Chapter 9

A two-echelon supply chain system with Poisson demand and partial backorders

9.1 Introduction

We consider a two-echelon supply chain consists of a single supplier and many identical retailers, demanding for a consumable item at the supplier. The inventory control policy is assumed to be continuous review ***(R, Q)*** policy in all installations. We assume the demand process to be independent Poisson and the unsatisfied demands are partially backordered in the retailers.

Our motivation for the study of partial backorder control in the retailers (lower echelon) stems from the observations on real world inventory systems. When a firm is a sole supplier or when there is a lack of substitutions or competitors, retailer may prefer to backorder unfilled demands. Lost sales occur when the customers prefer not to wait for the next replenishment or when the firm decides to buy similar items from competitors to satisfy demands and maintain customer loyalty. However as pointed out by Peterson and Silver [84, pg.253] in most practical situations, one finds a combination of these two extreme policies where some of the excess demands are backordered and the rest are lost during a stockout. Additional motivation for this study stems from consulting experience with inventory control systems in the chemical industry. There, the partial backorder policy is implemented in the following manner:

Out of stock items are first backordered (because there is no a priori knowledge of how many items will be short during the lead time). When the quantity of the backordered items reaches a certain limit the future demands are lost until the replenishment order is fulfilled. Here, the partial backorder policy is implemented in the identical retailers using an explicit control parameter $\boldsymbol{b}$ which is the maximum allowable number of backorders during lead time. The unfilled demands beyond b are lost. Here, control variable $\boldsymbol{b}$ provides a real time control of the amount of backorders allowed in the retailers.

To the best of our knowledge, no model exists for two-echelon supply chain with partially backorder control in the retailers and Poisson demands under general batch ordering policy. So this chapter fulfills the gap. The rest of this chapter is organised as follows.

In next section, we review two special cases which will be used later. In section 9.4, we approximate the demand process at the supplier and lead time at the retailers. In section 9.5, we derive the total cost function of the supply chain. Section 9.6 provides several numerical examples and we obtain a lot of observations. Finally, section 9.7 concludes the chapter.

9.2 Problem description

We consider a two-echelon supply chain system in which a single supplier and a number of identical retailers are controlted by continuous review policy $(\boldsymbol{R},\boldsymbol{Q})$. We implement the partial backorder policy with general batch ordering policy in all the identical retailers and it provides pure backorder policy when $\boldsymbol{b}$ = oo and the lost sales policy when $\boldsymbol{b}$ = 0. The transportation time of each order placed by the retailers is assumed to be constant. The lead time for replenishing the supplier orders from an external warehouse is assumed to be constant and we assume the unsatisfied retailer's orders to be backordered in the supplier and all backlogged orders are filled according to a First In First Out (FIFO) - policy. The reorder point and batch size of the supplier are assumed to be integer multiples of the retailer's identical batch size.

We assume $\boldsymbol{Q}_r$ (the batch size of a retailer) is determined with a known replenishment cost at both supplier and retailers as many papers such as Axsater [9,10]

Deuermeyer and Schwarz [33] and Svoronos and Zipkin [109] have done before to simplify the problem. The objective is to find the optimal reorder points of all installations and optimal backorder limit b^* at the identical retailers by minimizing the total holding costs (of the supplier and retailers) and the stockout costs of the retailer. Here, in addition to the notations as in chapters 1 and 8, we consider the following notations. L_0 is the constant lead time of the supplier replenishment orders at the warehouse and $\hat{\pi}_r$ is the penalty cost per unit of backorder at a retailer.

9.3 Review of two special cases

The sections 9.3.1 and 9.3.2 would be used at later in our approximation. So we just refer these before going to the core part of the model development.

9.3.1 Exact solution for the backordering problem with Poisson demands

Considering a single echelon inventory system with continuous review policy, reorder point of R and batch size of Q, constant lead time for replenishing orders, demand generated by a Poisson process and backordered demand during a stockout, Hadley and Whitin [49] developed formula for the average stock level $D(Q, R)$ and for the average stockout level $B(Q, R)$. Assuming linear unit costs of holding and stockout, they obtained the related annual cost. We briefly review their results and introduce the parameters that they have used in their formulae since we will use them later.

$$B(Q,R) = \frac{1}{Q}\left[\beta(R) - \beta(R+Q)\right] \tag{222}$$

$$\beta(v) = \frac{(\lambda L)^2}{2} P(v-1;\lambda L) - (\lambda L)vP(v;\lambda L) + \frac{v(v+1)}{2}P(v+1;\lambda L) \tag{223}$$

$$D(Q,R) = \frac{Q+1}{2} + R - \lambda L + B(Q,R) \tag{224}$$

$$P(x;\lambda L) = \sum_{i=x}^{\infty} \frac{e^{-\lambda L}(\lambda L)^i}{i!}, \quad x = 0,1,2,3....$$

where

λ Demand rate (mean of Poisson demand distribution),

L Constant lead time.

9.3.2 Exact solution for the partially backordering problem with Poisson demand and constant lead time

Consider a single echelon inventory system (see Fig.28) with continuous review control policy, reorder point of R and batch size of Q. Demands are assumed to be Poisson with rate $\lambda > 0$ with one unit demanded at a time. A demand which arrives at time t is satisfied immediately if the inventory level $I(t) > 0$, it is backordered if $I(t) \leq 0$ and amount of shortage is less than or equal to b. All shortages beyond b are lost.

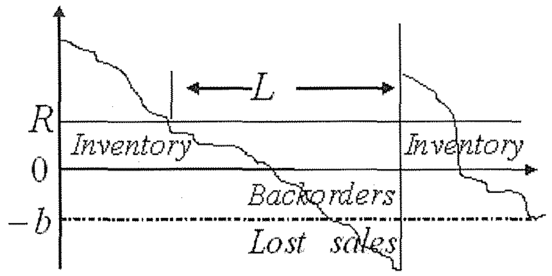

Figure 28: Partial backorder policy with control parameter b

Here cycle time is the time between two successive reorder times. When the replenishment of size Q arrives at a fixed lead time L, all the backorders are fulfilled. This is possible because we assume that almost one order outstanding is allowed i.e., the accumulated backorder up to the replenishment time L, $B(L) \leq b$ and $Q \geq R + b + 1$. Gad Rabinowitz et al. [37] derived the expressions for expected accumulated inventory in a cycle, expected number of lost sales in a cycle and expected number of units backordered in a cycle. We just review the results and use them later.

