Chapter 7

Evaluation of supply chain structures using a delay product differentiation process

7.1 Introduction

Delay product differentiation process is nothing but a late customization referring to postponement of orders in a supply chain until the supply chain is cost effective. Hewlett - Packard Development company have saved money by adopting the postponement strategy [71]. It is important to control and maintain the inventories of perishable items for the modern corporation. Since most of the goods are non-instantaneous deteriorating, we have implemented postponement strategy in a retailer of the supply chain involving the flow of the said items from one stage to another stage. Postponement is one of central features of mass customization. It has been reported that postponement strategy is highly successful in a wide range of industries that require high differentiation e.g., high-tech industry, food industry and fashion industry etc.

All the previous models in this area have not considered the case for non-instantaneous deteriorating items and a time proportional partial backlogging rate in a retailer of the supply chain. However, in real life, most of the goods are non-instantaneous deteriorating (e.g., vegetables, fruits, fishes and so on). These items would have a span of maintaining quality or the original condition, namely, during that period, there was no deterioration occurs. But deterioration starts after that period. We call this phenomenon as 'non-instantaneous deteriorating items' in literature.

Besides the assumption of pure backordering during shortages is not always applicable to real life situations. For instance some customers are willing to wait for backorder and others would return to buy from other sellers. In some inventory systems such as fashionable commodities, the length of the waiting time for next replenishment would determine whether the backlogging rate will be accepted or not. Therefore we assume that the backlogging rate is variable and dependent on the waiting time for the next replenishment.

To the best of our knowledge, no work exists for studying the interaction between inventory and postponement in a supply chain with the flow of non-instantaneous deteriorating items and partial backlogging of shortages. In this chapter, we wish to investigate whether or not the postponement system can outperform the independent system with non-instantaneous deteriorating items. We formulate two models to describe independent system and postponement system. We also give an algorithm to derive the optimal ordering strategies. We also investigate the effect of product deterioration on the total cost of the retailer and on inventory replenishment policies. Some numerical examples are provided to illustrate the theoretical results. We show that postponement strategy can give a lower total average cost under certain circumstances with perishable items.

This chapter is organised as follows. In the next section, we describe the problem undertaken. Section 7.3 provides mathematical model formulation of the problem considered. In section 7.4, postponement and independent systems are described clearly. Numerical examples are presented in section 7.5. Section 7.6 concludes the chapter.

7.2 Problem description

We consider a supply chain in which supplier supplies the items to the retailer and the retailer fulfills market customers' demand. The m number of end products are manufactured from the same raw material or semi-manufactured products. The end products belong to the same product category, but they have slight differences and the customization process can be delayed after ordering. The retailer can order rn end-products independently in an independent system. However, the retailer may order the material or semi-manufactured product and finish the customization itself, i.e., customization is postponed after ordering. The ordering decisions can be combined. This can be viewed as a postponement system. For example retailers of a soft-drink supplier can order concentrated syrup and mix it with carbonated water in house to make different soda products for sale at their retail stores. In this case, retailers make only one decision to acquire the concentrated syrup rather than making different decisions to acquire different products marketed by the soft-drink supplier.

We assume that the demand rates of the end-products are independent and constant. The unsatisfied demands (due to shortages) are partially backlogged at the retailer. We formulate two EOQ models to describe the independent system and postponement system. In the first model, retailer orders the m end-products independently with different schedules. So there are m EOQ decisions. However in the second model customization is postponed after ordering and the ordering decisions can be combined; so that a single EOQ decision is made. This process can be viewed as a postponement strategy. In addition to the notations in chapter 1, we describe the following notations.

i	end product i, i = 1,2,3,...,m
λ_i	demand rate of end-product i
c	unit production cost per unit time, $c > 0$
h	unit holding cost per unit time, $h > 0$
$\hat{\pi}$	unit backorder cost for end-products, $\hat{\pi} > 0$
π	unit cost for lost sales, $\pi > 0$
t_1	the time up to which the inventory is positive in a cycle
t_d	the length of time in which the end-product has no deterioration
TC	total average cost per unit time for ordering and keeping m end-products in an independent system
TCP	total average cost per unit time for ordering and keeping m end-products in a postponement system.

In addition, the following assumptions are made at the retailer of a supply chain.

1. The replenishment rate is infinite and the lead time is zero.

2. The end-product demand rates λ_i are deterministic and constant.

3. Shortages are allowed to occur. It is assumed that only a fraction of demand is backlogged. Besides, longer the waiting time is smaller the backlogging rate will be. We take this fraction as $\frac{1}{1+\delta(T-t)}$, $t_1 \le t \le T$ where the backlogging parameter δ is a positive constant.

4. All the end-products are produced from the same type of raw materials and the ratio of raw material to end-product is 1:1.

5. The distribution of the deterioration time of the items follows the exponential distribution with parameter θ, i.e., a constant rate of deterioration.

6. Deterioration of raw materials and end-product is considered only after they have been received into inventory and there is no replacement of deteriorated inventory.

