CHAPTER 4

ECG’S QRS, P AND T POINTS DETECTION ALGORITHM USING WAVELET TRANSFORM

4.1 Introduction

Mathematical transformations are applied to a signal to obtain further information, which is not readily available in its original time-domain form and which is more useful to the application at hand. Fourier Transform (FT) is the oldest and foremost of all transforms used in the history of signal processing. Joseph Fourier (1770-1830) first introduced the remarkable idea of expansion of a function in terms of a trigonometric series. In other words, he presented a new technique to decompose a signal into complex exponential functions of different frequencies. For a continuous signal $x(t)$, the Fourier Transformation is defined as follows:

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} \, dt \quad (4.1)$$

The analysis coefficients, also called spectra $X(f)$, are computed as inner products of the signal with sinusoidal basis functions of infinite duration. The trigonometric kernel $\exp(-j2\pi ft)$, used here, oscillates indefinitely, and hence, the localized information contained in the signal $x(t)$ gets lost. While the spectrum $X(f)$ shows the overall strength with which any frequency $f$ is contained in the signal $x(t)$, it does not generally provide asymptotic information about the time-localization of spectral components. The analysis coefficients $X(f)$ define the notion of global frequency $f$ in a signal. However, time domain and frequency domain constitute two alternative ways of looking at a signal. Although FT allows a passage from one domain to the other, it does not allow a combination
of the two. This method enables us to investigate problems either in the time domain or in the frequency domain, but not simultaneously in both. Fourier transform theory has been very useful for analyzing harmonic signals, or signals for which there is no need for local information. Fourier analysis is therefore an effective tool for studying stationary signals, that is, signals with time-independent frequency content. However, many of the practically encountered signals, like the ECG and most of the physiological signals, are non-stationary. A complete analysis of non-stationary signals requires a joint time-frequency representation. The basic idea of time-frequency representations of signals is to map a one-dimensional signal of time, $x(t)$, into a two-dimensional function of time and frequency. Thus, they combine time-domain and frequency-domain analysis to yield a potentially more revealing picture of the temporal localization of a signal’s spectral components. In order to incorporate both time and frequency localization properties in FT, Dennis Gabor in 1946 [99] first introduced the windowed Fourier Transform or Short Time Fourier Transform (STFT). His major idea was to use a time-localization window function $g(t - \tau)$ for extracting local information from the Fourier transform of a signal. The parameter $\tau$ corresponds to the position of the window in time. $\tau$ is kept on varying to translate the window until the whole of time-domain is covered. The width of this window must be less than or equal to the segment of the signal where stationary condition is valid, that is, the distribution of the samples in that segment is similar to the distribution of the samples in any other segment.

$$STFT(\tau, f) = \int_{-\infty}^{\infty} x(t)g(t - \tau)e^{-j2\pi ft} \, dt$$ (4.2)

Although STFT overcomes the drawback of Fourier Transform apparently, it has got a serious problem related to the resolution in time and frequency. The root of this problem is related to Heisenberg’s Uncertainty Principle [20,100,101], according to which exact time-frequency representation of a signal is not possible. Therefore, one can never know precisely which of the spectral components exists at what instants of time. What we can know is the time interval during
which a certain band of frequency exists. A broader window gives better frequency resolution and poor time resolution. On the contrary, the time resolution can be improved at the cost of frequency resolution with shorter window. Once the window is chosen for STFT, the resolution in time and frequency domain gets fixed. However, many signals encountered in our practical life, especially in our case the ECG, requires a more flexible approach regarding this resolution. Wavelet transform (WT) was developed to overcome this fixed resolution problem of STFT. The Multi Resolution Approach (MRA) in time and frequency domain, also called Multi resolution Signal Analysis, is the heart of WT. The basis of FT is sinusoidal waves of infinite, periodic smooth and predictable duration. On the other hand, WT decomposes signal into a set of compactly supported basis functions called wavelets or small waves, obtained from a single prototype mother wavelet by means of dilation and translation. On the contrary to FT, wavelets are a periodic, irregular and localized waves of finite energy. They have their energy concentrated in time or space and are suitable to analyse a transient signal, which contains a high degree of non periodic components and a higher magnitude of high frequencies than its harmonic contents, like ECG signals.