Expected time during which the unsatisfied demand are lost

$$\hat{T} = LP(R + b; \lambda L) - \frac{R + b}{\lambda} P(R + b + 1; \lambda L) \tag{225}$$

The expected cycle length

$$T = \frac{Q}{\lambda} + \hat{T} \tag{226}$$

The expected number of cycles is

$$\frac{\lambda}{Q+\lambda\hat{T}} \tag{227}$$

Average number of lost sales incurred per unit time

$$L(Q,R,b) = \frac{\lambda}{Q+\lambda\hat{T}}(\lambda\hat{T}) \tag{228}$$

Average number of backorders incurred per unit time

$$\begin{aligned} B(Q,R,b) = {} & \frac{\lambda}{Q+\lambda\hat{T}}(\lambda L\left[P(R;\lambda L) - P(R+b-1;\lambda L)\right] - RP(R+1;\lambda L) \\ & + (R+b)P(R+b;\lambda L)) \end{aligned} \tag{229}$$

Average stock level

$$\begin{aligned} D(Q,R,b) \; = \; & \frac{\lambda}{Q+\lambda\hat{T}}\left[\frac{Q(Q+1)}{2\lambda} + \frac{QR}{\lambda} - QL + \frac{b(b-1)-2Q(R+b)}{2\lambda}P(R+b;\lambda L)\right. \\ & + QL\,P(R+b-1;\lambda L) + RL\left[P(R+b-1;\lambda L) - P(R;\lambda L)\right] \\ & + \frac{\lambda L^2}{2}\left[P(R-1;\lambda L) - P(R+b-2;\lambda L)\right] \\ & \left. + \frac{R(R+1)}{2\lambda}\left[P(R+1;\lambda L) - P(R+b;\lambda L)\right]\right] \end{aligned} \tag{230}$$

If b = 0, the equations (225), (228) and (230) reduce to the expressions for lost sales model given by Hadley and Whitin [49] (section 4.11).

9.4 Approximations in the lower and higher echelon

In this section, we are going to approximate the demand process in the higher echelon and retailer's lead time in the lower echelon.

9.4.1 Approximating the demand process in the higher echelon

The average number of cycles in each retailer is given by Eq. (227) of section 9.3.2 when the unsatisfied demand is partially backordered during a stock out, stochastic process generating demands is Poisson and lead time is constant in all the retailers. From Eq. (227), the average number of cycles in a retailer is

$$T^{-1} = \frac{\lambda}{Q+\lambda\left[LP(R+b;\lambda L) - \frac{R+b}{\lambda}P(R+b+1;\lambda L)\right]} \tag{231}$$

since a batch size of Q is ordered in each cycle, the mean rate of demand (from this inventory system to a higher echelon) will be T^{-1} in terms of the identical batch size of Q.

When we consider pure backorder policy in all the identical retailers and the number of retailers in the model is large, the arrival process of the orders in the supplier can be well approximated by a Poisson process with mean rate $\sum_{i=1}^{\infty}(\lambda_i/Q)$ (λ_i is the demand rate at a retailer i and Q is the identical batch size of the retailers) as Moinzadeh and Lee [75]. Muckstadt [78], Deuermeyer and Schwarz [33], Albin [3] and Zipkin [119] used this approximation. When we consider lost sales policy in all the retailers and the number of identical retailers (n) is large, the arrival process of the orders in the supplier can be well approximated by a Poisson process with mean rate

$$\frac{n\lambda_r}{Q_r + \lambda_r\left[L_r P(R_r; \lambda_r L_r) - \frac{R_r}{\lambda_r} P(R_r + 1; \lambda_r L_r)\right]}$$

In our model, we are implementing partial backordering policy in all the identical retailers instead of backordered or lost sales alone during a stockout.

Using the spirit of their nice approximations, we can approximate the demand process in the supplier (higher echelon) when the partially backorder control is maintained during a stockout at all identical retailers (lower echelon). The average number of cycles in each retailer is T^{-1} as given in Eq.(231). Since a batch size of Q_r is ordered in each cycle, the mean rate of demand (from this inventory system to a higher echelon) will be T^{-1} in terms of the identical batch size Q_r. We assume the number of retailers is large, extending for the case of partially backordering policy in all retailers, the arrival process of the orders in the supplier can be well approximated by a Poisson process with mean rate

$$\frac{n\lambda_r}{Q_r + \lambda_r\left[L_r P(R_r + b; \lambda_r L_r) - \frac{R_r + b}{\lambda_r} P(R_r + b + 1; \lambda_r L_r)\right]} \tag{232}$$

in terms of the identical batch size of Q_r. Clearly when we have, b $= 0$, our approximation reduces to Seifbarghy and Akbari Jokar [96] approximation. Also, if b $= \infty$ and for the case of non-identical retailer with demand rate λ_i (Poisson process), then our approximation reduces to Moinzadeh and Lee [75] approximation. Hence our approximation is valid for the case of partially backordering policy in all the identical retailers.

9.4.2 Retailers lead time approximation

Retailers at the lower echelon of the model experience independent Poisson demand processes. Unsatisfied demand during a stock out is partially backordered. The maximum number of backorders allowed to accumulate during a cycle is b and the unfilled demands beyond b are lost. The minimum lead time for each retailer is the transportation time for their orders from supplier to a retailer. The lead time of the retailer is approximated as the sum of transportation time and expected time lapsed (waiting time) due to stockout in the supplier while the retailer is ordering. In our model, we are assuming constant transportation time and so we have to analyze the waiting time distribution for the retailers. This waiting time does not have any clear distribution.

