Chapter 6

Two-echelon trade credit in a supply chain with perishable items when demand depends on both selling price and credit period

6.1 Introduction

Today's research is interested in focusing on supply chain models which have real life applications. In real life business via share marketing, trade credit financing becomes a powerful tool to improve sales and profits in an industry. In practice, suppliers/ retailers allow a fixed period to settle the payment without penalty for their retailers/ customers to increase sales and reduce on-hand stock. This permissible delay in payments reduces the cost of holding stock because it reduces the amount of capital invested in stock for the duration of the permissible period. During the delay period (i.e. credit period) the retailer can accumulate revenue on sales and earn interest on that revenue via share market investment or banking business.

If we consider a supply chain problem for perishable items under two-echelon trade credit policy, it is very essential to consider the impact of both selling price and credit period on retailer's demand. Because, the large piles of customer goods in a supermarket are often associated with a price cut to induce more sales, as well as profits. Also, the marginal effect of credit period on sales is proportional to the unrealized potential of the market demand (Jaggi et al. [60]). Thus the retailer's demand becomes a function of both the selling price and credit period. Mostly, the selling items are perishable such as fruits, fresh fishes, gasoline, photographic films, etc. The effect of time is even more critical for the goods such as food stuff and cigarettes. Stocks are often replenished at certain production rate which is seldom infinite. Even for purchased items, when supply arrives at the warehouse, it may take days for receiving department to completely transfer the supply into storage room.

The effect of above situations imposed us to establish an EPQ inventory model for perishable items when the retailer's demand is a function of both credit period and selling price under two-echelon trade credit financing. To the best of our knowledge, this research is the first to incorporate both the two-echelon trade credits and demand which depends on selling price and credit period.

This chapter is organised as follows. In the next section, we describe the supply chain problem undertaken. In section 6.3, mathematical model relevant to the problem is developed. Section 6.4 provides the solution procedure to find optimal policy through the proposed algorithms. In section 6.5, we show that Jaggi et al. [60] model is a particular case of our model. Computational analysis is done in section 6.6. Finally, section 6.7 concludes this chapter.

6.2 Problem description

We consider supplier-retailer supply chain in which supplier fulfills the retailer's demand and retailer fulfills market customer's demands. The retailer of the supply chain replenishes the inventory at finite rate (P) from the supplier. P is known and uniform. The market customers purchase the items from the retailer. The selling items are perishable such as fruits, fresh fishes, gasoline, photographic films, etc. The supplier provides the retailer a full trade credit period (M). The retailer offers its customers full permissible delay period N, hence the retailer earns its revenue from N to $T+N$. The retailer's demand is a function of both selling price and credit period (N). The retailer's trade credit period (M) offered by the supplier is not necessarily longer than the customer's trade credit period (N) offered by the retailer. Here, we consider the possible three cases : $N \leq M \leq T+N$, $N \leq T+N \leq M$, $M \leq N \leq T+N$.

In addition to the notations described in chapter 1, the following notations have been used at the retailer of a supply chain. s is the unit selling price per item of good quality (decision variable). N is the permissible delay period in payment for the customer offered by the retailer (decision variable). $\lambda(s, N)$ is the annual demand, as a function of both s and N. P is the annual replenishment rate. t_1 is the time at which the production stops in a cycle. T is cycle time in years (decision variable). $TP(s, T, N)$ is the the annual total profit. We use the notations $\lambda(s, N)$ and λ interchangeably throughout this chapter. The following assumptions are made in deriving the model.

1. The demand, $\lambda(s, N)$, is a marginally increasing function with respect to N and downward sloping function of price s.

2. The replenishment rate is finite and $P > \lambda$.

3. The gross profit $(s - c)\lambda(s, N)$ is concave.

4. The time to deterioration of a product follows an exponential distribution with parameter θ, i.e. the deterioration rate is a constant fraction of the on-hand inventory.

5. Before the settlement of an account, the retailer can use sales revenue to earn the interest with an annual rate I_e up to the end of period M. At time $t = M$, the credit is settled and the retailer starts to pay the interest at rate I_k for the items in stock.

6. The retailer offers its customers a permissible delay period N, hence the retailer earns its revenue from N to $T + N$, not from 0 to T (cycle time).

7. Time horizon is infinite.

8. Inventory holding cost is charged only on the amount of undecayed stock.

9. Shortages are not allowed and lead time is negligible.

Under these conditions, we determine optimal credit period (N^*), selling price (s^*), and replenishment time (T^*) in order to maximize the retailer's profit.

6.3 Model formulation

We first model the demand function as below. For a given selling price $s > 0$, the marginal effect of credit period on sales is proportional to the unrealized potential of market demand without any delay. Under this assumption, the demand $\lambda(s, N)$ can be defined in the following two ways. First, the demand $\lambda(s, N)$ is represented by the partial differential equation,

$$\frac{\partial \lambda(s, N)}{\partial N} = r[\alpha(s) - \lambda(s, N)] \tag{149}$$

where

$\alpha(s)$ maximum demand over the planning horizon when the selling price is s,

r saturation rate of demand and $0 \leq r < 1$.

Secondly, $\lambda(s, N)$ can also be represented by the difference equation,

$$\lambda(s, N+1) - \lambda(s, N) = r[\alpha(s) - \lambda(s, N)] \tag{150}$$

The solutions to Eqs.(149) and (150) can be found by using the initial condition that when $N = 0$, $\lambda(s, N)$ is $\beta(s)$. That is, when there is no permissible delay period offered to the customers from the retailer, the demand is a function of selling price alone. This is a trivial case. We let $\beta(s)$ such that $\beta(s) < \alpha(s)$ for any value of $s > 0$. Here both $\alpha(s)$ and $\beta(s)$ are any non-negative, continuous, convex, decreasing functions of the selling price in $[0, s_u]$, where s_u is an extremely large number.

The solutions to Eqs.(149) and (150) are

Type 1:

$$\lambda(s, N) = \alpha(s) - [\alpha(s) - \beta(s)]e^{-rN} \tag{151}$$

and

Type 2:

$$\lambda(s, N) = \alpha(s)[1 - (1 - r)^N] + \beta(s)(1 - r)^N \tag{152}$$

respectively. The rest of our theory is fittest for both the Type 1 and Type 2 demand functions.

Due to the finite replenishment rate, the inventory level gradually increases from time $t = 0$ and it reaches the maximum at $t = t_1$. Production then stops at $t = t_1$

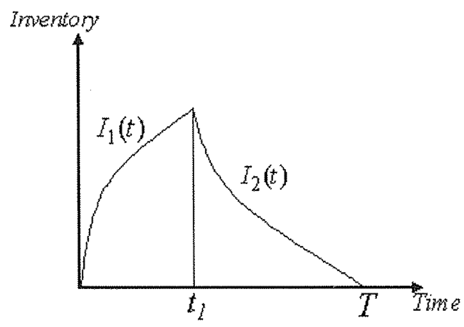

Figure 12: Inventory level at time t

and the inventory gradually depletes to zero at the end of the cycle $t = T$ due to deterioration and consumption. The graphical representation of this inventory system is clearly depicted in Fig. 12. The objective is to determine the optimal values for s, T and N (namely, s^*, T^* and N^*, respectively) such that the annual profit is maximized. From the above assumptions and notations, we know that the inventory level $I(t)$ at time t satisfies the following differential equations:

$$\frac{dI(t)}{dt} = P - \lambda(s, N) - \theta I(t), \text{ if } 0 \leq t \leq t_1 \tag{153}$$

and

$$\frac{dI(t)}{dt} = -\lambda(s, N) - \theta I(t), \qquad \text{if } t_1 \leq t \leq T \tag{154}$$

with the boundary conditions $I(0) = 0$ and $I(T) = 0$. Consequently the solutions for Eqs.(153) and (154) are given by

$$I(t) = \begin{cases} I_1(t) & \text{if } 0 \leq t \leq t_1 \\ I_2(t) & \text{if } t_1 \leq t \leq T \end{cases} \tag{155}$$

where

$$I_1(t) = \frac{P - \lambda(s, N)}{\theta}(1 - e^{-\theta t}) \tag{156}$$

$$I_2(t) = \frac{\lambda(s, N)}{\theta}(e^{\theta(T-t)} - 1) \tag{157}$$

By using the condition that $I_1(t) = I_2(t)$ at $t = t_1$, we obtain

$$t_1(s,T,N) = \frac{1}{\theta}\log\left[1 + \frac{\lambda(s,N)}{P}(e^{\theta T} - 1)\right] \tag{158}$$

We use the notations $t_1(s,T,N)$ and t_1 interchangeably throughout this chapter.