7.3 Model formulation

The inventory system goes like this: I_{max} units of item arrive at the inventory system at the beginning of each cycle. During the time interval $[0, t_d]$, the inventory level decreasing owing to constant demand rate. The inventory level is dropping to zero due to demand and deterioration during the time interval $[t_d, t_1]$. Then shortage interval keeps to the end of the current cycle. The whole process is repeated. The behavior of the inventory system is depicted in Fig.26.

As described above the inventory level $I(t)$ of an end-product at time t is governed by the following differential equation.

$$\frac{dI(t)}{dt} = \begin{cases} -\lambda & \text{if } 0 \le t \le t_d \\ -\lambda - \theta I(t) & \text{if } t_d \le t \le t_1 \\ \frac{-\lambda}{1+\delta(T-t)} & \text{if } t_1 \le t \le T \end{cases} \tag{188}$$

with the boundary conditions $I(0) = I_{max}$, $I(t_1) = 0$, where t_1 is the time upto which the inventory level is positive in a cycle and T is the cycle length.

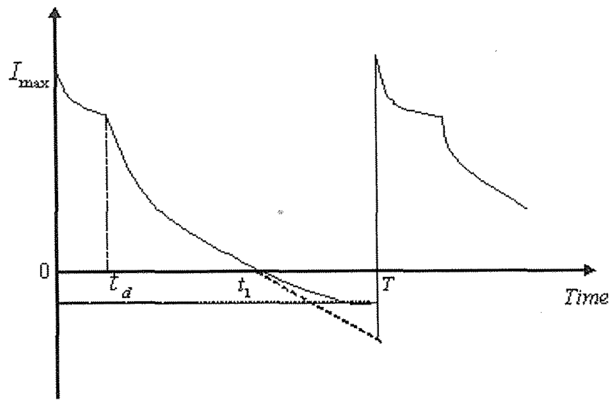

Figure 26: Graphical representation of the inventory system

The solution of Eq.(188) is

$$I(t) = \begin{cases} I_1(t) & \text{if } 0 \le t \le t_d \\ I_2(t) & \text{if } t_d \le t \le t_1 \\ I_3(t) & \text{if } t_1 \le t \le T \end{cases} \tag{189}$$

where

$$\begin{aligned} I_1(t) &= \frac{\lambda}{\theta}\left[e^{\theta(t_1-t_d)} - \theta(t-t_d) - 1\right] \\ I_2(t) &= \frac{\lambda}{\theta}\left[e^{\theta(t_1-t)} - 1\right] \\ I_3(t) &= \frac{-\lambda}{\delta}\left[\ \ln[1+\delta(T-t_1)] - \ \ln[1+\delta(T-t)]\right] \end{aligned}$$

where ln(.) is natural logarithm.

Based on Eq.(189), we obtain the following costs over the planning horizon.

(1) Purchase cost $= c\lambda$.

(2) Ordering cost $= \frac{A}{T}$

(3) Holding cost

$$\begin{aligned} &= \frac{h}{T}\left[\int_0^{t_d} I_1(t)\ \mathrm{dt} + \int_{t_d}^{t_1} I_2(t)\ \mathrm{dt}\right] \\ &= \frac{\lambda}{T}\left[\frac{ht_d}{\theta}\left[e^{\theta(t_1-t_d)} - 1\right] + \frac{ht_d^2}{2} + \frac{h}{\theta^2}\left[e^{\theta(t_1-t_d)} - \theta(t_1-t_d) - 1\right]\right] \end{aligned}$$

(4) Cost of deteriorated units

$$
\begin{aligned}
&= \frac{c}{T}\left[I_2(t_d) - \int_{t_d}^{t_1} \lambda \ \mathrm{dt}\right] \\
&= \frac{c\theta\lambda}{\theta^2 T}\left[e^{\theta(t_1-t_d)} - \theta(t_1-t_d) - 1\right]
\end{aligned}
$$

(5) Shortage cost due to backlogging

$$
\begin{aligned}
&= \frac{\hat{\pi}}{T}\int_{t_1}^{T} [-I_3(t)] \ \mathrm{dt} \\
&= \frac{\hat{\pi}\lambda}{\delta T}\left[(T-t_1) - \frac{1}{\delta} \ \ln\left[1+\delta(T-t_1)\right]\right]
\end{aligned}
$$

(6) Shortage cost due to lost sale

$$
\begin{aligned}
&= \frac{\pi}{T}\int_{t_1}^{T} \lambda\left[1 - \frac{1}{1+\delta(T-t)}\right] \ \mathrm{dt} \\
&= \frac{\pi\lambda}{T}\left[(T-t_1) - \frac{1}{\delta} \ \ln\left[1+\delta(T-t_1)\right]\right]
\end{aligned}
$$