Waiting time $\bar{w}$ is zero if the retailer's orders incur when the supplier is in the state of fulfilling the orders and is positive if the retailer's orders incur when the supplier is out of stock. By Little's formula in queueing theory

$$\text{Waiting time} = \frac{\text{Queue length}}{\text{Rate of arrival}} \tag{233}$$

Therefore the waiting time of the retailers due to stockout in the supplier is

$$\frac{\text{Average number of backorders in the supplier}}{\text{Rate of arrival in the supplier}}$$

Based on the approximation of demand process at the supplier described in section 9.4.1, the supplier behaves just like an inventory system of type described in section 9.3.1 and we can use formula in Eq.(222) to find average number of backorders in the supplier. But Eq. (222) is valid when the customer demands occur one at a time. If we assume that batch size (Q_0) and reorder point (R_0) of the supplier are integer multiples of identical batch size of the retailer (Q_r), then we can use the Eq. (222). Hence the average waiting time of the orders placed by identical retailers is

$$\bar{w} = \frac{B(Q_0/Q_r, R_0/Q_r)}{\lambda_0} \tag{234}$$

where

$$\begin{aligned} \lambda_0 &= \frac{n\lambda_r}{Q_r + \lambda_r \hat{T}_r} \\ \hat{T}_r &= L_r P(R_r + b; \lambda_r L_r) - \frac{R_r + b}{\lambda_r} P(R_r + b + 1; \lambda_r L_r) \end{aligned} \tag{235}$$

Based on our approximation, the approximated lead time of the retailer is sum of transportation time (L_r) and the average waiting time $\bar{w}$ due to stock out in the supplier. It can be used for evaluating the costs incurred in the retailers.

9.5 Total cost of a two-echelon supply chain

Let C_r be the total cost incurred in a retailer and C_0 be the total cost incurred in the supplier. Based on the results of the previous sections, the total cost (TC) of the two-echelon inventory system is $C_0 + nC_r$. The supplier cost consists of just holding cost as follows

$$C_0 = h_0 D(Q_0/Q_r, R_0/Q_r)Q_r \tag{236}$$

In the above formula $D(Q_0/Q_r, R_0/Q_r)$ is the average stock level in the supplier and it is obtained using Eq. (224). Noting that Q_0 should be an integer multiple of Q_r, we have

$$D(Q_0/Q_r, R_0/Q_r) = \frac{1}{2}\left[\frac{Q_0}{Q_r} + 1\right] + \frac{R_0}{Q_r} - \lambda_0 L_0 + B(Q_0/Q_r, R_0/Q_r) \tag{237}$$

The average backorder level $B(Q_0/Q_r, R_0/Q_r)$ in the supplier is obtained using Eqs. (222) and (223) from section 9.3.1. Noting again that Q_0 should be an integer multiple of Q_r, we have

$$B(Q_0/Q_r, R_0/Q_r) = \frac{1}{Q_0/Q_r}\left[\beta(R_0/Q_r) - \beta\left(\frac{R_0 + Q_0}{Q_r}\right)\right] \tag{238}$$

where

$$\beta(v) = \frac{(\lambda_0 L_0)^2}{2}P(v-1;\lambda_0 L_0) - (\lambda_0 L_0)vP(v;\lambda_0 L_0) + \frac{v(v+1)}{2}P(v+1;\lambda_0 L_0) \tag{239}$$

Knowing λ_0 and $B(Q_0/Q_r, R_0/Q_r)$, we can determine $\bar{w}$ from Eq. (240) as explained in section 9.4.2,

$$\bar{w} = \frac{B(Q_0/Q_r, R_0/Q_r)}{\lambda_0} \tag{240}$$

We observe that $\bar{w}$ is dependent on λ_0 and λ_0 itself depending on the expected time during which the retailer is in out of stock. Since the stochastic distribution of lead time for a retailer is not clear, $\hat{T}_r$ does not have any exact form in this complicated model. We therefore approximate it by the constant lead time formula in section

9.4.2 as given in Eq.(235). Our experience shows that the effect of this approximation is negligible on λ_0. However, our numerical results will show how accurate this approximation is.

Now, we consider retailer cost as a sum of expected holding cost, expected cost incurred due to lost sales and expected backordered cost. Let D_r be the average stock level in a retailer when the lead time of a retailer is $L_r + \bar{w}$. Let E_r be the average number of lost sales incurred per unit time in a retailer when the lead time of a retailer is $L_r + \bar{w}$. Let B_r be the average number of backorders incurred per unit time in a retailer when the lead time of a retailer is $L_r + \bar{w}$. Hence the total cost incurred at each retailer is

