Chapter 5

Dominant retailer's optimal policy in a supply chain with, perishable items under advance payment scheme and two-echelon trade credits

5.1 Introduction

Now-a-days, the retailing industry is increasingly dominated by large centrally powered retailers. In a supply chain, the retailer has the dominant power of controlling or influencing another member's decisions. Raju and Zhang [88] summed up three salient characteristics of the dominant retailer: first, he has the ability to offer customers unprecedented opportunity for shipping and manufacturers effective promotional services; second, he is frequently the largest distributor for the supplier; third, he is frequently the price leader, for instance, the sales through Wal-Mart accounted for 19 % of P & G's total sales in 2008, 40 % of Tandy's and over 45 % for many other large suppliers. Thus a retailer has the power to dictate prices of products. However, dominance of a retailer does not mean that he can decide prices and ordering quantities arbitrarily. So it is useful to propose a model, that is expected to help the retailer, in order to compute optimal price discounting policy not only to minimize the expected costs but also to make relationship sustainable.

In recent competitive market, retailers have various ways to encourage sales acting on customers with price discounts, slotting fees, buy downs, and so on. The effects of these marketing tools on sales are filtered by the retailer's sales attitude and aptitude. It is normally observed that the retailers often try to stimulate the demand by offering price discounts. Price discounts offer economic benefits to consumers, influence consumers' beliefs about the brand, and arouse positive feelings and emotions within consumers, which will increase consumers' brand awareness and purchase intentions. In real life, there are situations in which the retailer requires advance payment when an order from a customer is placed. Further, if a customer gives an advance payment (AP) then he may get some price discount on the ordering quantity. For examples, in bricks and tiles factories, the retailer announce price discount offer under AP scheme prior to the selling season. In India, to improve liquidity, a policy decision taken by the Maharashtra state co-operative cotton growers' federation is to give price discount to the customers who are making a payment (full or partial) in advance for the cotton. Paying a certain percentage of the total purchase cost per cycle as an AP to the retailer, the retailer can earn the interest on the amount of money via share market business or banking business. The customers can also save their money if they are sure about the necessity of a product. Thus the AP scheme is a real life phenomenon and it has a crucial impact on the inventory policies.

Perishable products are commonly found in commerce and industry. Sometimes the rate of deterioration is low, for items such as bricks, tiles, steel, hardware, glassware and toys, to cause consideration of deterioration in the determination of economic lot sizes. However, during winter season the spoilage rate of bricks should be considered significantly. Moreover, the items such as fruits, fresh fishes, perfumes, alcohol, gasoline and photographic films deteriorate rapidly over time, which can not be ignored in the decision making process of ordering lot size.

Recently, two-echelon trade credit policy is implemented by several authors in the literature. To the best of our knowledge, this research is the first to incorporate both the AP scheme and two-echelon trade credits into an EOQ-model for perishable items. Here, we consider partial trade credit financing at the dominant retailer. We not only find the optimal replenishment policy but also the price discounting policy by minimizing the dominant retailer's total cost under the AP scheme. We also present numerical examples incorporating with several managerial insights.

This chapter is organised as follows. In the next section, we describe the supply chain problem undertaken. In section 5.3, mathematical model relevant to the considered problem is developed. In the section 5.4, we find the optimal solution for cycle time when discount rate is fixed. Section 5.5 illustrates how to find the optimal policy when discount rate is endogeneous. In section 5.6, the proposed problem is illustrated by several numerical examples and sensitivity analysis is performed for various inventory parameters. Finally, section 5.7 concludes the chapter.

5.2 Problem description

We consider a supply chain consisting of a supplier and many retailers. Supplier fulfills retailers' demands and retailers inturn fulfill their market customers' demand. Among the retailers, we mainly focus on the dominant retailer who attracts customers through some promotional efforts such as trade credit option and advance payment scheme etc. The supplier provides the dominant retailer a full trade credit period (M) for payments whereas the dominant retailer offers his customers partial trade credit (up to time N) to his customers. The dominant retailer also offers his customers price discounts if the customers register their orders with advance payment prior to sales period. As soon as the items are arrived to the inventory, the priority will be given to the customers who use AP scheme. The remaining customers are likely to use trade credit option only.

In addition to the notations in chapter 1, the following notations have been used. λ_1 is the demand rate at the dominant retailer (without loss of generality, we let retailer 1 as the dominant retailer) of a supply chain; λ_2 is demand rate at the other retailers; η is the rate of discount in retail price under AP scheme, decision variable; $\beta_1(\eta)$ is the fraction of retailer 1's customers who use AP scheme and $0 < \beta_1(\eta) < 1$; $\beta_2(\eta)$ is the fraction of other retailers' customers who use AP scheme and $0 < \beta_2(\eta) < 1$; δ is a fraction of amount (per unit) payable as an AP by the customers under AP scheme, $0 \leq \delta \leq 1$; $TC(\eta, T)$ is annual total relevant cost, which is a function of both η and T. We use the notations $\beta_i(\eta)$ and β_i for $i = 1, 2$ interchangeably.

We describe various assumptions at the retailer of a supply chain.

1. The supply chain consists of a supplier supplying a single product to the retailers.

2. Units start deteriorating the moment they are received into an inventory. The time to deterioration of a product follows an exponential distribution with parameter θ, i.e, the deterioration rate is a constant fraction of the on-hand inventory.

3. Demand rates, λ_1 and λ_2, are known constants. Shortages are not allowed to occur in order to make more profit. The retailer may replenish inventory when it does not drop to zero yet.

4. The dominant retailer also attracts the other retailers' customers with price discount offer at rate η under AP scheme. The fraction (say, δ) of the purchasing cost should be paid as an AP by the customers to the dominant retailer if they use AP scheme prior to selling period.

5. As and when the items are arrived to the inventory, the demand is fulfilled first to the customers who use AP scheme.

6. All customers, including the customers who use AP scheme, should settle their payments at the end of the time $t = N$.

7. During the selling period, the dominant retailer 1 offers the partial trade credit to his customers who are not availing AP scheme. Hence these customers must make a partial payment to the retailer 1 when the items are sold. The remaining balance must be paid at the time N. That is, the retailer can accumulate interest earned at the rate I_e from these customers' payment.

8. M need not be greater than N, since the retailer can also earn interest during the time $0 \leq t \leq N$ from the customers' partial payment (during the selling period) and from the advance payment of the customers under AP scheme.

9. The supplier offers the full trade credit period (M) to the retailer. When $T \geq M$, the account is settled at the time $t = M$, the retailer pays off all units sold and keeps his profits and starts paying for the interest charges on the items in stock with rate I_k. When $T \leq M$, the account is settled at $t = M$ and the retailer does not need to pay any interest charge.

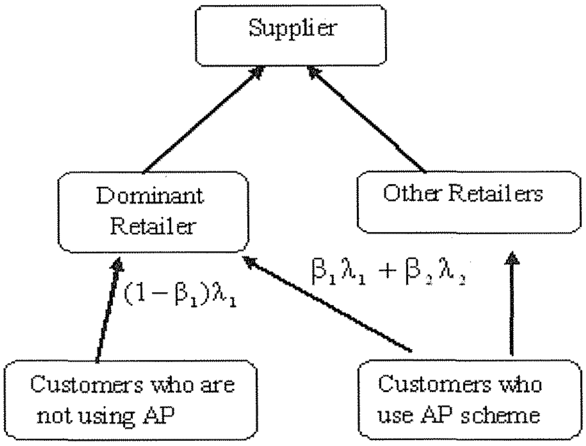

Figure 9: A two-echelon supply chain with AP scheme

10. Time horizon is infinite and lead time is zero.

11. Inventory holding cost is charged only on the amount of undecayed stock.

Under the above scenario, we not only find the optimal replenishment policy but also the price discounting policy by minimizing the dominant retailer's costs under the AP scheme.