The total average cost per unit time for ordering and keeping the end-product as follows:

$$
\begin{aligned}
C(t_1, T/\theta) &= \text{Purchase cost} + \text{Ordering cost} + \text{ Holding cost} \\
& \quad + \text{Cost of deteriorated items} + \text{Shortage costs} \\
&= c\lambda + \frac{\lambda}{T}\left[\frac{A}{\lambda} + \frac{ht_d}{\theta}\left(e^{\theta(t_1-t_d)} - 1\right) + \frac{ht_d^2}{2} + \frac{h+c\theta}{\theta^2}\left(e^{\theta(t_1-t_d)} - \theta(t_1-t_d) - 1\right)\right. \\
& \quad \left. + \frac{\hat{\pi}+\delta\pi}{\delta}\left(T - t_1 - \frac{1}{\delta} \ \ln[1+\delta(T-t_1)]\right)\right] \qquad (190)
\end{aligned}
$$

The necessary conditions for the minimum value of $C(t_1, T/\theta)$ are

$$
\frac{\partial C}{\partial t_1} = \frac{\lambda}{T}\left[ht_d e^{\theta(t_1-t_d)} + \frac{h+c\theta}{\theta}\left(e^{\theta(t_1-t_d)} - 1\right) + \frac{\hat{\pi}+\delta\pi}{\delta}\left(-1 + \frac{1}{1+\delta(T-t_1)}\right)\right] = 0 \qquad (191)
$$

$$
\begin{aligned}
\frac{\partial C}{\partial T} &= \frac{\lambda}{T^2}\left[\frac{\hat{\pi}+\delta\pi}{\delta}\left(\frac{(T-t_1)(\delta t_1 - 1)}{1+\delta(T-t_1)} + \frac{1}{\delta} \ \ln[1+\delta(T-t_1)]\right) - \frac{A}{\lambda} - \frac{ht_d}{\theta}\left(e^{\theta(t_1-t_d)} - 1\right) - \frac{ht_d^2}{2}\right. \\
& \quad \left. - \frac{h+c\theta}{\theta^2}\left(e^{\theta(t_1-t_d)} - \theta(t_1-t_d) - 1\right)\right] = 0 \qquad (192)
\end{aligned}
$$

For notational convenience, let $\xi_1 = \frac{h+c\theta}{\theta}$, $\xi_2 = \frac{\hat{\pi}+\delta\pi}{\delta}$ and $\xi_3 = ht_d + \xi_1$. Clearly $\xi_1 > 0, \xi_2 > 0$ and $\xi_3 > 0$.

Then Eqs. (191) and (192) become

$$
T = t_1 + \frac{\xi_3 e^{\theta(t_1-t_d)} - \xi_1}{\delta\left[\xi_1 + \xi_2 - \xi_3 e^{\theta(t_1-t_d)}\right]} \qquad (193)
$$

and

$$\xi_2\left[\frac{(T-t_1)(\delta t_1-1)}{1+\delta(T-t_1)}+\frac{1}{\delta}\ \ln[1+\delta(T-t_1)]\right]-\frac{A}{\lambda}-\frac{\xi_3}{\theta}\left[e^{\theta(t_1-t_d)}-1\right]$$
$$-\frac{ht_d^2}{2}+\xi_1(t_1-t_d)=0 \tag{194}$$

Substituting Eq.(193) into Eq.(194), we have

$$\frac{\delta t_1-1}{\delta}\left[\xi_3 e^{\theta(t_1-t_d)}-\xi_1\right]-\frac{\xi_2}{\delta}\ \ln\left[\frac{\xi_1}{\xi_2}+1-\frac{\xi_3}{\xi_2}e^{\theta(t_1-t_d)}\right]-\frac{A}{\lambda}-\frac{\xi_3}{\theta}\left[e^{\theta(t_1-t_d)}-1\right]$$
$$-\frac{ht_d^2}{2}+\xi_1(t_1-t_d)=0 \tag{195}$$

Lemma 7.1. If $\frac{A}{\lambda}+\frac{\xi_2}{\delta}\ \ln\left[\frac{\xi_1-\xi_3}{\xi_2}+1\right]+\frac{\delta t_d-2}{2\delta}(\xi_1-\xi_3)\geq 0$, then the point $(t_1^*>0, T^*>0)$ that solves (193) and (195) simultaneously exists and is unique. The point $(t_1^*>0, T^*>0)$ is also the unique global optimum for the problem $\mathbf{min}\{C(t_1,T/\theta):0<t_1<T<\infty\}$.

Proof. Similar to the proof of Theorem 1 in Wu et al. [115] . □

Thus t_1^* can be uniquely determined as a function of θ, say $t_1^*=t(\theta)$ and T^* can be uniquely determined as a function of θ say $T^*=T(\theta)$. This also implies that $C(t_1^*,T^*/\theta)=C(t(\theta),T(\theta)/\theta)$.

Theorem 7.1. $\hat{C}(\theta)\triangleq C(t_1^*,T^*/\theta)$ is an increasing and continuous function of θ in $[0,+\infty)$ and

$$\lim_{\theta\to 0}\hat{C}(\theta)\cong c\lambda+\sqrt{\frac{2A\lambda h(\hat{\pi}+\delta\pi)}{(h+\hat{\pi}+\delta\pi)}}$$

.