$$C_r = h_r D_r + \pi_r E_r + \hat{\pi}_r B_r \tag{241}$$

where

$$\begin{aligned}
D_r &= \frac{\lambda_r}{Q_r + \lambda_r \hat{T}_r'} \Bigg[\frac{Q_r(Q_r+1)}{2\lambda_r} + \frac{Q_r R_r}{\lambda_r} - Q_r(L_r + \bar{w}) \\
&\quad + \frac{b(b-1) - 2Q_r(R_r+b)}{2\lambda_r} P(R_r + b; \lambda_r(L_r+\bar{w})) \\
&\quad + Q_r(L_r+\bar{w})\, P(R_r+b-1; \lambda_r(L_r+\bar{w})) + R_r(L_r+\bar{w})[P(R_r+b-1;\lambda_r(L_r+\bar{w})) \\
&\quad - P(R_r;\lambda_r(L_r+\bar{w}))] + \frac{\lambda_r(L_r+\bar{w})^2}{2}[P(R_r-1;\lambda_r(L_r+\bar{w})) - P(R_r+b-2;\lambda_r(L_r+\bar{w}))] \\
&\quad + \frac{R_r(R_r+1)}{2\lambda_r}[P(R_r+1;\lambda_r(L_r+\bar{w})) - P(R_r+b;\lambda_r(L_r+\bar{w}))] \Bigg] \qquad (242) \\
E_r &= \frac{\lambda_r}{Q_r + \lambda_r \hat{T}_r'} \Big[\lambda_r(L_r+\bar{w}) P(R_r+b;\lambda_r(L_r+\bar{w})) - (R_r+b)P(R_r+b+1;\lambda_r(L_r+\bar{w})) \Big] \\
&\qquad (243) \\
B_r &= \frac{\lambda_r}{Q_r + \lambda_r \hat{T}_r'} \Big[\lambda_r(L_r+\bar{w})\,[P(R_r;\lambda_r(L_r+\bar{w})) - P(R_r+b-1;\lambda_r(L_r+\bar{w}))] \\
&\quad - R_r P(R_r+1;\lambda_r(L_r+\bar{w})) + (R_r+b)P(R_r+b;\lambda_r(L_r+\bar{w})) \Big] \qquad (244) \\
\hat{T}_r' &= (L_r+\bar{w})P(R_r+b;\lambda_r(L_r+\bar{w})) - \frac{R_r+b}{\lambda_r} P(R_r+b+1;\lambda_r(L_r+\bar{w})) \qquad (245)
\end{aligned}$$

It is clear that we should find R_0^*, R_r^*, b^* where

R_0^* – Optimal reorder point in the supplier.

R_r^* – Optimal reorder point in the identical retailers.

b^* – Optimal backorder limit in the retailers.

These optimal values are found by minimizing the total cost TC which is a non-linear function of R_0, R_r and b by the known batch order size in all installations. Since there should not be more than one order outstanding in each retailer at any time, we should consider the constraint $Q_r \geq R_r + b + 1 > 0$. Further more, since there are n retailers in our model and none of them can have more than one order outstanding, we have $R_0 \geq -nQ_r$. This is because if $R_0 < -nQ_r$, then the reorder point is never reached in the supplier. Therefore, from the above explanations, we can easily find optimum value of the total cost TC. Summarizing the above, our optimization problem is

$$\text{Minimize } TC(R_0, R_r, b)$$

$$\text{subject to}$$

$$Q_r \geq R_r + b + 1$$

$$R_0 \geq -nQ_r$$

$$R_r > 0 \text{ and } b \geq 0$$

9.6 Numerical illustration

We have designed a set of 27 numerical problems. We developed a problem set which offered a reasonable range of model parameters. It is necessary to mention that the optimal reorder points of all installations and optimal backorder limit 'b' were found. We also simulated each numerical problem 20 times (having 20 runs), for the optimal reorder points and b obtained from the approximated model, using GPSS/H simulation software. Different starting number seeds were employed for each problem. The simulation time length of each run considered is sufficient for the system to reach a steady state. This is also clear from the standard deviations of the total system cost. The cost error is obtained by the following relation:

$$\text{Cost error} = \frac{|\text{simulated total cost - approximated total cost}|}{\text{simulated total cost}}$$

We also report retailers fill rate (fraction of time with positive stock on hand). This can be obtained from the relation:

$$\text{Fill rate} = 1 - \frac{\text{(Average number of backorders + average number of lost sales) per unit time}}{\text{simulated total cost}}$$

The above relation has been employed in the approximation and also in the simulation model to find fill rates.

The numerical problems are as in Table 14. We consider different values of π_r to assess the accuracy of the approximations for various fill rates obtained. We fix $\hat{\pi}_r = 20$ (naturally, $\hat{\pi}_r \leq \pi_r), h_0 = h_r = 1, L_0 = L_r = 1$. The number of retailers, n, is considered is considered 10 and 20 (a large number enough to approximate the demand distribution as Poisson in the supplier as Moinzadeh and Lee [75] and Mehdi Seifbarghy and Akbari Jokar [96]) to compare the results when the number of retailers is changed.

Table 14 : Input data

No	π_r	Q_0	Q_r	λ_r	No	π_r	Q_0	Q_r	λ_r
1	25	16	8	0.5	15	50	32	8	1.5
2	25	16	8	1.0	16	50	64	8	0.5
3	25	16	8	1.5	17	50	64	8	1.0
4	25	32	8	0 5	18	50	64	8	1.5
5	25	32	8	1.0	19	100	16	8	0.5
6	25	32	8	1.5	20	100	16	8	1.0
7	25	64	8	0.5	21	100	16	8	1.5
8	25	64	8	1.0	22	100	32	8	0.5
9	25	64	8	1.5	23	100	32	8	1.0
10	50	16	8	0.5	24	100	32	8	1.5
11	50	16	8	1.0	25	100	64	8	0.5
12	50	16	8	1.5	26	100	64	8	1.0
13	50	32	8	0.5	27	100	64	8	1.5
14	50	32	8	1.0					

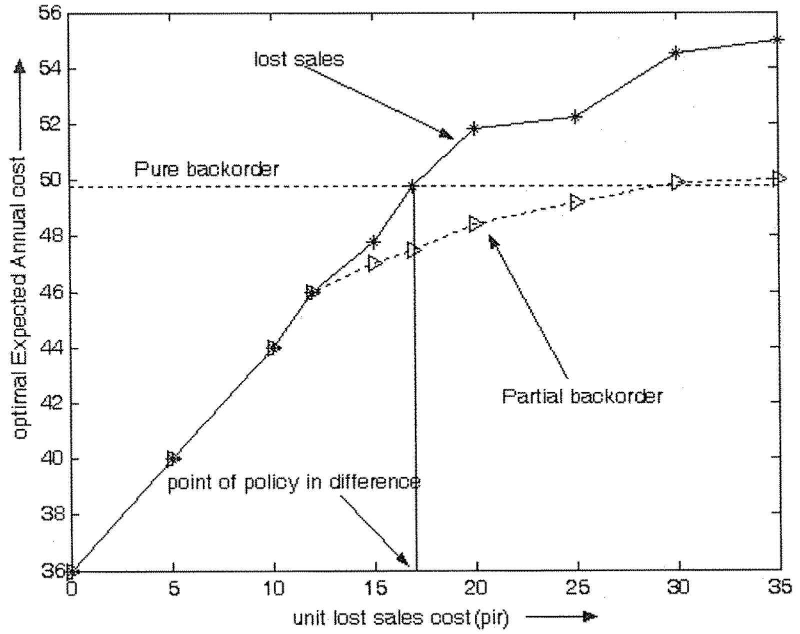

Figure 29: Expected annual cost for three policies when Q_0 = 32, Q_r = 4, λ_r = 0.5, $h_0 = h_r = 1$, $L_r = 1$,$\hat{\pi}_r = 0$

