \left[\int_0^1 (x_l - y_l)^2 \, d\alpha + \int_0^1 (x_r - y_r)^2 \, d\alpha\right]^{1/2}$$

We have to find an interval $I_m(\tilde{X})$ which is nearest to $\tilde{X}$ with respect to metric m. Let it be $I_m(\tilde{X})= [I_{m_l}, I_{m_r}]$.
So our objective is

$$\text{Minimize } m(\tilde{X}, I_m(\tilde{X})) = \left[\int_0^1 (x_l - I_{m_l})^2 \, d\alpha + \int_0^1 (x_r - I_{m_r})^2 \, d\alpha\right]^{1/2} \tag{249}$$

with respect to I_{m_l} and I_{m_r}.
The necessary conditions for the minimization problem are

$$\begin{aligned} \frac{\partial}{\partial I_{m_l}} m(I_{m_l}, I_{m_r}) &= -2\int_0^1 x_l d\alpha + 2I_{m_l} \quad \text{and} \\ \frac{\partial}{\partial I_{m_r}} m(I_{m_l}, I_{m_r}) &= -2\int_0^1 x_r d\alpha + 2I_{m_r} \end{aligned}$$

Solving the first order conditions, we have the optimum interval,

$$I_m(\tilde{X}) = \left[\int_0^1 x_l \, d\alpha, \int_0^1 x_r \, d\alpha\right]$$

The second order sufficient condition is satisfied i.e.,

$$\frac{\partial^2}{\partial I_{m_l}^2} m(I_{m_l}, I_{m_r}) \frac{\partial^2}{\partial I_{m_r}^2} m(I_{m_l}, I_{m_r}) - \left(\frac{\partial^2}{\partial I_{m_l} \partial I_{m_r}} m(I_{m_l}, I_{m_r})\right)^2 = 4 > 0.$$

Hence $I_m(\tilde{X})$ is the closest interval to $\tilde{X}$ with respect to the metric m.

Let $\tilde{X} =(x_1, x_2, x_3)$ be a fuzzy number. If $\tilde{X}$ is a triangular fuzzy number (TFN), then $x_l = x_1 + \alpha(x_2 - x_1)$ and $x_r = x_3 + \alpha(x_2 - x_3)$. By the above nearest interval approximation method,

$$\begin{aligned} I_{m_l} &= \frac{1}{2}(x_1 + x_2) \\ I_{m_r} &= \frac{1}{2}(x_2 + x_3) \end{aligned}$$

Therefore the nearest interval number corresponding to $\tilde{X}$ as a TFN is $[(x_1+x_2)/2, (x_2+x_3)/2]$. Similarly when $\tilde{X}$ is a parabolic fuzzy number then $x_l = x_2 - (x_2 - x_1)\sqrt{1-\alpha}$ and $x_r = x_2 - (x_2 - x_3)\sqrt{1-\alpha}$. Following the procedure stated above, the interval number is $[(2x_1 + x_2)/3, (x_2 + 2x_3)/3]$.

10.2 Modeling the supply chain system in an uncertain environment

10.2.1 Notations

In addition to the notations in chapter 9, let us introduce the following notations. Here the wavy bar $\tilde{}$ indicates the fuzzification of the parameter

$\tilde{\lambda}_r$ demand rate at a retailer and $\tilde{\lambda}_r = (\lambda_{r1}, \lambda_{r2}, \lambda_{r3})$

$\tilde{\lambda}_0$ demand rate at the supplier

$\tilde{L}_r$ the lead time for deliveries from the supplier to a retailer and $\tilde{L}_r = (L_{r1}, L_{r2}, L_{r3})$

$\tilde{L}_0$ the lead time of the supplier replenishment orders at the supplier and $\tilde{L}_0 = (L_{01}, L_{02}, L_{03})$

$\tilde{h}_r$ holding cost per unit per unit time at a retailer and $\tilde{h}_r = (h_{r1}, h_{r2}, h_{r3})$

$\tilde{h}_0$ holding cost per unit per unit time at the supplier and $\tilde{h}_0 = (h_{01}, h_{02}, h_{03})$

$\tilde{\pi}_p$ penalty cost per unit of backordered demand at a retailer and $\tilde{\pi}_p = (\pi_{p1}, \pi_{p2}, \pi_{p3})$

$\tilde{\pi}_r$ penalty cost per unit of lost sale at a retailer and $\tilde{\pi}_r = (\pi_{r1}, \pi_{r2}, \pi_{r3})$.