Proof. Using the power series of e^x, we have

$$C(t_1,T/\theta)=c\lambda+\frac{\lambda}{T}\left[\frac{A}{\lambda}+ht_d(t_1-t_d)\sum_{n=1}^{\infty}\frac{(\theta(t_1-t_d))^{n-1}}{n!}+\frac{ht_d^2}{2}\right.$$
$$\left.+(h+c\theta)(t_1-t_d)^2\sum_{n=2}^{\infty}\frac{(\theta(t_1-t_d))^{n-2}}{n!}+\frac{\hat{\pi}+\delta\pi}{\delta}\left(T-t_1-\frac{1}{\delta}\ \ln[1+\delta(T-t_1)]\right)\right] \tag{196}$$

For fixed value of $t_1>0$ and $T>0$, it is obvious that $C(t_1,T/\theta)$ is an increasing function of θ for $\theta \geq 0$. If $\theta_1 < \theta_2$, we have

$$\hat{C}(\theta_2) = C(t_1(\theta_2), T(\theta_2)/\theta_2) > C(t_1(\theta_2), T(\theta_2)/\theta_1)$$
$$\geq C(t_1(\theta_1), T(\theta_1)/\theta_1) = \hat{C}(\theta_1)$$

Thus, $\hat{C}(\theta)$ is an increasing function of θ in $[0, +\infty)$.

From Eqs.(193) and (195), let

$$\begin{aligned} f_1(t_1, T/\theta) &= \frac{\delta t_1 - 1}{\delta}\left[\xi_3 e^{\theta(t_1 - t_d)} - \xi_1\right] - \frac{\xi_2}{\delta}\ln\left[\frac{\xi_1}{\xi_2} + 1 - \frac{\xi_3}{\xi_2}e^{\theta(t_1-t_d)}\right] - \frac{A}{\lambda} - \frac{\xi_3}{\theta}\left[e^{\theta(t_1-t_d)} - 1\right] \\ &\quad -\frac{ht_d^2}{2} + \xi_1(t_1 - t_d) \\ f_2(t_1, T/\theta) &= T - t_1 - \frac{\xi_3 e^{\theta(t_1-t_d)} - \xi_1}{\delta\left[\xi_1 + \xi_2 - \xi_3 e^{\theta(t_1-t_d)}\right]} \end{aligned}$$

We have,

$$\begin{aligned} \frac{\partial f_1}{\partial t_1} &= \theta\xi_3 e^{\theta(t_1-t_d)}\left[t_1 + \frac{\xi_3 e^{\theta(t_1-t_d)} - \xi_1}{\delta\left[\xi_1 + \xi_2 - \xi_3 e^{\theta(t_1-t_d)}\right]}\right] > 0 \text{ (since } T > 0 \text{ and from Eq.(193))} \\ \frac{\partial f_1}{\partial T} &= 0 \\ \frac{\partial f_2}{\partial t_1} &= -1 - \frac{\xi_2\xi_3\theta e^{\theta(t_1-t_d)}}{\delta(\xi_1 + \xi_2 - \xi_3 e^{\theta(t_1-t_d)})^2} \text{ (finite)} \\ \frac{\partial f_2}{\partial T} &= 1 \end{aligned}$$

It is clear that

$$\begin{vmatrix} \frac{\partial f_1}{\partial t_1} & \frac{\partial f_1}{\partial T} \\ \frac{\partial f_2}{\partial t_1} & \frac{\partial f_2}{\partial T} \end{vmatrix} = \frac{\partial f_1}{\partial t_1} > 0.$$

From the implicit function theorem (in Real analysis), we know that $t_1(\theta), T(\theta)$ are continuous functions of θ in $[0, +\infty)$ respectively. Moreover $C(t_1, T/\theta)$ is a continuously differentiable real valued function for $0 < t_1 < T$ and $\theta \geq 0$. Thus $\hat{C}(\theta)$ is also a continuous function of θ in $[0, +\infty)$. Let

$$f(t_1, T) = \frac{\lambda}{T}\left[\frac{A}{\lambda} + ht_d(t_1 - t_d) + \frac{ht_d^2}{2} + \frac{h}{2}(t_1 - t_d)^2 + \frac{\xi_2}{\delta}\left[\delta(T - t_1) - \ln[1 + \delta(T - t_1)]\right]\right]$$

Because $\hat{C}(\theta)$ is continuous in $[0, +\infty)$, we have

$$\begin{aligned}
\lim_{\theta \to 0} \hat{C}(\theta) &= \hat{C}(0) \\
&= \min_{0<t_1<T<\infty} \{c\lambda + f(t_1, T)\} \\
&= c\lambda + \min_{0<t_1<T<\infty} \{f(t_1, T)\} \\
&\cong c\lambda + \sqrt{\frac{2Ah\delta\xi_2\lambda}{h+\delta\xi_2}} \\
&= c\lambda + \sqrt{\frac{2A\lambda h(\hat{\pi}+\delta\pi)}{(h+\hat{\pi}+\delta\pi)}} \quad \square
\end{aligned}$$