5.3 Model formulation

In the dominant retailer 1, $(1-\beta_1)\lambda_1$ is the annual demand of the customers who are not using the AP scheme (see fig.9). $\lambda_1\beta_1+\lambda_2\beta_2$ is the annual demand of the customers who use AP scheme with price discount, where $\lambda_2\beta_2$ is the increased demand from other retailers' customers who use AP scheme at the dominant retailer.

The inventory situation at the dominant retailer is described as follows. During the time period $[0, T]$, the inventory level is decreased owing to demand rate $(1-\beta_1)\lambda_1$ as well as deterioration process. The inventory level of the dominant retailer at time t can be described by the following differential equation:

$$\frac{dI(t)}{dt} = -(1-\beta_1)\lambda_1 - \theta I(t), \quad 0 \le t \le T \tag{120}$$

with the boundary condition $I(t) = 0$ at time $t = T$. The solution to Eq.(120) is

$$I(t) = \frac{(1-\beta_1)\lambda_1}{\theta}\left[e^{\theta(T-t)} - 1\right], \quad 0 \le t \le T \tag{121}$$

Consequently, the ordering quantity for each cycle (including for the customers under AP scheme) is

$$Q = (\beta_1\lambda_1 + \beta_2\lambda_2)T + \frac{(1-\beta_1)\lambda_1}{\theta}\left[e^{\theta T} - 1\right] \tag{122}$$

We now derive the annual ordering cost (OC), annual stock-holding cost (HC) and annual cost for the deteriorated items (DC) at the dominant retailer.

(1) Annual ordering cost $= A/T$.

(2) Annual stock-holding cost (excluding interest charges)

$$\begin{aligned} &= \frac{h}{T}\int_0^T \frac{(1-\beta_1)\lambda_1}{\theta}\left[e^{\theta(T-t)} - 1\right] dt \\ &= \frac{h(1-\beta_1)\lambda_1}{T\theta^2}\left[e^{\theta T} - \theta T - 1\right] \end{aligned}$$

(3) Annual cost for the deteriorated items

$$\begin{aligned} &= \frac{c}{T}[Q - (1-\beta_1)\lambda_1 T - (\beta_1\lambda_1 + \beta_2\lambda_2)T] \\ &= \frac{c\theta(1-\beta_1)\lambda_1}{T\theta^2}\left[e^{\theta T} - \theta T - 1\right] \end{aligned}$$

The above three costs are same in both the cases $M < N$ and $M \ge N$. In order to calculate interest payable and interest earned by the retailer 1, we have to consider two situations: (1) $M < N$ and (2) $M \ge N$.

5.3.1 Total cost incurred at the dominant retailer 1 when $M < N$

According to assumption (9), there are two cases to be considered for interest payable (IP).

Case 1. $M \le T$

In this case,

$$\begin{aligned} IP &= \frac{cI_k}{T}\int_M^T I(t)dt \\ &= \frac{cI_k(1-\beta_1)\lambda_1}{T\theta^2}\left[e^{\theta(T-M)} - \theta(T-M) - 1\right] \end{aligned}$$

Case 2. $M \ge T$

In this case, IP $= 0$.

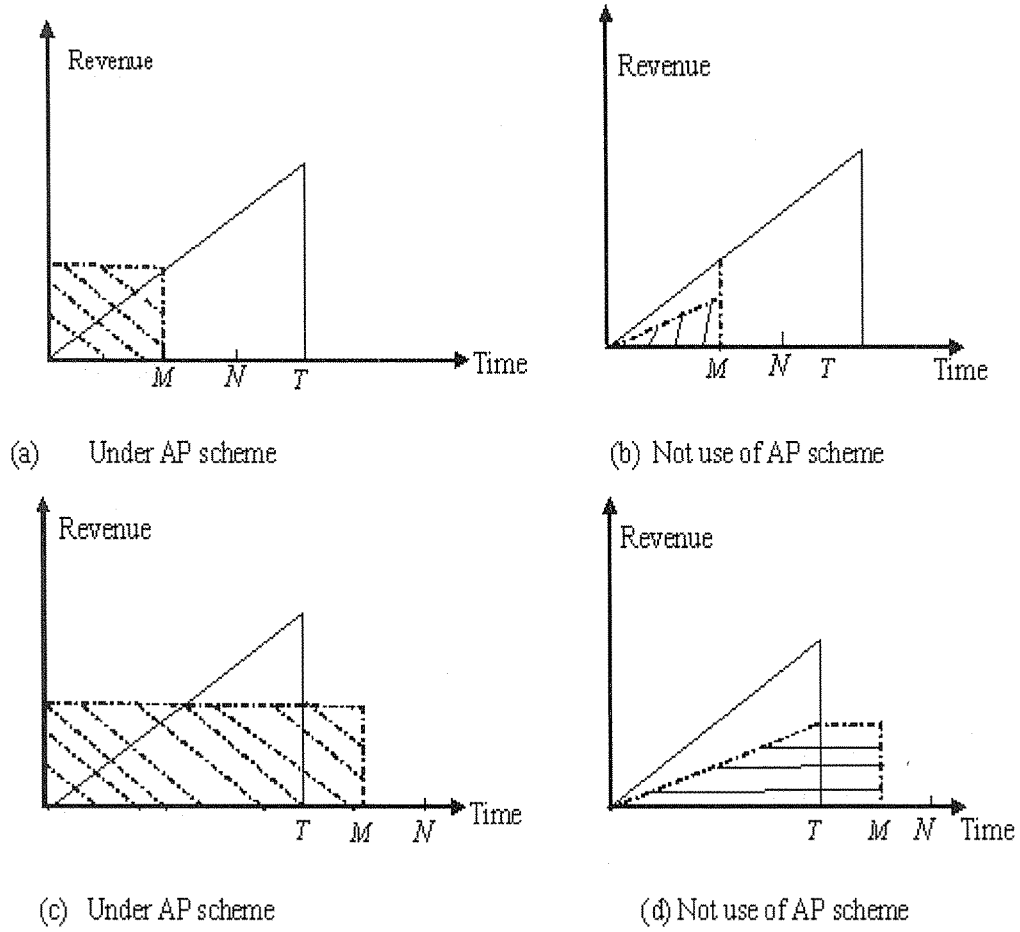

Figure 10: The total accumulation of interest earned in a cycle when $M < N$: (a) when $M < T$, (b) when $M \geq T$.

The annual interest earned by the dominant retailer 1 can be calculated in two ways. Firstly, during the sales period, the interest (IE_1) is earned from the revenue of selling items (not use of AP scheme). Secondly, interest (IE_2) is earned from the advance payment of the customers who use AP scheme. According to assumption (7), there are two cases to be considered as in interest payable.

