Chapter 8

A two-echelon supply chain with Poisson demand and stochastic lead time

8.1 Introduction

We consider a two-echelon supply chain in which the supplier and the retailers follow a continuous review (R, Q) replenishment policy: whenever the inventory stock drops to the reorder level R, a lot size Q units is ordered. The benefit of this replenishment policy is that it smooths the workload at the distribution center and provides the truck drivers with a simple, fixed schedule. Given the benefits of the replenishment policy, it is not surprising that it is also used in other industries such as food and tobacco. In a different context, the replenishment policy has been suggested for its ability to reduce the so-called bull whip effect (Lee et al. [70]).

Most of the researches on two-echelon inventory models under continuous review (R, Q) policy (e.g., Axsätor [12]) assume that lead time is deterministic and constant. But, we treat the lead time (interval between placement of an order and arrival of goods) at the supplier as explicitly random, while the demand follows the Poisson process with known rate. The assumption of Poisson process to model the demand for both high value capital goods and spare parts is reasonable because in both cases demands will almost certainly for a discrete number of units.

Our work is motivated by situations where lead time, while random, can in fact be influenced by the decision maker (Cakanyildirim [15]). As one example, the inventory manager of a JIT system could invest in decreasing the lead time in a stochastic order sense. This is the case where (symmetrically) a rebate would be given to the customer who accepts an above - average lead time. In another case, if we consider machine failure in the third echelon (a production inventory system - warehouse) then the time to repair (service time) the machines follows a probabilistic distribution. So the lead time for the orders from the supplier (second echelon) has a probabilistic distribution. Consequently, the lead time for the retailer orders at the supplier is changed and this is also considered in our approximation. We will obtain general results and then specialize to the Gamma distributed lead times at the supplier.

To the best of our knowledge, no work has been done in the case of lost sales in retailers and stochastic lead time in the supplier with policy of batch ordering in all installations. So we fulfill the gap in this chapter.

The chapter is organized as follows. In the next section, we describe the problem undertaken. In section 8.3, we approximate demand process at the supplier and lead time at the retailer. In section 8.4, total cost of the supply chain system is obtained. Numerical results are presented in section 8.5. Finally, section 8.6 concludes the chapter.

8.2 Problem description

We consider a two-echelon supply chain system in which the supplier and n number of identical retailers are controlled by continuous review policy (R, Q) with general batch orders. The unsatisfied customer demand in the retailers are lost. The lead times for the supplier replenishment orders are stochastic. We consider independent Poisson demands with constant transportation times for the retailers. The unsatisfied retailers orders are backordered at the supplier. The objective here is to find optimal reorder points at all sites.

The batch replenishment quantity Q is fixed for all installations, having been determined by packaging and handling constraints as in many papers such as Axsater [9,10], Deuermeyer and Schwarz [33] and Svoronos and Zipkin [105]. For example Q may represent the number of units packed into a carton or shrink-wrapped onto a pallet; the carton or pallet represents the product unit for purchase, storage, handling and shipment purposes.

Unsatisfied demand at a retailer is lost. Technically, this may mean either that a demand is lost to the system or that is expedited (satisfied, but by means external to the normal replenishment system). Orders on the supplier are met on a first come, first served basis. We assume that $0 < R \leq Q$ for all sites with the consequence that not more than one order from any given retailer may be outstanding at any time. The technical reason for making this assumption is that the continuous review lost sales model with $1 < Q < R$ has no known analytical solution. Our assumption is reasonable if Q is much greater than the mean demand in lead time at a retailer. This, in turn, is reasonable if we are considering a relatively slow-moving product. The assumption, that it is a slow-moving product, is consistent with the use of a continuous review model to approximate what, in real life, will almost certainly be a periodic (daily) review setting. The practical context is therefore in the control of relatively slow-moving, but possibly high value, goods in a retail chain or in the control of low-usage but important spare parts in a parts network (where the importance of the part means that demand during a stockout has to be expedited).