10.2.2 A continuous review two-echelon supply chain model under partial backordering with crisp parameters

Here, we shortly describe the problem undertaken in chapter 9. In the next sub-section, we convert it into fuzzy optimization problem. We consider a supply chain consisting of a supplier at the higher echelon and many retailers at the lower echelon. Each site follows a continuous review (R, Q) policy which means that whenever inventory stock level drops to R (reorder point), Q units are ordered. Each retailer fulfills the demands of the customers and he partially backlogs the shortages during stock out period. We implement partial backlogging at a retailer using the control parameter 'b', i.e., shortages at the retailer are backordered upto b and future shortages are lost in a cycle. This is depicted in Fig.28 (chapter 9). Supplier satisfies demands of the retailers and shortages at the supplier are backordered. The lead times and system costs are assumed to be constant in this section. The demands are in Poisson stream with constant rate at all sites. The batch replenishment quantity Q is fixed for all installations, having been determined by packaging and handling constraints. For example Q may represent the number of units packed into a carton or shrink-wrapped a pallet - the carton or pallet representing the product unit of purchase, storage, handling and shipment purposes.

The cycle time at a retailer is $\frac{Q_r+w}{\lambda_r}$ where w is the expected number of shortages (lost sales and backorders) during stockout at the retailer and it is expressed by the following equation.

$$w = \lambda_r L_r P(R_r + b; \lambda_r L_r) - (R_r + b)P(R_r + b + 1; \lambda_r L_r)$$

where

$$P(x; \lambda L) = \sum_{i=x}^{\infty} \frac{e^{-\lambda L}(\lambda L)^i}{i!}$$

Thus, average number of cycles is $\frac{\lambda_r}{Q_r+w}$. Since a batch size of Q_r is ordered in each cycle at a retailer, the mean rate of demand (from this retailer to supplier) will be $\frac{Q_r\lambda_r}{Q_r+w}$. If the number of retailers is n, then demand process in the supplier is a Poisson process with demand rate $\lambda_0 = \frac{n\lambda_r}{Q_r+w}$ in terms of Q_r.

From the above discussion, supplier follows the continuous review (R_0, Q_0) policy, under Poisson stream with rate $\frac{n\lambda_r}{Q_r+w}$. Let $D(Q_0, R_0)$ and $B(Q_0, R_0)$ be the average inventory level and average stockout level at the supplier respectively. D_r ,E_r and B_r are the respective average inventory level, average number of lost sales and average number of backorders at a retailer.

The total cost of the system consists of holding cost (in the supplier and retailers) and shortage cost in the retailers. The cost incurred at the supplier

$$C_0 = h_0 D(Q_0, R_0) \tag{250}$$

where

$$
\begin{aligned}
D(Q_0, R_0) &= \frac{Q_0+1}{2} + R_0 - \lambda_0 L_0 + B(Q_0, R_0) \\
B(Q_0, R_0) &= \frac{1}{Q_0}\left[\beta(R_0) - \beta(R_0+Q_0)\right] \\
\beta(v) &= \frac{(\lambda_0 L_0)^2}{2} P(v-1; \lambda_0 L_0) - (\lambda_0 L_0) v P(v; \lambda_0 L_0) + \frac{v(v+1)}{2} P(v+1; \lambda_0 L_0) \\
\lambda_0 &= \frac{n\lambda_r}{Q_r + w} \\
w &= \lambda_r L_r P(R_r + b; \lambda_r L_r) - (R_r + b) P(R_r + b + 1; \lambda_r L_r) \\
P(x; \lambda L) &= \sum_{i=x}^{\infty} \frac{e^{-\lambda L}(\lambda L)^i}{i!}
\end{aligned}
$$