Theorem 7.2. If

$$\frac{\delta t_d - 1}{\delta}[\xi_3 - \xi_1] - \frac{\xi_2}{\delta} \ln\left[\frac{\xi_1 - \xi_3}{\xi_2} + 1\right] - \frac{A}{\lambda} - \frac{ht_d^2}{2} > 0$$

then the function $C(t_1, T/\theta)$ has a minimum value at the point (t_1^*, T^*) where $t_1^* = t_d$ and

$$T^* = t_d + \frac{ht_d}{(\hat{\pi}+\delta\pi) - \delta h t_d}$$

Proof. We have

$$\frac{\xi_3 e^{\theta(t_1 - t_d)} - \xi_1}{\delta\left[\xi_1 + \xi_2 - \xi_3 e^{\theta(t_1 - t_d)}\right]} > 0$$

since $T > t_1$ and from Eq.(193).

Now the numerator

$$\xi_3 e^{\theta(t_1-t_d)} - \xi_1 = \left(ht_d e^{\theta(t_1-t_d)} + \xi_1\left[e^{\theta(t_1-t_d)} - 1\right]\right) > 0$$

Therefore the denominator should be greater than zero. Hence

$$\delta\left[\xi_1 + \xi_2 - \xi_3 e^{\theta(t_1-t_d)}\right] > 0$$

Since $\delta > 0$ the above inequality becomes

$$t_1 < t_d + \frac{1}{\theta} \ln\left[\frac{\xi_1+\xi_2}{\xi_3}\right] \equiv t_1^b (say)$$

Let

$$Z(x) = \frac{\delta x - 1}{\delta}\left[\xi_3 e^{\theta(x-t_d)} - \xi_1\right] - \frac{\xi_2}{\delta} \ln\left[\frac{\xi_1}{\xi_2} + 1 - \frac{\xi_3}{\xi_2} e^{\theta(x-t_d)}\right] - \frac{A}{\lambda} - \frac{\xi_3}{\theta}\left[e^{\theta(x-t_d)} - 1\right] - \frac{ht_d^2}{2} + \xi_1(x - t_d)$$

$$\frac{dZ(x)}{dx} = \theta\xi_3 e^{\theta(x-t_d)}\left[x + \frac{\xi_3 e^{\theta(x-t_d)} - \xi_1}{\delta\left[\xi_1+\xi_2-\xi_3 e^{\theta(x-t_d)}\right]}\right] > 0 \text{ for } x \geq t_d \geq 0.$$

$\frac{dZ(x)}{dx} > 0$ implies that $Z(x)$ is strictly increasing function of x in the interval $[t_d\ t_1^b)$. Now

$$Z(t_d) = \frac{\delta t_d - 1}{\delta}[\xi_3 - \xi_1] - \frac{\xi_2}{\delta}\ln\left[\frac{\xi_1 - \xi_3}{\xi_2} + 1\right] - \frac{A}{\lambda} - \frac{ht_d^2}{2}$$

and therefore $Z(t_d) > 0$ by given condition. That implies $Z(x) > 0,\ \forall\, x \in [t_d, t_1^b)$. Thus

$$\frac{\partial C(t_1, T)}{\partial T} = \frac{\lambda}{T^2}Z(t_1) > 0, \quad \forall\ t_1 \in [t_d, t_1^b)$$

which implies that $C(t_1, T)$ is a strictly increasing function of T. Thus $C(t_1, T)$ has minimum value when T is minimum. From Eq.(193), when $t_1 = t_d$, T has minimum value

$$t_d + \frac{ht_d}{(\hat{\pi} + \delta\pi) - \delta ht_d}$$

Therefore $C(t_1, T)$ has minimum value at the point (t_1^*, T^*) where $t_1^* = t_d$ and

$$T^* = t_d + \frac{ht_d}{(\hat{\pi} + \delta\pi) - \delta ht_d} \quad \square$$

Because t_1^*, T^* cannot be determined in a closed-form from Eqs.(193) and (195), we want to determine them using the following algorithm 7.1.

Algorithm 7.1

Step 1. Obtain the value of t_1^* by solving Eq.(195).

Step 2. Compute T^* from Eq.(193).

Step 3. The value of $C(t_1^*, T^*)$ can be obtained from Eq.(190).

7.4 Postponement and independent systems

Let t_i be the time upto which the inventory of product i is positive in a cycle. T_i is total cycle time for end-product i, $T_i > 0$. $C(t_i, T_i)$ is the total average cost per unit time for ordering and keeping end-product i.