The use of a Poisson process to model the demand for both high value capital goods and spare parts is reasonable because in both cases demands will almost certainly be for a single unit. The low demand rate assumption also lends credence to the modified base stock policy in operation at the supplier. There is an implicit assumption that there is a regular and frequent delivery service (covering a range of products) between the warehouse and the supplier and between the the supplier and each retailer.

In addition to notations in chapter 1, let us introduce the following notations

λ_r demand rate at a retailer

λ_0 demand rate at the supplier

L_0 random variable representing the lead time of the supplier replenishment orders at the warehouse

h_r holding cost per unit per unit time at the retailer

h_0 holding cost per unit per unit time at the supplier

π_r penalty cost per unit of lost sale at a retailer

C_r cost per unit time of a retailer in steady state

C_0 cost per unit time of the supplier in steady state

The results which are needed for solving the proposed model are presented in the sections 8.2.1 and 8.2.2. The objective is to find the optimal reorder points by minimizing the holding costs and the stockout costs in the supply chain.

8.2.1 Exact solution for the inventory system with Poisson demand, stochastic lead time and backorders

Consider an inventory system with continuous review policy (R, Q). Here we assume the following

1. Poisson process generates the times between demands. The mean rate of demand is λ units.
2. Units are demanded one at a time.
3. Unsatisfied demand during stockout is backordered.
4. Order quantity(Q) and reorder point(R) are discrete.
5. Procurement lead time L (random variable) has probabilistic density function $g(L)$
6. Lead times are independent random variables and the number of replenishment orders is not allowed to be more than one at any time.

If $g(L)$ is the lead time density function, the marginal distribution of lead time demand is

$$h(x) = \int_0^\infty p(x; \lambda L) g(L) \text{ dL} \tag{202}$$

where $p(x; \lambda L) = e^{-\lambda L}(\lambda L)^x/x!, x = 0, 1, 2, ...$. In steady state, each of the inventory positions $R + j, j = 1, 2, ..., Q$ has the same probability $1/Q$ (Hadley and Whitin [49]). Consider the system at time t with on hand inventory x units. Thus, if the inventory position of the system was $R+j$ at time $t-L$, the probability that there are x units at time t is equivalent to the probability that $R+j-x$ units were demanded in the lead time L if $R+j-x \geq 0$ and is zero otherwise.

Therefore, steady state probability that x units on hand at any time t is

$$\begin{aligned}
\psi_1(x) &= \sum_{j=1}^{Q} \frac{1}{Q} \int_{0}^{\infty} p(R+j-x;\lambda L)g(L)dL \\
&= \frac{1}{Q}\sum_{j=1}^{Q} h(R+j-x) \ \ for \ \ 0 \leq x \leq R+1 \qquad (203) \\
\psi_1(x) &= \sum_{j=x-R}^{Q} \frac{1}{Q} \int_{0}^{\infty} p(R+j-x;\lambda L)g(L)dL \\
&= \frac{1}{Q}\sum_{j=x-R}^{Q} h(R+j-x) \ \ for \ \ R+1 \leq x \leq R+Q \qquad (204)
\end{aligned}$$

where $\int_{0}^{\infty} p(R+j-x;\lambda L)g(L)dL = h(R+j-x)$, is the marginal distribution of lead time demand. If the inventory position is in the state $R+j$ at time $t-L$, the probability that there are y backorders at time t is

$$\int_{0}^{\infty} p(R+j+y;\lambda L)g(L)dL = h(R+j+y) \qquad (205)$$

Thus, the steady state probability $\psi_2(y)$ that there are y backorders at time t is

$$\psi_2(y) = \frac{1}{Q}\sum_{j=1}^{Q} h(R+j+y), \quad y \geq 0 \qquad (206)$$

Expected number of backorders $B(Q,R)$ at any time t

$$B(Q,R) = \sum_{y=0}^{\infty} y\psi_2(y) \qquad (207)$$

Using the Eqs. (203) and (204), the expected on hand inventory at any time t

$$D(Q,R) = \sum_{x=0}^{R+Q} x\psi_1(x) \qquad (208)$$

We have calculated exact formula for all the above equations in appendix D when the lead time is exponentially distributed as a special case of gamma distribution.