4.2 Multi resolution Signal Analysis

A signal can be viewed as the sum of a coarse part and a detailed part. The smooth part reflects the main features of the signal, therefore called the approximation signal, whereas the faster fluctuations represent the details of the signal. The separation of a signal into two parts is determined by the resolution, with which the signal is analyzed, i.e., by the scale below which no details can be discerned. A progressively better approximation of the signal is obtained by increasing the resolution so that finer and finer details are included in the smooth part [3]. The approximation of a signal $x(t)$ at scale $j$ is de-noted as $x_j(t)$. At the next scale $j + l$, the approximation signal $x_{j+1}(t)$ is composed of $x_j(t)$ and the details $y_j(t)$ at that level such that

$$x_{j+1}(t) = x_j(t) + y_j(t)$$  (4.3)
By adding more and more detail to $x_j(t)$ arrived, as the resolution approaches infinity, at a dyadic multi resolution representation of the original signal $x(t)$ which involves a smooth part and the sum of different details,

$$x(t) = x_j(t) + \sum_{k=j}^{\infty} y_k(t)$$ \hspace{1cm} (4.4)

Unlike STFT which has a constant resolution at all times and frequencies, WT uses a Multi-Resolutonal Approach (MRA), i.e. varying temporal resolution for different spectral components, which can be clarified as follows. Lower or narrower scales (higher frequencies) mean lesser ambiguity in time, i.e. good time resolution. Higher scales (lower frequencies) have wider support, leading to more ambiguity in time, or in other words, poor temporal resolution [103-107]. Figure 4.1 compares the resolution for four different representations of the same signal.

The original time-domain signal has got no time resolution problem, since we know the value of the signal at every instant of time. In the Fourier transformed version, there is no resolution problem in the frequency domain, i.e. we know precisely what frequencies exist. Conversely, the frequency resolution in time domain and time resolution in Fourier domain are zero, since we have no information about them. For the two bottom diagrams, each box represents an equal area of the time-frequency plane, but different sized boxes giving different proportion to time and frequency.

All the boxes are of same size for STFT, i.e. the time and frequency resolutions are constant all over the time-frequency plane. For wavelet transform, at low frequencies (high scales), the height of the boxes are shorter (which corresponds to better frequency resolution, since there is less ambiguity regarding the value of the exact frequency), but their widths are longer (which correspond to poor time resolution, since there is more ambiguity regarding the value of the exact time). At higher frequencies (low scales), the width of the boxes decreases, i.e. the time resolution gets better, and height of the boxes increases, i.e. the frequency resolution gets poorer.[107]
4.3 WAVELET TRANSFORM (WT)

4.3.1 Continuous Wavelet Transform (CWT)
To analyze the signal in different sizes, it is must to have time frequency components with different time sets. Wavelet transformation is a linear operation that decomposes a signal into components that
appear at different scales [4]. Let \( \psi(t) \) be a real or complex valued function in \( L^2(R) \). The function is said to be wavelet if and only if its Fourier transform (FT) \( \hat{\psi}(\omega) \) satisfies

\[
\int_{-\infty}^{+\infty} |\hat{\psi}(\omega)/(\omega)| = c < \infty \quad (4.5)
\]

This admissibility condition implies that \( \int_{-\infty}^{+\infty} \psi(t) = 0 \) which means the \( \psi(t) \) is oscillatory and its area is zero.