In the independent system the raw materials are ordered independently (i.e., without postponement). The total average cost for ordering and keeping the m end-products in an independent system is

$$
\begin{aligned}
TC(\theta) &= \sum_{i=1}^{m} C(t_i, T_i/\theta) \\
&= \sum_{i=1}^{m} \Bigg[c\lambda_i + \frac{\lambda_i}{T_i}\Bigg(\frac{A}{\lambda_i} + \frac{ht_d}{2}\big(e^{\theta(t_i - t_d)} - 1\big) + \frac{ht_d^2}{2} + \frac{h + c\theta}{\theta^2}\big(e^{\theta(t_i - t_d)} - \theta(t_i - t_d) - 1\big) \\
&+ \frac{\hat{\pi} + \delta\pi}{\delta}\big(T_i - t_i - \frac{1}{\delta}\ln[1 + \delta(T_i - t_i)]\big)\Bigg)\Bigg]
\end{aligned} \tag{197}
$$

In the form of postponement system, all the raw materials are ordered together (i.e., postponing the customization process) and the demand rate is $\hat{\lambda} = \lambda_1 + \lambda_2 + ... + \lambda_m$. The total average cost for ordering and keeping the m end-products is given by (excluding the customization cost)

$$
\begin{aligned}
TCP(\hat{t}, \hat{T}/\theta) = c\hat{\lambda} + \frac{\hat{\lambda}}{\hat{T}}\Bigg(\frac{A}{\hat{\lambda}} + \frac{ht_d}{2}\big(e^{\theta(\hat{t} - t_d)} - 1\big) + \frac{ht_d^2}{2} + \frac{h + c\theta}{\theta^2}\big(e^{\theta(\hat{t} - t_d)} - \theta(\hat{t} - t_d) - 1\big) \\
+ \frac{\hat{\pi} + \delta\pi}{\delta}\big(\hat{T} - \hat{t} - \frac{1}{\delta}\ln[1 + \delta(\hat{T} - \hat{t}\,)]\big)\Bigg)
\end{aligned} \tag{198}
$$

The difference in the optimal cost of independent system and postponement system is defined as $z^* = TCP^*(\theta) - TC^*(\theta)$.

Theorem 7.3. The postponement system can give a lower total average cost than the independent system.

Proof. We have to prove that $TCP^*(\theta) < TC^*(\theta)$. Because $\hat{C}(\theta), = C(t^*, T^*/\theta)$, is continuous in $[0, +\infty)$, we have from theorem 7.1

$$
\lim_{\theta \to 0} TCP^*(\theta) = TCP^*(0) = c\hat{\lambda} + \sqrt{\frac{2A\delta h\xi_2\hat{\lambda}}{h + \delta\xi_2}} \tag{199}
$$

$$
\lim_{\theta \to 0} TC^*(\theta) = TC^*(0) = c\sum_{i=1}^{m}\lambda_i + \sum_{i=1}^{m}\sqrt{\frac{2A\delta h\xi_2\lambda_i}{h + \delta\xi_2}} \tag{200}
$$

Now

$$
TCP^*(0) - TC^*(0) = \sqrt{\frac{2A\delta h\xi_2}{h + \delta\xi_2}}\left[\sqrt{\hat{\lambda}} - \sum_{i=1}^{m}\sqrt{\lambda_i}\right] \tag{201}
$$

We now prove that $\sqrt{\hat{\lambda}} - \sum_{i=1}^{m}\sqrt{\lambda_i} < 0$ by induction on m. For $m = 2$, let $A = \sqrt{\lambda_1 + \lambda_2}$ and $B = \sqrt{\lambda_1} + \sqrt{\lambda_2}$. $A^2 - B^2 = -2\sqrt{\lambda_1\lambda_2} < 0$ for $\lambda_1, \lambda_2 > 0$. Hence

$A - B < 0$, since $A + B > 0$. So the inequality is true for $m = 2$. By induction, we assume the inequality for $m - 1$.

Now

$$\left[\sum_{i=1}^{m} \lambda_i\right]^{\frac{1}{2}} = \left[\sum_{i=1}^{m-1} \lambda_i + \lambda_n\right]^{\frac{1}{2}} < \left[\sum_{i=1}^{m-1} \lambda_i\right]^{\frac{1}{2}} + \sqrt{\lambda_m} < \sum_{i=1}^{m-1} \sqrt{\lambda_i} + \sqrt{\lambda_m}$$

Hence

$$\sqrt{\hat{\lambda}} - \sum_{i=1}^{m} \sqrt{\lambda_i} < 0.$$

Hence by the definition of continuity, there exists a $\bar{\theta} > 0$ such that for any $0 < \theta \leq \bar{\theta}$, $TCP^*(\theta) < TC^*(\theta)$. □

Remark 7.1.

When θ is small, we can obtain the approximate optimum total cost as

$$c\lambda + \sqrt{\frac{2A\delta(h + c\theta)\xi_2\lambda}{h + c\theta + \delta\xi_2}}.$$

From this we can find that deterioration effectively adds on additional component to the h i.e., from h to $h + c\theta$. Then

$$z^* = \sqrt{\frac{2A\delta(h + c\theta)\xi_2}{h + c\theta + \delta\xi_2}} \left[\sqrt{\hat{\lambda}} - \sum_{i=1}^{m} \sqrt{\lambda_i}\right] < 0.$$

From this we can observe that cost is reduced in the postponement system than in the independent system.