8.2.2 Exact solution for the inventory system with Poisson demand, constant transportation time and lost sales

Considering a single echelon inventory system with continuous review control policy, reorder point of R and batch size of Q, constant transportation time for replenishing orders, demand generated by a Poisson process, lost demand during a stockout and $R < Q$. Hadley and Whitin [49] developed formula for the average stock level (D) and for the average number of lost sales incurred per unit time (E). Assuming linear unit costs of holding and stockout, they obtained the related annual cost. We briefly review their results and introduce the parameters they have used in their formulae since we use them in our approximation later.

$$E = \frac{\lambda}{Q+\lambda\hat{T}}(\lambda\hat{T}) \tag{209}$$

$$D = \frac{\lambda}{Q+\lambda\hat{T}}\left[\frac{Q(Q+1)}{2\lambda} + \frac{QR}{\lambda} - QL\right] + \frac{Q}{\lambda}E \tag{210}$$

$$T = \frac{Q+\lambda\hat{T}}{\lambda} \tag{211}$$

$$\hat{T} = LP(R;\lambda L) - \frac{R}{\lambda}P(R+1;\lambda L)$$

$$P(x;\lambda L) = \sum_{i=x}^{\infty}\frac{e^{-\lambda L}(\lambda L)^i}{i!}, \quad x = 0,1,2,\ldots.$$

where $\hat{T}$ is the expected length of stock out time per cycle.

8.3 Approximations in the higher and lower echelons

In this section we are going to approximate the demand process in the supplier and the lead time in the retailers.

8.3.1 Demand analysis in the supplier

The average number of cycles per unit time in a continuous review inventory system when demand is lost during a stockout is $T^{-1} = \frac{\lambda}{Q+\lambda\hat{T}}$ (Hadley and Whitin [49]) without any special assumption concerning the nature of the stochastic processes generating demands and lead times except to assume that they do not change with time and that units are demanded one at a time. Since a batch size of Q is ordered in each cycle, the mean rate of demand (from this inventory to a higher echelon) will be $\frac{\lambda}{Q+\lambda\hat{T}}$ in terms of the identical batch size Q. As Moinzadeh and Lee [75] mentioned when the stockout is backordered in the retailers and the demand process at each retailer is Poisson, the arrival process of orders at the supplier is a superposition of n arrival processes in the case of one supplier and n retailers, each inter arrival time is Erlang distributed with shape parameter Q.

When the number of retailers in the model is large, the arrival process can be well approximated by the Poisson process with mean rate $\sum_{i=1}^{n} \frac{\lambda_i}{Q}$ (λ_i is the demand rate at the retailer i and Q is identical batch size of the retailers). They also stress that such an approximation has been used or suggested by Muckstadt [78], Deuermeyer and Schwarz [33], Albin [3] and Zipkin [118]. Using the spirit of this nice approximation and extending it for the case of lost sales (with identical retailers) when the number of retailers in the model is large, the arrival process can be well approximated by the Poisson process with mean rate $\frac{n\lambda_r}{Q_r+\lambda_r\hat{T}_r}$ in terms of the identical batch size of Q_r noting that $\hat{T}_r$ is the expected length of time per cycle that a retailer is in out of stock. The accuracy of this approximation for lost sales case is assessed in the numerical examples.

8.3.2 Approximating the retailers lead time

As mentioned before, retailers at the lower echelon of the model experience independent Poisson demand processes. Demand during a stockout is assumed to be lost. Each order i.e., placed at the supplier by each retailer will have a minimum lead time equal to the transportation time. Since some of the orders are placed when there is a stockout at the supplier, the lead time may be more than just the transportation time of the orders from the supplier into the retailer. Second an additional waiting time which results from a stockout does not have any clear distribution and we just know that it is zero when orders don't incur stockouts in the supplier and has a positive value when they are backordered in the supplier.