The wavelet transform of a function \( y(t) \in L^2(R) \) at dilation \( a \) and translation \( b \) is

\[
w(a,b) = \int_{-\infty}^{+\infty} y(t) \frac{1}{\sqrt{|a|}} \psi \ast \left( \frac{t-b}{a} \right) dt \quad (4.6)
\]

\( w(a,b) \) - Wavelet transform of signal \( y(t) \), \( \psi(t) \) - Basic wavelet function (mother), \( \psi \ast \left( \frac{t-b}{a} \right) \) - Basic wavelet function, \( a \) - Dilation parameter, \( b \) - Translation parameter,* Indicates complex conjugate, \( \frac{1}{\sqrt{|a|}} \) - Keeps energy constant for all values of \( a \). If \( a \) is greater than one the wavelet function \( \psi(t) \) is stretched along the time axis. For \( 'a' \) less than one, the function \( \psi(t) \) contracts, and if \( 'a' \) is negative, \( \psi(t) \) flips in the time axis. Where the factor \( \frac{1}{\sqrt{|a|}} \) serves the purpose of energy normalization of the wavelet across various scales, that is, assuring that all scaled functions have equal energy. Thus, the wavelet is contracted for \( a < 1 \), whereas it is expanded for \( a > 1 \).[110],[111].

The contraction of a wavelet to a smaller scale makes it more localized in time, while the corresponding frequency response is shifted to higher frequencies and the bandwidth is increased to become less localized in frequency; the reverse behavior is obtained when the wavelet is expanded in time. High scale gives a gross or global picture of the signal, whereas low scale corresponds to a detailed view. Similarly, in terms of frequency, low frequencies correspond to global information of a signal that usually spans the entire signal, whereas high frequencies correspond to detailed information of a transient pattern in the signal having relatively short duration. That is why scaling
conveys a notion of something reciprocal to the frequency. Scaling, as a mathematical operation, either dilates or compresses a signal [111-115].

4.3.2 The Dyadic Wavelet Transform (DyWT)

The CWT is a two-dimensional function $w(a,b)$ which is highly redundant. That is, CWT assigns a value to the continuum of points on the translation-scale plane taking long computing time. Therefore, it is necessary to discretize the scaling and translation parameters $s$ and $\tau$ according to a suitably chosen sampling grid. Dyadic Wavelet Transform (DyWT) is based on sampling the translation-scale plane by using dyadic sampling of the two parameters

$$a = 2^j, \quad b = k2^{-j}$$

Where $j$ and $k$ are both integers. Accordingly, the discretized wavelet function is defined by

$$\psi_{j,k}(t) = 2^{j/2}\psi(2^j t - k) \quad (4.7)$$

Inserting 4.7 into the CWT in 4.6, the dyadic wavelet transform (DyWT) is obtained

$$DyWT^\psi_x (j,k) = w_{j,k} = \int_{-\infty}^{\infty} x(t) \psi_{j,k}(t) dt = 2^{j/2} \int_{-\infty}^{\infty} x(t) \psi(2^j t - k) dt \quad (4.8)$$

It can be shown that with dyadic sampling it is still possible to exactly reconstruct $x(t)$ from the coefficients $w_{j,k}$ resulting from discretization of the CWT; a coarser sampling grid cannot reconstruct $x(t)$ [12]. From Nyquist’s rule, we know that at higher scale (i.e. lower frequencies) the sampling rate
can be reduced. In other words, if the translation scale plane needs to be sampled with a sampling rate of \( N_1 \) at scale \( s_1 \), the same plane can be sampled with a sampling rate of \( N_2 \) at scale \( s_2 \), where \( s_1 < s_2 \) (corresponding to frequencies \( f_1 > f_2 \)) and \( N_1 < N_2 \) [112]. The actual relationship between \( N_1 \) and \( N_2 \) is, \( N_2 = (S_1/S_2) N_1 \) or \( N_2 = (f_2/f_1)N_1 \).

Therefore, at lower frequencies, the sampling rate can be reduced, saving a considerable amount of computation time. The original signal is retrieved by the inverse DyWT, or the wavelet series expansion

\[
x(t) = \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} w_{j,k} \psi_{j,k}(t) = \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} 2^{j/2} w_{j,k} \psi(2^j t - k) \quad (4.9)
\]

where \( \psi_{j,k}(t) \) is a set of ortho normal basis functions. The wavelet series expansion is more flexible since it is the sum over two indices which are related to scaling and translation of the basic functions \( w_{j,k}(t) \). Although DyWT has got computational efficiency over CWT, still it provides a high degree of redundancy as far as data reconstruction is concerned. This redundancy, on the other hand consumes a significant amount of computational resources. This is the reason to construct digital filters for wavelet implementation. i.e. the Discrete Wavelet Transform (DWT), for discrete time signals, which is amazingly faster in operation.