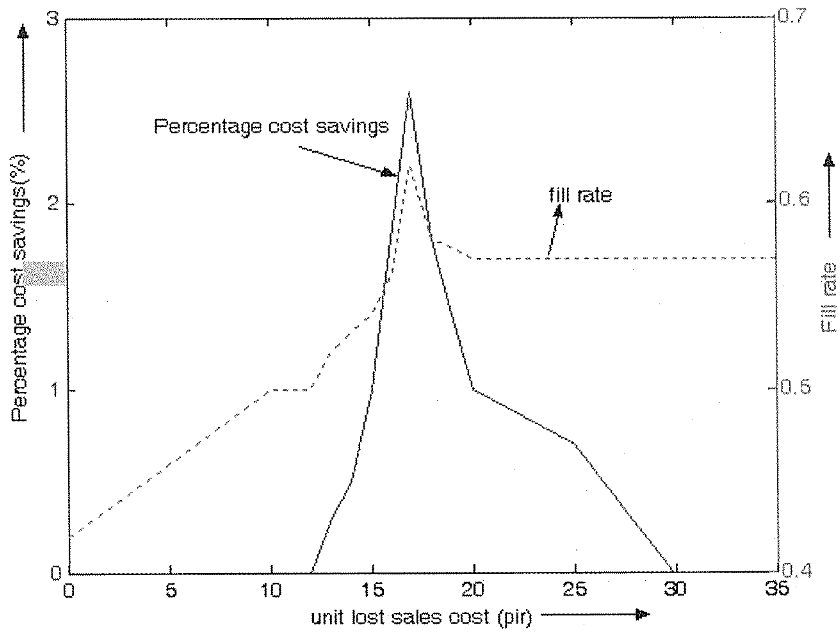

Figure 30: Percentage of cost saving due to partial backordering policy and fill rates when Q_0 = 32, Q_r = 4, λ_r = 0.5, $h_0 = h_r = 1$, $L_r = 1$,$\hat{\pi}_r = 0$

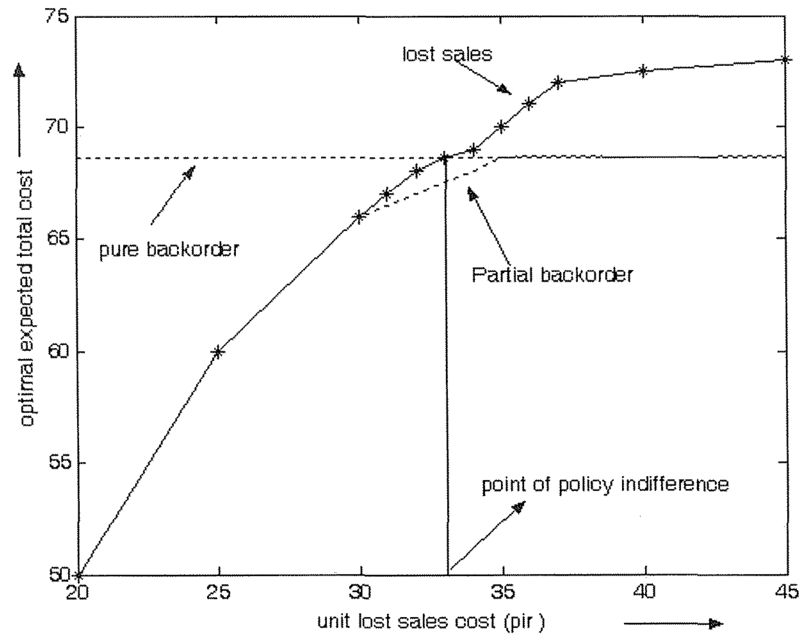

Figure 31: Expected annual cost for three policies when Q_0 = 32, Q_r = 4, λ_r = 0.5, $h_0 = h_r = 1$, $L_r = 1, \hat{\pi}_r = 20$

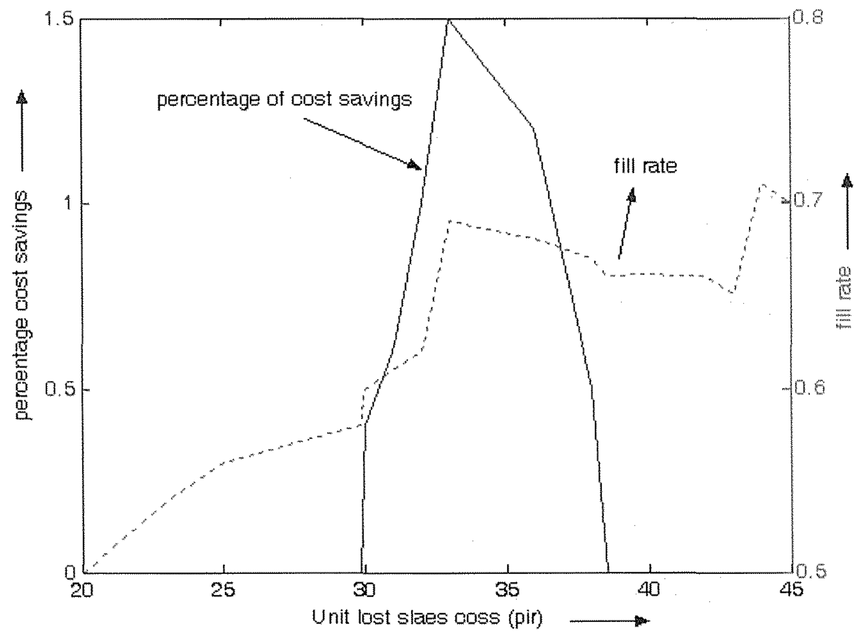

Figure 32: Percentage of cost saving due to partial backordering policy and fill rates when Q_0 = 32, Q_r = 4, λ_r = 0.5, $h_0 = h_r = 1$, $L_r = 1, \hat{\pi}_r = 20$