Based on the approximation of demand at the supplier described in section 8.3.1, the supplier behaves just like an inventory system of type described in section 8.2.1. From Little's formula in queuing theory (as Anderson and Melchiors [4]) used it in their approximation, we can use expression for the average stockout level given by Eq. (207) to obtain the average waiting time of each retailer order as given by Eq.(212). It should be noticed that Eq. (207) is valid when customer demands occur one at a time. Since each retailer orders a batch size of Q_r, we can still use the formula (207), if we make the additional assumption that the batch size (Q_0) and reorder point (R_0) of the supplier are integer multiples of identical batch size of the retailer (Q_r). The average waiting time of the orders placed by identical retailers,

$$\bar{w} = \frac{B\left(\frac{Q_0}{Q_r}, \frac{R_0}{Q_r}\right)}{\lambda_0} \tag{212}$$

$$\lambda_0 = \frac{n\lambda_r}{Q_r + \lambda_r \hat{T}_r} \tag{213}$$

In the above formula, $\bar{w}$ is the average waiting time of the orders placed by the identical retailers and $\hat{T}_r$ (expected time during which the retailer is out of stock). Based on our approximation $\bar{w}$ is added to the transportation time of each retailer to make the approximate constant lead time for the orders and it can be used for evaluating the retailers costs (holding and stockout costs). λ_0 is the mean rate of demand in the supplier. Eq.(213) follows directly from the result in section 8.3.1.

8.4 Total cost of a two-echelon supply chain

The total cost of the supply chain consisting of a supplier and n number of identical retailers is

$$TC = C_0 + nC_r \tag{214}$$

The holding cost at the supplier C_0 is

$$h_0 D\left(\frac{Q_0}{Q_r}, \frac{R_0}{Q_r}\right) Q_r \tag{215}$$

Knowing λ_0 and $B\left(\frac{Q_0}{Q_r}, \frac{R_0}{Q_r}\right)$, we can determine

$$\bar{w} = \frac{B\left(\frac{Q_0}{Q_r}, \frac{R_0}{Q_r}\right)}{\lambda_0} \tag{216}$$

An important point is that $\bar{w}$ is dependent on λ_0 and λ_0 itself depends on $\hat{T}_r$ (expected time during which the retailer is out of stock).

Each retailer cost consists of shortage cost and holding cost as follows

$$C_r = \pi_r E_r + h_r D_r \tag{217}$$

In the above equation, E_r is the average number of lost sales incurred per unit time in a retailer and D_r is the average stock level in a retailer. Based on the approximation in section 8.3.2, $L_r + \bar{w}$ is the lead time of a retailer's orders. Hence, we have

$$E_r = \frac{\lambda_r}{Q_r + \lambda_r \hat{T}'_r}(\lambda_r \hat{T}'_r) \tag{218}$$

$$D_r = \frac{\lambda_r}{Q_r + \lambda_r \hat{T}'_r}\left[\frac{Q_r(Q_r+1)}{2\lambda_r} + \frac{Q_r R_r}{\lambda_r} - Q_r(L_r + \bar{w})\right] + \frac{Q_r}{\lambda_r}E_r \tag{219}$$

where

$$\hat{T}'_r = (L_r + \bar{w})P(R_r; \lambda_r(L_r + \bar{w})) - \frac{R_r}{\lambda_r}P(R_r + 1; \lambda_r(L_r + \bar{w})) \tag{220}$$

$\hat{T}'_r$ is the average length of time per cycle for which a retailer is out of stock when the lead time is assumed to be the constant value $L_r + \bar{w}$.