The cost incurred at a retailer

$$C_r = h_r D_r + \pi_p B_r + \pi_r E_r \tag{251}$$

where

$$
\begin{aligned}
D_r &= \frac{\lambda_r}{Q_r + w}\left[\frac{Q_r(Q_r+1)}{2\lambda_r} + \frac{Q_r R_r}{\lambda_r} - Q_r L_r + \frac{b(b-1) - 2Q_r(R_r+b)}{2\lambda_r} P(R_r+b; \lambda_r L_r)\right. \\
&\quad + Q_r L_r\, P(R_r+b-1; \lambda_r L_r) + RL\left[P(R_r+b-1; \lambda_r L_r) - P(R_r; \lambda_r L_r)\right] \\
&\quad + \frac{\lambda_r L_r^2}{2}\left[P(R_r-1; \lambda_r L_r) - P(R_r+b-2; \lambda_r L_r)\right] \\
&\quad \left. + \frac{R_r(R_r+1)}{2\lambda_r}\left[P(R_r+1; \lambda_r L_r) - P(R_r+b; \lambda_r L_r)\right]\right] \\
B_r &= \frac{\lambda_r}{Q_r + w}\left[\lambda_r L_r\left[P(R_r; \lambda_r L_r) - P(R_r+b-1; \lambda_r L_r)\right] - R_r P(R_r+1; \lambda_r L_r)\right. \\
&\quad \left. + (R_r+b) P(R_r+b; \lambda_r L_r)\right] \\
E_r &= \frac{\lambda_r w}{Q_r + w}
\end{aligned}
$$

The total cost of the supply chain system

$$TC(R_0, R_r, b) = C_0 + nC_r \tag{252}$$

Since there should not be more than one order outstanding in each retailer, we should consider the constraint $Q_r \geq R_r + b + 1 > 0$. Since there are n retailers in our model and none of them can have more than one order outstanding, we have $R_0 \geq$

$-nQ_r$. This is because if R_0 <-nQ_r then the reorder point is never reached in the supplier. Summarising the above, our non-linear optimization problem is

$$\begin{aligned}&\text{Minimize } TC(R_0, R_r, b)\\ &\text{subject to}\\ &Q_r \geq R_r + b + 1, R_0 \geq -nQ_r, R_r > 0, b \geq 0.\end{aligned}$$

10.2.3 Proposed supply chain model in a fuzzy environment

Considering holding cost, shortage cost, lead time and demand rate as fuzzy parameters, the fuzzy total cost function becomes

$$\widetilde{TC}(R_0, R_r, b) = \widetilde{C}_0 + n\widetilde{C}_r \tag{253}$$

The cost incurred at the supplier

$$\widetilde{C}_0 = \widetilde{h}_0 D(Q_0, R_0) \tag{254}$$

where

$$\begin{aligned}
D(Q_0, R_0) &= \frac{Q_0 + 1}{2} + R_0 - \widetilde{\lambda}_0\widetilde{L}_0 + B(Q_0, R_0)\\
B(Q_0, R_0) &= \frac{1}{Q_0}\left[\beta(R_0) - \beta(R_0 + Q_0)\right]\\
\beta(v) &= \frac{(\widetilde{\lambda}_0\widetilde{L}_0)^2}{2}P(v-1; \widetilde{\lambda}_0\widetilde{L}_0) - (\widetilde{\lambda}_0\widetilde{L}_0)vP(v; \widetilde{\lambda}_0\widetilde{L}_0) + \frac{v(v+1)}{2}P(v+1; \widetilde{\lambda}_0\widetilde{L}_0)\\
\widetilde{\lambda}_0 &= \frac{N\widetilde{\lambda}_r}{Q_r + w}\\
w &= \widetilde{\lambda}_r\widetilde{L}_rP(R_r + b; \widetilde{\lambda}_r\widetilde{L}_r) - (R_r + b)P(R_r + b + 1; \widetilde{\lambda}_r\widetilde{L}_r)\\
P(x; \widetilde{\lambda}\widetilde{L}) &= \sum_{i=x}^{\infty}\frac{e^{-\widetilde{\lambda}\widetilde{L}}(\widetilde{\lambda}\widetilde{L})^i}{i!}
\end{aligned}$$

The cost incurred at a retailer

$$\widetilde{C}_r = \widetilde{h}_rD_r + \widetilde{\pi}_pB_r + \widetilde{\pi}_rE_r \tag{255}$$