6.3.1 Determination of total profit function

We now derive the equations for sales profit, ordering cost, holding cost, deterioration cost, interest payable and interest earned. These components are evaluated as follows:

(1) **Sales profit** (SP)

The total sales profit is given by $(s-c)\lambda(s,N)$

(2) **Ordering cost**(OC)

The annual ordering cost is A/T

(3) **Holding cost** (HC)

Annual stock holding cost (excluding interest charges)

$$= \frac{h}{T}\int_0^T I(t)\ \mathrm{dt}$$

$$= \frac{h}{\theta^2 T}\left[(P - \lambda(s,N))[e^{-\theta t_1} + \theta t_1 - 1] + \lambda(s,N)[e^{\theta(T-t_1)} - \theta(T-t_1) - 1]\right]$$

(3) **Deteriorating cost**

Annual deteriorating cost (DC)

$$= \frac{c}{T}Pt_1 - c\lambda(s,N)$$

(4) **Interest Payable (IP)**

Based on the values of T, N and M, there are three cases to be considered: (i) $N \leq M \leq T+N$, (ii) $N \leq T+N \leq M$ and (iii) $M \leq N \leq T+N$.

Case 1. $N \leq M \leq T+N$.

Here, the retailer sells the units upto the end of his permissible delay period M. The items in stock are charged at interest rate I_k by the supplier starting from time M to $T+N$. As a result, the interest payable per unit time is

$$\frac{cI_k\lambda(s,N)}{\theta T}\int_M^{T+N}(e^{\theta(T+N-t)} - 1)\ \mathrm{dt} = \frac{cI_k\lambda(s,N)}{\theta^2 T}\left[e^{\theta(T+N-M)} - \theta(T+N-M) - 1\right] \tag{159}$$

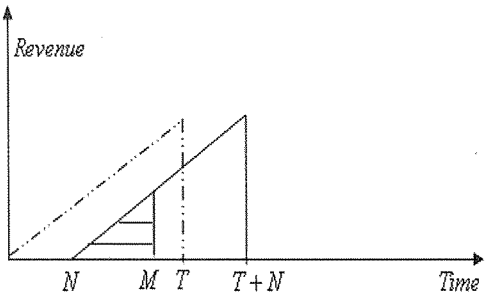

Figure 13: Total amount of interest earned when $N \le M \le T + N$

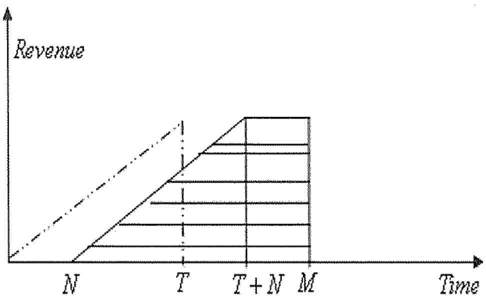

Figure 14: Total amount of interest earned when $N \le T + N \le M$

Case 2. $N \le T + N \le M$.

In this case, there is no interest payable by the retailer.

Case 3. $M \le N \le T + N$.

The retailer pays interest on full replenished quantity for a period from M to N and on average stock held during the cycle length. As a result, the annual interest payable is

$$\begin{aligned} &\frac{cI_k}{T}\left[\int_M^N Pt_1 \, \mathrm{dt} + \int_N^{T+N} \frac{\lambda(s,N)}{\theta}\left[e^{\theta(T+N-t)} - 1\right] \mathrm{dt}\right] \\ &\quad = \frac{cI_k}{\theta^2 T}\left[\theta^2 Pt_1(N-M) + \lambda(s,N)(e^{\theta T} - \theta T - 1)\right] \end{aligned} \tag{160}$$

(5) **Interest Earned (IE)**

Same as the interest payable, there are three cases to be considered.

Case 1. $N \leq M \leq T + N$, shown in Fig.13 (as in Jaggi et al.[60])

During the period from N to M, the retailer earns interest I_e per dollar. Therefore the annual interest earned is

$$\frac{sI_e\lambda(s,N)(M-N)^2}{2T} \tag{161}$$

Case 2. $N \leq T + N \leq M$, shown in Fig.14 (as in Jaggi et al.[60])

In this case, the retailer earns interest on average sales revenues received during the period from N to $T+N$ and on full sales revenue from a period from $T+N$ to M. As a result the annual interest earned is

$$\frac{sI_e}{T}\left[\frac{\lambda(s,N)T^2}{2} + \lambda(s,N)T(M-T-N)\right] = sI_e\lambda(s,N)[M-N-T/2] \tag{162}$$

Case 3. $M \leq N \leq T + N$.

In this case, there is no interest earned by the retailer..

The retailer's annual profit is

$$\begin{aligned} TP(s,T,N) &= \text{Sales profit} - \text{Ordering cost} - \text{Holding cost} \\ &\quad -\text{Cost of deteriorated items} - \text{Interest payable} + \text{Interest earned}, \\ &= (SP) - (OC) - (HC) - (DC) - (IP) + (IE) \end{aligned} \tag{163}$$

From the above arguments, the annual profit is given by

$$TP(s,T,N) = \begin{cases} TP_1(s,T,N) & \text{if } N \leq M \leq T+N \\ TP_2(s,T,N) & \text{if } N \leq T+N \leq M \\ TP_3(s,T,N) & \text{if } M \leq N \leq T+N \end{cases} \tag{164}$$

where

$$\begin{aligned} TP_1(s,T,N) &= (s-c)\lambda(s,N) - \frac{A}{T} - \frac{h}{T\theta^2}\Big[(P-\lambda(s,N))(e^{-\theta t_1} + \theta t_1 - 1) \\ &\quad +\lambda(s,N)(e^{\theta(T-t_1)} - \theta(T-t_1) - 1)\Big] - \frac{cPt_1}{T} + c\lambda(s,N) \\ &\quad -\frac{cI_k\lambda(s,N)}{T\theta^2}\left[e^{\theta(T+N-M)} - \theta(T+N-M) - 1\right] + \frac{sI_e\lambda(s,N)(M-N)^2}{2T} \end{aligned} \tag{165}$$

$$
\begin{aligned}
TP_2(s,T,N) &= (s-c)\lambda(s,N) - \frac{A}{T} - \frac{h}{T\theta^2}\Big[(P-\lambda(s,N))(e^{-\theta t_1}+\theta t_1 - 1) \\
&\quad + \lambda(s,N)(e^{\theta(T-t_1)} - \theta(T-t_1) - 1)\Big] - \frac{cPt_1}{T} + c\lambda(s,N) \\
&\quad + sI_e\lambda(s,N)[M-N-T/2] \qquad (166) \\
TP_3(s,T,N) &= (s-c)\lambda(s,N) - \frac{A}{T} - \frac{h}{T\theta^2}\Big[(P-\lambda(s,N))(e^{-\theta t_1}+\theta t_1 - 1) \\
&\quad + \lambda(s,N)(e^{\theta(T-t_1)} - \theta(T-t_1) - 1)\Big] \\
&\quad - \frac{cI_k}{T\theta^2}\left[\theta^2 Pt_1(N-M) + \lambda(s,N)(e^{\theta T} - \theta T - 1)\right] \qquad (167)
\end{aligned}
$$

After some significant simplifications, we have

$$
\begin{aligned}
TP_1(s,T,N) &= (s-c)\lambda(s,N) - \frac{A}{T} + \frac{(h+c\theta)}{\theta}\left[\lambda(s,N) - \frac{Pt_1}{T}\right] \\
&\quad - \frac{cI_k\lambda(s,N)}{T\theta^2}\left[e^{\theta(T+N-M)} - \theta(T+N-M) - 1\right] + \frac{sI_e\lambda(s,N)(M-N)^2}{2T} \qquad (168) \\
TP_2(s,T,N) &= (s-c)\lambda(s,N) - \frac{A}{T} + \frac{(h+c\theta)}{\theta}\left[\lambda(s,N) - \frac{Pt_1}{T}\right] + sI_e\lambda(s,N)[M-N-T/2] \\
&\qquad (169) \\
TP_3(s,T,N) &= (s-c)\lambda(s,N) - \frac{A}{T} + \frac{(h+c\theta)}{\theta}\left[\lambda(s,N) - \frac{Pt_1}{T}\right] \\
&\quad - \frac{cI_k}{T\theta^2}\left[\theta^2 Pt_1(N-M) + \lambda(s,N)(e^{\theta T} - \theta T - 1)\right] \qquad (170)
\end{aligned}
$$

6.4 Solution procedures

To find the optimal solution, say (s^*, T^*, N^*), for $TP(s,T,N)$, the following procedures are considered.