It is clear that we should find the optimal values of the reorder points of all installations through minimizing TC which is nonlinear function with integer variables R_0 and R_r . It is necessary to state that the reorder point of a retailer bound by zero and Q_r i.e., $0 < R_r < Q_r$. Since there should not be more than one order outstanding in each retailer at any time and this constraint satisfies this condition for a continuous review inventory system with lost sales (Hadley and Whitin [49]). Furthermore, since there are n retailers in our model and none of them can have more than one order outstanding, we have $R_0 \geq -nQ_r$. This is because if $R_0 < -nQ_r$, then the reorder point is never reached in the supplier.

8.5 Numerical Results

We design a set of 27 numerical problems. It is necessary to find the optimal reorder points of the supplier and retailer. Our optimization problem is to minimize total cost TC (non-linear) subject to $0 < R_r < Q_r$ and $R_0 \geq -nQ_r$. We also simulated each numerical problem 10 times (having 10 runs), for the optimal reorder point obtained from the approximate model using GPSS/H simulation software. The simulation time length of each run is 11000 unit times with 10000 unit times as a "run in period".

Different starting random number seeds were employed for each problem. All of the results show that this length of time is sufficient for the system to reach a steady state. This is also clear from standard deviation of the total system cost. The cost error is obtained by the following relation

$$\text{cost error} = \frac{|\text{ simulated total cost - approximated total cost}|}{\text{simulated total cost}}$$

We also report the retailers' service levels. This can be obtained through Hadley and Whitin [49] as

$$\text{service level} = 1 - \frac{\text{average number of lost sales per unit time}}{\text{average demand per unit time}}$$

The above relation has been employed in the porposed model and also in the simulation model to find the service levels. We assume that the lead time for the supplier replenishment orders, L_0, has a gamma distribution $\gamma(L_0; \alpha, \delta) = \frac{\delta(\delta L_0)^\alpha e^{-\delta L_0}}{\gamma(\alpha+1)}$, α is an integer and $\delta > 0$.

If demand (x) is Poisson distributed and the lead time (L_0) has a gamma distribution then the marginal distribution of lead time demand $h(x)$ has a negative binomial distribution (Hadley and Whitin [49])

$$b_N(x, \alpha+1, \frac{\delta}{\delta+\lambda}) = \frac{(\alpha+x)!}{\alpha! x!}\left(\frac{\delta}{\delta+\lambda}\right)^{\alpha+1}\left(\frac{\lambda}{\lambda+\delta}\right)^{x} \quad x = 0, 1, 2, 3... \tag{221}$$

where $\alpha > -1$ (an integer) and $\delta > 0, \lambda > 0$. We have presented the numerical results for the case $\alpha = 0$, i.e, lead times are assumed to be exponentially distributed. Hence, marginal distribution of lead time demand $h(x)$ is $\left(\frac{\delta}{\delta+\lambda}\right)\left(\frac{\lambda}{\delta+\lambda}\right)^x$ which is geometric distribution. In appendix D, we explicitly calculated all the formula in section 8.2.1 for the case of exponentially distributed lead times in the supplier. From Eq. (215) and appendix D, we can find C_0. The formula for C_r can be used from Eq.(217).

Table 10: Input data

No	π_r	Q_0	Q_r	λ_r	No	π_r	Q_0	Q_r	λ_r
1	25	16	8	0.5	15	50	32	8	1.5
2	25	16	8	1.0	16	50	64	8	0.5
3	25	16	8	1.5	17	50	64	8	1.0
4	25	32	8	0.5	18	50	64	8	1.5
5	25	32	8	1.0	19	100	16	8	0.5
6	25	32	8	1.5	20	100	16	8	1.0
7	25	64	8	0.5	21	100	16	8	1.5
8	25	64	8	1.0	22	100	32	8	0.5
9	25	64	8	1.5	23	100	32	8	1.0
10	50	16	8	0.5	24	100	32	8	1.5
11	50	16	8	1.0	25	100	64	8	0.5
12	50	16	8	1.5	26	100	64	8	1.0
13	50	32	8	0.5	27	100	64	8	1.5
14	50	32	8	1.0					