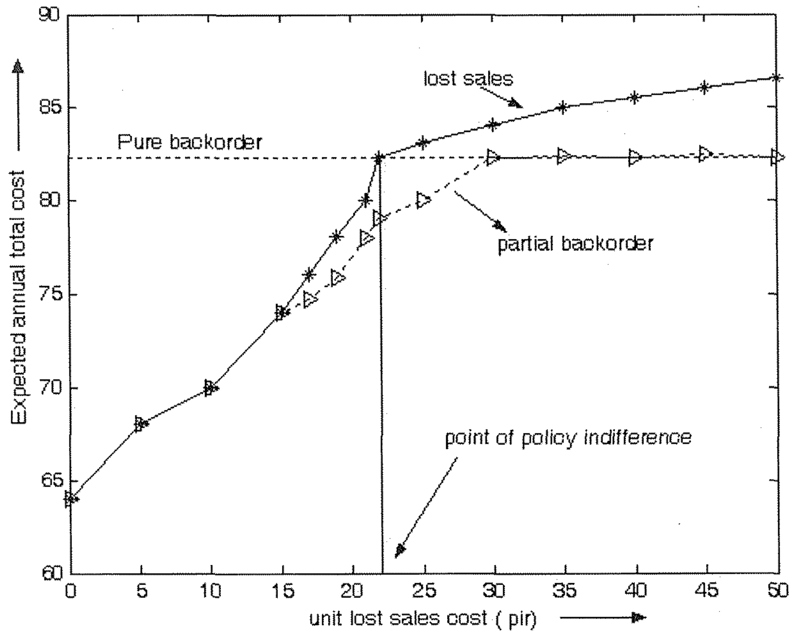

Figure 33: Expected annual cost for three policies when Q_0 = 32, Q_r = 4, λ_r = 0.5, h_0 = 1, $h_r = 10$, $L_r = 1, \hat{\pi}_r = 0$

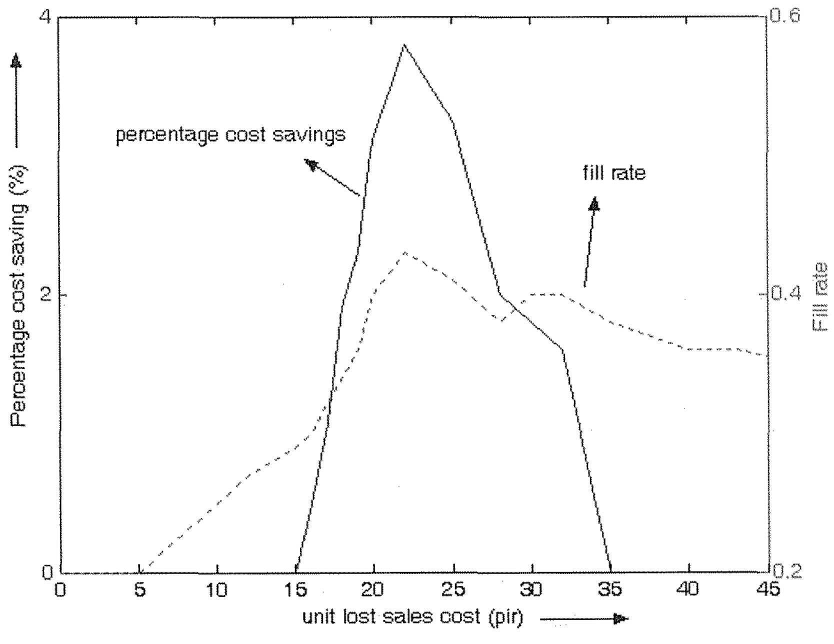

Figure 34: Percentage of cost saving due to partial backordering policy and fill rates when Q_0 = 32, Q_r = 4, λ_r = 0.5, h_0 = 1, $h_r = 10$, $L_r = 1, \hat{\pi}_r = 0$

Table 15

Total cost,service level, and waiting time results when π_r =25
(Appr = Appoximated, Simu = Simulated, AWT = Average waiting time)