7.5 Numerical examples

7.5.1 Example 1

Let us consider an inventory system with the following data: $A = \$250$ per order, $h = \$1.75$ per unit, $c = \$5$ per unit, $\hat{\pi} = \$3$ per unit, $\pi = \$2$ per unit, $\delta = 2$, $\lambda = 600$ units, $t_d = 0.0833$ and $\theta = 0.2$.

We now study the effect of changes in the system parameters $A, h, \theta,$ and λ on the optimal length of inventory interval with positive inventory t_1^*, the optimal length of order cycle T^* and the minimum total cost $\hat{C}(\theta)$. The sensitivity analysis is performed by changing each of the said parameters by $+50\%$, $+25\%$, -25% and -50% taking one parameter at a time and keeping the remaining parameters unchanged. The results are shown in Table 6.

On the basis of the results of Table 6, the following observations can be made

1. The values of t_1^*, T^* and $\hat{C}(t_1^*, T^*)$ increase with the value of parameter A. This implies that if ordering cost per order could be reduced effectively, the total cost per unit time could be minimized.

2. t_1^* and T^* decrease whereas $\hat{C}(t_1^*, T^*)$ increases with the increase of h, θ, and λ.

Table 6: Effect of changes in various parameters of the inventory system.

Parameter	% change	% change in		
		t_1^*	T^*	$\hat{C}(t_1^*, T^*)$
A	+50	19.54	26.20	22.06
	+25	10.38	13.47	11.66
	−25	-12.07	-14.63	-13.44
	−50	-26.78	-31.23	-29.69
h	+50	-0.32	-0.41	3.64
	+25	-0.16	-0.20	1.82
	−25	0.16	0.21	-1.82
	−50	0.33	0.41	-3.64
λ	+50	-16.61	-19.88	22.31
	+25	-9.49	-11.58	11.78
	−25	13.55	17.78	-13.55
	−50	35.27	50.40	-29.95
θ	+50	-21.89	-8.77	8.77
	+25	-12.72	-5.23	4.96
	−25	19.02	8.42	-6.76
	−50	51.40	24.08	-16.61

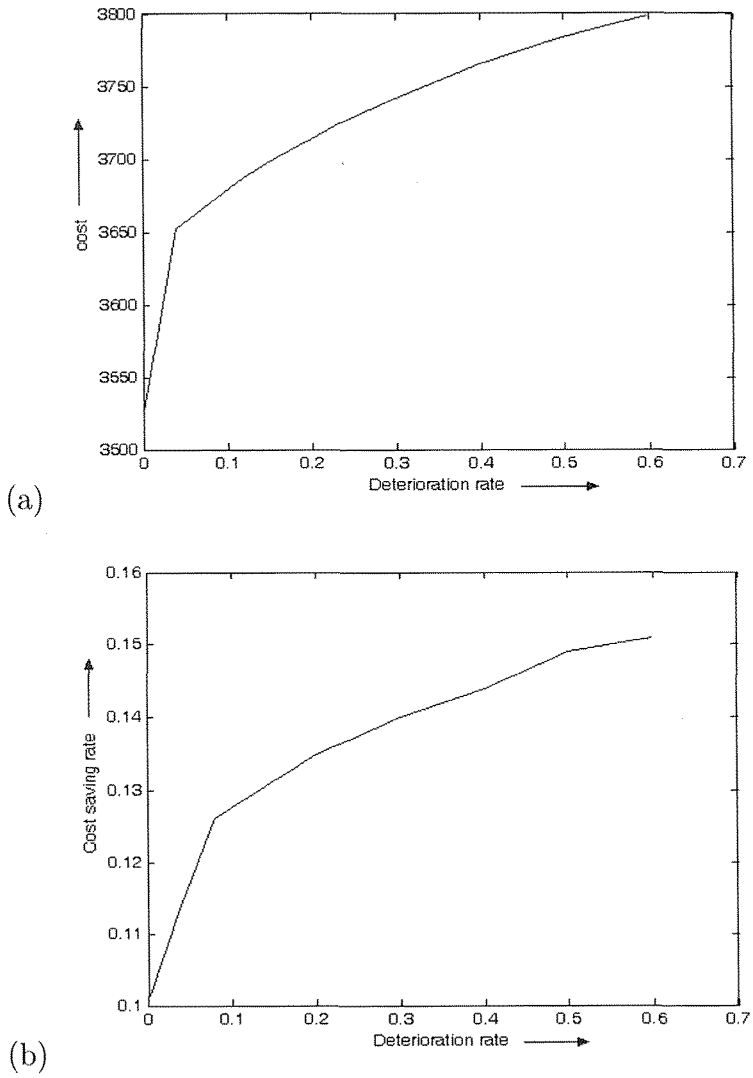

Figure 27: (a) Impact of deterioration rate on the total cost. (b) Effect of deterioration rate in cost savings.