The input data for the numerical problems are as in Table 10. There are different stockout costs in the table to assess the accuracy of the approximation for various service level obtained. The number of retailers is considered 10 and 20 (a large enough number to approximate the demand distribution as Poisson in the supplier) as in Moinzadeh and Lee [75] to compare the results when number of retailers is changed. The holding costs of the supplier and retailers per unit per unit time are assumed to be 1, $h_0 = h_r = 1$, the transportation time of retailers $L_r = 1$ and $\delta = 0.5$. The total cost and the service level results are as in Tables 11 to 13. The table shows the result when the stockout cost (π_r) is 25,50 and 100. Each table also shows the result for the case of 10 and 20 retailers.

As can be seen from Tables 11 to 13, the errors in the total cost and service level are very small in comparison with the simulation values. Further the mean error (of total cost, average waiting time and service level) decreases when the number of retailer (n) increases in each table. From Tables 11 and 12, when Q_0 increases, total cost TC decreases whereas in Table 13 total cost increases because of relatively high penalty cost (π_r) comparing to π_r= 25 and π_r= 50. We have also reported the average waiting time of retailer orders in last three columns of Tables 11 to 13 as a criterion to assess the accuracy of the approximation of Poisson demand distribution at the supplier. When L_0 follows a gamma distribution $\gamma(L_0; c-1, \frac{c}{\tau})$ for $\tau > 0$ and $c > 0$ (an integer), the marginal distribution of lead time demand h(x) becomes $b_N(x; c, \frac{c}{c+\lambda\tau}) = \binom{c+x-1}{c}(\frac{c}{c+\lambda\tau})^c(\frac{\lambda\tau}{c+\lambda\tau})^x$, x = 0,1,2,.. . The limiting case of c $\rightarrow \infty$ corresponds to the constant lead time for the supplier replenishment orders in the supplier. It implies that the lead time demand has Poisson distribution with mean $\lambda\tau$ and variance $\lambda\tau$.

Table 11

Total cost,service level, and waiting time results when $\pi_r = 25$
(Appr = Appoximated, Simu = Simulated, AWT = Average waiting time)

No.	Appr			simu		Error(%)	Servicelevel(%)		Error(%)	AWT		Error(%)
	R_0	R_r	Total cost	Mean of total cost	St.dev		Appr	Simu		Appr	Simu	
1	-32	5	48.9653	50.86	0.10	3.7	89.91	91.23	1.5	10.072	9.8270	2.5
2	-24	5	64.4602	65.12	0.05	1.01	93.49	93.79	0.3	4.3601	3.9603	10.1
3	-8	6	75.3070	76.04	0.08	0.96	97.11	97.21	0.1	2.7895	2.7850	0.16
4	-40	5	45.9048	46.09	0.07	0.40	85.04	84.88	0.2	11.685	10.1372	15.3
5	-40	7	60.9651	61.00	0.12	0.06	93.46	92.87	0.6	5.9728	5.6423	5.9
6	-24	7	74.0229	74.86	0.05	1.12	96.92	96.03	0.9	3.3303	3.1307	6.4
7	-48	7	45.2557	45.98	0.09	1.58	86.38	85.83	0.6	14.755	12.8761	14.6
8	-48	7	59.3805	60.13	0.06	1.25	91.33	90.79	0.6	6.4413	6.4412	0
9	-32	7	72.7095	73.69	0.03	1.33	96.44	96.23	0.2	3.4387	3.0375	13.2
for the	case of	10	retailers		mean	1.27	92.23	92.10	0.6	-	-	7.56
					St.devi	1.03	4.45	4.42	0.4	-	-	6.01
1	-24	2	105.7417	106.24	0.06	0.46	92.45	92.44	0.01	4.3631	4.2531	2.59
2	0	3	130.3332	131.02	0.05	0.52	94.95	94.08	0.92	2.5267	2.4678	10.84
3	8	5	149.3572	150.2	0.02	0.61	96.30	95.09	1.27	2.4652	2.2241	24.03
4	-56	5	101.1934	102.14	0.07	0.93	87.93	87.36	0.65	10.747	10.2423	10.84
5	-24	4	129.6748	130.46	0.1	0.60	95.13	95.12	0.01	3.2368	2.9831	8.50
6	-16	6	149.7579	150.08	0.09	0.21	96.63	95.36	1.33	2.8971	2.7018	7.23
7	-72	7	97.6604	98.26	0.13	0.61	82.99	82.90	0.11	15.931	15.6708	1.66
8	-56	6	127.0281	128.12	0.06	0.85	94.31	94.10	0.22	4.9720	4.4817	10.94
9	-40	7	148.9100	148.98	0.08	0.05	96.31	95.89	0.44	3.4686	3.0616	13.29
for the	case of	20	retailers		mean	0.54	93.0	92.48	0.55	-	-	7.224
					St.devi	0.28	4.63	4.43	0.52	-	-	4.20