Lemma 6.1. If $f(t)$ is a continuous function on (a,b) and if $\frac{df}{dt}$ is non-increasing, then f is concave.

6.4.1 Determination of the optimal replenishment cycle length T for the given s and N

Case 1. $N \leq M \leq T + N$

For the given values of s and N, the first derivative of $TP_1(T|s,N)$ with respect to T is

$$\frac{dTP_1(T|s,N)}{dT} = \frac{Z_1(T|s,N)}{T^2}$$

where

$$Z_1(T|s,N) = \left(A - \frac{sI_e\lambda(s,N)(M-N)^2}{2}\right) - \frac{(c\theta+h)P}{\theta}\left[T\frac{\partial t_1}{\partial T} - t_1(s,T,N)\right] - \frac{cI_k\lambda(s,N)}{\theta^2}\left[\theta T e^{\theta(T+N-M)} - e^{\theta(T+N-M)} + \theta(N-M) + 1\right] \tag{171}$$

The optimal value of T, say $T_1^*(s,N)$, can be found by solving the equation $Z_1(T|s,N) = 0$. It is easy to obtain the following derivative

$$\frac{dZ_1(T|s,N)}{dT} = -\left[\frac{(c\theta+h)P}{\theta}\left(T\frac{\partial^2 t_1}{\partial T^2}\right) + cI_k\lambda(s,N)Te^{\theta(T+N-M)}\right] < 0,$$

since $\frac{\partial^2 t_1}{\partial T^2} > 0$ (it is easy to verify). Hence $Z_1(T|s,N)$ is non-increasing on $(0,\infty)$ and so $\frac{dTP_1(T|s,N)}{dT}$ is non-increasing. From Lemma 6.1, $TP_1(T|s,N)$ is a concave function on $(0,\infty)$. On the other hand, we have

$$\lim_{T\to\infty} Z_1(T|s,N) = -\infty < 0$$

and

$$Z_1(0) = \left(A - \frac{sI_e\lambda(s,N)(M-N)^2}{2}\right) - \frac{cI_k\lambda(s,N)}{\theta^2}\left[1 + \theta(N-M) - e^{\theta(N-M)}\right]$$

Since $1+\theta(N-M) < e^{\theta(N-M)}$, $Z_1(0) > 0$ if $\left(A - \frac{sI_e\lambda(s,N)(M-N)^2}{2}\right) > 0$. So we restrict our attention to the condition of $\left(A - \frac{sI_e\lambda(s,N)(M-N)^2}{2}\right) > 0$, in the rest of our mathematical analysis. Hence, we have

$$\lim_{T\to\infty} Z_1(T|s,N) = -\infty < 0 \text{ and } Z_1(0) > 0.$$

Based upon the above arguments, the intermediate value theorem yields that the optimal solution $T_1^*(s,N)$, not only exists but also is unique.

The similar procedure as described in case 1 can be applied to the remaining two cases.

Case 2. $N \le T + N \le M$

For the given values of s and N, we have

$$\frac{dTP_2(T|s,N)}{dT} = \frac{Z_2(T|s,N)}{T^2}$$

where

$$Z_2(T|s,N) = \left(A - \frac{sI_e\lambda(s,N)T^2}{2}\right) - \frac{(c\theta+h)P}{\theta}\left[T\frac{\partial t_1}{\partial T} - t_1(s,T,N)\right] \tag{172}$$

The optimal value of T, say $T_2^*(s,N)$, can be found by solving the equation $Z_2(T|s,N) = 0$. Now, we have

$$\frac{dZ_2(T|s,N)}{dT} = -sI_e\lambda(s,N)T - \left[\frac{(c\theta+h)PT}{\theta}\frac{\partial^2 t_1}{\partial T^2}\right] < 0,$$

Hence, $Z_2(T|s,N)$ is non-increasing on $(0,\infty)$ and so $\frac{dTP_2(T|s,N)}{dT}$ is non-increasing. From Lemma 6.1, $TP_2(T|s,N)$ is a concave function on $(0,\infty)$. On the other hand, we have

$$\lim_{T\to\infty} Z_2(T|s,N) = -\infty < 0 \text{ and } Z_2(0) > 0.$$

Based upon the above arguments, the intermediate value theorem yields that the optimal solution $T_2^*(s,N)$, not only exists but also is unique.

Case 3. $M \le N \le T+N$

For the given values of s and N, we have

$$\frac{dTP_3(T|s,N)}{dT} = \frac{Z_3(T|s,N)}{T^2}$$

where

$$Z_3(T|s,N) = A - \frac{P}{\theta}\left[(c\theta+h) + cI_k\theta(N-M)\right]\left[T\frac{\partial t_1}{\partial T} - t_1(s,T,N)\right] - \frac{cI_k\lambda(s,N)}{\theta^2}\left[\theta Te^{\theta T} - e^{\theta T} + 1\right] \tag{173}$$

The optimal value of T, say $T_3^*(s,N)$, can be found by solving the equation $Z_3(T|s,N) = 0$. Now, we have

$$\frac{dZ_3(T|s,N)}{dT} = -\frac{P}{\theta}\left[(h+c\theta) + cI_k\theta(N-M)\right]T\frac{\partial^2 t_1}{\partial T^2} - cI_k\lambda(s,N)Te^{\theta T} < 0,$$

Hence, $Z_3(T|s,N)$ is non-increasing on $(0,\infty)$ and so $\frac{dTP_3(T|s,N)}{dT}$ is non-increasing. From Lemma 6.1, $TP_3(T|s,N)$ is a concave function on $(0,\infty)$. On the other hand, we have

$$\lim_{T\to\infty} Z_3(T|s,N) = -\infty < 0 \text{ and } Z_3(0) > 0.$$

Based upon the above arguments, the intermediate value theorem yields that the optimal solution $T_3^*(s,N)$, not only exists but also is unique.

Combining the above three cases we have the following Lemma 6.2.

Lemma 6.2 For the given values of s and N,

(1) if $A - \frac{sI_e\lambda(s,N)}{2}(M-N)^2 > 0$, then $T_1^*(s,N)$ is the unique optimal solution to the profit function $TP_1(T|s,N)$.

(2) $TP_2(T|s,N)$ has the unique optimal solution $T_2^*(s,N)$ on the non-negative interval $(0,\infty)$

(3) $TP_3(T|s,N)$ has the unique optimal solution $T_3^*(s,N)$ on the non-negative interval $(0,\infty)$

Let

$$\Delta_1(s,N) = A - \frac{sI_e\lambda(s,N)(M-N)^2}{2} - \frac{(c\theta+h)P}{\theta}\left[\frac{(M-N)\lambda(s,N)e^{\theta(M-N)}}{P+\lambda(s,N)(e^{\theta(M-N)}-1)} - \frac{1}{\theta}\log\left(1+\frac{\lambda(s,N)}{P}(e^{\theta(M-N)}-1)\right)\right] \tag{174}$$

and

$$\Delta_2(s,N) = A - \frac{P}{\theta}\left[(c\theta+h)+cI_k\theta(N-M)\right]\left[\frac{(M-N)\lambda(s,N)e^{\theta(M-N)}}{P+\lambda(s,N)(e^{\theta(M-N)}-1)} - \frac{1}{\theta}\log\left(1+\frac{\lambda(s,N)}{P}(e^{\theta(M-N)}-1)\right)\right] - \frac{cI_k\lambda(s,N)}{\theta^2}\left[\theta(M-N)e^{\theta(M-N)} - e^{\theta(M-N)}+1\right] \tag{175}$$

Using the values of $\Delta_1(s,N)$ and $\Delta_2(s,N)$, we have the following Theorem.

