Chapter 3

Analysis of partial trade credit financing in a supply chain by EPQ-based model

3.1 Introduction

In the classical EOQ models, it was assumed that the retailer must pay off as soon as the items are received. Usually, retailers are allowed a fixed time period before they settle the payment for the purchased items to the supplier. We term this period as **trade credit period.** Suppliers often offer trade credit as a marketing strategy to increase sales and reduce on-hand stock level. In addition, during the time of credit period, the retailer can accumulate revenue by selling the items and earn interest. A higher interest is charged if the payment is not settled at the end of the trade credit period. One level trade credit financing refers that the supplier would offer the retailer trade credit but the retailer would not offer the trade credit to his customers. Several interesting and relevant papers related to inventory models under trade credit financing exist in the literature.

Partial trade credit financing refers to paying partial amount for the purchased items as soon as the items are received and remaining amount should be settled at the end of trade credit period. To reduce default risks, in practice, a retailer frequently offers partial trade credit to its credit risk customers. Partial trade credit financing is one of the central features in supply chain management. Since in most business transactions, the one level trade credit financing is unrealistic, we want to investigate the situation in a supply chain in which the supplier is willing to provide the retailer a full trade credit period for payments and the retailer offers the partial trade credit to his customers. In practice, this partial trade credit financing at a retailer is more matched to real life supply chains. For example, in India, the TATA Company can delay the full amount of purchasing cost until the end of the delay period offered by his supplier. But the TATA Company only offers partial delay payment to his dealership on the permissible credit period.

In the classical inventory model, it was assumed that the products are obtained from an outside supplier and the entire lot size was delivered at the same time; that is, replenishment is done instantaneously. In real life, however, inventories are often replenished periodically at finite rate and replenishment rate is seldom infinite. In fact, when a product can be produced in-house, the replenishment rate is also the production rate, and hence it is finite. Even for the purchased items from the supplier, when supply arrives at the retailer, it may take days for receiving department to completely transfer the supply into storage room. Hence, the EPQ model should be the efficient model to deal with inventory management issues in a supply chain. It is considered to be one of the most popular inventory control models used in an industry.

To the best of our knowledge, there is no inventory model for a supply chain which incorporates both partial trade credit financing and EPQ-strategy at the retailer. So this chapter fulfills the gap in the literature.

This chapter is organised as follows. Section 3.2 describes the problem undertaken. Section 3.3 provides mathematical modeling for the problem considered. In section 3.4, we derive various previous results in the literature as the particular cases of our model. Section 3.5 illustrates the model with several numerical examples and sensitivity analysis is performed for various inventory parameters. Section 3.6 concludes this chapter.

3.2 Problem description

We consider supplier-retailer supply chain in which supplier fulfills the retailer's demand and retailer fulfills market customer's demands. The retailer of the supply chain replenishes the inventory of perfect items at finite rate (P) from the supplier. P is known and uniform. The market customers purchase the items from the retailer. Demand rate, λ, at the retailer is constant. The replenishment rate is greater than the demand rate. Retailer's inventory is building up at a constant rate $P - \lambda$ units per unit time during the time interval $[0, \frac{\lambda T}{P}]$ where T is the cycle time. No replenishments take place during the time interval $[\frac{\lambda T}{P}, T]$ and inventory is decreasing at the rate λ.

The supplier is willing to provide the retailer a full trade credit period (M) for payments and the retailer offers only the partial trade credit (upto time N) to his customers. Retailer's cycle time (T) may or may not exceed the credit period (M). In addition to the notations in chapter 1, here, we describe various assumptions at the retailer of a supply chain.

1. When $T \geq M$, the retailer settles his account to the supplier at time M for the units sold. During the time period $[0, M]$ the retailer accumulates revenue and keeps interest earned for other activities. After the time M, the retailer starts paying for the interest charges on the items in stock at rate I_k. When $T \leq M$, the account is settled at time M and he does not need to pay any interest on the stock.

2. The retailer offers partial payment option to his customers at the rate of α when the customer purchases an item. Each customer must pay off the remaining balance to the retailer at the end of period N. Hence the retailer can earn interest at rate I_e from the partial payment up to the time N.

3. The retailer's trade credit period (M) offered by the supplier is not necessarily longer than the customer's trade credit period (N) offered by the retailer.

4. Interest charge rate is not necessarily higher than the interest earned rate.

5. Time horizon is infinite.

6. Shortages are not allowed and lead time is negligible.

Under these conditions, we model the retailer's inventory system as a cost minimization problem to determine retailer's optimal ordering policies.

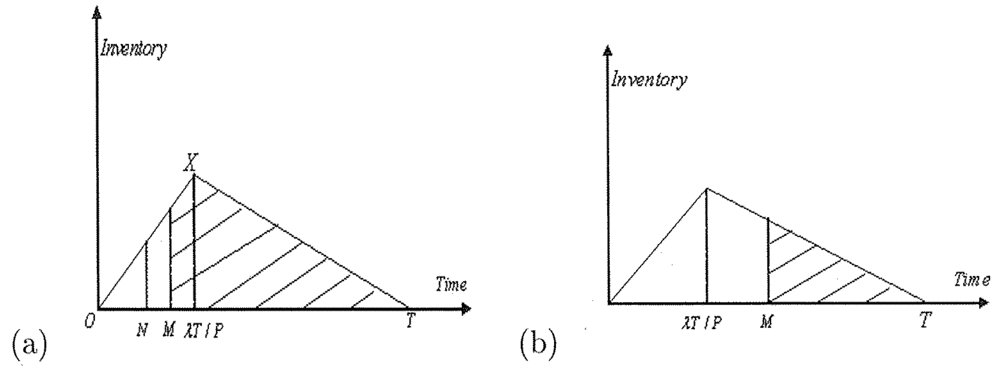

Figure 4: (a) Total accumulation of interest payable in a cycle when $PM/\lambda \leq T$, (b) Total accumulation of interest payable in a cycle when $M \leq T \leq PM/\lambda$

3.3 Mathematical models

From the values of N and M, we have two potential cases: (1) $M \geq N$ and (2) $M < N$. For the moment, the individual costs are now evaluated before they are grouped together.