where

$$
\begin{aligned}
D_r &= \frac{\widetilde{\lambda}_r}{Q_r+w}\Bigg[\frac{Q_r(Q_r+1)}{2\widetilde{\lambda}_r}+\frac{Q_rR_r}{\widetilde{\lambda}_r}-Q_r\widetilde{L}_r+\frac{b(b-1)-2Q_r(R_r+b)}{2\widetilde{\lambda}_r}P(R_r+b;\widetilde{\lambda}_r\widetilde{L}_r) \\
&\quad + Q_r\widetilde{L}_r\,P(R_r+b-1;\widetilde{\lambda}_r\widetilde{L}_r)+R_r\widetilde{L}_r\left[P(R+b-1;\widetilde{\lambda}_r\widetilde{L}_r)-P(R_r;\widetilde{\lambda}_r\widetilde{L}_r)\right] \\
&\quad +\frac{\widetilde{\lambda}_r\widetilde{L}_r^2}{2}[P(R_r-1;\widetilde{\lambda}_r\widetilde{L}_r)-P(R_r+b-2;\widetilde{\lambda}_r\widetilde{L}_r)] \\
&\quad +\frac{R_r(R_r+1)}{2\widetilde{\lambda}_r}[P(R_r+1;\widetilde{\lambda}_r\widetilde{L}_r)-P(R_r+b;\widetilde{\lambda}_r\widetilde{L}_r)]\Bigg] \\
B_r &= \frac{\widetilde{\lambda}_r}{Q_r+w}\Bigg[\widetilde{\lambda}_r\widetilde{L}_r\left[P(R_r;\widetilde{\lambda}_r\widetilde{L}_r)-P(R_r+b-1;\widetilde{\lambda}_r\widetilde{L}_r)\right]-R_rP(R_r+1;\widetilde{\lambda}_r\widetilde{L}_r) \\
&\quad +(R_r+b)P(R_r+b;\widetilde{\lambda}_r\widetilde{L}_r)\Bigg] \\
E_r &= \frac{\widetilde{\lambda}_r w}{Q_r+w}
\end{aligned}
$$

10.2.4 Multi-objective optimization problem of the proposed supply chain

Following Grzegorzewski [45], the fuzzy numbers are now transformed to interval numbers and the Eq. (253) is expressed as

$$\widetilde{TC}(R_0,R_r,b)\equiv[TC_l,TC_r].$$

(For calculation of TC_l and TC_r, see Appendix E.)

From section 10.1.1, our optimization problem can be stated as

$$\text{Minimize}\quad \{TC_l(R_0,R_r,b),TC_r(R_0,R_r,b)\} \tag{256}$$

Normally, the multi - objective optimization problem in Eq.(256), in the case of minimization problem, is formulated in a conservative sense as

$$\text{Minimize}\quad \{TC_l(R_0,R_r,b),TC_c(R_0,R_r,b),TC_r(R_0,R_r,b)\} \tag{257}$$

where $TC_c=(TC_l+TC_r)/2$

Hence the interval problem in Eq.(253) is represented as

$$\text{Minimize}\quad \{TC_l(R_0,R_r,b),TC_c(R_0,R_r,b),TC_r(R_0,R_r,b)\} \tag{258}$$

$$\text{subject to}\quad Q_r\ge R_r+b+1, R_0\ge -NQ_r, R_r>0, b\ge 0.$$

The formulation in Eq.(258) gives better approximate solutions than those obtained from Eq.(257). Moreover, by the formulation in Eq.(258), the decision maker does have the freedom to choose any one of the three functions - TC_l, TC_c, TC_r for minimization.

10.2.5 Membership functions for the fuzzy parameters

Now considering the imprecise nature of decision maker's (DM) judgement, DM may have different fuzzy or imprecise goals for each of the objective functions.

To derive the membership functions $\mu_{TC_l}, \mu_{TC_r}, \mu_{TC_c}$ for the objective functions TC_l, TC_r, TC_c respectively, we first calculate individual minimum (i.e., TC_l^{min}, TC_r^{min}, TC_c^{min}) and individual maximum (i.e.,$TC_l^{max}, TC_r^{max}, TC_c^{max}$) by a non-linear optimization technique.

With the help of individual minimum and maximum, the DM can select any one among the following three types of membership functions

(i) Linear membership functions

(ii) Quadratic membership functions

(iii) Exponential membership functions

The membership functions μ_{TC_l}, μ_{TC_r} and μ_{TC_c} for the corresponding objective functions TC_l, TC_r and TC_c may be written as

$$\mu_{TC_z} = \begin{cases} 1 & \text{if } TC_z \leq TC_z^1 \\ d_z & \text{if } TC_z^1 \leq TC_z \leq TC_z^0 \\ 0 & \text{if } TC_z \geq TC_z^0 \end{cases} \tag{259}$$

where TC_z^1 and TC_z^0 are to be chosen such that $TC_z^{min} \leq TC_z^1 \leq TC_z^0 \leq TC_z^{max}$ and d_z is a strictly monotonic decreasing continuous function of TC_z which may be linear or non-linear.