### 4.3.3 Discrete Wavelet Transform (DWT)

Since the transform is calculated on a computer and the ECG signals are represented as a sequence of samples, the continuous wavelet transform as described above cannot be applied to this problem. The
first problem is how to implement the transform on a computer. The discretization of the signal will be dealt with later in this section and for now it is assumed that the signals being transformed are continuous. What is needed to solve this problem is the discrete wavelet transform (DWT), which uses discrete values of the modification parameters. “A natural way to sample the parameters $a$ and $b$ is to use a logarithmic discretization of the $a$ scale and link this, in turn, to the size of steps taken between $b$ locations” [23].

The discrete form of the wavelet is shown in equation 4.10, where “$m$” controls the dilation and “$n$” controls the translation. In addition, $a_0$ is a fixed dilation step greater than 1 and $b_0$ is the location parameter, which must be greater than zero.

\[
\psi_{m,n}(t) = \frac{1}{\sqrt{|a|^{m}}} \psi\left(\frac{t-nb_0a_0}{a_0^m}\right) \quad (4.10)
\]

From the discrete wavelet in equation 4.10 it is now possible to formulate the DWT of a continuous signal as given in equation 4.11. The values that the transform returns are known as wavelet or detail coefficients.

\[
\psi_{m,n} = \int_{-\infty}^{+\infty} \frac{x(t)}{a_0^{-m/2}} \psi\left(a_0^{-m}t - nb_0\right) dt \quad (4.11)
\]

Two popular choices for the discrete wavelet parameters $a_0$ and $b_0$ are 2 and 1 respectively, a configuration that is known as the dyadic grid arrangement. Discrete dyadic wavelets are usually selected to be orthonormal, which means that the wavelets are both orthogonal and normalized to have unit energy. “This means that the information stored in a wavelet coefficient $\delta_{mn}$ is not repeated elsewhere and allows for the complete regeneration of the original signal without redundancy” [23]. The fact that the signal can be fully recovered is also referred to as the perfect reconstruction property. In order for the DWT to be implemented on a computer it is also necessary
to sample the continuous signal. For this purpose orthonormal dyadic discrete wavelets are associated with scaling functions. The scaling function is convolved with the continuous signal, as can be seen in equation 4.20, in order to produce approximation coefficients, which are weighted averages of the signal factored by $2^{m/2}$.

$$S_{m,n} = \int_{-\infty}^{\infty} x(t) \psi_{m,n}(t) dt$$ (4.20)

Through the approximation coefficients a discrete approximation of the signal at scale $m$ is obtained.

### 4.4 Decomposition of wavelet

#### 4.4.1 Implementation of The DWT Using Filter Banks

Multi-resolution analysis is the efficient calculation of the scaling and wavelet coefficients. This can be done with a set of recursive equations whose implementation involves well-known, basic signal processing operations (i.e., filtering and down- or up sampling)[116].
For Discrete time signals, the dyadic discrete wavelet transform (DyDWT) is equivalent, according to Mallet’s algorithm. This can be implemented by filter banks either using Mallat’s algorithm [20] or àtrous algorithm [117,118]. In Mallat algorithm, the signal should be down sampled after each filter to remove the signal redundancy of signal representation. It is time variant and reduces temporal resolution of the wavelet coefficients for increasing the scales.

In àtrous algorithm, it maintains time variant and temporal resolution at different constant scales, and it is uses same sampling rate in all scales. In filter bank techniques, the signal is passed through the filter, and is separated into two components low pass and high pass. The low pass component is the scaling or smoothing function (i.e. approximation) $S_{n,k}$, and the high pass component is the detail signal $D_{n,k}$. The detail level signals are used for multi resolution analysis [20].
which is used to decompose the ECG signal is shown in Figure 4.2. This approach is called as differentiator filter bank approach.

