Chapter - I

INTRODUCTION
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1.1. FLUID DYNAMICS

Fluid Mechanics is the study of fluids either in motion or at rest and the subsequent effects of the fluid upon the boundaries which may be either solid surfaces or interfaces with other fluids. Fluid Mechanics is an important branch of applied mathematics. This branch of science deals with the static, kinematic and dynamic aspects of fluids. The fluid is considered to be static if all particles of the fluid are at rest with respect to a reference coordinate system. The study of fluids in motion, where pressure forces are not considered, is called fluid kinematics and if the pressure forces are also considered for the fluids in motion that branch of science is called fluid dynamics.

Many applications of fluid mechanics make it one of the most vital and fundamental engineering and applied science studies. The flow of fluids in pipes and channels makes fluid mechanics such as pumps, compressors, heat exchanges, jet and rocket engines, and the like, makes fluid mechanics important to mechanical engineers.

For example, fluid mechanics and electromagnetic theory together are known as magneto hydrodynamics. New types of energy conversion devices and in the study of stellar and ionospheric phenomena magneto hydrodynamics are vital.

Dynamics, the study of motion of matter, may be divided into two parts – dynamics of rigid bodies and dynamics of non-rigid bodies. The latter is usually further divided into two general classifications which are elasticity (Solid elastic bodies) and fluid mechanics. In simple terms fluid is a substance, which cannot resist a shear force or stress without moving as a solid.

Fluid Dynamics is an important and rich branch of science and any living being in the universe cannot exist without fluid. The development of the fluid dynamics started in a quite early age of civilization. Archimedes formulated the basic principles of fluid dynamics which are true till date. Some applications of fluid dynamics are flowing in pipes and channels, irrigation and in the design of water
supply system, dam spillways, ships, submarines, air-crafts, rockets, wind mills, turbines, missiles, ice bergs, engines, filters, jets etc.,

1.2. CLASSIFICATION OF FLUIDS

Fluid can exist in three states (i) solid (ii) liquid (iii) gaseous. But a large number of solids are not fluids as they cannot flow. Some solids of non-crystalline structure such as plastics and glassy substances can be made to flow when subjected to high pressure for a long time. They can be considered fluids as relative motion occurs between molecules under high stress. Such substances when heated do not melt suddenly at a fixed temperature, but gradually become softer and more pliable. Though fluid can exist in any of three states, the important distinction between a fluid and a solid is a fluid begins to flow even if it is imperceptible due to the action of the small net shear force and it continues to flow as long as the shear forces are applied.

A liquid has a definite volume and it takes the shape of the vessel containing it. It would occupy the vessel fully or partially depending on its content and it can have a free surface. But a gas has no definite shape and it would expand and occupy the vessel fully and it cannot have a free surface. The volume of a liquid varies very slightly due to change in temperature and pressure. This variation is so small that for all practical purposes, it is negligible and hence a liquid is considered as incompressible. But a gas undergoes considerable change in volume to change in its temperature and pressure. Hence gases are compressible fluids.

The liquid and gaseous states make up the fluid state behavior which can be divided into three types i) Statics; ii) Kinematics; iii) Dynamics. In the first case, the fluid elements are, at least with respect to one another and thus, free of shearing stresses. The Kinematics of fluids deals with a study of the translation, rotation and rate of deformation of fluid particle.

The real fluids can be divided into several categories based on their rheological behavior. If shearing stress is linearly proportional to the rate of strain, the fluid is named a Newtonian fluid. If the relationship between the shearing stress and the rate of strain is non-linear, the fluid is called a non-Newtonian fluid. Newtonian behavior has been observed in all gases and in liquids or solutions of materials of low molecular weight. It is speculated that fluids which are normally considered to be
Newtonian may show some non-Newtonian behavior. The study of non-Newtonian fluid dynamics is important in connection with manufacturing of plastic, performance of lubricants, application of paints, processing of food, and movement of biological fluids. Most of the biologically important fluids contain higher molecular weight components and are, therefore, non-Newtonian. The unusual properties of polymer melts and solutions, together with the desirable attributes of many polymeric solids, have given rise to the world-wide industry of polymer processing.

In recent years, considerable efforts have been usefully devoted to the study of flow non-Newtonian fluids because of their practical and fundamental importance associated with many technological applications.

Non-Newtonian materials are commonly divided into three groups as given below

- **Time-independent Fluids**
- **Time-dependent Fluids**
- **Visco-elastic Fluids**

**Time-independent Non-Newtonian Fluids** are those for which fluids the rate of strain at a given point mainly depends on the instantaneous shear stress at that point. These materials are sometimes referred to as “Non-Newtonian Viscous fluids” or “purely viscous fluids”. A convenient classification can be made based on the yield stress of these fluids.