Linear membership function (Type - 1)

The linear membership function is as follows:

$$\mu_{TC_z} = \begin{cases} 1 & \text{if } TC_z \leq TC_z^1 \\ 1 - \frac{TC_z - TC_z^1}{P_z} & \text{if } TC_z^1 \leq TC_z \leq TC_z^0 \\ 0 & \text{if } TC_z \geq TC_z^0 \end{cases} \tag{260}$$

where TC_z^1 and TC_z^0 are to be chosen such that $TC_z^{min} \leq TC_z^1 \leq TC_z^0 \leq TC_z^{max}$ and $P_z = TC_z^0 - TC_z^1$ is the tolerance of zth objective function TC_z (Fig. 35).

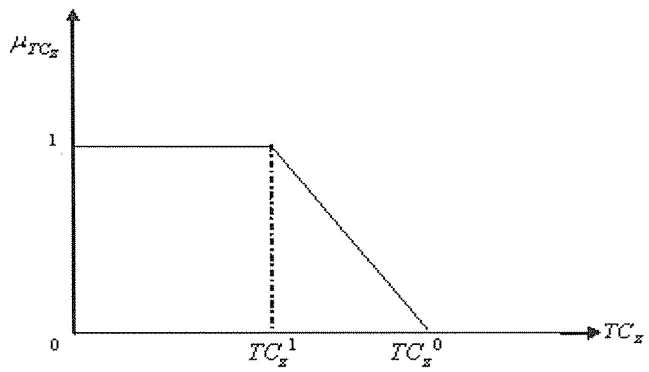

Figure 35: Pictorial representation of linear μ_{TC_z}

Quadratic membership function (Type - 2)

The quadratic membership function is as follows:

$$\mu_{TC_z} = \begin{cases} 1 & \text{if } TC_z \leq TC_z^1, \\ 1 - \left(\frac{TC_z - TC_z^1}{P_z}\right)^2 & \text{if } TC_z^1 \leq TC_z \leq TC_z^0 \\ 0 & \text{if } TC_z \geq TC_z^0 \end{cases} \tag{261}$$

where TC_z^1 and TC_z^0 are to be chosen such that $TC_z^{min} \leq TC_z^1 \leq TC_z^0 \leq TC_z^{max}$ and $P_z = TC_z^0 - TC_z^1$ is the tolerance of zth objective function TC_z (Fig. 36).

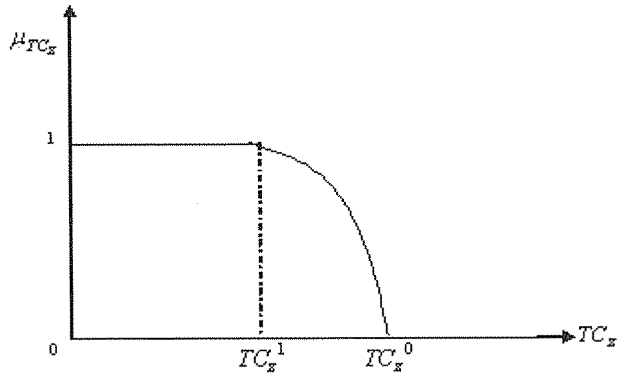

Figure 36: Pictorial representation of quadratic μ_{TC_z}

Exponential membership function (Type - 3)

The exponential membership function is as follows:

$$\mu_{TC_z} = \begin{cases} 1 & \text{if } TC_z \leq TC_z^1, \\ \alpha_z \left[1 - e^{-\beta_z \left(\frac{TC_z - TC_z^1}{P_z} \right)} \right], & \text{if } TC_z^1 \leq TC_z \leq TC_z^0 \\ 0 & \text{if } TC_z \geq TC_z^0 \end{cases} \tag{262}$$

The constants $\alpha_z > 1$ and $\beta_z > 0$ can be determined by asking the DM to specify the three points $TC_z^1, TC_z^{0.5}, TC_z^0$ such that$TC_z^{min} \leq TC_z^1 \leq TC_z^{0.5} \leq TC_z^0 \leq TC_z^{max}$ and $P_z = TC_z^0 - TC_z^1$ is the tolerance of zth objective function TC_z (Fig. 37).