Case 1. $M \leq T$, shown in Fig 10 (a)

Here, we have

$$\begin{aligned} IE_1 &= \frac{sI_e\alpha}{T}\int_0^M (1-\beta_1)\lambda_1 t \, \mathrm{dt} \\ &= \frac{sI_e\alpha(1-\beta_1)\lambda_1 M^2}{2T} \end{aligned}$$

$$\text{and } IE_2 = sI_e\delta(1-\eta)(\beta_1\lambda_1+\beta_2\lambda_2)M$$

Case 2. $M \geq T$, shown in Fig.10 (b)

Here, we have

$$\begin{aligned} IE_1 &= \frac{sI_e\alpha}{T}\left[\int_0^T (1-\beta_1)\lambda_1 t \, \mathrm{dt} + \int_T^M (1-\beta_1)\lambda_1 T \, \mathrm{dt}\right] \\ &= sI_e\alpha(1-\beta_1)\lambda_1[M-T/2] \\ \text{and } IE_2 &= sI_e\delta(1-\eta)(\beta_1\lambda_1+\beta_2\lambda_2)M \end{aligned}$$

The annual total cost incurred at the dominant retailer,

$$\begin{aligned} TC(\eta,T) &= \text{Ordering cost} + \text{ Holding cost} + \text{ Cost of deteriorated items} \\ &\quad + \text{Interest payable} - \text{ Interest earned} \\ &= \text{(OC)+(HC)+(DC)+(IP)} - (IE_1+IE_2) \end{aligned}$$

From the above arguments,

$$TC(\eta,T) = \begin{cases} TC_1(\eta,T) & \text{if } M \leq T \\ TC_2(\eta,T) & \text{if } 0 < T \leq M \end{cases} \tag{123}$$

where

$$\begin{aligned} TC_1(\eta,T) &= \frac{A}{T} + \frac{(c\theta+h)(1-\beta_1)\lambda_1}{T\theta^2}\left[e^{\theta T}-\theta T-1\right] + \frac{cI_k(1-\beta_1)\lambda_1}{T\theta^2}\left[e^{\theta(T-M)}-\theta(T-M)-1\right] \\ &\quad -sI_e\left[\frac{(1-\beta_1)\lambda_1\alpha M^2}{2T} + (1-\eta)\delta(\lambda_1\beta_1+\lambda_2\beta_2)M\right] \end{aligned} \tag{124}$$

$$\begin{aligned} TC_2(\eta,T) &= \frac{A}{T} + \frac{(c\theta+h)(1-\beta_1)\lambda_1}{T\theta^2}\left[e^{\theta T}-\theta T-1\right] \\ &\quad -sI_e\left[(1-\beta_1)\lambda_1\alpha(M-T/2) + (1-\eta)\delta(\lambda_1\beta_1+\lambda_2\beta_2)M\right] \end{aligned} \tag{125}$$

Since $TC_1(\eta, M) = TC_2(\eta, M)$, $TC(\eta, T)$ is continuous and well defined.

5.3.2 Total cost incurred at the dominant retailer when $M \geq N$

According to assumption (9), there are three cases to be considered for interest payable (IP).

Case 1. $M \leq T$

In this case,

$$\begin{aligned} IP &= \frac{cI_k}{T}\int_M^T I(t)\ \mathrm{dt} \\ &= \frac{cI_k(1-\beta_1)\lambda_1}{T\theta^2}[e^{\theta(T-M)} - \theta(T-M) - 1] \end{aligned}$$

Case 2. $N \leq T \leq M$

Here, IP = 0

Case 3. $T \leq N$

In this case, IP = 0

According to assumption (7), there are three cases that occur in costs of interest earned per year. Similar to the case $M < N$, we calculate IE_1 and IE_2 as follows.

Case 1: $M \leq T$, shown in Fig.11 (a)

In this case,

$$\begin{aligned} IE_1 &= \frac{sI_e}{T}\left[\alpha\int_0^N (1-\beta_1)\lambda_1 t\ \mathrm{dt} + \int_N^M (1-\beta_1)\lambda_1 t\ \mathrm{dt}\right] \\ &= \frac{sI_e(1-\beta_1)\lambda_1}{2T}[M^2 - (1-\alpha)N^2] \\ \text{and } IE_2 &= sI_e(1-\eta)(\lambda_1\beta_1 + \lambda_2\beta_2)(M - (1-\delta)N) \end{aligned}$$

Case 2: $N \leq T \leq M$, shown in Fig.11 (b)

Here, we have

$$\begin{aligned} IE_1 &= \frac{sI_e}{T}\left[\alpha\int_0^N (1-\beta_1)\lambda_1 t\ \mathrm{dt} + \int_N^T (1-\beta_1)\lambda_1 t\ \mathrm{dt} + \int_T^M (1-\beta_1)\lambda_1 T\ \mathrm{dt}\right] \\ &= \frac{sI_e(1-\beta_1)\lambda_1}{2T}[2MT - (1-\alpha)N^2 - T^2] \\ \text{and } IE_2 &= sI_e(1-\eta)(\lambda_1\beta_1 + \lambda_2\beta_2)(M - (1-\delta)N) \end{aligned}$$

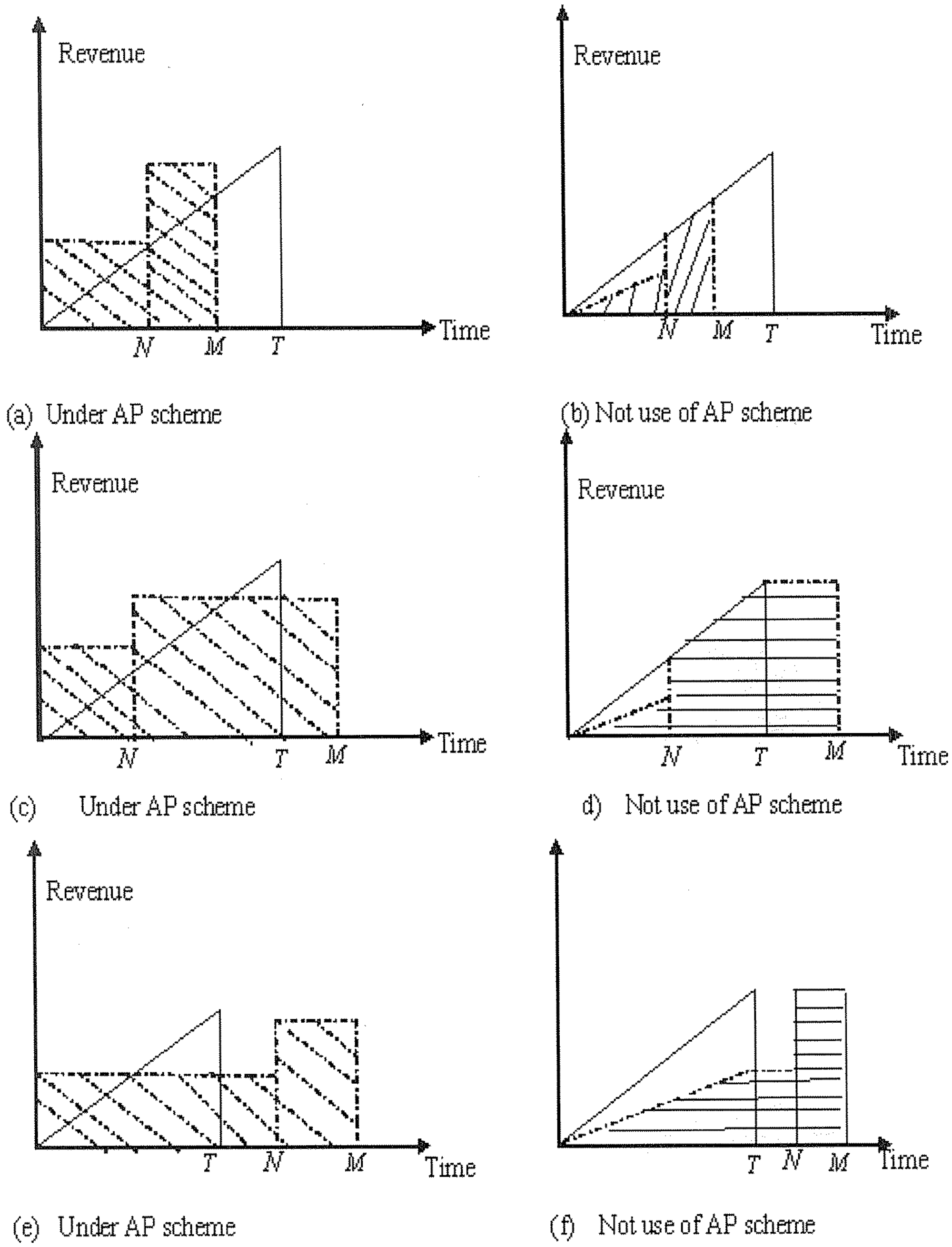

Figure 11: Total amount of interest earned when $M \geq N$: (a) when $M \leq T$, (b) when $N < T < M$ and (c) when $T \leq N$.