Table 12
Total cost,service level, and waiting time results when $\pi_r = 50$
(Appr = Appoximated, Simu = Simulated, AWT = Average waiting time)

No.	Appr			simu		Error(%)	Servicelevel(%)		Error(%)	AWT		Error(%)
	R_0	R_r	Total cost	Mean of total cost	St.dev		Appr	Simu		Appr	Simu	
10	-24	4	58.2812	59.085	0.08	1.36	96.08	96.12	0.04	6.0401	5.8872	2.5972
11	-16	5	73.0847	73.00	0.06	0.11	97.29	95.98	1.36	3.4000	3.4	0
12	0	6	83.9679	83.44	0.03	0.63	98.47	96.99	1.52	2.4127	2.1246	13.5602
13	-32	4	56.9205	57.03	0.10	0.19	94.02	93.98	0.04	6.9361	5.9312	16.9426
14	-32	7	71.7219	72.12	0.12	0.55	98.00	98.44	0.45	4.5520	4.6139	1.3416
15	-16	7	83.7105	83.69	0.02	0.02	98.56	98.21	0.36	2.8397	2.8600	0.7098
16	-40	5	56.4761	56.98	0.01	0.88	93.68	92.87	0.87	8.6417	7.9816	8.2703
17	-40	7	70.1410	70.14	0.07	0.00	97.21	98.22	1.03	4.8866	4.0120	21.7996
18	-24	7	83.3872	82.99	0.11	0.48	98.33	96.89	1.49	2.9253	2.0125	45.3565
for the	case of	10	retailers		mean	0.47	96.85	96.41	0.8	-	-	12.28
					St.devi	0.45	1.87	1.93	0.6	-	-	14.67
10	-8	2	118.9945	119.48	0.07	0.41	97.31	98.12	0.83	2.7139	2.9450	7.8472
11	0	4	145.5960	145.55	0.03	0.03	97.83	97.23	0.62	2.5276	2.4960	1.2660
12	8	6	166.7538	166.08	0.04	0.20	98.31	97.89	0.43	2.4654	2.6950	8.5195
13	32	3	117.7283	117.29	0.08	0.37	96.18	96.01	0.18	4.5525	4.4460	2.3954
14	-8	4	145.4031	145.17	0.09	0.16	97.75	96.95	0.83	2.5531	2.0527	24.3776
15	0	6	166.3021	166.59	0.12	0.17	98.27	97.97	0.31	2.4769	2.4778	0.0363
16	-40	3	116.6509	117.67	0.13	0.86	95.38	94.98	0.42	4.8873	4.7783	2.2811
17	-32	5	144.8940	145.86	0.15	0.66	97.59	97.50	0.09	3.2986	3.2668	0.9734
18	-24	7	166.0226	167.34	0.07	0.79	98.34	98.20	0.14	2.9219	2.5316	15.4171
for the	case of	20	retailers		mean	0.41	97.44	97.21	0.43	-	-	7.0126
					St.devi	0.29	1.02	1.08	0.28	-	-	8.1931