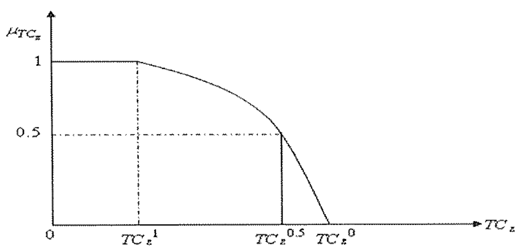

Figure 37: Pictorial representation of exponential μ_{F_K}

10.2.6 Fuzzy decision making method

After determining the different linear/ non-linear membership functions (MF) for each of the objective functions, following Zimmermann [117], the problem in Eq.(258) can be formulated as

$$\begin{aligned}
&\text{Max} \quad \lambda \\
&\quad \text{subject to} \\
&\quad\quad \lambda \leq \mu_{TC_l}, \lambda \leq \mu_{TC_c}, \lambda \leq \mu_{TC_r}, \\
&\quad\quad Q_r \geq R_r + b + 1, R_0 \geq -NQ_r, R_r > 0, b \geq 0, \\
&\quad\quad 0 \leq \lambda \leq 1.
\end{aligned} \tag{263}$$

with the help of two different types of membership functions given by Eqs.(260) and (261), the above problem can be restated for a particular choice of DM as

$$\begin{aligned}
&\text{Max} \quad \lambda \\
&\quad \text{subject to} \\
&\quad\quad \lambda \leq 1 - \frac{TC_l - TC_l^1}{P_l}, \text{ if the MF of first objective} \in \text{Type-1}, \\
&\quad\quad \lambda \leq 1 - \frac{TC_c - TC_c^1}{P_c}, \text{ if the MF of second objective} \in \text{Type-1}, \\
&\quad\quad \lambda \leq 1 - \left(\frac{TC_r - TC_r^1}{P_r}\right)^2, \text{ if the MF of third objective} \in \text{Type-2}, \\
&\quad\quad Q_r \geq R_r + b + 1, R_0 \geq -NQ_r, R_r > 0, b \geq 0. \\
&\quad\quad 0 \leq \lambda \leq 1.
\end{aligned} \tag{264}$$

Here DM selects the above membership functions for the corresponding objective functions. Then the above problem can be solved by a non-linear optimization technique and optimal solution of λ, say λ^* is obtained.

Now after obtaining λ^*, the DM selects the most important objective function from among the objective functions $TC_l, TC_c,$ and TC_r. Here TC_r is selected as DM would like to minimize his worst case. Then the problem becomes (for $\lambda = \lambda^*$)

$$\begin{aligned}
&\text{Min } TC_r \\
&\quad \text{subject to} \\
&\qquad TC_l \leq m_l,\ TC_c \leq m_c,\ TC_r \leq m_r, \\
&\qquad Q_r \geq R_r + b + 1, R_0 \geq -NQ_r, R_r > 0, b \geq 0, \\
&\qquad 0 \leq \lambda \leq 1.
\end{aligned} \tag{265}$$

where

$$\begin{aligned}
m_l &= TC_l^1 + P_l(1-\lambda^*), \text{ if the MF of first objective} \in \text{Type-1} \\
m_c &= TC_c^1 + P_c(1-\lambda^*), \text{ if the MF of second objective} \in \text{Type-1} \\
m_r &= TC_r^1 + P_r\sqrt{(1-\lambda^*)}, \text{ if the MF of third objective} \in \text{Type-2}
\end{aligned}$$

10.2.7 Pareto-optimal solutions

Now, after deriving the optimum decision variables, Pareto-optimality test is performed according to Sakawa [91]. Let the decision variables R_0^*, R_r^*, b^* and the optimum values, $TC_l^* = TC_l(R_0^*, R_r^*, b^*), TC_r^* = TC_r(R_0^*, R_r^*, b^*)$ and $TC_c^* = TC_c(R_0^*, R_r^*, b^*)$ are obtained from Eq.(265). With these values, the following problem is solved using a non-linear optimization technique:

$$\begin{aligned}
&\text{Minimize } \Delta = \delta_l + \delta_c + \delta_r \\
&\quad \text{subject to} \\
&\qquad TC_l + \delta_l = TC_l^*,\ TC_c + \delta_c = TC_c^*,\ TC_r + \delta_r = TC_r^*, \\
&\qquad \delta_l, \delta_c, \delta_r \geq 0, \\
&\qquad Q_r \geq R_r + b + 1, R_0 \geq -NQ_r, R_r > 0, b \geq 0, \\
&\qquad 0 \leq \lambda \leq 1.
\end{aligned} \tag{266}$$

The optimal solutions of Eq.(266), say $\bar{R}_0, \bar{R}_r, \bar{b}, \overline{TC}_l, \overline{TC}_c,$ and $\overline{TC}_r$ are called the strong Pareto-optimal solutions of the problem in Eq.(265) provided Δ is very small; Otherwise it is weak Pareto-optimum.