The down samplers after each filter in Figure 4.2a remove the redundancy of the signal representation. As side effects, they make the signal representation time-variant, and reduce the temporal resolution of the wavelet coefficients for increasing scales. To keep the time-invariance and the temporal resolution at different scales, we use the same sampling rate in all scales, what is achieved by removing the decimation stages and interpolating the filter impulse responses of the previous scale. This algorithm, called à trous algorithm [117], is shown in Figure 4.2b. Using this algorithm, the equivalent frequency response for the $k$th scale is

$$Q_k (e^{j\omega}) = G(e^{j\omega}), \quad k = 1$$

$$Q_k (e^{j\omega}), \prod_{i=0}^{k-2} H(e^{j2^{i}\omega}) \quad k \geq 2$$

(4.21)

4.5 Choice of wavelet

The Wavelet we used for our analysis is quadratic spline. It is the first derivative of Gaussian smoothing function. The wavelet and smoothing function are shown in Figure 4.3. This wavelet has been used by many researchers already [27,30, 38]. The wavelet is defined by

$$\psi(\omega) = j(\omega) \frac{\sin (\omega/A)}{\omega/4}$$

(4.22)

This wavelet has a center frequency of 120 Hz and bandwidth of 240Hz. The QRS complex is 6-30 Hz, noise and motion artifacts is within 0-5 Hz. The ECG signal is passed through the differentiator filter bank shown in Figure 4.3 for the first four scales only, because QRS complex have maximum power in the fourth level [34].
Figure 4.4  a) Smoothing function and b) Cubic Spline Wavelet

Figure 4.5 Equivalent filter response

For this prototype wavelet, the filters $H(z)$ and $G(z)$ to implement DWT is shown in Figure 4.2 [119]

$$H(e^{j\omega}) = e^{j\omega/2} (\cos \omega/2)^3$$
\[ G(e^{j\omega}) = 4j e^{j\omega} (\sin \omega / 2)^1 \]  

(4.23)

Which are FIR filters with impulse response

\[ h[n] = \frac{1}{8} [\delta[n + 2] + 3\delta[n + 1] = 3\delta[n] + \delta[n - 1]] \]

\[ g[n] = 2\{\delta[n + 1] - \delta[n]\} \]  

(4.24)

Using atrou algorithm, equation (4.20) and (4.23) the frequency response of the equivalent filter for the wavelet transform at different scales is drawn in Figure 4.5 with a sampling frequency of 250 Hz. and their 3-dB band width is shown in Table 4.1, as the analysis filters have linear phase, the outputs of filters can be realigned in order to present the same delay with respect the original ECG. It is a differentiator filter-bank approach with the filter responses for the equation 4.21.

<table>
<thead>
<tr>
<th>Scale</th>
<th>3db bandwidth</th>
</tr>
</thead>
<tbody>
<tr>
<td>S=2^1</td>
<td>62.5~ 125Hz</td>
</tr>
<tr>
<td>S=2^2</td>
<td>18~58.5 Hz</td>
</tr>
<tr>
<td>S=2^3</td>
<td>8~27 Hz</td>
</tr>
<tr>
<td>S=2^4</td>
<td>4~13 Hz</td>
</tr>
<tr>
<td>S=2^5</td>
<td>2~6.5 Hz</td>
</tr>
</tbody>
</table>

**TABLE 4.1**

3-DB BANDWIDTH OF FILTER AT DIFFERENT SCALES

### 4.6 Algorithm

#### 4.6.1 Wavelet Maxima as multi scale edges

The zero-crossings and local extrema of any WT, as shown by Mallat[120], are shift-invariant. This means that a translation in a signal will cause the zero-crossings and local extrema in the WT of that signal to be translated by the same amount.
Consider a wavelet constructed in such a way that it is the $p$-th order derivative of a smoothing function. In particular, if the wavelet is the first order derivative of a smoothing function, the local extrema of the WT by such a wavelet characterize sharp variations in a signal at multiple scales. Equivalently, if the wavelet is the second order derivative of the smoothing function, the zero-crossings of the WT convey the same information. Such observations can be made with mathematical rigor. Let $\psi_1(x)$ and $\psi_2(x)$ be, respectively, the first and second order derivatives of $\phi(x)$ i.e.