7.5.2 Example 2

In order to study how deterioration rate affects the optimal cost of the inventory, the sensitivity analysis is performed. We consider the same data as in Example 1 except the value of θ. The values of the deterioration rate varies as follows: (0, 0.04, 0.08, 0.12, 0.16, 0.20, 0.24, 0.30, 0.40, 0.50, 0.60). We derive the results as shown in Table 7 and Fig.27 (a) from which the following results can be made:

- $T(\theta)$ and $t_1(\theta)$ are decreasing functions of θ in $[0, +\infty)$.
- θt_1^* is less sensitive on θ. The reason is that $t_1(\theta)$ is a decreasing function of θ.
- $\hat{C}(\theta)$ is an increasing and concave function of θ in $[0, +\infty)$.

Table 7. Impact of deterioration rate on inventory replenishment policies.

θ	0	0.04	0.08	0.12	0.16	0.20	0.24	0.30	0.40	0.50	0.60
t_1^*	0.483	0.560	0.525	0.495	0.468	0.446	0.425	0.399	0.364	0.335	0.312
θt_1^*	0	0.022	0.042	0.059	0.075	0.090	0.102	0.120	0.143	0.168	0.187
T^*	0.793	0.785	0.759	0.737	0.719	0.703	0.690	0.672	0.650	0.633	0.620
$\hat{C}(\theta)$	3526	3652	3670	3687	3701	3714	3726	3742	3765	3784	3799

7.5.3 Example 3

We study the effects of changes of A, h, λ and θ on the percentage change of cost difference between the independent system and postponement system. As in example 1, we consider the same data except the value of λ. Here we assume that there are eleven end products. For the 11 products, we assume that $\lambda_1 = 550, \lambda_2 = 560, \lambda_3 = 570, \lambda_4 = 580, \lambda_5 = 590, \lambda_6 = 600, \lambda_7 = 610, \lambda_8 = 620, \lambda_9 = 630, \lambda_{10} = 640, \lambda_{11} = 650$. We obtain the sensitivity results as in Table 8 by changing the values of A, h, λ and θ by $+50\%, +25\%, -25\%$ and -50% taking one parameter at a time and keeping the remaining parameters unchanged.

From the table 8, we see that the postponement system outperforms the independent system when the values of A, h, λ and θ increase.

Table 8: Effect of changes in various parameters in the implementation of postponement strategy

Parameter	% change	% change in		
		$\hat{t}^*$	$\hat{T}^*$	Cost saving z^*
A	+50	19.33	21.58	21.07
	+25	10.15	11.27	11.19
	−25	-24.98	-27.24	-28.92
	−50	-11.48	-12.61	-13.03
h	+50	-3.05	-3.37	3.42
	+25	-1.54	-1.70	1.86
	−25	1.57	1.73	-2.21
	−50	3.16	3.49	-4.76
λ	+50	-15.71	-17.21	23.09
	+25	-9.05	-9.95	12.19
	−25	13.30	14.80	-14.03
	−50	35.61	40.13	-31.00
θ	+50	-13.86	-9.05	10.86
	+25	-8.13	-5.33	6.14
	−25	12.56	8.31	-8.31
	−50	34.85	23.29	-20.29

Table 9: Impact of deterioration rate on the difference between the two systems.

θ	0	0.04	0.08	0.12	0.16	0.20	0.24	0.30	0.40	0.50	0.60
$-z^*$	4312	4406	4590	4728	4854	4968	5071	5211	5409	5574	5713
$\lvert z^*/TC^*\rvert$	0.101	0.114	0.126	0.129	0.132	0.135	0.137	0.140	0.144	0.149	0.151

7.5.4 Example 4

In order to study how various deterioration rates affect the difference in cost between the postponement system and independent system, we take λ_i value for i^{th} end-product as in numerical example 3. Other data are as in example 1 except the value of θ. We vary the θ value as in numerical example 2. We obtain the results of analysis with these parameters and it is shown in Table 9 and Fig.27(b), from which the following observations can be made:

- The absolute value of z^*/TC^* becomes larger when the deterioration rate becomes larger.
- The absolute value of z^* becomes larger when the deterioration rate becomes larger.
- The postponement system yields good savings in the total average cost.

These observations imply that the larger the deterioration rate is, the more cost-effective the postponement strategy is.

7.5.5 Practical implications

The proposed postponement strategy on a retailer of the supply chain can be implemented in the inventory control of 'non-instantaneous deteriorating items' such as food items, electronic components, fashionable commodities and others. A Hewlett - Packard Development company have saved money by adopting the postponement strategy [71].

7.6 Conclusion

We have developed EOQ - based models with 'non-instantaneous deteriorating' items to evaluate the impact of a postponement strategy on the retailer in a supply chain. It is shown that the postponement strategy outperforms the independent strategy. Our numerical examples showed that the difference between the two strategies will become larger when θ becomes larger. We assumed that the deterioration rate of raw materials is the same as that of end-products. But in real life the deterioration rate of raw materials, such as Integrated Circuit chips, is often smaller than that of the end-products. So postponement system can yield more savings in total cost in practice.