No.	Appr			simu		Error(%)	Servicelevel(%)		Error(%)	AWT		Error(%)
	R_0	R_r	Total cost	Mean of total cost	St.dev		Appr	Simu		Appr	Simu	
1	-32	5	48.9653	50.86	0.10	3.7	89.91	91.23	1.5	10.072	9.8270	2.5
2	-24	5	64.4602	65.12	0.05	1.01	93.49	93.79	0.3	4.3601	3.9603	10.1
3	-8	6	75.3070	76.04	0.08	0.96	97.11	97.21	0.1	2.7895	2.7850	0.16
4	-40	5	45.9048	46.09	0.07	0.40	85.04	84.88	0.2	11.685	10.1372	15.3
5	-40	7	60.9651	61.00	0.12	0.06	93.46	92.87	0.6	5.9728	5.6423	5.9
6	-24	7	74.0229	74.86	0.05	1.12	96.92	96.03	0.9	3.3303	3.1307	6.4
7	-48	7	45.2557	45.98	0.09	1.58	86.38	85.83	0.6	14.755	12.8761	14.6
8	-48	7	59.3805	60.13	0.06	1.25	91.33	90.79	0.6	6.4413	6.4412	0
9	-32	7	72.7095	73.69	0.03	1.33	96.44	96.23	0.2	3.4387	3.0375	13.2
for the	case of	10	retailers		mean	1.27	92.23	92.10	0.6	-	-	7.56
					St.devi	1.03	4.45	4.42	0.4	-	-	6.01
1	-24	2	105.7417	106.24	0.06	0.46	92.45	92.44	0.01	4.3631	4.2531	2.59
2	0	3	130.3332	131.02	0.05	0.52	94.95	94.08	0.92	2.5267	2.4678	10.84
3	8	5	149.3572	150.2	0.02	0.61	96.30	95.09	1.27	2.4652	2.2241	24.03
4	-56	5	101.1934	102.14	0.07	0.93	87.93	87.36	0.65	10.747	10.2423	10.84
5	-24	4	129.6748	130.46	0.1	0.60	95.13	95.12	0.01	3.2368	2.9831	8.50
6	-16	6	149.7579	150.08	0.09	0.21	96.63	95.36	1.33	2.8971	2.7018	7.23
7	-72	7	97.6604	98.26	0.13	0.61	82.99	82.90	0.11	15.931	15.6708	1.66
8	-56	6	127.0281	128.12	0.06	0.85	94.31	94.10	0.22	4.9720	4.4817	10.94
9	-40	7	148.9100	148.98	0.08	0.05	96.31	95.89	0.44	3.4686	3.0616	13.29
for the	case of	20	retailers		mean	0.54	93.0	92.48	0.55	-	-	7.224
					St.devi	0.28	4.63	4.43	0.52	-	-	4.20

Table 16

Total cost,service level, and waiting time results when π_r =50
(Appr = Appoximated, Simu = Simulated, AWT = Average waiting time)

No.	Appr			simu		Error(%)	Servicelevel(%)		Error(%)	AWT		Error(%)
	R_0	R_r	Total cost	Mean of total cost	St.dev		Appr	Simu		Appr	Simu	
10	-24	4	58.2812	59.085	0.08	1.36	96.08	96.12	0.04	6.0401	5.8872	2.5972
11	-16	5	73.0847	73.00	0.06	0.11	97.29	95.98	1.36	3.4000	3.4	0
12	0	6	83.9679	83.44	0.03	0.63	98.47	96.99	1.52	2.4127	2.1246	13.5602
13	-32	4	56.9205	57.03	0.10	0.19	94.02	93.98	0.04	6.9361	5.9312	16.9426
14	-32	7	71.7219	72.12	0.12	0.55	98.00	98.44	0.45	4.5520	4.6139	1.3416
15	-16	7	83.7105	83.69	0.02	0.02	98.56	98.21	0.36	2.8397	2.8600	0.7098
16	-40	5	56.4761	56.98	0.01	0.88	93.68	92.87	0.87	8.6417	7.9816	8.2703
17	-40	7	70.1410	70.14	0.07	0.00	97.21	98.22	1.03	4.8866	4.0120	21.7996
18	-24	7	83.3872	82.99	0.11	0.48	98.33	96.89	1.49	2.9253	2.0125	45.3565
for the	case of	10	retailers		mean	0.47	96.85	96.41	0.8	-	-	12.28
					St.devi	0.45	1.87	1.93	0.6	-	-	14.67
10	-8	2	118.9945	119.48	0.07	0.41	97.31	98.12	0.83	2.7139	2.9450	7.8472
11	0	4	145.5960	145.55	0.03	0.03	97.83	97.23	0.62	2.5276	2.4960	1.2660
12	8	6	166.7538	166.08	0.04	0.20	98.31	97.89	0.43	2.4654	2.6950	8.5195
13	32	3	117.7283	117.29	0.08	0.37	96.18	96.01	0.18	4.5525	4.4460	2.3954
14	-8	4	145.4031	145.17	0.09	0.16	97.75	96.95	0.83	2.5531	2.0527	24.3776
15	0	6	166.3021	166.59	0.12	0.17	98.27	97.97	0.31	2.4769	2.4778	0.0363
16	-40	3	116.6509	117.67	0.13	0.86	95.38	94.98	0.42	4.8873	4.7783	2.2811
17	-32	5	144.8940	145.86	0.15	0.66	97.59	97.50	0.09	3.2986	3.2668	0.9734
18	-24	7	166.0226	167.34	0.07	0.79	98.34	98.20	0.14	2.9219	2.5316	15.4171
for the	case of	20	retailers		mean	0.41	97.44	97.21	0.43	-	-	7.0126
					St.devi	0.29	1.02	1.08	0.28	-	-	8.1931

Table 17

Total cost,service level, and waiting time results when π_r =100
(Appr = Appoximated, Simu = Simulated, AWT = Average waiting time)