$$\psi_1(x) = \frac{d\phi(x)}{dx}$$ (4.25)

$$\psi_2(x) = \frac{d^2\phi(x)}{dx^2}$$ (4.26)

where $\phi(x)$ is smoothing function. The dyadic WT's of a function $f(x)$ with these two wavelets are given as

$$W_{2^j}^1 f(x) = f \ast \left(2^j \frac{d\phi_{2^j}}{dx}\right)(x)$$ (4.28)

$$W_{2^j}^2 f(x) = f \ast \left(2^{2j} \frac{d^2\phi_{2^j}}{dx^2}\right)(x)$$ (4.29)

Because differentiation and convolution commute, (5.3) and (5.4) can be rewritten as,

$$W_{2^j}^1 f(x) = 2^j \frac{d}{dx} \left(f \ast \phi_{2^j}\right)(x) = \left(2^j \frac{df}{dx}\right) \ast \phi_{2^j}$$ (4.30)
and

\[ W^2_{2^j}f(x) = 2^j \frac{d^2}{dx^2} (f \ast \phi_{2^j}) = (2^j \frac{d^2f}{dx^2}) \ast \phi_{2^j}(x) \]  \hspace{1cm} (4.31)

(4.30) and (4.31) can be explained as differentiation of the signal followed by smoothing at multiple scales. This is the design philosophy of many edge detectors in computational vision.

Mallat and Zhong [20,104] further pointed out that some local minima and zero-crossings of the WTs given by (4.30) and (4.31) do not correspond to sharp variation points in the signal; they are inflection points of \((f \ast \phi_{2^j})\). This situation is illustrated by Figure 4.6 with simulated signal. On the other hand, the local maxima of \(W^1_{2^j}f(x)\) are sharp variation points in the signal observed at multiple scales. The local maxima of \(W^1_{2^j}f(x)\) are called modulus maxima of the WT and interpreted as multiscale edges [104]. For simplicity, we shall call them "wavelet maxima". We also use the term "multiscale edges" interchangeably for "wavelet maxima".
4.6.1.1 Wavelet Modulus Maxima (WTMM)

To detect the extrema and inflection points in the waveform, first derivative of the square modulus of the signal has to be considered, by approximating the derivative of square modulus of DWT, it can be established by connection between local extrema of $|\psi(2^j, b_1)|$ and inflection points and local extrema of signal being analyzed. Maxima and minima points can be found by partly overlaying each other. To detect the extremal points the following method is used. It is based on local maxima of DWT modulus. A local maximum of the DWT modulus located at time $b_2$ at scale $2^j$ is defined as

$$|\psi(2^j, b_1)| < |\psi(2^j, b_2)| < |\psi(2^j, b_3)|$$  \hspace{1cm} (4.25)
Where $b_1 < b_2 < b_3$, $b_1 \rightarrow b_2$ and $b_2 \rightarrow b_3$ (are adjoining samples). If inequality in the equation is applied on all $2^j$ s, adjoining ($2^j$, $b_2$) pairs compose separate lines called maximum curves. The maximum curve coincides with minor extrema function.

To detect local minimum of DWT modulus located at time $b_2$ at scale $2^j$ is defined similar to the above equation with reversing the inequalities. If the inequality equation is applied on all $2^j$ s adjoining ($2^j$, $b_2$) pairs compose separated lines called minimum curves [120].

### 4.6.2 QRS Algorithm
The sample simulated ECG signal with noise and artifacts are drawn in the figure 4.7. As exemplified by (a), monophasic waves produce a positive maximum-negative minimum pair along the scales, with a zero crossing between them. Each sharp change in the signal is associated to a line of maxima or minima across the scales. In wave (b), which simulates a QRS complex, it can be observed that the small Q and S wave peaks have zero crossings associated in the WT, mainly at scales $2^1$ and $2^2$. P or T-like waves (c) have their major component at scales $2^4$ to $2^5$, whereas artifacts like (d) produce isolated maximum or minimum lines which can be easily discarded. If the signal is contaminated with high-frequency noise (e), the most affected scales are $2^1$ and $2^2$, being higher scales essentially immune to this sort of noise. Baseline wander (f) affects only at scales higher than $2^4$.