Case 1. $M \geq N$

1. Annual ordering cost is A/T.

2. Excluding interest charges, the stock-holding cost per cycle is equal to h multiplied by area of the triangle OXT in Fig. 4(a). Hence, annual stock holding cost is

$$\begin{aligned} \frac{h}{T}\left[\frac{T(P-\lambda)(\lambda T/P)}{2}\right] &= \frac{\lambda T h}{2}\left(1-\frac{\lambda}{P}\right) \\ &= \frac{\lambda T h \rho}{2} \text{ where } \rho = 1-\frac{\lambda}{P} \end{aligned}$$

3. **Interest payable**

Based on the values of M (the time at which the retailer must pay the supplier to avoid interest charge), T (cycle time), $\lambda T/P$ (production time) and N (the time at which the customers should pay the retailer to avoid interest charge), we have the following three possible sub-cases: (a) $M \leq PM/\lambda \leq T$, (b) $M \leq T \leq PM/\lambda$, (c) $N \leq T \leq M$ and (d) $T \leq N$. Now let us discuss the interest payable in each sub-case.

(a) When $M \leq PM/\lambda \leq T$,

the retailer must finance for all the items sold after M. Therefore, the interest payable per cycle is I_k multiplied by the area of the shaded region in Fig. 4(a). Hence, the annual amount of interest payable is

$$\frac{cI_k}{T}\left[\frac{\lambda T^2\rho}{2} - \frac{(P-\lambda)M^2}{2}\right] = \frac{cI_k\rho}{T}\left[\frac{\lambda T^2 - PM^2}{2}\right]$$

(b) When $M \leq T \leq PM/\lambda$,

the interest payable per cycle is I_k multiplied by the area of the shaded region in Fig. 4(b). Hence, the annual amount of interest payable is

$$= \frac{cI_k}{T}\left[\frac{\lambda(T-M)^2}{2}\right]$$

(c) When $N \leq T \leq M$,

the annual interest payable amount is zero since credit period M exceeds the cycle time T.

(d) When $T \leq N$,

there is no annual interest payable amount.

4. **Interest earned**

Similar to interest payable, there are four cases that occur in interest earned per year.

(a) When $M \leq PM/\lambda \leq T$

by the time M, retailer has two sources to accumulate revenue in an account that earns I_e per dollar per year: (1) from the portion of partial payment

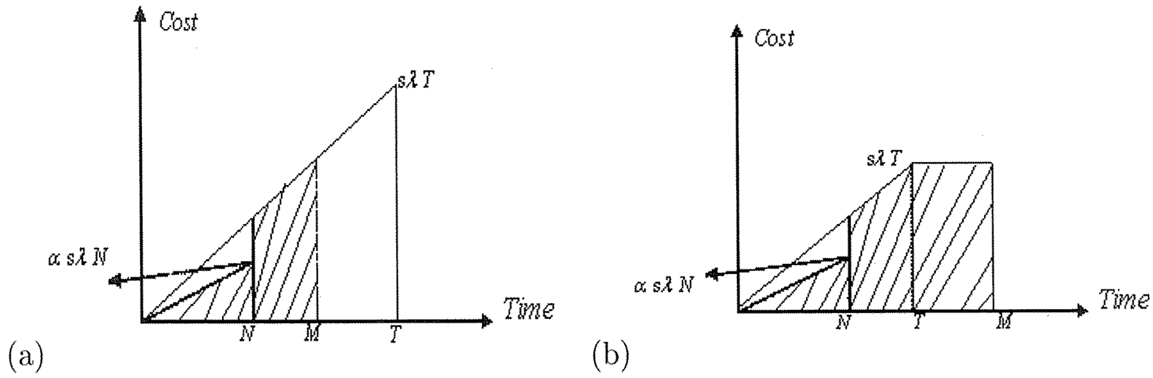

Figure 5: (a) Total amount of interest earned in a cycle when $PM/\lambda \leq T$ (or) $M \leq T \leq PM/\lambda$, (b) Total amount of interest earned in a cycle when $N \leq T \leq M$

(starting from 0 to N) and (2) from the portion of immediate payment (from N to M). Therefore, the interest earned per cycle is I_e multiplied by the area of the shaded region in Fig. 5(a). Hence, the annual interest earned is

$$\frac{sI_e}{T}\left[\frac{\alpha\lambda N^2}{2}+\frac{(\lambda N+\lambda M)(M-N)}{2}\right]=\frac{sI_e\lambda}{2T}[M^2-(1-\alpha)N^2]$$

(b) When $M \leq T \leq PM/\lambda$,

similar to the above case, the annual interest earned is

$$\frac{sI_e\lambda}{2T}[M^2-(1-\alpha)N^2]$$

(c) When $N \leq T \leq M$,

the interest earned per cycle is I_e multiplied by the area of the shaded region in Fig. 5(b). Hence, the annual interest earned is

$$\frac{sI_e}{T}\left[\frac{\alpha\lambda N^2}{2}+\frac{(\lambda N+\lambda T)(T-N)}{2}+\lambda T(M-T)\right]=\frac{sI_e\lambda}{2T}[2MT-(1-\alpha)N^2-T^2]$$

(d) When $T \leq N$

the interest earned per cycle is I_e multiplied by the area of the shaded region in Fig. 6. Hence, the annual interest earned is

$$\frac{sI_e}{T}\left[\frac{\alpha\lambda T^2}{2}+\alpha\lambda T(N-T)+\lambda T(M-N)\right]=sI_e\lambda\left[M-(1-\alpha)N-\frac{\alpha T}{2}\right]$$