**Time-dependent non-Newtonian Fluids**

Time-dependent fluids are those for which the shear rate is a function of both the magnitude and the duration of shear and possibly of the time lapse between consecutive applications of shear stress. These fluids are classified into Thixotropic fluids and Rheopectic fluids.

**Visco-elastic Fluids**

These materials exhibit both elastic properties and fluid properties. Examples of visco-elastic fluids are soap solutions, Napalm and similar Jellies, polymers and polymer melts such as nylon, and many polymer solutions. The visco-elastic fluids exhibit normal stress effects. When a steady visco-elastic fluid flows through a circular tube, the normal stresses in three mutually perpendicular directions are
unequal. But when a purely viscous fluid flows through the same tube, the normal stresses in three directions are same and equal to pressure. Further, the visco-elastic fluids exhibit Weissenberg effect. Several mathematical models have been proposed for visco-elastic fluids. These models are made from more or less complex combinations of a series of springs and dashpots.

Some types of Fluids

Flows can be studied in three types

Steady and Unsteady Flows

A flow in which the viscous like velocity, pressure and density at any point do not change with time is said to be a steady flow. For steady flow if \( u \) is the velocity at a point then

\[
\frac{\partial u}{\partial t} = 0
\]  

(1.2.1)

A flow in which this parameter depends on time is called unsteady flow.

Laminar and Turbulent Flow

Laminar flow is a kind of flow in which fluid particles trace out a definite curve and curves traced out by any two various particles do not intersect each other is called Laminar flow.

On the contrary to the laminar in which a piece does not trace out a particular curve and the curves traced by the fluid particles intersect each other is said to be turbulent flow. The irregular fluctuation is superimposed on the main system.

In this laminar flow, the fluids are moving in smooth layers or lamina. There is comparatively less mixing and accordingly the velocity gradients are little and shear stresses are very low. The thickness of the laminar boundary layer increases with distance from the beginning of the boundary layer and decrease with Reynolds number \( Re \).
Compressible and Incompressible Flows

The gases are compressible and their density changes with temperature and pressure. On the other hand, liquids are rather difficult to compress and for all practical purposes these may be considered incompressible fluids.

1.3 FLUID PROPERTIES

Pressure, Density and Viscosity

When a fluid is contained in a vessel, it exerts a force at each point of the inner surface of the vessel. Such a force per unit area is known as pressure. Mathematically, the pressure at a point in the fluid can be defined as

\[ p = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} \]  

(1.3.1)

The density represents the mass of fluid per unit volume. Mathematically, the density \( \rho \) at a point is given by

\[ \rho = \lim_{\Delta \tau \to 0} \frac{\Delta m}{\Delta \tau} \]  

(1.3.2)

The specific weight is closely related to the density. It represents the weight of the fluid per unit volume. The specific weight \( \gamma = \rho g \).

The specific volume of a fluid is the fluid occupied by a unit mass and is clearly the reciprocal of the density. If a fluid requires a large variation in pressure to produce some appreciable variation in the density refers to incompressible fluid and the remaining fluids refer to compressible.

The viscosity of a fluid is a very important property in the analysis of liquid behavior and the motion near a solid boundary. It is the result of the intermolecular forces exerted as layers of fluid attempt to slide by one another. The relation between the shearing stress and the transverse velocity gradient is given by \( \tau_{yx} = \mu \frac{\partial u}{\partial y} \). The constant of proportionality \( \mu \) in this equation is the coefficient of viscosity. Due to a shearing stress, a viscous fluid produces resistance to the body moving through it as well as between the particles of the fluid itself. Water and air are treated inviscid whereas syrup and heavy oil are treated as viscous fluids.
Temperature

Suppose two bodies of different heat content are brought into contact while isolated from all other bodies, there is flow of heat until the bodies reach thermal equilibrium. In other words, when two bodies are thermal, all changes in properties of the bodies cease. Under this condition they are said to have a property in common. This property is called temperature. Scales of temperature are referred in terms of certain measurable physical properties such as the volumetric expansion of mercury. The magnitude of temperature can be related to the molecular activity resulting from heat transfer.

Thermal Conductivity

The well known Fourier's heat conduction law states that the conductive heat flow per unit area $q_n$ is proportional to temperature decrease per unit distance in a direction to the area through which the heat is flowing. Mathematically the Fourier's heat conduction law is

$$ q_n = -k \frac{\partial T}{\partial n}. $$

Specific Heat

The inherent thermal properties of a flowing gas become important when the energies are considered. The Specific Heat is such a quantity. The Specific Heat of the fluid is the amount of a unit mass of the fluid by one degree. The value of specific heat depends on two well known processes the constant volume process and the constant pressure process. The specific heats of above processes are designated by $C_v$ and $C_p$ respectively. Thus $C_v = \frac{\partial E}{\partial T}$ and $C_p = \frac{\partial H}{\partial T}$. The ratio of specific heats are given by $r = \frac{C_v}{C_p}$.