Case 3: $T \le N$, shown in Fig.11 (c)

In this case,

$$\begin{aligned} IE_1 &= \frac{sI_e}{T}\left[\alpha\int_0^T (1-\beta_1)\lambda_1 t \text{ dt} + \alpha\int_T^N (1-\beta_1)\lambda_1 T \text{ dt} + \int_N^M (1-\beta_1)\lambda_1 T \text{ dt}\right] \\ &= sI_e(1-\beta_1)\lambda_1[M-(1-\alpha)N-\alpha T/2] \end{aligned}$$

$$\text{and } IE_2 = sI_e(1-\eta)(\lambda_1\beta_1+\lambda_2\beta_2)(M-(1-\delta)N)$$

From the above arguments, the annual total cost for the dominant retailer can be expressed as

$$TC(\eta,T) = \begin{cases} TC_3(\eta,T) & \text{if } M \le T \\ TC_4(\eta,T) & \text{if } N \le T \le M \\ TC_5(\eta,T) & \text{if } T \le N \end{cases} \tag{126}$$

where

$$\begin{aligned} TC_3(\eta,T) &= \frac{A}{T} + \frac{(c\theta+h)(1-\beta_1)\lambda_1}{T\theta^2}\left[e^{\theta T}-\theta T-1\right] + \frac{cI_k(1-\beta_1)\lambda_1}{T\theta^2}\left[e^{\theta(T-M)}-\theta(T-M)-1\right] \\ &\quad -sI_e\left[\frac{(1-\beta_1)\lambda_1}{2T}[M^2-(1-\alpha)N^2]+(1-\eta)(\lambda_1\beta_1+\lambda_2\beta_2)(M-(1-\delta)N)\right] \end{aligned} \tag{127}$$

$$\begin{aligned} TC_4(\eta,T) &= \frac{A}{T} + \frac{(c\theta+h)(1-\beta_1)\lambda_1}{T\theta^2}\left[e^{\theta T}-\theta T-1\right] - sI_e\left[\frac{(1-\beta_1)\lambda_1}{2T}[2MT-(1-\alpha)N^2-T^2]\right. \\ &\quad \left.+(1-\eta)(\lambda_1\beta_1+\lambda_2\beta_2)(M-(1-\delta)N)\right] \end{aligned} \tag{128}$$

$$\begin{aligned} TC_5(\eta,T) &= \frac{A}{T} + \frac{(c\theta+h)(1-\beta_1)\lambda_1}{T\theta^2}\left[e^{\theta T}-\theta T-1\right] - sI_e\left[(1-\beta_1)\lambda_1[M-(1-\alpha)N-\alpha T/2]\right. \\ &\quad \left.+(1-\eta)(\lambda_1\beta_1+\lambda_2\beta_2)[M-(1-\delta)N]\right] \end{aligned} \tag{129}$$

Since $TC_3(\eta,M) = TC_4(\eta,M)$ and $TC_4(\eta,N) = TC_5(\eta,N)$, $TC(\eta,T)$ is continuous and well defined.

5.4 Unique optimal solution T^* when η is fixed

5.4.1 When $M < N$

The first order partial derivative of TC_1 with respect to T is

$$\frac{\partial TC_1(T)}{\partial T} = \frac{1}{T^2}\Bigg[-A + \frac{(c\theta + h)(1-\beta_1)\lambda_1}{\theta^2}\left[\theta T e^{\theta T} - e^{\theta T} + 1\right] + \frac{cI_k(1-\beta_1)\lambda_1}{\theta^2}\left[\theta T e^{\theta(T-M)} - e^{\theta(T-M)} - \theta M + 1\right] + \frac{sI_e\alpha(1-\beta_1)\lambda_1 M^2}{2}\Bigg] \tag{130}$$

For a fixed value of η, let

$$\begin{aligned} f_1(T) = -A &+ \frac{(c\theta + h)(1-\beta_1)\lambda_1}{\theta^2}\left[\theta T e^{\theta T} - e^{\theta T} + 1\right] \\ &+ \frac{cI_k(1-\beta_1)\lambda_1}{\theta^2}\left[\theta T e^{\theta(T-M)} - e^{\theta(T-M)} - \theta M + 1\right] + \frac{sI_e\alpha(1-\beta_1)\lambda_1 M^2}{2} \end{aligned} \tag{131}$$

We observe that $f_1(T)$ and $\frac{\partial TC_1(T)}{\partial T}$ have same sign and domain. The first order derivative of $f_1(T)$ with respect to T,

$$f_1^{'}(T) = (1-\beta_1)\lambda_1(c\theta + h)Te^{\theta T} + cI_k(1-\beta_1)\lambda_1 Te^{\theta(T-M)} > 0$$

Hence $f_1(T)$ is increasing on $[M, \infty)$ and $\lim_{T\to\infty} f_1(T) = \infty > 0$. Suppose that $\frac{\partial TC_1}{\partial T}_{|T=M} < 0$, we have $f_1(M) < 0$ then $f_1(T) = 0$ has the unique solution $T_1^*(\eta)$ on $[M, \infty)$ by Intermediate value theorem. Suppose that $\frac{\partial TC_1}{\partial T}_{|T=M} \geq 0$, we have $f_1(M) \geq 0$. Since $f_1(T)$ is increasing on $[M, \infty)$, we have $f_1(T) \geq f_1(M) \geq 0$ for $T \geq M$. Hence, $\frac{\partial TC_1}{\partial T} \geq 0$ and so TC_1 is increasing on $[M, \infty)$. So $T_1^* = M$.

In a similar argument as above and using the first order partial derivative of TC_2 with respect to T, the optimal replenishment time $T_2^*(\eta)$ can be found by solving the Eq. (132).

$$-A + \frac{(c\theta + h)(1-\beta_1)\lambda_1}{\theta^2}\left[\theta T e^{\theta T} - e^{\theta T} + 1\right] + \frac{sI_e\alpha(1-\beta_1)\lambda_1 T^2}{2} = 0 \tag{132}$$

Let $\Delta_1(\eta) = (1-\beta_1)\lambda_1 M^2(h + c\theta + sI_e\alpha)$, then we have the following theorem.

Theorem 5.1 When $M < N$,

(a) $\Delta_1(\eta) \leq 2A$ implies that $T^* = T_1^*(\eta)$,

(b) $\Delta_1(\eta) > 2A$ implies that $T^* = T_2^*(\eta)$.

Proof. Please refer to appendix B1.