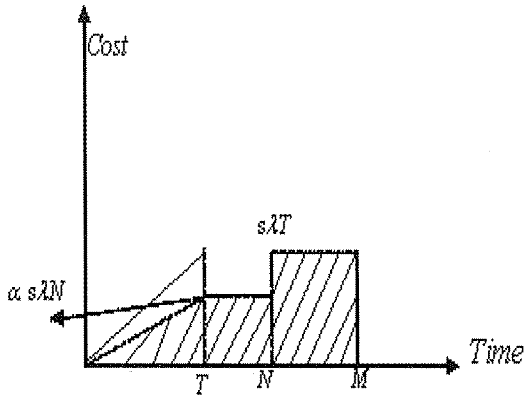

Figure 6: Total amount of interest earned in a cycle when $T \le N$

The annual total cost incurred at the retailer

$TC(T)$ = Setup cost + Holding cost + Interest payable - Interest earned

Therefore,

$$TC(T) = \begin{cases} TC_1(T) & \text{if } M \le PM/\lambda \le T \\ TC_2(T) & \text{if } M \le T \le PM/\lambda \\ TC_3(T) & \text{if } N \le T \le M \\ TC_4(T) & \text{if } 0 < T \le N \end{cases} \tag{6}$$

where

$$TC_1(T) = \frac{A}{T} + \frac{\lambda T h \rho}{2} + \frac{cI_k\rho}{T}\left[\frac{\lambda T^2 - PM^2}{2}\right] - \frac{sI_e\lambda}{2T}[M^2 - (1-\alpha)N^2] \tag{7}$$

$$TC_2(T) = \frac{A}{T} + \frac{\lambda T h \rho}{2} + \frac{cI_k}{T}\left[\frac{\lambda(T-M)^2}{2}\right] - \frac{sI_e\lambda}{2T}[M^2 - (1-\alpha)N^2] \tag{8}$$

$$TC_3(T) = \frac{A}{T} + \frac{\lambda T h \rho}{2} - \frac{sI_e\lambda}{2T}[2MT - (1-\alpha)N^2 - T^2] \tag{9}$$

$$TC_4(T) = \frac{A}{T} + \frac{\lambda T h \rho}{2} - sI_e\lambda\left[M - (1-\alpha)N - \frac{\alpha T}{2}\right] \tag{10}$$

Since $TC_1(PM/\lambda) = TC_2(PM/\lambda), TC_2(M) = TC_3(M)$ and $TC_3(N) = TC_4(N)$, $TC(T)$ is continuous and well-defined. All $TC_1(T), TC_2(T), TC_3(T)$ and $TC_4(T)$ are defined on $T > 0$. Taking first and second derivatives with respect to T in the Eqs. (7) - (10), we have,

$$TC_1'(T) = -\left[\frac{2A - cI_k\rho PM^2 - sI_e\lambda[M^2-(1-\alpha)N^2]}{2T^2}\right] + \lambda\rho\left(\frac{h+cI_k}{2}\right) \quad (11)$$

$$TC_1''(T) = \frac{2A - cI_k\rho PM^2 - sI_e\lambda[M^2-(1-\alpha)N^2]}{T^3} \quad (12)$$

$$TC_2'(T) = -\left[\frac{2A + cI_k\lambda M^2 - sI_e\lambda[M^2-(1-\alpha)N^2]}{2T^2}\right] + \lambda\left(\frac{h\rho+cI_k}{2}\right) \quad (13)$$

$$TC_2''(T) = \frac{2A + \lambda M^2(cI_k - sI_e) + sI_e\lambda(1-\alpha)N^2]}{T^3} > 0 \quad (14)$$

$$TC_3'(T) = -\left[\frac{2A + sI_e\lambda(1-\alpha)N^2}{2T^2} + \frac{\lambda}{2}(h\rho + sI_e)\right] \quad (15)$$

$$TC_3''(T) = \frac{2A + s\lambda(1-\alpha)N^2 I_e}{T^3} > 0 \quad (16)$$

$$TC_4'(T) = \frac{-A}{T^2} + \lambda\left[\frac{h\rho + s\alpha I_e}{2}\right] \quad (17)$$

$$TC_4''(T) = \frac{2A}{T^3} > 0 \quad (18)$$

Eqs. (14), (16) and (18) imply that $TC_2(T), TC_3(T)$ and $TC_4(T)$ are strictly convex on $T > 0$. Eq.(12) implies that $TC_1(T)$ is strictly convex on $T > 0$ when $2A - cI_k\rho PM^2 - sI_e\lambda[M^2-(1-\alpha)N^2] > 0$. Furthermore, we have $TC_1'(PM/\lambda) = TC_2'(PM/\lambda), TC_2'(M) = TC_3'(M)$ and $TC_3'(N) = TC_4'(N)$. Therefore $TC(T)$ is strictly convex on $T > 0$ when $2A - cI_k\rho PM^2 - sI_e\lambda[M^2-(1-\alpha)N^2] > 0$.

3.3.1 Optimal cycle time T^* for the case $M \geq N$

For optimality, we have $TC_i'(T) = 0$ for all $i = 1, 2, 3, 4$. Solving the equations, we obtain

$$T_1^* = \sqrt{\frac{2A - cI_k\rho PM^2 - sI_e\lambda[M^2-(1-\alpha)N^2]}{\lambda\rho(h+cI_k)}} \quad (19)$$

if $2A - cI_k\rho PM^2 - sI_e\lambda[M^2-(1-\alpha)N^2] > 0$

$$T_2^* = \sqrt{\frac{2A + cI_k\lambda M^2 - sI_e\lambda[M^2-(1-\alpha)N^2]}{\lambda(h\rho+cI_k)}} \quad (20)$$

$$T_3^* = \sqrt{\frac{2A + sI_e\lambda(1-\alpha)N^2}{\lambda(h\rho+sI_e)}} \quad (21)$$

$$T_4^* = \sqrt{\frac{2A}{\lambda(h\rho+s\alpha I_e)}} \quad (22)$$

Eq.(19) gives the optimal value T_1^* when $T \geq PM/\lambda$. We substitute Eq.(19) into $T_1^* \geq PM/\lambda$, then we obtain that

$T_1^* \geq PM/\lambda$ if and only if $-2A - cI_k\lambda M^2 + (PM/\lambda)^2\lambda(h\rho + cI_k) + sI_e\lambda[M^2-(1-\alpha)N^2] \leq 0$.