No.	Appr			simu		Error(%)	Servicelevel(%)		Error(%)	AWT		Error(%)
	R_0	R_r	Total cost	Mean of total cost	St.dev		Appr	Simu		Appr	Simu	
19	-16	4	65.2063	65.20	0.07	0	97.72	97.71	0.01	3.8002	3.3861	12.2294
20	-8	5	79.0698	79.91	0.14	1.05	98.91	99.18	0.27	2.7143	2.6381	2.8884
21	0	6	91.2639	92.06	0.09	0.87	99.38	98.38	1.02	2.4127	2.6706	9.6570
22	-32	4	65.3507	63.01	0.16	3.71	97.08	98.64	1.58	6.9360	6.187	12.1060
23	-24	7	79.8206	78.87	0.07	1.20	98.62	97.15	1.51	3.5372	3.7361	5.3237
24	-8	7	92.1206	93.13	0.32	1.09	99.33	98.16	1.19	2.4524	2.4127	1.6455
25	-32	5	66.1115	65.98	0.31	0.19	97.54	98.10	0.57	5.2455	5.5524	5.5273
26	-32	7	79.9007	80.13	0.26	0.28	99.22	98.18	1.05	3.7762	3.1267	20.7727
27	-16	7	92.7014	93.01	0.24	0.33	99.22	99.22	0	2.5200	2.0010	25.9370
for the	case of	10	retailers		mean	0.47	98.56	98.30	0.80	-	-	10.6763
					St.devi	1.15	0.88	0.66	0.61	-	-	8.1983
19	-8	2	129.5360	130.43	0.09	0.69	98.33	97.87	0.47	2.2235	2.6706	16.7416
20	0	5	158.2907	157.98	0.18	0.2	99.19	99.19	0	2.5277	2.5004	1.0918
21	8	7	180.8318	181.83	0.23	0.55	99.31	98.63	0.69	2.4655	2.5412	2.9789
22	32	3	130.2640	129.87	0.15	0.30	98.15	99.16	1.02	3.5373	3.7613	5.9554
23	-8	5	159.0180	160.28	0.15	0.79	99.16	98.78	0.38	2.5532	2.222	14.9055
24	0	7	181.3604	182.64	0.12	0.70	99.29	98.30	1.00	2.4770	2.4753	0.0687
25	-40	3	130.5269	129.87	0.16	0.51	97.76	96.97	0.81	3.7764	3.4784	8.5672
26	-32	5	159.4098	158.6	0.17	0.51	99.10	99.18	0.08	2.5962	2.6396	1.6442
27	-24	7	182.2687	80.87	0.09	0.77	99.26	99.27	0.01	2.4963	2.0647	20.9038
for the	case of	20	retailers		mean	0.55	98.84	98.59	0.49	-	-	8.09
					St.devi	0.21	0.59	0.77	0.41	-	-	7.67

The total cost and fill rate results are as in Tables 15 to 17. The tables show that the results when $\pi_r = 25$, 50,100. Each table shows the results for the cases of 10 and 20 retailers. As can be seen from Tables 15 to 17, (i) the errors in the total cost and the fill rates are very small in comparison with the simulation values. Furthermore, (ii) the mean error (of total cost, average waiting time and fill rate) decreases when the number of retailer (n) increases in each table. In each Table, (iii) as Q_0 increases approximated total cost TC decreases since h_0 is low value and higher Q_0 in the supplier will reduce the stockout time at the retailers. In each table, (iv) the errors in the approximate waiting time are small in comparison with the simulation values. From above information, we can conclude that our numerical results do well for the optimization with minimum error.

Secondly, we fix the following parameter $Q_0 = 32, Q_r = 4, \lambda_r = 0.5, h_0 = h_r = 1, L_0 = L_r = 1$ and $\hat{\pi}_r = 0$. Now we compare the resulting optimal costs of the three policies: the pure backorder policy, the lost sales policy and the partial backordering policy at the identical retailers. The lost sales policy is identical to our partial backorder policy when $b = 0$. The exact formulation of pure backorder model is obtained from Hadley and Whitin [49]. In our partial backorder model, when b is sufficiently large, it will be defined to the pure backorder model with at most one order outstanding at the retailers. So we expect the optimal total cost of our partial backorder policy (with large b) to be greater than or equal to that of Hadley and Whitin [49]. The partial backordering policy is implemented at the lower echelon using a control parameter 'b' which provides the pure backorder policy when b is sufficiently large and lost sales policy when $b = 0$.

In Figs. 29 and 30, the unit lost sales cost of the retailer (π_r) varies from 0 to 35. If we consider only the pure backorder and lost sales policy at the retailers, the point of policy indifference between the optimal of these two policies occurs around $\pi_r = 17$ (See Fig.29). As expected, Figs. 29 and 30 showed that in the neighborhood of the point of policy indifference, the optimal partial backorder policy is superior to the other two policies. The maximum cost savings is 2.65 % (See. Fig.30) (When compared to the better of the lost sales and pure backorder policies) and it occurs at the policy of indifference. The magnitude of cost savings is realized with a reasonable fill rate (β = 0.62). To the left of the policy indifference, the optimal lost sales policy is the best (because of low values of π_r) and to the right of it, the optimal pure backorder policy is best. For $\pi_r \leq 12$ the optimal partial backorder policy is the same as the optimal lost sales policy. From Fig.31, we found that the expected annual cost of partial backorder policy is slightly higher than that of pure backorder for $\pi_r \geq 30$. The reason for this is that the pure backorder policy of Hadley and Whitin [49] allows more than one order outstanding.

Next, the same parameters as above are used except the value $\hat{\pi}_r = 20$. We vary the $\hat{\pi}_r$ from 20 to 45. The behaviour of three policies is shown in Fig. 31. The point of policy indifference occurs around $\pi_r = 33$. At the point of policy indifference, the cost saving is 1.5% with fill rate ($\beta = 0.69$). Here, the fill rate is higher and cost saving is smaller than that in the previous example (see Fig.32). Finally, we construct an example with the same cost parameters as those in Fig. 29, except now $h_r = 10$. From Figs. 33 and 34, we can see that the partial backorder policy achieves a cost saving of over 3.8% with a low fill rate ($\beta = 0.43$). It is obvious that if partial backorder policy is to make a big difference, the average annual shortage (lost and backordered) must be sufficiently large, which is indicated by the low value of fill rate.

9.7 Conclusion

This chapter has presented a means of studying a steady state behavior of a two level supply chain consisting of one supplier and several identical retailers with continuous review at all installations. A proposed model takes into consideration of partial backordering policy during a stockout with general batch ordering in the retailers since all of the existing papers in this field had assumed demand during a stockout to be purely backordered or lost sales alone. We have approximated demand distribution as Poisson in the supplier and each retailer's lead time using Little's formula from queueing theory. We saw that the partial backorder policy using the explicit control parameter b is used in the practice of inventory control systems.

From the numerical examples provided, we observe that optimal partial backorder policy at the retailers is better than both the optimal pure backorder and lost sales policies with reasonable fill rates. We compared our approximated results with the simulated results for 27 numerical problems and the error levels are consistent with those of reported for similar approximations by other researches in this area. The real time implementation of the partial backorder policy at the retailers is easy as long as the level of backordering is tracked. After it reaches b, the unfilled demands can be covered by emergency orders.