Theorem 6.1. For the fixed values of s and N,

(1) if $\Delta_1(s,N) \geq 0$, then $T^*(s,N) = T_1^*(s,N)$.

(2) if $\Delta_1(s,N) \leq 0$, then $T^*(s,N) = T_2^*(s,N)$.

(3) if $\Delta_2(s,N) \geq 0$ and $M-N<0$ then $T^*(s,N) = T_3^*(s,N)$.

Proof. Please refer to appendix C for details.□

6.4.2 Optimal selling price s^* for the given value of T and N

For any given values of T and N, the optimal value of s can be determined by solving the first order necessary condition (i.e., $\frac{\partial TP}{\partial s}=0$) and examining the second order sufficient condition for concavity (i.e., $\frac{\partial^2 TP}{\partial s^2}<0$).

Case 1. $N \le M \le T+N$

For the given values of T and N, the first order and second order partial derivatives of $TP_1(s,T,N)$ with respect to s are given by

$$\begin{aligned}\frac{\partial TP_1}{\partial s} &= \left[(s-c)\frac{\partial \lambda}{\partial s}+\lambda\right]+\frac{I_e(M-N)^2}{2T}\left[s\frac{\partial \lambda}{\partial s}+\lambda\right]+\frac{(h+c\theta)}{\theta}\left[\frac{\partial \lambda}{\partial s}-\frac{P}{T}\frac{\partial t_1}{\partial s}\right]\\ &- \frac{cI_k}{T\theta^2}\left[e^{\theta(T+N-M)}-\theta(T+N-M)-1\right]\frac{\partial \lambda}{\partial s} \qquad (176)\end{aligned}$$

and

$$\begin{aligned}\frac{\partial^2 TP_1}{\partial s^2} &= \left[(s-c)\frac{\partial^2 \lambda}{\partial s^2}+2\frac{\partial \lambda}{\partial s}\right]+\frac{I_e(M-N)^2}{2T}\left[s\frac{\partial^2 \lambda}{\partial s^2}+2\frac{\partial \lambda}{\partial s}\right]+\frac{(h+c\theta)}{\theta}\left[\frac{\partial^2 \lambda}{\partial s^2}-\frac{P}{T}\frac{\partial^2 t_1}{\partial s^2}\right]\\ &- \frac{cI_k}{T\theta^2}\left[e^{\theta(T+N-M)}-\theta(T+N-M)-1\right]\frac{\partial^2 \lambda}{\partial s^2} \qquad (177)\end{aligned}$$

Case 2. $N \le T+N \le M$

For the given values of T and N, the first order and second order partial derivatives of $TP_2(s,T,N)$ with respect to s are given by

$$\frac{\partial TP_2}{\partial s} = \left[(s-c)\frac{\partial \lambda}{\partial s}+\lambda\right]+I_e(M-N-T/2)\left[s\frac{\partial \lambda}{\partial s}+\lambda\right]+\frac{(h+c\theta)}{\theta}\left[\frac{\partial \lambda}{\partial s}-\frac{P}{T}\frac{\partial t_1}{\partial s}\right] \qquad (178)$$

and

$$\frac{\partial^2 TP_2}{\partial s^2} = \left[(s-c)\frac{\partial^2 \lambda}{\partial s^2}+2\frac{\partial \lambda}{\partial s}\right]+I_e(M-N-T/2)\left[s\frac{\partial^2 \lambda}{\partial s^2}+2\frac{\partial \lambda}{\partial s}\right]+\frac{(h+c\theta)}{\theta}\left[\frac{\partial^2 \lambda}{\partial s^2}-\frac{P}{T}\frac{\partial^2 t_1}{\partial s^2}\right] \qquad (179)$$

Case 3. $M \le N \le T+N$

For the given values of T and N, the first order and second order partial derivatives of $TP_3(s,T,N)$ with respect to s are given by

$$\begin{aligned}\frac{\partial TP_3}{\partial s} &= \left[(s-c)\frac{\partial \lambda}{\partial s}+\lambda\right]+\frac{(h+c\theta)}{\theta}\left[\frac{\partial \lambda}{\partial s}-\frac{P}{T}\frac{\partial t_1}{\partial s}\right]\\ &-\frac{cI_k}{T\theta^2}\left[\theta^2 P\frac{\partial t_1}{\partial s}(N-M)+\frac{\partial \lambda}{\partial s}(e^{\theta T}-\theta T-1)\right] \qquad (180)\end{aligned}$$

and

$$\begin{aligned}\frac{\partial^2 TP_2}{\partial s^2} &= \left[(s-c)\frac{\partial^2 \lambda}{\partial s^2}+2\frac{\partial \lambda}{\partial s}\right]+\frac{(h+c\theta)}{\theta}\left[\frac{\partial^2 \lambda}{\partial s^2}-\frac{P}{T}\frac{\partial^2 t_1}{\partial s^2}\right]\\ &-\frac{cI_k}{T\theta^2}\left[\theta^2 P\frac{\partial^2 t_1}{\partial s^2}(N-M)+\frac{\partial^2 \lambda}{\partial s^2}(e^{\theta T}-\theta T-1)\right] \qquad (181)\end{aligned}$$

Note that $TP_i(s|N,T)$ (i=1,2,3) are continuous functions of s over the compact set $[0,s_u]$, where s_u is an extremely large number. Hence $TP_i(s|N,T)$ (i=1,2,3) has a maximum value. It is clear that $TP_i(s|N,T)$ (i=1,2,3) is not maximum if $s = 0$ or s_u. As a result, the optimal s must be an interior point between 0 and s_u. This implies that Eqs.(176), (178) and (180) have at least one solution. If the solution is unique, then it is the optimal solution (i.e. the Eqs. (176), (178) and (180) are necessary and sufficient conditions). Otherwise, we have to find the one at which the second order partial derivative is less than zero. Although we are not able to prove the uniqueness of the solution, the numerical examples 1 and 2 show that a unique solution exists for the considered optimization problem. We can easily calculate the Hessain matrix of TP as a negative definite although it is not straightforward to find a closed form solution when both the price and replenishment time are decision variables.

Following the solution approach given in Chang et al. [20], the solution of $(s - c)\frac{\partial \lambda}{\partial s} + \lambda(s) = 0$, say s_l, is the lower bound for the optimal selling price s^*. The solution s_l has been taken as a initial value for s in algorithm 6.1. To find the optimal solution (s^*, T^*, N^*), we run algorithm 6.1 and algorithm 6.2 simultaneously using Matlab 7.0.

In order to find the optimal solutions, the following algorithms are used.

Algorithm 6.1

To find the optimal value of s and T for a given value of N, we adopt the following steps.

1. Let $j = 1$. Find s_l by solving the equation $(s - c)\frac{\partial \lambda}{\partial s} + \lambda(s) = 0$.

2. Compute $\Delta_1(s_l, N)$ and $\Delta_2(s_l, N)$. Using Theorem 6.1, find optimal cycle time which belongs to the set $\{T_1^*, T_2^*, T_3^*\}$. Let T_{ij} be the corresponding T_i^*, $i \in \{1, 2, 3\}$. Let $s_{ij} = s_l$.

3. Solve the equation $\frac{\partial TP_i}{\partial s} = 0$ for s by using the value $T = T_{ij}$. Find $\hat{s}_i$ such that $\frac{\partial^2 TP_i}{\partial s^2}|_{(T=T_{ij}, s=\hat{s}_i)} < 0$ and let $s_{i,j+1} = \hat{s}_i$.

4. Solve the equation $\frac{\partial TP_i}{\partial T} = 0$ for T by using the value $s = s_{i,j+1}$ and let the solution be $T_{i,j+1}$.