Eq.(20) gives the optimal value T_2^* when $M \leq T \leq PM/\lambda$ so that $M \leq T_2^* \leq PM/\lambda$. We substitute Eq.(20) in $M \leq T_2^* \leq PM/\lambda$, then we obtain that

$T_2^* \leq PM/\lambda$ if and only if $-2A - cI_k\lambda M^2 + (PM/\lambda)^2\lambda(h\rho + cI_k) + sI_e\lambda[M^2 - (1-\alpha)N^2] \geq 0.$

and

$$M \leq T_2^* \text{ if and only if } -2A + \lambda M^2 h\rho + sI_e\lambda[M^2 - (1-\alpha)N^2] \leq 0$$

Similarly, Eq.(21) gives the optimal value T_3^* when $N \leq T \leq M$ so that $N \leq T_3^* \leq M$. We substitute Eq.(21) in $N \leq T_3^* \leq M$, then we obtain that

$$T_3^* \leq M \text{ if and only if } -2A + \lambda M^2 h\rho + sI_e\lambda[M^2 - (1-\alpha)N^2] \geq 0$$

and

$$T_3^* \geq N \text{ if and only if } -2A + \lambda N^2[h\rho + s\alpha I_e] \leq 0$$

Finally, Eq.(22) gives the optimal value T_4^* when $T \leq N$ so that $T_4^* \leq N$. We substitute Eq.(22) in $T_4^* \leq N$, then we obtain that

$$T_4^* \leq N \text{ if and only if } -2A + \lambda N^2[h\rho + s\alpha I_e] \geq 0$$

From the above discussions, we let

$$\Delta_1 = -2A - cI_k\lambda M^2 + (PM/\lambda)^2\lambda(h\rho + cI_k) + sI_e\lambda[M^2 - (1-\alpha)N^2] \quad (23)$$

$$\Delta_2 = -2A + \lambda M^2 h\rho + sI_e\lambda[M^2 - (1-\alpha)N^2] \quad (24)$$

$$\text{and } \Delta_3 = -2A + \lambda N^2[h\rho + s\alpha I_e] \quad (25)$$

Eqs. (23), (24) and (25) imply that $\Delta_1 \geq \Delta_2 \geq \Delta_3$. From the above arguments, we obtain the following Theorem 3.1.

Theorem 3.1 When $M \geq N$,

(A) If $\Delta_1 \leq 0$, then $T^* = T_1^*$ and $TC^*(T) = TC_1(T_1^*)$

(B) If $\Delta_1 \geq 0$ and $\Delta_2 \leq 0$, then $T^* = T_2^*$ and $TC^*(T) = TC_2(T_2^*)$

(C) If $\Delta_2 \geq 0$ and $\Delta_3 \leq 0$, then $T^* = T_3^*$ and $TC^*(T) = TC_3(T_3^*)$

(D) If $\Delta_3 \geq 0$, then $T^* = T_4^*$ and $TC^*(T) = TC_4(T_4^*)$.

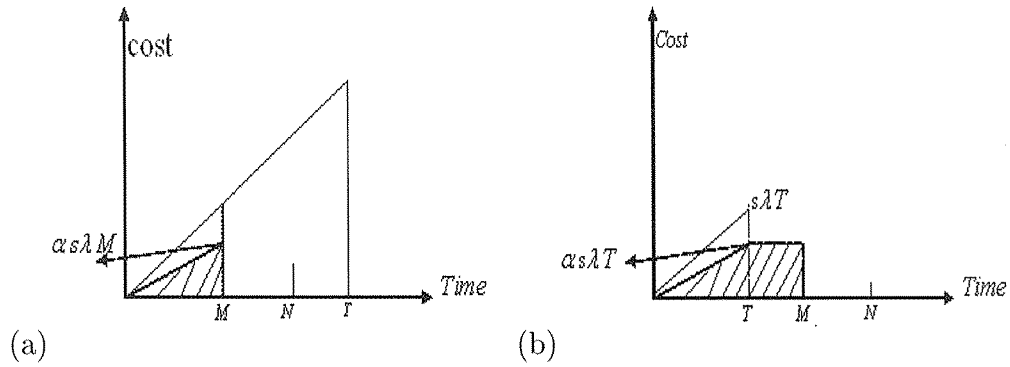

Figure 7: (a) Total amount of interest earned in a cycle when $PM/\lambda \leq T$ (or) $M \leq T$, (b) Total amount of interest earned in a cycle when $T \leq M$

Case 2. $M < N$

1. Annual ordering cost is A/T.

2. Excluding interest charges, annual stock-holding cost is $\frac{\lambda Th\rho}{2}$

3. **Interest payable**

   According to assumption (1), there are three sub-cases that occur in costs of interest payable for the items kept in stock per year.

   (a) When $PM/\lambda \leq T$,
   similar to sub-case (a) under the condition $M > N$, the annual interest payable

   $$= \frac{cI_k\rho}{T}\left[\frac{\lambda T^2 - PM^2}{2}\right]$$

   (b) When $M \leq T \leq PM/\lambda$,
   similar to sub-case (b) under the condition $M > N$, the annual interest payable

   $$= \frac{cI_k}{T}\left[\frac{\lambda(T-M)^2}{2}\right]$$

   (c) When $M \geq T$
   The annual interest payable $= 0$

4. **Interest earned**

According to assumption (2), there are three sub-cases that occur in interest earned per year.