5.4.2 When $M \geq N$

The first order partial derivative of TC_3 with respect to T is given by

$$\begin{aligned}\frac{\partial TC_3}{\partial T} &= -A/T^2 + \frac{(c\theta + h)(1-\beta_1)\lambda_1}{\theta^2 T^2}\left[\theta T e^{\theta T} - e^{\theta T} + 1\right] + \frac{sI_e(1-\beta_1)\lambda_1}{2T^2}[M^2 - (1-\alpha)N^2] \\ &+ \frac{cI_k(1-\beta_1)\lambda_1}{\theta^2 T^2}\left[\theta T e^{\theta(T-M)} - e^{\theta(T-M)} - \theta M + 1\right]\end{aligned} \tag{133}$$

For a fixed value of η, let

$$\begin{aligned}f_2(T) = &- A + \frac{(c\theta + h)(1-\beta_1)\lambda_1}{\theta^2}\left[\theta T e^{\theta T} - e^{\theta T} + 1\right] + \frac{sI_e(1-\beta_1)\lambda_1}{2}[M^2 - (1-\alpha)N^2] \\ &+ \frac{cI_k(1-\beta_1)\lambda_1}{\theta^2}\left[\theta T e^{\theta(T-M)} - e^{\theta(T-M)} - \theta M + 1\right]\end{aligned} \tag{134}$$

We observe that $f_2(T)$ and $\frac{\partial TC_3(T)}{\partial T}$ have same sign and domain. The first order derivative of $f_2(T)$ with respect to T,

$$f_2'(T) = (1-\beta_1)\lambda_1(c\theta + h)Te^{\theta T} + cI_k(1-\beta_1)\lambda_1 Te^{\theta(T-M)} > 0$$

Hence $f_2(T)$ is increasing on $[M, \infty)$ and $\lim_{T\to\infty} f_2(T) = \infty > 0$. Suppose that $\frac{\partial TC_3}{\partial T}_{|T=M} < 0$, we have $f_2(M) < 0$ then $f_2(T) = 0$ has the unique solution $T_3^*(\eta)$ on $[M, \infty)$ by Intermediate value theorem. Suppose that $\frac{\partial TC_3}{\partial T}_{|T=M} \geq 0$, we have $f_2(M) \geq 0$. Since $f_2(T)$ is increasing on $[M, \infty)$, we have $f_2(T) \geq f_2(M) \geq 0$ for $T \geq M$. Hence, $\frac{\partial TC_3}{\partial T} \geq 0$ and so TC_3 is increasing on $[M, \infty)$. So $T_3^* = M$.

The first order partial derivative of TC_4 with respect to T is

$$\begin{aligned}\frac{\partial TC_4}{\partial T} &= -A/T^2 + \frac{(c\theta + h)(1-\beta_1)\lambda_1}{\theta^2 T^2}\left[\theta T e^{\theta T} - e^{\theta T} + 1\right] \\ &\quad - \frac{sI_e(1-\beta_1)\lambda_1}{2T^2}[(1-\alpha)N^2 - T^2]\end{aligned} \tag{135}$$

Let

$$\begin{aligned}f_3(T) &= -A + \frac{(c\theta + h)(1-\beta_1)\lambda_1}{\theta^2}\left[\theta T e^{\theta T} - e^{\theta T} + 1\right] \\ &\quad - \frac{sI_e(1-\beta_1)\lambda_1}{2}[(1-\alpha)N^2 - T^2]\end{aligned} \tag{136}$$

We observe that $f_3(T)$ and $\frac{\partial TC_4(T)}{\partial T}$ have same sign and domain. The first order derivative of $f_3(T)$ with respect to T,

$$f_3'(T) = (1-\beta_1)\lambda_1(c\theta + h)Te^{\theta T} + sI_e(1-\beta_1)\lambda_1 T > 0.$$

Hence $f_3(T)$ is increasing on $(0, \infty)$ and so $\frac{\partial TC_4}{\partial T}$ is increasing on $(0, \infty)$. Therefore TC_4 is convex on $(0, \infty)$

The first and second order partial derivatives of TC_5 w.r.t T is

$$\frac{\partial TC_5}{\partial T} = -A/T^2 + \frac{(c\theta + h)(1-\beta_1)\lambda_1}{\theta^2 T^2}\left[\theta T e^{\theta T} - e^{\theta T} + 1\right] + \frac{sI_e\alpha(1-\beta_1)\lambda_1}{2} \quad (137)$$

$$\frac{\partial^2 TC_5}{\partial T^2} = \frac{2A}{T^3} + \frac{(c\theta + h)(1-\beta_1)\lambda_1}{\theta^2 T^3}\left[(\theta T)^2 e^{\theta T} - 2\theta T e^{\theta T} + 2e^{\theta T} - 2\right] > 0 \quad (138)$$

since the function $(x^2 - 2x + 2)e^x - 2$ is increasing on $(0, \infty)$. Hence TC_5 is convex on $(0, \infty)$.

Let

$$\Delta_2(\eta) = (1-\beta_1)\lambda_1[(c\theta + h)M^2 + sI_e(M^2 - (1-\alpha)N^2)]$$

$$\Delta_3(\eta) = (1-\beta_1)\lambda_1[c\theta + h + sI_e\alpha]N^2$$

then we have the following theorem.

Theorem 5.2 When $M \geq N$,
(a) $\Delta_2(\eta) \leq 2A$ implies that $T^* = T_3^*(\eta)$,
(b) $\Delta_3(\eta) < 2A < \Delta_2(\eta)$ implies that $T^* = T_4^*(\eta)$,
(c) $\Delta_3(\eta) \geq 2A$ implies that $T^* = T_5^*(\eta)$.

Proof. Please refer to appendix B2.

5.5 The optimal policy, (η^*, T^*), when η is endogenous

The fraction of the dominant retailer's customers who use AP scheme and fraction of other customers who switch to dominant retailer under AP scheme increase as the rate of discount, η, increases. So we let $\beta_1(\eta) = a\eta$ and $\beta_2(\eta) = b\eta$; where a, b are any two positive constants. It is easy to verify that $\frac{\partial^2 TC_i}{\partial \eta^2} > 0$ for $i = 1, 2, 3, 4, 5$. Hence TC is a convex function of η and so there exists a optimum solution η^* which minimizes the cost.

5.5.1 When $M < N$

The optimal values $\eta_1(T)$ and $\eta_2(T)$ are obtained by solving the equations: $\frac{\partial TC_i}{\partial \eta} = 0, i = 1, 2.$

$$\eta_1^*(T) = \frac{1}{2sI_e\delta(a\lambda_1+b\lambda_2)M}\left[\frac{a\lambda_1(c\theta+h)}{T\theta^2}\left[e^{\theta T}-\theta T-1\right]+\frac{a\lambda_1 cI_k}{T\theta^2}\left[e^{\theta(T-M)}-\theta(T-M)-1\right]\right.$$
$$\left.-sI_e\left(\frac{a\lambda_1\alpha M^2}{2T}-\delta(a\lambda_1+b\lambda_2)M\right)\right] \tag{139}$$

$$\eta_2^*(T) = \frac{1}{2sI_e\delta(a\lambda_1+b\lambda_2)M}\left[\frac{a\lambda_1(c\theta+h)}{T\theta^2}\left[e^{\theta T}-\theta T-1\right]-sI_e\left(a\lambda_1\alpha(M-T/2)-\delta(a\lambda_1+b\lambda_2)M\right)\right] \tag{140}$$