Table 13

Total cost,service level, and waiting time results when $\pi_r = 100$
(Appr = Appoximated, Simu = Simulated, AWT = Average waiting time)

No.	Appr			simu		Error(%)	Servicelevel(%)		Error(%)	AWT		Error(%)
	R_0	R_r	Total cost	Mean of total cost	St.dev		Appr	Simu		Appr	Simu	
19	-16	4	65.2063	65.20	0.07	0	97.72	97.71	0.01	3.8002	3.3861	12.2294
20	-8	5	79.0698	79.91	0.14	1.05	98.91	99.18	0.27	2.7143	2.6381	2.8884
21	0	6	91.2639	92.06	0.09	0.87	99.38	98.38	1.02	2.4127	2.6706	9.6570
22	-32	4	65.3507	63.01	0.16	3.71	97.08	98.64	1.58	6.9360	6.187	12.1060
23	-24	7	79.8206	78.87	0.07	1.20	98.62	97.15	1.51	3.5372	3.7361	5.3237
24	-8	7	92.1206	93.13	0.32	1.09	99.33	98.16	1.19	2.4524	2.4127	1.6455
25	-32	5	66.1115	65.98	0.31	0.19	97.54	98.10	0.57	5.2455	5.5524	5.5273
26	-32	7	79.9007	80.13	0.26	0.28	99.22	98.18	1.05	3.7762	3.1267	20.7727
27	-16	7	92.7014	93.01	0.24	0.33	99.22	99.22	0	2.5200	2.0010	25.9370
for the	case of	10	retailers		mean	0.47	98.56	98.30	0.80	-	-	10.6763
					St.devi	1.15	0.88	0.66	0.61	-	-	8.1983
19	-8	2	129.5360	130.43	0.09	0.69	98.33	97.87	0.47	2.2235	2.6706	16.7416
20	0	5	158.2907	157.98	0.18	0.2	99.19	99.19	0	2.5277	2.5004	1.0918
21	8	7	180.8318	181.83	0.23	0.55	99.31	98.63	0.69	2.4655	2.5412	2.9789
22	32	3	130.2640	129.87	0.15	0.30	98.15	99.16	1.02	3.5373	3.7613	5.9554
23	-8	5	159.0180	160.28	0.15	0.79	99.16	98.78	0.38	2.5532	2.222	14.9055
24	0	7	181.3604	182.64	0.12	0.70	99.29	98.30	1.00	2.4770	2.4753	0.0687
25	-40	3	130.5269	129.87	0.16	0.51	97.76	96.97	0.81	3.7764	3.4784	8.5672
26	-32	5	159.4098	158.6	0.17	0.51	99.10	99.18	0.08	2.5962	2.6396	1.6442
27	-24	7	182.2687	80.87	0.09	0.77	99.26	99.27	0.01	2.4963	2.0647	20.9038
for the	case of	20	retailers		mean	0.55	98.84	98.59	0.49	-	-	8.09
					St.devi	0.21	0.59	0.77	0.41	-	-	7.67

8.6 Conclusion

In this chapter, we have developed a two echelon supply chain system with one supplier and several identical retailers. The unsatisfied demand in the retailers is lost and control policy is continuous review. The unsatisfied demand in the supplier is backordered. The main point of the model is to consider stochastic lead time for supplier orders and lost sales during stockout in the retailers, since most of the previous studies have considered constant lead time at all installtions.

We have approximated demand distribution as Poisson in the supplier and each retailer's lead time by a constant and obtained the average waiting time of retailers orders using Little's formula from queuing theory. We compared our numerical results with simulated values for 27 numerical problems for the case of 10 and 20 retailers. The error levels are consistent with those of reported for similar approximations by other researchers in this field.