(a) When $PM/\lambda \leq T$

the amount of interest earned per cycle is I_e multiplied by the area of the shaded region in Fig. 7(a). Hence, the annual interest earned is

$$\frac{sI_e\lambda\alpha M^2}{2T}$$

(b) When $M \leq T \leq PM/\lambda$

the amount of interest earned per cycle is I_e multiplied by the area of the shaded region in Fig. 7(a). Hence, the annual interest earned is

$$\frac{sI_e\lambda\alpha M^2}{2T}$$

(c) When $M \geq T$

the amount of interest earned per cycle is I_e multiplied by the area of the shaded region in Fig. 7(b). Hence, the annual interest earned is

$$sI_e\left[\frac{\alpha\lambda T^2}{2} + \alpha\lambda T(M-T)\right]/T = sI_e\lambda[\alpha M - \alpha T/2]$$

The total cost incurred at the retailer, $TC(T)$, is

$$TC(T) = \begin{cases} TC_5(T) & \text{if } PM/\lambda \leq T \\ TC_6(T) & \text{if } M \leq T \leq PM/\lambda \\ TC_7(T) & \text{if } M \geq T \end{cases} \tag{26}$$

where

$$TC_5(T) = \frac{A}{T} + \frac{\lambda Th\rho}{2} + \frac{cI_k\rho}{T}\left[\frac{\lambda T^2 - PM^2}{2}\right] - \frac{sI_e\lambda\alpha M^2}{2T} \tag{27}$$

$$TC_6(T) = \frac{A}{T} + \frac{\lambda Th\rho}{2} + \frac{cI_k}{T}\left[\frac{\lambda(T-M)^2}{2}\right] - \frac{sI_e\lambda\alpha M^2}{2T} \tag{28}$$

$$TC_7(T) = \frac{A}{T} + \frac{\lambda Th\rho}{2} - sI_e\lambda[\alpha M - \alpha T/2] \tag{29}$$

Since $TC_5(PM/\lambda) = TC_6(PM/\lambda)$ and $TC_6(M) = TC_7(M)$, $TC(T)$ is continuous and well-defined. All $TC_5(T), TC_6(T)$ and $TC_7(T)$ are defined on $T > 0$. Taking first and second derivatives with respect to T in the Eqs. (27)-(29), we have

$$TC_5'(T) = -\left[\frac{2A - cI_k\rho PM^2 - sI_e\alpha\lambda M^2}{2T^2}\right] + \lambda\rho\left(\frac{h + cI_k}{2}\right) \quad (30)$$

$$TC_5''(T) = \frac{2A - cI_k\rho PM^2 - sI_e\lambda\alpha M^2}{T^3} \quad (31)$$

$$TC_6'(T) = -\left[\frac{2A + cI_k\lambda M^2 - sI_e\lambda\alpha M^2}{2T^2}\right] + \lambda\left(\frac{h\rho + cI_k}{2}\right) \quad (32)$$

$$TC_6''(T) = \frac{2A + \lambda M^2(cI_k - \alpha sI_e)}{T^3} > 0 \quad (33)$$

$$TC_7'(T) = \frac{-A}{T^2} + \lambda\left[\frac{h\rho + s\alpha I_e}{2}\right] \quad (34)$$

$$TC_7''(T) = \frac{2A}{T^3} > 0 \quad (35)$$

Eqs. (33) and (35) imply that $TC_6(T)$ and $TC_7(T)$ are strictly convex on $T > 0$ and Eq.(31) implies that $TC_5(T)$ is strictly convex on $T > 0$ if $2A - cI_k\rho PM^2 - sI_e\lambda\alpha M^2 > 0$. Furthermore, we have $TC_5'(PM/\lambda) = TC_6'(PM/\lambda)$ and $TC_6'(M) = TC_7'(M)$. Eq.(26) implies that TC(T) is strictly convex on $T > 0$ when $2A - cI_k\rho PM^2 - sI_e\lambda\alpha M^2 > 0$.

3.3.2 Optimal cycle time T^* for the case $M < N$

From the optimality condition, we have $TC_i'(T) = 0$ for all $i = 5, 6, 7$. Solving these equations, we obtain

$$T_5^* = \sqrt{\frac{2A - cI_k\rho PM^2 - sI_e\lambda\alpha M^2}{\lambda\rho(h + cI_k)}} \quad (36)$$

$$T_6^* = \sqrt{\frac{2A + cI_k\lambda M^2 - sI_e\lambda\alpha M^2}{\lambda(h\rho + cI_k)}} \quad (37)$$

$$T_7^* = \sqrt{\frac{2A}{\lambda(h\rho + s\alpha I_e)}} \quad (38)$$

Eq.(36) gives the optimal value T_5^* when $T \geq PM/\lambda$; so that $T_5^* \geq PM/\lambda$. We substitute Eq.(36) in $T_5^* \geq PM/\lambda$ then we obtain that

$$T_5^* \geq PM/\lambda \text{ if and only if } -2A + \lambda M^2(s\alpha I_e - cI_k) + (PM/\lambda)^2\lambda(h\rho + cI_k) \leq 0.$$

Eq.(37) gives the optimal value T_6^* when $M \leq T \leq PM/\lambda$; so that $M \leq T_6^* \leq PM/\lambda$. We substitute Eq.(37) in $M \leq T_6^* \leq PM/\lambda$ then we obtain that

$$T_6^* \leq PM/\lambda \text{ if and only if } -2A + \lambda M^2(s\alpha I_e - cI_k) + (PM/\lambda)^2\lambda(h\rho + cI_k) > 0.$$

and

$$M \leq T_6^* \text{ if and only if } -2A + \lambda M^2 h\rho + sI_e\alpha\lambda M^2 \leq 0$$

Similarly, Eq.(38) gives the optimal value T_7^* when $T \leq M$; so that $T_7^* \leq M$. We substitute Eq.(38) in $T_7^* \leq M$ then we obtain that

$$T_7^* \leq M \text{ if and only if } -2A + \lambda M^2[h\rho + s\alpha I_e] \geq 0$$

Furthermore, we let

$$\Delta_4 = -2A + \lambda M^2(s\alpha I_e - cI_k) + (PM/\lambda)^2\lambda(h\rho + cI_k) \quad (39)$$

$$\Delta_5 = -2A + \lambda M^2(h\rho + s\alpha I_e) \quad (40)$$

Eqs.(39) and (40) imply that $\Delta_4 \geq \Delta_5$. From the above arguments, we obtain the following Theorem 3.2.