5. If $|T_{ij} - T_{i,j+1}| < \epsilon$ and $|s_{ij} - s_{i,j+1}| < \epsilon$, where ϵ is any small positive number, then $s^* = s_{i,j+1}$ and $T^* = T_{i,j+1}$. Otherwise let j = j+1, go to step 3.

Algorithm 6.2

For finding the optimal solution (s^*, T^*, N^*), the following steps can be followed.

1. Let $N = 1$ and $TP_i(s^{(0)}, T^{(0)}, 0) = 0$ for $i = 1, 2, 3$.

2. Determine the optimal solution $(s^{(N)}, T^{(N)})$ from Algorithm 6.1.

3. If $N \leq M \leq T^{(N)} + N$ then calculate $TP_1(s^{(N)}, T^{(N)}, N)$; else go to step 5.

4. If $TP_1(s^{(N-1)}, T^{(N-1)}, N-1) > TP_1(s^{(N)}, T^{(N)}, N)$ then the optimum solution, say (s^*, T^*, N^*), is $(s^{(N-1)}, T^{(N-1)}, N-1)$ and $TP^* = TP_1(s^*, T^*, N^*)$. Otherwise, let $N = N + 1$ and go to step 2.

5. If $N \leq T^{(N)} + N \leq M$ then calculate $TP_2(s^{(N)}, T^{(N)}, N)$; else go to step 7.

6. If $TP_2(s^{(N-1)}, T^{(N-1)}, N-1) > TP_2(s^{(N)}, T^{(N)}, N)$ then the optimum solution, say (s^*, T^*, N^*), is $(s^{(N-1)}, T^{(N-1)}, N-1)$ and $TP^* = TP_2(s^*, T^*, N^*)$. Otherwise, let $N = N + 1$ and go to step 2.

7. If $M \leq N \leq T^{(N)} + N$ then calculate $TP_3(s^{(N)}, T^{(N)}, N)$.

8. If $TP_3(s^{(N-1)}, T^{(N-1)}, N-1) > TP_3(s^{(N)}, T^{(N)}, N)$ then the optimum solution, say (s^*, T^*, N^*), is $(s^{(N-1)}, T^{(N-1)}, N-1)$ and $TP^* = TP_3(s^*, T^*, N^*)$. Otherwise, let $N = N + 1$ and go to step 2.

6.5 Particular case

Here, we let $\theta \to 0$, $P \to \infty$ and demand to be a function of credit period alone i.e., s is fixed. When the deterioration is ignored, there is no cost due to deteriorated units. If the selling price (s) is fixed then the demand is $\lambda(N)$. By the above parametric considerations, we have $\lim_{\theta \to 0^+} t_1 = 0$. Adopting above conditions, Eqs. (165), (166) and (167) become as follows:

$$TP_1(T, N) = (s-c)\lambda - \frac{A}{T} - \frac{h\lambda T}{2} - \frac{cI_k\lambda(T+N-M)^2}{2T} + \frac{sI_e\lambda(M-N)^2}{2T} \quad (182)$$

$$TP_2(T, N) = (s-c)\lambda - \frac{A}{T} - \frac{h\lambda T}{2} + sI_e\lambda(M-N-T/2) \quad (183)$$

$$TP_3(T, N) = (s-c)\lambda - \frac{A}{T} - \frac{h\lambda T}{2} - cI_k\lambda(N-M+T/2) \quad (184)$$

The Eqs. (182), (183) and (184) are consistent with Eqs. (10), (12) and (14) in Jaggi et al. [60] (where $IC = h$).

Consequently, we obtain that

$$
\begin{aligned}
Z_1(T,N) &= A - \frac{sI_e\lambda(M-N)^2}{2} - \frac{h\lambda T^2}{2} - \frac{cI_k\lambda}{2}(T+N-M)(T-N+M) \quad (185)\\
Z_2(T,N) &= A - \frac{sI_e\lambda T^2}{2} - \frac{h\lambda T^2}{2} \quad (186)\\
Z_3(T,N) &= A - \frac{h\lambda T^2}{2} - \frac{cI_k\lambda T^2}{2} \quad (187)
\end{aligned}
$$

From Eqs.(185), (186) and (187), we obtain the following,

$$
\begin{aligned}
\Delta_1(N) &= A - \frac{\lambda(M-N)^2}{2}[sI_e + h]\\
\Delta_2(N) &= A - \frac{\lambda(M-N)^2}{2}[h + cI_k]
\end{aligned}
$$

From the above, Theorem 6.1 becomes

Theorem 6.2. For the fixed values N,
(1) if $\Delta_1(N) \geq 0$, then $T^*(N) = T_1^*(N)$.
(2) if $\Delta_1(N) \leq 0$, then $T^*(N) = T_2^*(N)$.
(3) if $\Delta_2(N) \geq 0$ and $M - N < 0$ then $T^*(N) = T_3^*(N)$.

These results of Theorem 6.2 are identical to Theorem 1 in Jaggi et al. [60]. Thus, Jaggi et al. [60] can be treated as a special case of our model.

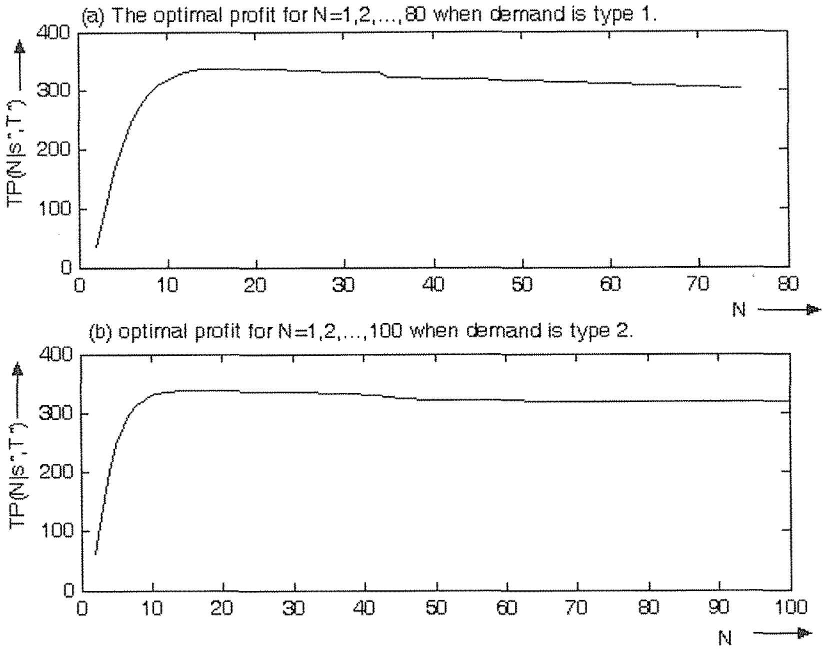

Figure 15: The optimal total profit for various values of N in type-1 and type-2 demands.

6.6 Computational analysis

The purposes of the numerical analysis are as follows:

1. To obtain the optimal solutions for two types of demand functions.
2. To use sensitivity analysis to highlight the influence of model parameters.

6.6.1 Numerical Examples

Example 1

We consider the demand type-1, i.e., $\lambda(s,N) = \alpha(s) - [\alpha(s) - \beta(s)]e^{-rN}$. Let $\alpha(s) = 80 - 1.21s, \beta(s) = 30 - 1.21s, r = 0.35$, production rate $P = 100, \theta = 0.01, A =$ \$1000, $M = 30$ days, $h =$ \$4.5 per unit, $c =$ \$30 per unit, $I_e = 25\%$ per year, $I_k = 15\%$ per year.