10.3 Numerical Examples

Depending upon the backlogging parameter b at a retailer, the following three different scenarios may arise.

Scenario-I. When $0 < b < \infty$, the supply chain considers partial backlogging at a retailer during stockout.

Scenario-II. When $b = 0$, the supply chain considers lost sales alone at a retailer during shortages.

Scenario-III. When $b = \infty$, we consider only backordered demand during shortages at a retailer.

To illustrate the proposed supply chain model, following input data are considered: $n = 5$, $\widetilde{\lambda}_r = (0.5,\ 1,\ 1.5)$, $\widetilde{L}_r$=(1, 1.5, 2), $\widetilde{L}_0$=(1, 1.5, 2), $Q_r = 8$, $Q_0 = 32$, $\widetilde{h}_r =$ (1, 1.8, 2.5), $\widetilde{h}_0 = (1.5,\ 2,\ 2.5)$, $\widetilde{\pi}_r = (25,\ 30,\ 35)$, $\widetilde{\pi}_p = (20,\ 25,\ 30)$.

10.3.1. Case - I (when fuzzy parameters are TFN)

Considering the above fuzzy parameters as Triangular fuzzy numbers (TFN), the nearest interval approximations according to Grzegorzewski [45] are

$\widetilde{\lambda}_r \equiv [0.75, 1.25]$, $\widetilde{L}_r \equiv [1.25, 1.75]$, $\widetilde{L}_0 \equiv [1.25, 1.75]$, $\widetilde{h}_r \equiv [1.4, 4.3]$, $\widetilde{h}_0 \equiv [1.75, 2.25]$, $\widetilde{\pi}_r \equiv [27.5, 32.5]$, $\widetilde{\pi}_p \equiv [22.5, 27.5]$.

Following Eqs.(263) and (265), the optimization problem in Eq.(258) is solved and the results are presented in Tables 18 and 19. Let, with the above values, the membership functions of the objective functions be formed for the types as per Table 20. Let, at the beginning, analysis be performed to find optimum λ with the membership functions TC_l, TC_c as linear (Type-1) and TC_r as Quadratic (Type - 2). The optimum value λ is presented in Table 21. With this value of λ^*, the objective function TC_r is optimized and the optimum results are in Table 22.

Table 18. Individual minimum and maximum of objective functions:

Objective functions	Minimum			Maximum		
	Scenario I	Scenario II	Scenario III	Scenario I	Scenario II	Scenario III
TC_l	60.6551	114.5984	168.5480	88.8573	137.3885	197.2668
TC_c	96.2663	136.2378	184.4993	121.9979	156.6242	208.8418
TC_r	133.3176	157.8772	200.4506	155.1385	175.8610	277.416

Table 19

Input data for TC_z^1,TC_z^0 :

	TC_l^1	TC_l^0	TC_c^1	TC_c^0	TC_r^1	TC_r^0
Scenario - I	60.6551	88.6551	96.2663	121.2663	133.3176	154.3176
Scenario - II	114.5984	136.5984	136.2378	156.2378	157.8772	174.8772
Scenario - III	168.5480	196.5480	184.4993	238.548	200.4506	279.4506

Table 20

Possible values of MF for objective functions:

Objective functions	Type of membership functions
TC_l	Type - 1 or Type - 2 or Type - 3
TC_c	Type - 1 or Type - 2 or Type - 3
TC_r	Type - 1 or Type - 2 or Type - 3

Table 21

Optimal value of λ

Maximum λ	Scenario - I	Scenario - II	Scenario - III
λ^*	0.890	0.956	0.999

Table 22

Optimal results when TC_r is chosen as the most important objective function:

	R_0^*	R_r^*	b^*	$[TC_l^*, TC_r^*]$	TC_c^*
Scenario - I	-8	2	3	[64.9187, 139.0658]	101.9923
Scenario - II	-32	4	-	[115.5985, 160.8773]	138.2379
Scenario - III	16	5	-	[171.5481, 206.4507]	188.9994

Table 23

Pareto optimal results:

	$\bar{R}_0$	$\bar{R}_r$	$\bar{b}$	$[\overline{TC}_l, \overline{TC}_r]$	$\overline{TC}_c$	Δ
Scenario - I	-8	2	3	[64.9187, 139.0658]	101.9923	0.00014
Scenario - II	-32	4	-	[115.5985, 160.8761]	138.2379	0.00010
Scenario - III	16	5	-	[171.5481, 206.4507]	188.9994	0.00005

The results obtained from Table 22 are tested for Pareto-optimality and Pareto-optimal results are given in Table 23.