Theorem 3.2. When $M < N$,

(A) If $\Delta_5 \geq 0$, then $T^* = T_7^*$ and $TC^*(T) = TC_7(T_7^*)$

(B) If $\Delta_5 \leq 0$ and $\Delta_4 \geq 0$, then $T^* = T_6^*$ and $TC^*(T) = TC_6(T_6^*)$

(D) If $\Delta_4 \leq 0$, then $T^* = T_5^*$ and $TC^*(T) = TC_5(T_5^*)$.

3.4 Particular cases

By substituting some inventory parameters with certain values in our model, the models developed by Chung and Haung [26], Haung [52] and Goyal [46] become particular cases to our problem.

3.4.1 Chung and Haung's model [26]

Consider that $M \geq 0, s = c, N = 0$ and $\alpha = 0$. It means that the supplier offers full trade credit to his retailer but the retailer does not offer trade credit to his customer. From Eqs.(7) to (9), we have

$$TC_8(T) = \frac{A}{T} + \frac{\lambda T h\rho}{2} + \frac{cI_k\rho}{T}\left[\frac{\lambda T^2 - PM^2}{2}\right] - \frac{cI_e\lambda M^2}{2T} \text{if } T \geq PM/\lambda \quad (41)$$

$$TC_9(T) = \frac{A}{T} + \frac{\lambda T h\rho}{2} + \frac{cI_k}{T}\left[\frac{\lambda(T-M)^2}{2}\right] - \frac{cI_e\lambda M^2}{2T} \text{ if } M \leq T \leq PM/\lambda \quad (42)$$

$$TC_{10}(T) = \frac{A}{T} + \frac{\lambda T h\rho}{2} - \frac{cI_e}{T}[\lambda T^2/2 + \lambda T(M-T)] \text{ if } T \leq M \quad (43)$$

From the optimality conditions, we have

$$TC_i'(T_i^*) = 0 \text{ for } i = 8, 9, 10$$

where

$$T_8^* = \sqrt{\frac{2A + \lambda M^2 c(I_k - I_e) - PM^2 cI_k}{\lambda\rho(h + cI_k)}} \quad (44)$$
$$\text{if } 2A + \lambda M^2 c(I_k - I_e) - PM^2 cI_k > 0,$$

$$T_9^* = \sqrt{\frac{2A + \lambda M^2 c(I_k - I_e)}{\lambda(h\rho + cI_k)}} \quad (45)$$

$$T_{10}^* = \sqrt{\frac{2A}{\lambda(h\rho + cI_e)}} \quad (46)$$

Eq.(6) is modified as follows:

$$TC(T) = \begin{cases} TC_8(T) & \text{if } PM/\lambda \leq T \\ TC_9(T) & \text{if } M \leq T \leq PM/\lambda \\ TC_{10}(T) & \text{if } M \geq T \end{cases} \quad (47)$$

Eq.(47) is consistent with Eqs.(6 a-c) in Chung and Haung's model [26]. Eqs.(23) and (24) can be modified as

$$\Delta_1 = -2A + \frac{M^2}{\lambda}[P(P-\lambda)h + cI_k(P^2 - \lambda^2) + cI_e\lambda^2] \quad (48)$$
$$\text{and } \Delta_2 = -2A + \lambda M^2(h\rho + cI_e) \quad (49)$$

respectively. If we let $\bar{\Delta}_1 = -2A + \frac{M^2}{\lambda}[P(P-\lambda)h + cI_k(P^2 - \lambda^2) + cI_e\lambda^2]$ and $\bar{\Delta}_2 = -2A + \lambda M^2(h\rho + cI_e)$ then Theorem 3.1 can be modified as follows.

Theorem 3.3.

(A) If $\bar{\Delta}_1 \leq 0$ then $TC(T^*) = TC(T_8^*)$ and $T^* = T_8^*$

(B) If $\bar{\Delta}_1 > 0$ and $\bar{\Delta}_2 \leq 0$, then $TC(T^*) = TC(T_9^*)$ and $T^* = T_9^*$

(C) If $\bar{\Delta}_2 \geq 0$ then $TC(T^*) = TC(T_{10}^*)$ and $T^* = T_{10}^*$

The above Theorem 3.3 has been discussed in Theorem 3 of Chung and Haung's model [26]. Hence, Chung and Haung's model [26] is a particular case of our model.

3.4.2 Haung's model [52]

When $P \to \infty$ (i.e., EOQ strategy), $M \geq N, s = c$ and $\alpha = 0$ (it means that the retailer also offers full trade credit to his customer), we have from Eqs. (7) to (9)

$$\begin{aligned} TC_{11}(T) &= \frac{A}{T} + \frac{\lambda Th}{2} + \frac{cI_k\lambda(T-M)^2}{2T} - \frac{cI_e\lambda(M^2-N^2)}{2T} \quad &(50)\\ TC_{12}(T) &= \frac{A}{T} + \frac{\lambda Th}{2} - cI_e\lambda[2MT - N^2 - T^2]/2T \quad &(51)\\ TC_{13}(T) &= \frac{A}{T} + \frac{\lambda Th}{2} - cI_e\lambda(M-N) \quad &(52)\end{aligned}$$