Using the information of local maxima, minima and zero crossings at different scales, the algorithm identifies the significant points in the following steps: 1) detection of QRS complexes; 2) detection and identification of the QRS individual waves (Q, R, S, R’).
4.6.2.1 QRS Detection Algorithm

The algorithm departs from the position given by the detector, \( n_{qrs} \), which must be flanked by a pair of maximum moduli with opposite signs at scale \( 2^2 \), namely at \( n_{\text{pre}} \) and \( n_{\text{post}} \). The delineator looks before \( n_{\text{pre}} \) and after \( n_{\text{post}} \) for significant maxima of \( |W_{2^2} x(n)| \) accounting for other adjacent slopes within the QRS complex. To consider a local maximum modulus as significant, it must exceed the threshold, \( \gamma_{QRS1} \) or \( \gamma_{QRS2} \) respectively for previous or subsequent waves. The zero crossings between these significant slopes at scale \( 2^1 \) are assigned to wave peaks, and labeled depending on the sign and the sequence of the maximum moduli. The algorithm considers any possible QRS morphology with three or less waves (QRS, RSR', QR, RS, R, and QS complexes), and includes protection measures, based on time interval and sign rules, to reject notches in waves and anomalous deflections in the ECG signal.

From MIT – BIH arrhythmia data base, ECG signals are taken to analyze the developed algorithm. A sample ECG signal is shown in the Figure 4.8. This analysis was conducted with different set ECG episodes from 1 sec to 5 sec with a difference of 0.5 sec. For each length the dataset was randomly divided and tested. we have used quadratic Spline Wavelet with centre frequency 250 Hz, the ECG data is resampled at the rate of 250 Hz. First we have chosen 1024 samples as first window. The wavelet transform has been computed for four levels \( 2^1, 2^2, 2^3 \) and \( 2^4 \) for ECG signal. Then wavelet modulus maxima points are identified by fixing 60% of the maximum value. The QRS signal is having maximum energy in level \( 2^4 \) (0.1 – 30 Hz) [121]. After eliminating all redundant maximum points, the zero crossing of wavelet transform at level \( 2^1 \) between the positive maximum and negative minimum pair is marked as a QRS [27],[38].
Wavelet modulus maximum from the QRS is marked as R peak of entire window. From that each R peaks are identified by local maximum principle. From that R peak, 0.06% of its maximum value is searched towards left and the first point is marked as ‘Q’ point in the search window of level 2. To fix ‘S’ point, 0.09% of the R peak is searched towards right and the last point in the search window is marked as ‘S’ point. QRS width is measured and stored as reference for the rest of the signal. Then second window is selected for identifying next R peak, Q and S points by using the same method of wavelet modulus maxima. The distance between first and second R wave is referred to as R- R interval. It is stored as reference point for R-R interval. The normal value of R-R interval is 0.8 sec to 1 sec and QRS width is 0.05 sec to 0.10 sec. If the stored R-R interval and QRS width exceed the normal value, it will omit that particular peak and search for the next. In case of any baseline
wandering, the wavelet will remove the wandering signals and give actual peaks. If any high P wave is identified, by comparing with reference points it will be eliminated. If R-R interval is different from the normal value, it indicates a cardiac abnormality and can be diagnosed easily. Figure 4.9 shows the decomposition of ECG signal in different scales levels. Figure 4.10 shows the R peak of the window from R peak; with a predefined threshold values Q and S points are identified. The threshold point will be varied according to the signal variation. This threshold point variation is obtained from QRS point through adaptive threshold variation technique [122]. Algorithm is tested in Matlab version 7.0 [123]