In Table 23, the values of Δ are quite small and hence, the optimum results in Table 22 are strong Pareto-optimum and can be accepted. Still, if the decision-maker/ practitioner is not satisfied with the outputs, he may perform the above analysis again by re-choosing the membership functions for TC_l, TC_c and TC_r as linear, quadratic and exponential (say). If the second time analysis does not also give the desired result, the DM may perform the analysis with the other possible different combinations of the membership functions and can select the most suitable optimum solution for his industry for implementation.

10.3.2 Case - II (when fuzzy parameters are PFN)

Considering the fuzzy parameters as Parabolic fuzzy numbers (PFN), the nearest interval approximations are

$\widetilde{\lambda}_r \equiv [0.67, 1.33]$, $\widetilde{L}_r \equiv [1.17, 1.83]$, $\widetilde{L}_0 \equiv [1.27, 2.27]$, $\widetilde{h}_r \equiv [1.67, 2.33]$, $\widetilde{h}_0 \equiv [1.67, 2.33]$, $\widetilde{\pi}_r \equiv [26.67, 33.33]$, $\widetilde{\pi}_p \equiv [21.3, 28.3]$.

Table 24

Individual minimum and maximum of objective functions:

Objective functions	Minimum			Maximum		
	Scenario I	Scenario II	Scenario III	Scenario I	Scenario II	Scenario III
TC_l	56.7540	107.8668	159.6900	85.2100	115.3128	212.3452
TC_c	99.6022	141.5797	235.6333	123.9282	154.8141	347.1646
TC_r	142.4504	175.2926	307.9366	162.6460	194.6460	481.9840

Table 25

Input data for TC_z^1,TC_z^0 :

	TC_l^1	TC_l^0	TC_c^1	TC_c^0	TC_r^1	TC_r^0
Scenario - I	56.7540	84.7540	99.6022	121.9689	142.4504	161.4504
Scenario - II	107.8668	113.8668	141.5797	154.5797	175.2926	193.2926
Scenario - III	159.6900	211.6900	235.6333	345.8133	307.9366	480.9366

Table 26

Optimal value of λ

Maximum λ	Scenario - I	Scenario - II	Scenario - III
λ^*	0.978	0.998	0.999

Table 27

Optimal results when TC_r is chosen as the most important objective function:

	R_0^*	R_r^*	b^*	$[TC_l^*, TC_r^*]$	TC_c^*
Scenario - I	-24	2	2	[58.3588, 145.2327]	101.7958
Scenario - II	0	4	-	[110.8668, 177.2927]	144.0798
Scenario - III	8	1	-	[163.5788, 309.9866]	236.7827

Table 28

Pareto optimal results:

	$\bar{R}_0$	$\bar{R}_r$	$\bar{b}$	$[\bar{TC}_l, \bar{TC}_r]$	$\bar{TC}_c$	Δ
Scenario - I	-24	2	2	[58.3588, 145.2327]	101.7958	0.0009
Scenario - II	0	4	-	[110.8668, 177.2927]	144.0799	0.00017
Scenario - I	8	1	-	[163.5788, 309.9866]	236.7827	0.00007

Here again, the values of Δ are very small and hence the optimum values in Table 27 are strong Pareto-optimum and can be accepted if DM is satisfied. If DM is not satisfied, he may perform the analysis with different combinations of membership functions.

Here, for comparison, we consider the average value of TC_l and TC_r. From Tables 22 and 27, it is clear that in scenario - I, optimum value of center of interval cost function (i.e., TC_c) is lower than those in the other two scenarios. Therefore from this point of view, DM should prefer scenario-I to the others. Here it is also observed that by considering linear membership function for the fuzzy numbers (i.e., system cost parameters, lead time, etc.) rather than the non-linear ones, the cost incurred by DM is marginally less.

10.4 Conclusion

In this chapter, we have proposed a possibilistic decision model to determine the optimal inventory policies with unreliable or unavailable statistical data. Fuzzy numbers were used to model uncertain and flexible SC parameters. Fuzzy numbers are then approximated to an interval number. After that, the problem has been converted into multi-objective optimization problem where the objective functions, represented by left limit, right limit and centers of interval function, are minimized. To obtain the solution of the multi-objective optimization problem, the interactive fuzzy solution procedure has been used. In the numerical examples, different scenarios have been considered depending upon the stockout control parameter at a retailer.