After obtaining the optimal price discount, $\eta_i(T), i=1,2$, we now want to determine the optimal cycle time, T^*. Now we substitute $\eta_1(T)$ to $TC_1(\eta,T)$ and $\eta_2(T)$ to $TC_2(\eta,T)$. The optimal values $T_i^*, i=1,2$ can be obtained by solving the equations:

$$\frac{\partial TC_1(\eta_1(T),T)}{\partial T} = \frac{-A}{T^2}+\frac{(c\theta+h)\lambda_1}{\theta^2T^2}\left[(1-a\eta_1)T\left(\theta Te^{\theta T}-e^{\theta T}+1\right)-\left(1-a\eta_1+aT\frac{d\eta_1}{dT}\right)\right.$$
$$\left.\left(e^{\theta T}-\theta T-1\right)\right]+\frac{cI_k\lambda_1}{\theta^2T^2}\left[(1-a\eta_1)T\left(\theta Te^{\theta(T-M)}-e^{\theta(T-M)}-\theta M+1\right)\right.$$
$$\left.-\left(1-a\eta_1+aT\frac{d\eta_1}{dT}\right)\left(e^{\theta(T-M)}-\theta(T-M)-1\right)\right]$$
$$-sI_e\left[\frac{\alpha\lambda_1M^2}{2T^2}\left(a\eta_1-1-aT\frac{d\eta_1}{dT}\right)+\delta(1-2\eta_1)\frac{d\eta_1}{dT}(a\lambda_1+b\lambda_2)M\right]=0 \tag{141}$$

and

$$\frac{\partial TC_2(\eta_2(T),T)}{\partial T} = \frac{-A}{T^2}+\frac{(c\theta+h)\lambda_1}{\theta^2T^2}\left[(1-a\eta_2)T\left(\theta Te^{\theta T}-e^{\theta T}+1\right)-\left(1-a\eta_2+aT\frac{d\eta_2}{dT}\right)\right.$$
$$\left.\left(e^{\theta T}-\theta T-1\right)\right]-sI_e\left[\frac{\alpha\lambda_1}{2}\left(a\eta_2-1-2a\frac{d\eta_2}{dT}(M-T/2)\right)\right.$$
$$\left.+\delta(1-2\eta_2)\frac{d\eta_2}{dT}(a\lambda_1+b\lambda_2)M\right]=0 \tag{142}$$

The optimum points, T_i^*, are determined by verifying the second order partial derivatives $\frac{\partial^2 TC_i}{\partial T^2}>0$. Thus our objective is

Minimize$\{TC_i(\eta_i(T),T)\}$

subject to

$0<\eta_i(T)<1, i=1,2.$

Theorem 5.3 When $M<N$,

(a) If $sI_e\left[\frac{a\lambda_1\alpha M^2}{2T}+\delta(a\lambda_1+b\lambda_2)M\right]>\frac{(c\theta+h)a\lambda_1}{T\theta^2}\left[e^{\theta T}-\theta T-1\right]+\frac{cI_ka\lambda_1}{T\theta^2}\left[e^{\theta(T-M)}-\theta(T-M)-1\right]$ then there exists a unique optimal solution $\eta_1^*(T)$ to $\frac{\partial TC_1}{\partial\eta}=0$, which satisfies $0<\eta_1^*(T)<1$.

(b) If $sI_e\left[a\lambda_1\alpha(M-T/2)+\delta(a\lambda_1+b\lambda_2)M\right] > \frac{(c\theta+h)a\lambda_1}{T\theta^2}\left[e^{\theta T}-\theta T-1\right]$ then there exists a unique optimal solution $\eta_2^*(T)$ to $\frac{\partial TC_2}{\partial \eta}=0$, which satisfies $0<\eta_2^*(T)<1$.

Proof. Please refer to appendix B3.

Note that $TC_i(\eta,T)$ (i=1,2) are continuous functions over the compact set $[0,1]\times[0,T_{max}]$, where T_{max} is the upper limit of the cycle time T. Hence $TC_i(\eta,T)$ has a minimum value. It is clear that $TC_i(\eta,T)$ has no minimum if $\eta=0$ or 1. As a result, the optimal η and optimal T must be an interior point.

Based on the Theorem 5.3, the following procedure can be used to find optimal solutions. First, determine T_1^* by solving the Eq.(141) such that it satisfies the condition that $M\leq T$ and obtain η_1^* from Eq.(139) when $T=T_1^*$. Let $TC_1(T_1^*,\eta_1^*)$ be the optimum value of $TC_1(\eta,T)$. Secondly, find T_2^* by solving the Eq.(142) such that it satisfies the condition that $M>T$ and obtain η_2^* from Eq.(140) when $T=T_2^*$. Let $TC_2(T_2^*,\eta_2^*)$ be the optimum value of $TC_2(\eta,T)$. Then the optimum value of TC, TC^*, is equal to the minimum of $TC_1(T_1^*,\eta_1^*)$ and $TC_2(T_2^*,\eta_2^*)$. The corresponding (η^*,T^*) is the optimal policy.

5.5.2 When $M\geq N$

The optimal values $\eta_3(T),\eta_4(T)$ and $\eta_5(T)$ are obtained by solving the equations: $\frac{\partial TC_i}{\partial \eta}=0, i=3,4$ and 5 respectively.

$$\eta_3^*(T) = \frac{1}{2sI_e(a\lambda_1+b\lambda_2)(M-(1-\delta)N)}\left[\frac{a\lambda_1(c\theta+h)}{T\theta^2}\left[e^{\theta T}-\theta T-1\right]+\frac{a\lambda_1 cI_k}{T\theta^2}\right.$$
$$\left.\left[e^{\theta(T-M)}-\theta(T-M)-1\right]-sI_e\left(\frac{a\lambda_1}{2T}(M^2-(1-\alpha)N^2)-(a\lambda_1+b\lambda_2)[M-(1-\delta)N]\right)\right] \quad (143)$$

$$\eta_4^*(T) = \frac{1}{2sI_e(a\lambda_1+b\lambda_2)(M-(1-\delta)N)}\left[\frac{a\lambda_1(c\theta+h)}{T\theta^2}\left[e^{\theta T}-\theta T-1\right]\right.$$
$$\left.-sI_e\left(\frac{a\lambda_1}{2T}(2MT-(1-\alpha)N^2-T^2)-(a\lambda_1+b\lambda_2)[M-(1-\delta)N]\right)\right] \quad (144)$$

$$\eta_5^*(T) = \frac{1}{2sI_e(a\lambda_1+b\lambda_2)(M-(1-\delta)N)}\left[\frac{a\lambda_1(c\theta+h)}{T\theta^2}\left[e^{\theta T}-\theta T-1\right]\right.$$
$$\left.-sI_e\left(a\lambda_1(M-(1-\alpha)N-\alpha T/2)-(a\lambda_1+b\lambda_2)[M-(1-\delta)N]\right)\right] \quad (145)$$

After obtaining the optimal price discount, $\eta_i(T), i=3,4,5$, we now want to determine the optimal cycle time, T^*. Now we substitute $\eta_3(T)$ to $TC_3(\eta,T)$, $\eta_4(T)$ to $TC_4(\eta,T)$ and $\eta_5(T)$ to $TC_5(\eta,T)$. The optimal values $T_i^*, i=3,4,5$ can be obtained by solving the equations:

$$\begin{aligned}\frac{\partial TC_3(\eta_3(T),T)}{\partial T} &= \frac{-A}{T^2}+\frac{(c\theta+h)\lambda_1}{\theta^2T^2}\Bigg[(1-a\eta_3)T\left(\theta Te^{\theta T}-e^{\theta T}+1\right)-\left(1-a\eta_3+aT\frac{d\eta_3}{dT}\right)\\ &\left(e^{\theta T}-\theta T-1\right)\Bigg]+\frac{cI_k\lambda_1}{\theta^2T^2}\Bigg[(1-a\eta_3)T\left(\theta Te^{\theta(T-M)}-e^{\theta(T-M)}-\theta M+1\right)\\ &-\left(1-a\eta_3+aT\frac{d\eta_3}{dT}\right)\left(e^{\theta(T-M)}-\theta(T-M)-1\right)\Bigg]\\ &-sI_e\Bigg[\frac{\lambda_1}{2T^2}(M^2-(1-\alpha)N^2)\left(a\eta_3-1-aT\frac{d\eta_3}{dT}\right)\\ &+(1-2\eta_3)\frac{d\eta_3}{dT}(a\lambda_1+b\lambda_2)(M-(1-\delta)N)\Bigg]=0 \qquad (146)\end{aligned}$$