Figure 4.9 Decomposition of ECG signal using Atrou algorithm
4.6.3 P Wave Detection Algorithm

The process for multi-scale P wave detection and delineation is as follows: first of all, we define a search window for each beat, relative to the QRS position and depending on a recursively computed RR interval. Within this window, we look for local maxima of $|W_2^n|$. If at least two of them exceed the threshold, a P wave is considered to be present. In this case, the local maxima of WT with amplitude greater than $\gamma P$ are considered as significant slopes of the wave, and the zero crossings between them as the wave peaks. Depending on the number and polarity of the found maxima, if the P wave is not found in scale $2^4$, we repeat the above process over $|W_2^5|$. Attending to the loss of time resolution in the growing scales, the peak(s) of the P wave correspond to the zero crossing(s) at scale $2^3$, if they exist, or at the scale $2^4$ in which p wave was found. To identify the wave limits, we used the same criteria as for QRS onset and end, with thresholds $P_{on}$ and $P_{end}$ applied to scale $2^4$. 

Figure 4.10  R wave detection
The distance between first and second R wave is referred to as R-R interval. It is stored as reference point for R-R interval. The normal value of R—R interval is 120ms - 200 ms and QRS width is 0.2 sec - 0.4 sec [70]. The P wave is searched by going back 200 msec (maximum) from the first QRS in level 2. By applying small threshold and wavelet modulus maxima, P maximum is identified. Then we have selected a small window centered at this P wave with a width of 130 msec. To find the P on and P end, points are calculated by fixing a small threshold before P max and after P max first and last points of the window. The Width of P wave, P Q interval and R – R interval are noted as reference points. To identify the next P wave, with known R - R interval searching is done by the same procedure. The searching algorithm values vary depending on the variations of R-R and PQ interval values. If the stored R-R interval & QRS width exceed the normal value, it will omit that particular peak and search for the next. In case of any baseline wandering the wavelet will remove the wandering signals and give actual peaks. The P wave detection is shown in the Figure 4.11.[124].

Figure 4.11  P wave detection
4.6.4 T Wave Detection

The process for multi scale T wave detection and delineation is as follows: first of all, we define a search window for each beat, relative to the QRS position and depending on a recursively computed RR interval. Within this window, we look for local maxima of \(|W^2_4x[n]|\). If at least two of them exceed the threshold, a T wave is considered to be present. In this case, the local maxima of WT with amplitude greater than \(\gamma_T\) are considered as significant slopes of the wave, and the zero crossings between them as the wave peaks. Depending on the number and polarity of the found maxima, if the T wave is not found in scale 2^4, we repeat the above process over \(|W^2_{5}x[n]|\). Attending to the loss of time resolution in the growing scales, the peak(s) of the T wave correspond to the zero crossing(s) at scale 2^3, if they exist, or at the scale 2^k in which T wave was found. To identify the wave limits the same criteria is used for QRS onset and end, with thresholds Ton and Tend applied to scale 2^k.

The distance between first and second R wave is referred to as R–R interval. It is stored as reference point for R-R interval. The normal value of R—R interval is 120ms - 200 ms and QRS width is 0.2 sec - 0.4 sec [125]. The T wave is searched from the first QRS in level 2^4 until 180 msec (maximum). By applying small threshold and wavelet modulus maxima, T maximum is identified. Then we have selected a small window centered at this T wave with a width of 100 msec. To find the “T on” and “T end”, points are calculated by fixing a small threshold before T max and after T max first and last points respectively of the window. The Width of T wave, P Q interval and R – R interval are noted as reference points. To identify the next T wave, with known R - R interval searching is done by the same procedure. The searching algorithm values vary, according to the
variations of R-R and T interval values.

![T wave Detection](image)

Figure 4.12 T wave detection

If the stored R-R interval & QRS width exceed the normal value, it will omit that particular peak and search for the next. The detected T wave is shown in the Figure 4.12.

### 4.7 Summary

ECG signal is decomposed into different levels using wavelet transform, the QRS, P and T points are identified. The energy distribution is calculated in different levels and from that, ventricular arrhythmias are classified in next chapter.