The three dimensional graphs of total profit of the retailer are presented in Fig.17. The graphs reveal that for any given N (e.g., $N = 8,17,26$), there exists a corresponding

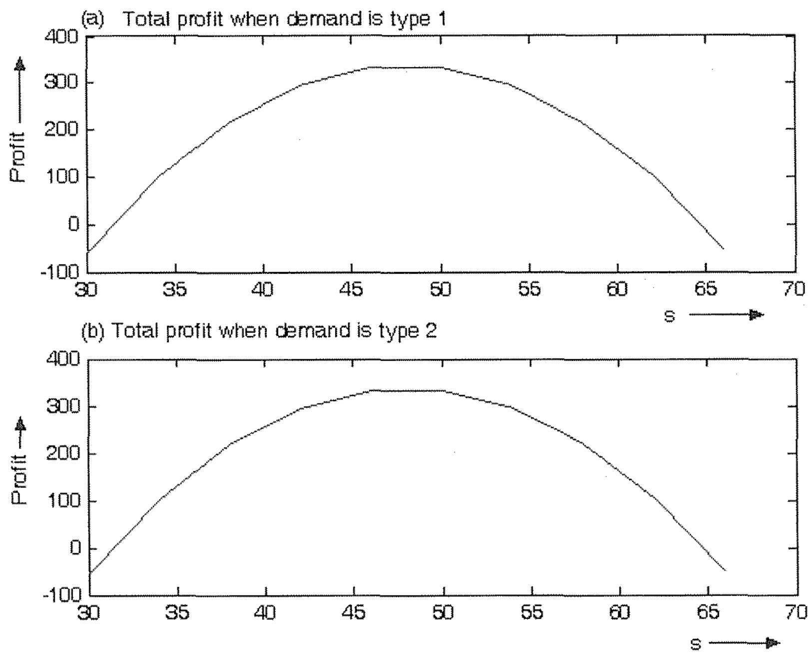

Figure 16: The optimal total profit for various values of s in type-1 and type-2 demands.

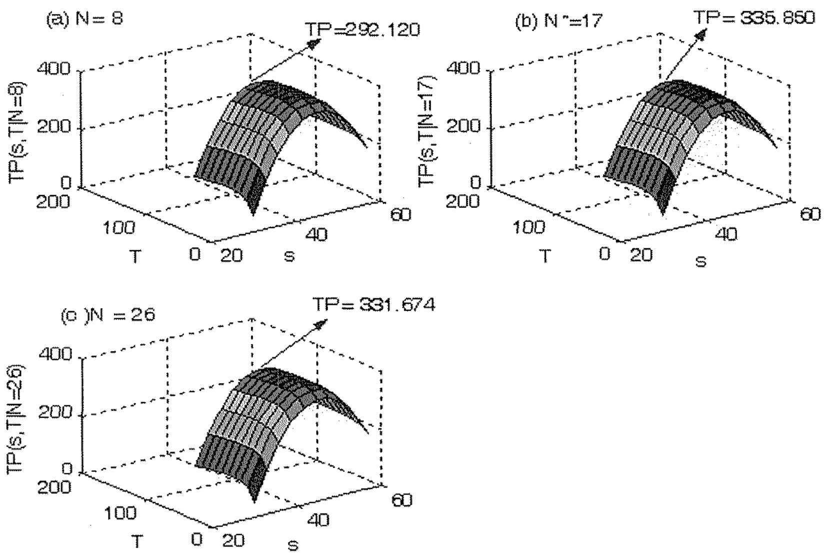

Figure 17: The total profit for any given N (e.g., N = 8, 17, and 26) when demand is of type-1.

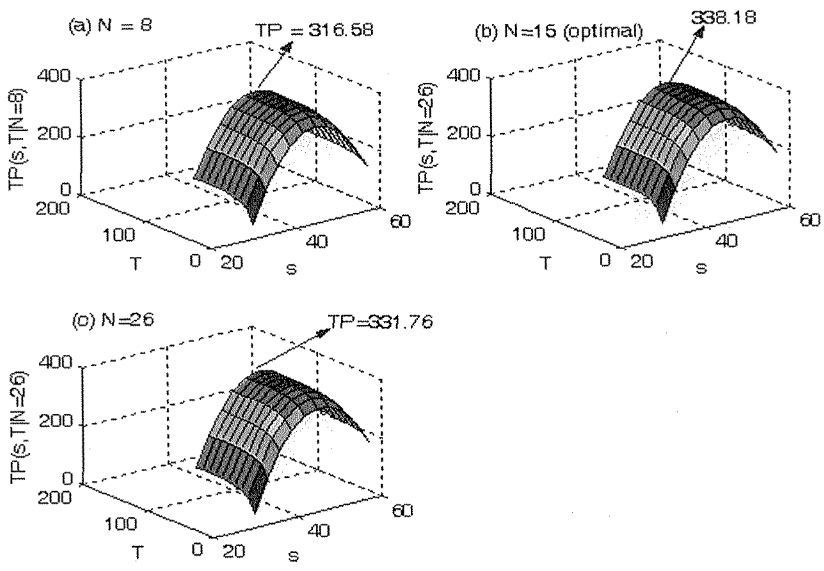

Figure 18: The total profit for any given N (e.g., N = 8, 15, and 26) when demand is of type-2.

optimal solution $(s^{(N)}, T^{(N)})$ which maximizes the total profit. Furthermore, we run the numerical results with values of N= 1,2,..,80. The numerical results indicate that there is a unique integer N which maximizes the value of $TP(N) = TP(N, s^{(N)}, T^{(N)})$, as shown in Fig.15(a). Also, Fig.16(a) shows that $TP(s|N^*, T^*)$ is strictly concave function of s. As a result, we are sure that the maximum obtained from the proposed algorithms is indeed the global maximum solution.

Using the Algorithms 6.1 and 6.2, in Matlab 7.0, we obtained the following optimal results: optimal cycle length (T^*) = 17.92 days, optimal credit period offered by the retailer to his customers, N^*, is 17 days, optimal selling price s^* = \$48.77 per unit and the optimal total profit is \$335.850.

Example 2

Here, we consider the same data set as in numerical example 1 but the demand function is type-2, i.e., $\lambda(s, N) = \alpha(s)[1-(1-r)^N]+\beta(s)(1-r)^N$. The three dimensional graphs of total profit of the retailer are presented in Fig.18. The graphs reveal that

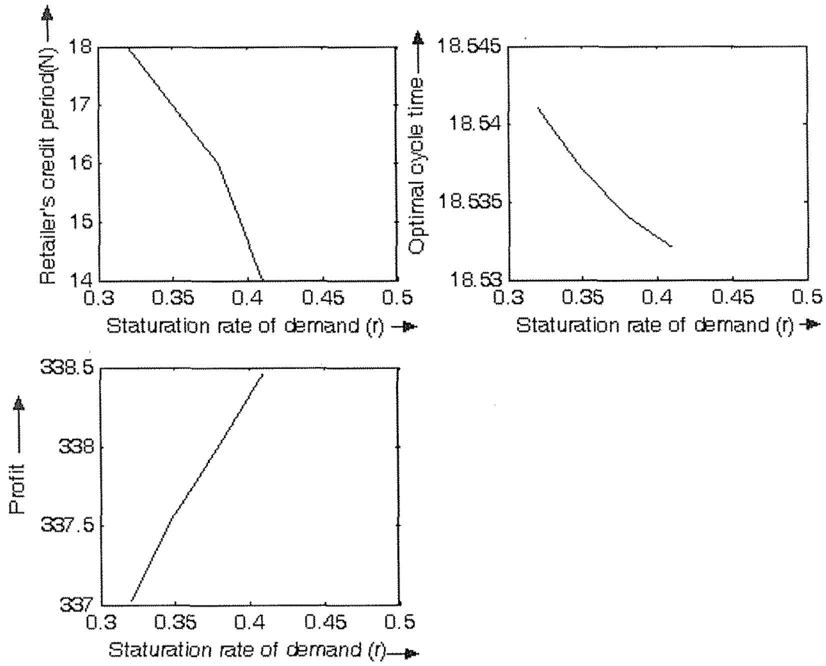

Figure 19: Effect of changing saturation rate of retailer's demand.

for any given N (e.g., N = 8,15,26), there exists a corresponding optimal solution $(s^{(N)}, T^{(N)})$ which maximizes the total profit. Furthermore, we run the numerical results with values of N= 1,2,..,100. The numerical results indicate that there is a unique integer N which maximizes the value of $TP(N) = TP(N, s^{(N)}, T^{(N)})$, as shown in Fig.15(b). Also, Fig.16(b) shows that $TP(s|N^*, T^*)$ is strictly concave function of s. As a result, we are sure that the optimum obtained from the proposed algorithms is indeed the global optimum solution for the type-2 demand function also.