$$\begin{aligned}\frac{\partial TC_4(\eta_4(T),T)}{\partial T} &= \frac{-A}{T^2}+\frac{(c\theta+h)\lambda_1}{\theta^2T^2}\Bigg[(1-a\eta_4)T\left(\theta Te^{\theta T}-e^{\theta T}+1\right)-\left(1-a\eta_4+aT\frac{d\eta_4}{dT}\right)\\ &\left(e^{\theta T}-\theta T-1\right)\Bigg]-sI_e\Bigg[\frac{(1-a\eta_4)\lambda_1}{2T^2}[(1-\alpha)N^2-T^2]\\ &-\frac{a\lambda_1}{2T}\frac{d\eta_4}{dT}\left(2MT-(1-\alpha)N^2-T^2)\right)\\ &+(1-2\eta_4)\frac{d\eta_4}{dT}(a\lambda_1+b\lambda_2)(M-(1-\delta)N)\Bigg]=0 \qquad (147)\end{aligned}$$

and

$$\begin{aligned}\frac{\partial TC_5(\eta_5(T),T)}{\partial T} &= \frac{-A}{T^2}+\frac{(c\theta+h)\lambda_1}{\theta^2T^2}\Bigg[(1-a\eta_5)T\left(\theta Te^{\theta T}-e^{\theta T}+1\right)-\left(1-a\eta_5+aT\frac{d\eta_5}{dT}\right)\\ &\left(e^{\theta T}-\theta T-1\right)\Bigg]-sI_e\Bigg[\frac{-(1-a\eta_5)\lambda_1\alpha}{2}-a\lambda_1\frac{d\eta_5}{dT}[M-(1-\alpha)N-\alpha T/2]\\ &+(1-2\eta_5)\frac{d\eta_5}{dT}(a\lambda_1+b\lambda_2)(M-(1-\delta)N)\Bigg]=0 \qquad (148)\end{aligned}$$

The optimum points, T_i^*, are determined by verifying the second order partial derivatives $\frac{\partial^2 TC_i}{\partial T^2}>0$. Thus our objective becomes

Minimize$\{TC_i(\eta_i(T),T)\}$

subject to

$0<\eta_i(T)<1, i=3,4,5.$

Theorem 5.4 When $M\geq N$

(a) If $sI_e\left[(a\lambda_1+b\lambda_2)(M-(1-\delta)N)\right]+\frac{a\lambda_1}{2T}[M^2-(1-\alpha)N^2]]$

$$>\frac{(c\theta+h)a\lambda_1}{T\theta^2}\left[e^{\theta T}-\theta T-1\right]+\frac{cI_k a\lambda_1}{T\theta^2}\left[e^{\theta(T-M)}-\theta(T-M)-1\right]$$

then there exists a unique optimal solution $\eta_3^*(T)$ to $\frac{\partial TC_3}{\partial\eta}=0$, which satisfies $0<\eta_3^*(T)<1$.

(b) If

$$sI_e\left[\frac{a\lambda_1}{2T}(2MT-(1-\alpha)N^2-T^2)+(a\lambda_1+b\lambda_2)(M-(1-\delta)N)\right] > \frac{(c\theta+h)a\lambda_1}{T\theta^2}\left[e^{\theta T}-\theta T-1\right]$$

then there exists a unique optimal solution $\eta_4^*(T)$ to $\frac{\partial TC_4}{\partial \eta}=0$, which satisfies $0<\eta_4^*(T)<1$.

(c) If

$$sI_e\left[a\lambda_1(M-(1-\alpha)N-\alpha T/2)+(a\lambda_1+b\lambda_2)(M-(1-\delta)N)\right] > \frac{(c\theta+h)a\lambda_1}{T\theta^2}\left[e^{\theta T}-\theta T-1\right]$$

then there exists a unique optimal solution $\eta_5^*(T)$ to $\frac{\partial TC_5}{\partial \eta}=0$, which satisfies $0<\eta_5^*(T)<1$.

Proof. It is similar to the theorem 5.3.

Note that $TC_i(\eta,T)$ (i=1,2,3) are continuous functions over the compact set $[0,1]\times[0,T_{max}]$, where T_{max} is the upper limit of the cycle time T. Hence $TC_i(\eta,T)$ has a minimum value. It is clear that $TC_i(\eta,T)$ has no minimum if $\eta=0$ or 1. As a result, the optimal solution (η^*,T^*) must be an interior point of the space $[0,1]\times[0,T_{max}]$.

Based on the Theorem 5.4, the following procedure can be used to find optimal solutions. First, determine T_3^* by solving the Eq.(146) such that it satisfies the condition that $M\leq T$ and obtain η_3^* from Eq.(143) when $T=T_3^*$. Let $TC_3(T_3^*,\eta_3^*)$ be the optimum value of $TC_3(\eta,T)$. Secondly, find T_4^* by solving the Eq.(147) such that it satisfies the condition that $N<T<M$ and obtain η_4^* from Eq.(144) when $T=T_4^*$. Let $TC_4(T_4^*,\eta_4^*)$ be the optimum value of $TC_4(\eta,T)$. Thirdly, find T_5^* by solving the Eq.(148) such that it satisfies the condition that $T\leq N$ and obtain η_5^* from Eq.(145) when $T=T_5^*$. Let $TC_5(T_5^*,\eta_5^*)$ be the optimum value of $TC_5(\eta,T)$. Then the optimum value of TC, TC^*, is equal to the minimum of $TC_3(T_3^*,\eta_3^*)$,$TC_4(T_4^*,\eta_4^*)$ and $TC_5(T_5^*,\eta_5^*)$ and the corresponding (η^*,T^*) is the optimal policy.

5.6 Numerical examples and managerial insights

5.6.1 Numerical examples

In order to illustrate the solution method, we consider the following examples.

Example 1. When $M < N$ and $M \leq T$

Let $A = \$100; s = \$15/\text{unit}; c = \$10/\text{unit}; h = \$0.1/\text{unit}; I_k = \$0.15; I_e = \$0.4; M = 0.15$ year; $N = 0.3$ year; $\delta = 0.4; \lambda_1 = 1500$ units; $\lambda_2 = 1500$ units; $\eta = 0.56; \alpha = 0.4; \theta = 0.1; \beta_1 = 0.52\eta$ and $\beta_2 = 0.51\eta$. Since $\Delta_1(\eta) = 83.72 < 200 = 2A$, by Theorem 5.1, we get $T^* = T_1 = 0.2541$ year, the total cost $TC_1 = 491.962$. If η is endogenous, the optimal cycle time $T^* = T_1 = 0.2086$ year, $\eta_1^* = 0.5512$, the ordering quantity $Q^* = 403.237$ units and the total cost $TC^* = \$\, 505.256$.

Example 2. When $M < N$ and $M \geq T$

Let $A = \$100; s = \$12/\text{unit}; c = \$10/\text{unit}; h = \$0.1/\text{unit}; I_k = \$0.15; I_e = \$0.4; M = 0.27$ year; $N = 0.3$ year; $\delta = 0.09; \lambda_1 = 4000$ units; $\lambda_2 = 4000$ units; $\eta = 0.56; \alpha = 0.09; \theta = 0.1; \beta_1 = 0.52\eta$ and $\beta_2 = 0.51\eta$. Since $\Delta_1(\eta) = 316.64 > 200 = 2A$, by Theorem 5.1, we get $T^* = T_2 = 0.2146$ year, the total cost $TC_2 = 485.343$. If η is endogenous, the optimal cycle time $T^* = T_2 = 0.2659$ year, $\eta_2^* = 0.5754$, the ordering quantity $Q^* = 1385.46$ units and the total cost $TC^* = \$\, 506.278$.