From the optimality conditions, we have

$$TC_i^{'}(T_i^*) = 0 \text{ for } i = 11, 12, 13$$

where

$$T_{11}^* = \sqrt{\frac{2A + c\lambda[M^2(I_k - I_e) + N^2 I_e]}{\lambda(h + cI_k)}} \quad (53)$$

$$T_{12}^* = \sqrt{\frac{2A + \lambda N^2 I_e}{\lambda(h + cI_e)}} \quad (54)$$

$$T_{13}^* = \sqrt{\frac{2A}{\lambda h}} \quad (55)$$

Eq.(6) is modified as follows:

$$TC(T) = \begin{cases} TC_{11}(T) & \text{if } M \leq T \\ TC_{12}(T) & \text{if } N \leq T \leq M \\ TC_{13}(T) & \text{if } 0 < T \leq N \end{cases} \quad (56)$$

Eq.(56) is consistent with Eq.(1 a-c) in Haung's model [52]. Eqs.(23) and (24) can be modified as

$$\begin{aligned} \Delta_1 &= -2A + M^2[h + cI_e] - c\lambda N^2 I_e \quad &(57)\\ \text{and } \Delta_2 &= -2A + \lambda N^2 h \quad &(58)\end{aligned}$$

respectively. If we let, $\bar{\Delta}_3 = \Delta_1$ and $\bar{\Delta}_4 = \Delta_2$ then Theorem 3.1 can be modified as follows.

Theorem 3.4.

(A) If $\bar{\Delta}_3 \leq 0$ then $TC(T^*) = TC(T_{11}^*)$ and $T^* = T_{11}^*$

(B) If $\bar{\Delta}_3 > 0$ and $\bar{\Delta}_4 \leq 0$, then $TC(T^*) = TC(T_{12}^*)$ and $T^* = T_{12}^*$

(C) If $\bar{\Delta}_4 \geq 0$ then $TC(T^*) = TC(T_{13}^*)$ and $T^* = T_{13}^*$

The above Theorem 3.4 has been discussed in Theorem 1 of Haung's model [52]. Hence, Haung's model is a particular case of our model.

3.4.3 Goyal's model [46]

When $P \to \infty$ (i.e., EOQ strategy), $N = 0, s = c$ and $\alpha = 0$ (it means that the retailer would not offer the delay period to his customer, that is one level trade credit), let

$$TC_{14}(T) = \frac{A}{T} + \frac{\lambda Th}{2} + \frac{cI_k\lambda(T-M)^2}{2T} - \frac{cI_e\lambda M^2}{2T} \tag{59}$$

$$TC_{15}(T) = \frac{A}{T} + \frac{\lambda Th}{2} - cI_e[\lambda T^2/2 + \lambda T(M-T)]/2T \tag{60}$$

From the optimality conditions, we have

$$TC_i^{'}(T_i^*) = 0 \text{ for } i = 14, 15 \tag{61}$$

where

$$T_{14}^* = \sqrt{\frac{2A + c\lambda M^2(I_k - I_e)}{\lambda(h + cI_k)}} \tag{62}$$

$$T_{15}^* = \sqrt{\frac{2A}{\lambda(h + cI_e)}} \tag{63}$$

Eq.(6) is modified as follows:

$$TC(T) = \begin{cases} TC_{14}(T) & \text{if } M \leq T \\ TC_{15}(T) & \text{if } 0 \leq T \leq M \end{cases} \tag{64}$$

Eq.(64) is consistent with Eqs.(1) and (4) in Goyal's model [46] respectively. Eq.(23) can be modified as $\Delta_1 = -2A + M^2[h + cI_e]$. If we let $\bar{\Delta} = \Delta_1$ then Theorem 3.1 can be modified as follows.

Theorem 3.5.

(A) If $\bar{\Delta} < 0$ then $T^* = T^*_{14}$

(B) If $\bar{\Delta} > 0$ then $T^* = T^*_{15}$

(C) If $\bar{\Delta} = 0$ then $T^* = T^*_{14} = T^*_{15} = M$

The above Theorem has been discussed in Theorem 1 of Goyal's model [46]. Hence, Goyal's model [46] is a particular case of our work.

3.5 Numerical examples and sensitivity analysis

In order to evaluate the proposed model, we exhibit 27 numerical problems for different parameters of α, N and s when $M \geq N$ and $M < N$. By using Theorems 3.1 and 3.2, we obtain optimal solutions.

3.5.1 When $M \geq N$

Consider the following data. Let P = 3000 units; λ= 2000 units; A = \$100 per order; c = \$14 per unit; h = \$7 per unit; I_k= 0.1 per dollar; I_e = 0.2 per dollar; M = 0.1 year. The optimal solutions are shown in Table 1 (in the next page) for various values of α, N and s.

3.5.2 When $M < N$

Let P = 6000 units; λ= 5000 units; A = \$ 43 per order; c = \$ 10 per unit; h = \$ 10 per unit; I_k= 0.1 per dollar; I_e = 0.2 per dollar; M = 0.05 year. The optimal solutions are shown in Table 2 (in the page 51) for various values of α, N and s.