Using the Algorithms 6.1 and 6.2, in Matlab 7.0, we obtained the following optimal results: optimal cycle length (T^*) = 17.90 days, optimal credit period offered by the retailer to his customers, N^*, is 15 days, optimal selling price s^* = \$48.70 per unit and the optimal total profit is \$338.18.

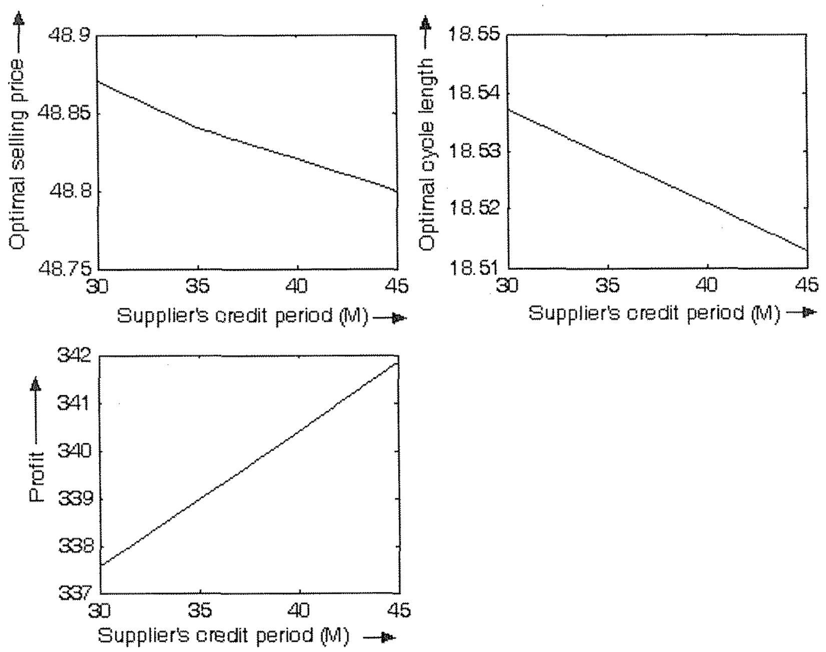

Figure 20: Effect of changing supplier's credit period.

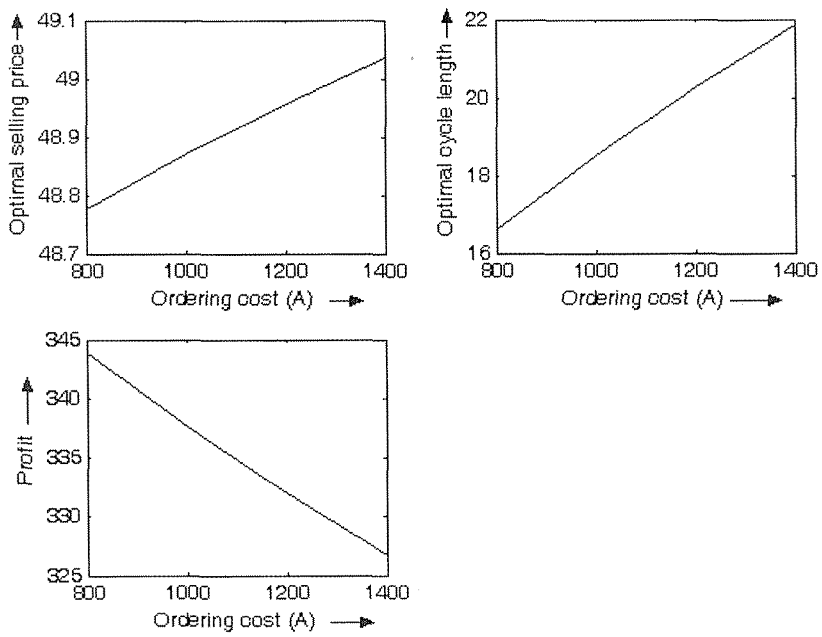

Figure 21: Effect of changing ordering cost.

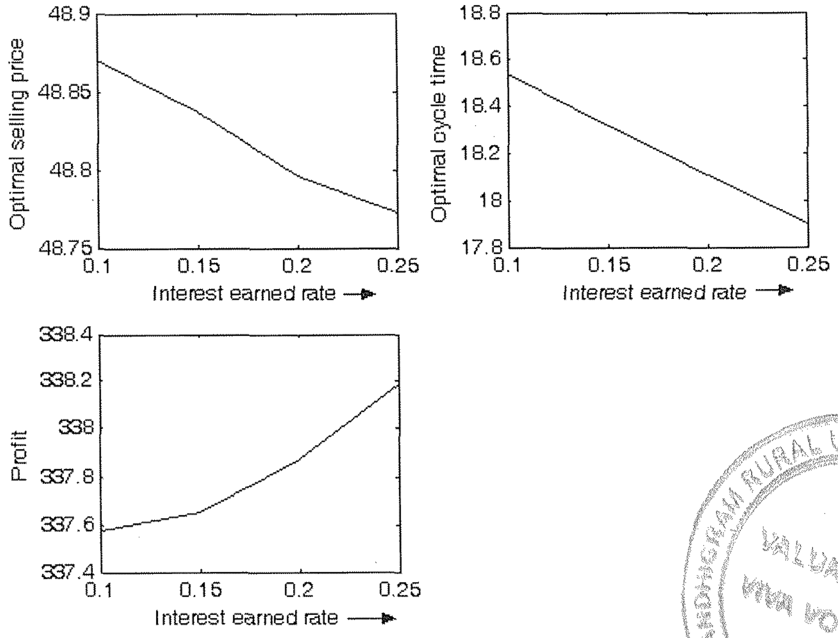

Figure 22: Effect of changing interest earned rate by the retailer.

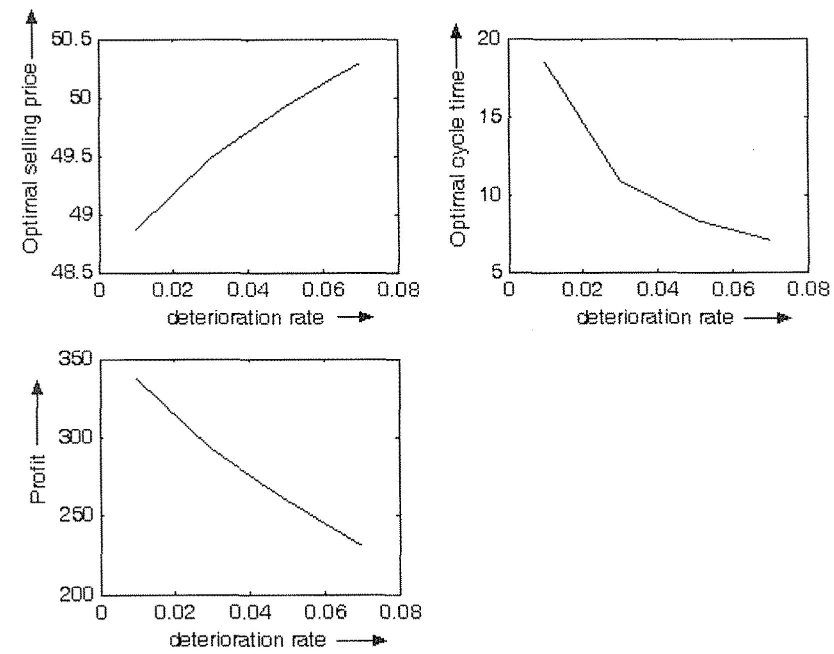

Figure 23: Effect of changing the deterioration rate.