Example 3. When $M \geq N$ and $M \leq T$

Let $A = \$80; s = \$10/\text{unit}; c = \$8/\text{unit}; h = \$0.1/\text{unit}; I_k = \$0.15; I_e = \$0.1; M = 0.25$ year; $N = 0.15$ year; $\delta = 0.02; \lambda_1 = 1500$ units; $\lambda_2 = 1500$ units; $\eta = 0.56; \alpha = 0.02; \theta = 0.1; \beta_1 = 0.52\eta$ and $\beta_2 = 0.51\eta$. Since $\Delta_2(\eta) = 102.81 < 200 = 2A$, by Theorem 5.2, we get $T^* = T_3 = 0.2968$ year, the total cost $TC_3 = 306.012$. If η is endogenous, the optimal cycle time $T^* = T_3 = 0.3385$ year, $\eta_3^* = 0.7609$, the ordering quantity $Q^* = 710.035$ units and the total cost $TC^* = \$\, 305.498$.

Example 4. When $M \geq N$ and $N < T < M$

Let $A = \$120; s = \$8/\text{unit}; c = \$5/\text{unit}; h = \$0.6/\text{unit}; I_k = \$0.15; I_e = \$0.1; M = 0.36$ year; $N = 0.15$ year; $\delta = 0.02; \lambda_1 = 2000$ units; $\lambda_2 = 2000$ units; $\eta = 0.56; \alpha = 0.02; \theta = 0.1; \beta_1 = 0.52\eta$ and $\beta_2 = 0.51\eta$. We have $\Delta_2(\eta) = 324.06$, $\Delta_3(\eta) = 35.59$ and

$\Delta_3(\eta) < 2A(= 240) < \Delta_2(\eta)$, from Theorem 5.2 we get $T^* = T_4 = 0.3137$ year, the total cost $TC_4 = 352.67$. If η is endogenous, the optimal cycle time $T^* = T_4 = 0.3506$ year, $\eta_4^* = 0.6041$, the ordering quantity $Q^* = 925.993$ units and the total cost $TC^* = \$\,357.789$.

Example 5. When $M \geq N$ and $T \leq N$

Let $A = \$80; s = \$8/$ unit; $c = \$6/$ unit; $h = \$2/$ unit; $I_k = \$0.15; I_e = \$0.1; M = 0.3$ year; $N = 0.18$ year; $\delta = 0.02; \lambda_1 = 6000$ units; $\lambda_2 = 6000$ units; $\eta = 0.56; \alpha = 0.02; \theta = 0.1; \beta_1 = 0.52\eta$ and $\beta_2 = 0.51\eta$. We have $\Delta_3(\eta) = 360.46$ and $\Delta_3(\eta) > 2A(= 160)$, from Theorem 5.2 we get $T^* = T_4 = 0.1199$ year, the total cost $TC_4 = 765.76$. If η is endogenous, the optimal cycle time $T^* = T_4 = 0.1635$ year, $\eta_4^* = 0.7935$, the ordering quantity $Q^* = 1382.939$ units and the total cost $TC^* = \$\,798.513$.

5.6.2 Effect of changing the inventory model parameters

It is important to discuss the influence of key model parameters on the optimal solutions. Here, we let the numerical data as in example 3. The sensitivity analysis is performed by varying different parameters and is given in Table 4.

Table 4: Sensitivity analysis for various inventory model parameters

parameter		T^*	η^*	Q^*	TC^*
δ	0.02	0.3384	0.7609	710.04	305.49
	0.04	0.3391	0.7549	709.93	304.74
	0.06	0.3398	0.7492	709.80	303.97
	0.08	0.3403	0.7438	709.67	303.19
α	0.02	0.3385	0.7609	710.04	305.49
	0.03	0.3382	0.7594	709.07	305.16
	0.04	0.3379	0.7580	708.12	304.83
	0.06	0.3374	0.7551	706.19	304.15
N	0.12	0.3342	0.6729	678.87	284.50
	0.14	0.3370	0.7259	698.12	298.58
	0.16	0.3398	0.8038	723.92	312.16
	0.18	0.3421	0.9249	759.85	323.82

M	0.25	0.3385	0.7609	710.03	305.49
	0.27	0.3378	0.6742	686.59	279.49
	0.29	0.3362	0.6108	667.17	251.48
	0.31	0.3336	0.5622	649.99	222.22
I_e	0.08	0.3430	0.8749	749.12	320.79
	0.10	0.3384	0.7609	710.04	305.49
	0.12	0.3325	0.6816	677.54	286.91
	0.14	0.3254	0.6224	648.59	266.06
θ	0.09	0.3441	0.7396	715.91	293.47
	0.11	0.3328	0.7812	703.78	317.14
	0.13	0.3218	0.8192	690.39	339.38
	0.15	0.3111	0.8539	676.11	360.40
h	0.08	0.3399	0.7556	711.67	302.56
	0.10	0.3384	0.7609	710.03	305.49
	0.12	0.3370	0.7660	708.37	308.40
	0.14	0.3356	0.7711	706.69	311.28
A	70	0.3216	0.7240	665.43	272.83
	75	0.3302	0.7428	688.19	289.45
	80	0.3384	0.7609	710.03	305.49
	85	0.3463	0.7782	731.06	321.00
s	9	0.3410	0.8121	728.47	313.65
	11	0.3356	0.7180	693.20	296.54
	13	0.3291	0.6500	662.75	276.73
	15	0.3215	0.5979	634.86	254.97

Based on the optimal results in Table 4, we observe the following managerial phenomena:

1. When the fraction of amount payable as an AP (or, the fraction of amount payable by the customers while purchasing items) is increasing, the dominant retailer will order less quantity and increases the order frequency. The dominant retailer can reduce his total cost by earning more interest under higher order frequency and

customer's advance payment (or, customer's partial payment while purchasing the items). Consequently, the retailer can reduce marginally the discount rate.

2. When the dominant retailer provides a longer credit period, the replenishment cycle time (T^*), price discount rate (η^*) and total cost (TC^*) will be increased. So the dominant retailer should not provide longer credit periods to his customers in order to reduce his total cost and so the loss.

3. When the supplier provides a longer credit period, the dominant retailer replenishes the goods more often; so the retailer will shorten the cycle time in order to take advantage of the longer credit period from the supplier. It is optimal to reduce the rate of discount since he can accumulate more interest earned during the credit period (M).

4. If the interest earned rate increases then the dominant retailer can shorten his replenishment cycle time and reduce the discount rate in order to manage his loss.

5. When the items are starting to deteriorate, it is optimal to rise marginally the discount rate in order to uplift the loss by stimulating sales through the increased discount.

6. It is reasonable that when the holding cost increases the dominant retailer will shorten the cycle time and increases marginally the discount rate to increase sales volume in an effort to maintain his gain whilst reducing the ordering quantity.

7. When ordering cost increases, it is reasonable that the dominant retailer lengthens the cycle time to reduce the frequency of replenishment and he marginally increases discount rate in order to attract more customers.

8. When the selling price increases, the dominant retailer will make less order quantity; he shortens the cycle time and reduces the discount rate in order to keep the system safety.

5.7 Conclusion

Though the advance payment scheme can be seen in practice, it has not been addressed till now in the area of two-echelon trade credits. So, in this chapter, we developed an EOQ model with perishable items incorporating both the advance payment scheme and the two-echelon trade credits. After formulating the model, we then developed solution procedures to determine the optimal price discounting and optimal cycle length for the dominant retailer. The managerial implications of numerical results are clear and provide suitable framework to assess the relative profitability.