Table 1: Optimal solutions when $M \geq N$

α	N	s	Δ_1	Δ_2	Δ_3	Theorem	T^*	$P(\frac{\lambda T^*}{P})$	TC^*
0.2	0.03	10	<0	<0	<0	1(A)	0.1631	326.1025	913.0871
		20	>0	<0	<0	1(B)	0.1435	287.0042	791.4825
		30	>0	<0	<0	1(B)	0.1250	249.9714	653.2267
	0.06	10	<0	<0	<0	1(A)	0.1677	335.4315	939.2082
		20	<0	<0	<0	1(A)	0.1518	303.5975	850.0729
		30	>0	<0	<0	1(B)	0.1382	276.3538	751.7209
	0.09	10	<0	<0	<0	1(A)	0.1752	350.4283	981.1993
		20	<0	<0	<0	1(A)	0.1679	335.7720	940.1617
		30	<0	<0	<0	1(A)	0.1602	320.4461	897.2491
0.5	0.03	10	<0	<0	<0	1(A)	0.1625	324.9176	909.7692
		20	>0	<0	<0	1(B)	0.1425	284.9812	783.9298
		30	>0	<0	<0	1(B)	0.1232	246.4752	640.1739
	0.06	10	<0	<0	<0	1(A)	0.1654	330.7999	926.2397
		20	>0	<0	<0	1(B)	0.1475	294.9576	821.1751
		30	>0	<0	<0	1(B)	0.1317	263.4930	703.7073
	0.09	10	<0	<0	<0	1(A)	0.1702	340.3779	953.0582
		20	>0	<0	<0	1(A)	0.1572	314.4156	880.3636
		30	>0	<0	<0	1(B)	0.1448	289.6426	801.3325
0.8	0.03	10	<0	<0	<0	1(A)	0.1619	323.7283	906.4392
		20	>0	<0	<0	1(B)	0.1415	282.9437	776.3232
		30	>0	<0	<0	1(B)	0.1215	242.9286	626.9333
	0.06	10	<0	<0	<0	1(A)	0.1631	326.1025	913.0871
		20	>0	<0	<0	1(B)	0.1435	287.0042	791.4825
		30	>0	<0	<0	1(B)	0.1250	249.9714	653.2267
	0.09	10	<0	<0	<0	1(A)	0.1650	330.0216	924.0606
		20	>0	<0	<0	1(B)	0.1468	293.6470	816.2822
		30	>0	<0	<0	1(B)	0.1306	261.2880	695.4753

Table 2: Optimal solutions when $M < N$

α	N	s	Δ_4	Δ_5	Theorem	T^*	$P(\frac{\lambda T^*}{P})$	TC^*
0.1	0.06	10	< 0	< 0	2(C)	0.0940	470.0097	861.6844
		30	< 0	< 0	2(C)	0.0911	455.2721	834.6656
		50	< 0	< 0	2(C)	0.0880	440.0413	806.7424
	0.08	10	< 0	< 0	2(C)	0.0940	470.0097	861.6844
		30	< 0	< 0	2(C)	0.0911	455.2721	834.6656
		50	< 0	< 0	2(C)	0.0880	440.0413	806.7424
	0.1	10	< 0	< 0	2(C)	0.0940	470.0097	861.6844
		30	< 0	< 0	2(C)	0.0911	455.2721	834.6656
		50	< 0	< 0	2(C)	0.0880	440.0413	806.7424
0.5	0.06	10	< 0	< 0	2(C)	0.0708	440.0413	806.7424
		30	> 0	< 0	2(B)	0.0520	354.1956	649.3587
		50	< 0	< 0	2(C)	0.0880	259.8076	442.8203
	0.08	10	< 0	< 0	2(C)	0.0880	440.0413	806.7424
		30	< 0	< 0	2(C)	0.0708	354.1956	649.3587
		50	> 0	< 0	2(B)	0.0520	259.8076	442.8203
	0.1	10	< 0	< 0	2(C)	0.0880	440.0413	806.7424
		30	< 0	< 0	2(C)	0.0708	354.1956	649.3587
		50	> 0	< 0	2(B)	0.0520	259.8076	442.8203
0.9	0.06	10	< 0	< 0	2(C)	0.0816	407.8770	747.7745
		30	> 0	> 0	2(A)	0.0493	246.6760	393.1772
		50	> 0	< 0	2(A)	0.0402	200.7797	108.3495
	0.08	10	< 0	< 0	2(C)	0.0816	407.8770	747.7745
		30	< 0	> 0	2(A)	0.0493	246.6760	393.1772
		50	> 0	> 0	2(A)	0.0402	200.7797	108.3495
	0.1	10	< 0	< 0	2(C)	0.0816	407.8770	747.7745
		30	> 0	> 0	2(A)	0.0493	246.6760	393.1772
		50	> 0	> 0	2(A)	0.0402	200.7797	108.3495

3.5.3 Sensitivity analysis

To study the effects of α, N and s on the optimal cycle time T^* and on the optimal total cost $TC(T^*)$, the following results are observed from Tables 1 and 2.

1. For fixed value of N and s, larger the value of α is smaller the value of optimal cycle time and lower the value of annual total cost.

2. For fixed α and s, larger the value of N is larger the value of optimal cycle time and higher the value of annual total cost when $M \geq N$; the optimal cycle time and the annual total cost are independent of N when $M < N$.

3. Finally, for fixed α and N, larger the value of s is smaller the value of optimal cycle time and smaller the value of annual total cost.

We obtain the following managerial phenomena from Tables 1 and 2.

- When the customer's fraction of the total amount owed payable at the time of placing an order (α) increases, the retailer will order less quantity and increase order frequency. The retailer can accumulate more interest under higher order frequency and higher customer's fraction of the total amount owed payable at the time of placing an order offered by the retailer.

- When the customer's trade credit period offered by the retailer increases, the retailer will order more quantity to accumulate more interest to compensate the loss of interest earned when longer trade credit period is offered to his customer under the condition of M $\geq$ N.

- When the unit selling price increases, the retailer will order less quantity to take the benefits of the trade credit more frequently.

3.6 Conclusion

In this chapter, we developed an EPQ-based inventory model to investigate retailer's decision making right in a supply chain under realistic conditions. First, the supplier provides the retailer a full trade credit period and the retailer adopts the partial trade credit option to his customers. Second, interest payable rate is not necessarily higher than the interest earned rate. These assumptions are consistent with economic senses. We developed two effective and easy-to-use theorems 3.1 and 3.2 which help the retailer of the supply chain to find optimal inventory policy when $M \geq N$ and $M < N$ respectively. Chung and Haung's model [26], Haung's model [52] and Goyal's model [46] are derived as particular cases of this proposed model. Numerical examples are given to illustrate all effective theorems and the results of sensitivity analysis are also consistent.