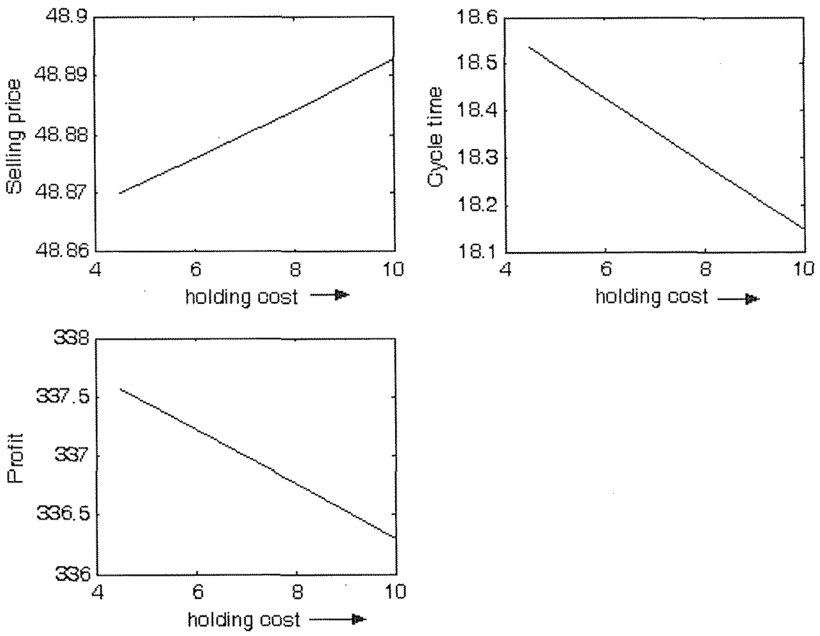

Figure 24: Effect of changing the holding cost.

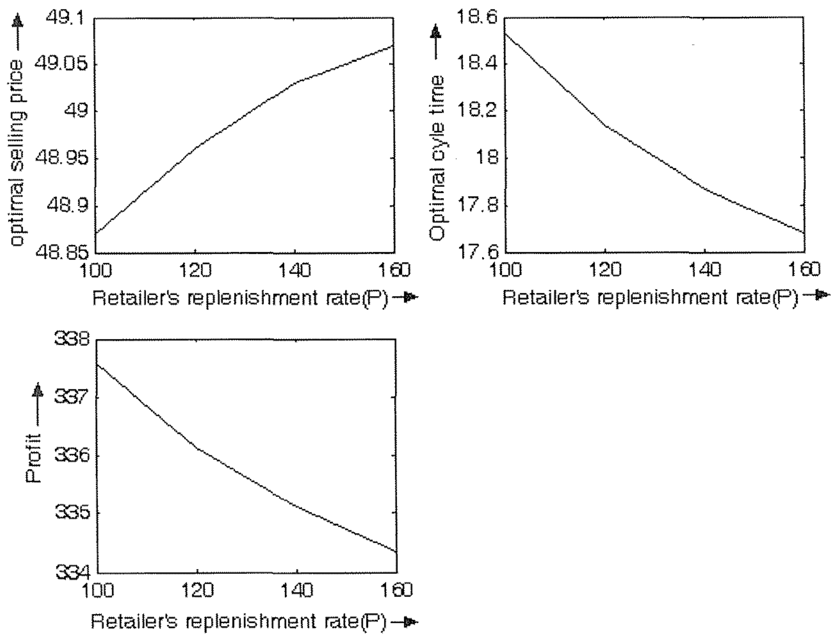

Figure 25: Effect of changing the replenishment rate.

Table 5: Sensitivity analysis for various inventory model parameters

parameter		N^*	T^*	s^*	TP^*
r	0.32	18	18.542	48.869	337.025
	0.35	17	18.537	48.869	337.578
	0.38	16	18.534	48.869	338.023
	0.41	14	18.532	48.857	338.464
M	30	17	18.537	48.870	337.578
	35	17	18.529	48.848	338.995
	40	17	18.521	48.825	340.414
	45	17	18.513	48.802	341.833
A	800	17	16.610	48.776	343.808
	1000	17	18.537	48.871	337.578
	1200	17	20.275	48.957	331.913
	1400	17	21.867	49.036	326.674
I_e	0.10	17	18.537	48.871	337.578
	0.15	16	18.320	48.837	337.653
	0.20	15	18.110	48.796	337.875
	0.25	15	17.904	48.772	338.188
P	100	17	18.537	48.870	337.578
	120	17	18.137	48.960	336.127
	140	17	17.869	49.029	335.104
	160	17	17.677	49.077	334.343
θ	0.01	17	18.537	48.869	337.578
	0.03	17	10.853	49.487	293.088
	0.05	17	8.370	49.927	259.300
	0.07	17	7.034	50.293	230.027
h	4.5	17	18.537	48.869	337.578
	6	17	18.428	48.876	337.220
	8	17	18.286	48.884	336.760
	10	17	18.147	48.893	336.302

6.6.2 Effect of changing the inventory model parameters

Here, we consider the demand function type-2 (one can also consider type-1). Let $\alpha(s) = 80 - 1.21s, \beta(s) = 30 - 1.21s, r = 0.35$, production rate $P = 100, \theta = 0.01, A =$ \$1000, $M = 30$ days, $h =$ \$4.5 per unit, $c =$ \$30 per unit, $I_e = 10\%$ per year, $I_k = 15\%$ per year. The sensitivity analysis is performed by varying different parameters and is given in Table 5.

It is important to discuss the influence of key model parameters on the optimal solutions. The effect of changing the parameters are shown graphically in Figs.(8) to (14). Based on these figures, we have the following comments.

1. It is observed that as r increases, N^* decreases and T^* marginally decreases; but TP^* increases. The optimal selling price remains at the threshold. It shows that retailer should offer lower credit period (N) to customers when the rate of saturation of demand (r) is higher. (fig. 19)

2. A higher value of M causes a higher value of TP^*, but lower value of s^* and T^*. It indicates the following managerial phenomena: when the supplier provides a longer credit period, the retailer replenishes the goods more often. In other words, the retailer will shorten the cycle time and reduce the selling price in order to take advantage of the longer credit period. (fig. 20)

3. As ordering cost, A, increases, the replenishment cycle time T^* significantly increases; but optimal selling price marginally increases. Keeping the credit period (N^*) at some threshold, the optimal profit decreases as A increases. It indicates the following managerial effect. If the ordering cost is higher, it is reasonable that the retailer lengthens the cycle time to reduce the frequency of replenishment and he marginally increases the selling price. (fig. 21)

4. As I_e increases, N^* decreases; but T^* and s^* are marginally decreasing. Profit TP^* increases as interest earned rate of the retailer increases. It implies that if the retailer increases his interest earned rate, then he can shorten his trade credit period with marginally less selling price. (fig. 22)

5. As the value of θ increases, TP^* and T^* decrease whereas s^* is increasing. That is, when the items are starting to deteriorate, it is optimal to rise marginally the selling price in order to manage the profit. (fig. 23)

6. When holding increases, it is seen that cycle length and profit decreases whereas the optimal selling price increases. So it is reasonable that when the holding cost increases the retailer will shorten the cycle time and increases the selling price in an effort to maintain his profit gained by keeping the threshold credit period (N^*). (fig. 24)

7. As replenishment rate increases, TP^* decreases; but there are marginal increases in selling price, s^*, under the optimal threshold value of N^*. So it is not advisable to increase the replenishment rate without the prior knowledege about the demands. (fig. 25)

6.6.3 When customer's payment exceeds the optimal credit period (N^*)

If a customer violates the payment condition, then the retailer can fix some another duration for payment with increased interest charges ($> I_k$). Using trade credit insurance, retailers can generally extend more credit to customers whilst reducing the risk of non-payment, thereby enabling sales growth without a corresponding increase in risk. Insurance can also enable the retailers economy to secure more favorable financing terms, as insured accounts receivable may be used as collateral.

6.7 Conclusion

The increased prominence of financial markets and their widespread use in pricing and in credit period allocation provides an opportunity for convergence between financial tools and operational problems. The latter is based mostly on a private valuation of profits and benefits associated with the trade credit policy. In this chapter, we first formulated an EPQ - based inventory model for perishable items under two-echelon trade credit policy with the assumptions that the market demand is sensitive to both the selling price and credit period offered by the retailer. After formulating the mathematical model, we then developed solution procedures to determine the best payment method, the optimal selling price and optimal cycle length for the retailer.

From the numerical examples 1 and 2, we examined that the proposed algorithms provide global optimum solutions for both types of demand functions. The managerial implications of numerical results are clear and provide a suitable framework to assess the relative profitability. Finally, Jaggi et al. [60] model becomes as a particular case to this model.