1.4. HEAT TRANSFER

Heat transfer plays a vital role in science and technology. It is the science which predicts temperature distributions, which may be the functions of both spatial coordinates and time, within the regions of matter. Heat transfer also predicts the rate at which energy is transferred across a surface of interest due to temperature gradients.
at the surface and temperature differences between different surfaces. There are many engineering applications that are encountered in a variety of disciplines.

Heat Flux

The heat transfer per unit area is called Heat Flux. If $q$ is the amount of heat transfer and $A$ is area normal to direction of the heat flow then, Heat Flux $Q = \frac{q}{A}$.

Heat may be produced as a by-product of a processing that heat in, if not removed, it causes damage to the equipment. Heat must be removed from an automobile engine, for example, a liquid coolant will cool the engine in the first step of heat removal. Then, air will cool the liquid coolant. Thus, the engineer plans for removal of the heat from where it is not wanted. Heat must be provided to residential and commercial buildings to maintain a comfortable environment. For economy, it is desirable to retard the rate at which heat is lost from buildings. Thus, the engineer is concerned with preventing loss of heat from the area where the heat should be kept. Super conducting magnets work properly at very low temperatures. In some cases, these magnets are located very near to high temperature, as in proposed designs for nuclear fusion plants. So, the engineer must prevent heat from entering the superconducting magnet at a faster rate. Hence, it is necessary that heat must be kept away from where it is not wanted.

Major segments of the chemical and metallurgical industries use components such as furnace, heat exchangers, condensers and vectors, where thermo fluid processes are at work. Aircraft and rockets owe their functioning to the fluid flow, heat transfer and chemical reaction. In the design of electrical machinery and electronic circuits, heat transfer is often the limiting factor. In the course of time, temperature, differences in a body are reduced by heat flowing from regions of higher temperature to those of lower temperature. Knowledge of the laws governing this process is of great importance because this knowledge affords means for channeling the flow of heat in a desired manner.

According to thermodynamic theory, such a heat engine consists in principle, of two heat reservoirs at different temperatures, with the engine that performs the work placed between them. The working medium frequently changes in the course of such process. The heat must then be exchanged between the individual media using
the smallest possible temperature drop. In steam power plants the heat is contained initially as internal energy in the combustion gases. In the steam boiler the heat is transferred to the steam. In the condenser the steam gives off its heat to the cooling water, and the cooling water passing through the cooling tower transfers this heat to the air. In internal-combustion engines of this type heat exchange does not exist, since the heat is produced by combustion directly in the working medium. Some of this heat is converted to work, and the waste heat as well the waste gases are jointly exhausted.

The pollution of the natural environment is largely caused by heat and mass transfer and so are storms, floods and fires. In case of changing weather conditions, the human body resorts to heat and mass transfer for its temperature control. This process of heat transfer and fluid flow seems to pervade in all aspects of our life.

All fields of engineering deal with problems of heat transfer and fluid flow, aerospace engineers are concerned with heat transfer in high-speed flow. Chemical engineers provide for input or removal of heat, depending on whether the processes are endothermic or exothermic. Civil engineers and architects must be concerned with heat transfer in the design and construction of buildings. Electrical engineers are concerned with proper operating temperatures of equipment such as computers and electronic devices. Mechanical engineers are concerned with many heat transfer situations, such as the automobile engines. Nuclear engineers are concerned with dynamic heat removal problems in fission reactors in addition to the fusion reactor. Thus, heat transfer is a basic engineering science of general concern to the engineering profession.

There are three basic mechanisms in the processes of heat transfer according to which heat can move from a high-temperature region to a low-temperature region.

CONDUCTION

Heat can move through a static body by interaction with the internal structure of the body. This process is called conduction. Heat transfer by conduction arises from temperature gradients within a material. This is considered the only mode of heat transfer with opaque solids. Suppose, a solid bar is surrounded by perfect insulation except at the left and right faces with temperatures $t_1$ and $t_2$ ($t_1 > t_2$). Experimentally it is found that if the conduct of the bar is not charged with time and if
there are no source of energy within the bar, there is a net energy transfer rate from, the left face to the right face. This is called a conduction heat transfer, and the direction of the net energy transfer is in accordance with the second law of thermodynamics.

The conduction in a solid is due to the energy transferred during collisions of adjacent molecules and the migration of free electrons. Conduction is a gas, due to collisions of the molecules which are in continuous random motion. Under steady state conditions, it is found that the heat transfer rate is directly proportional to the cross sectional area and the temperature difference, and it is inversely proportional to the path length which is the direction of heat flow. This Fourier's law of conduction is used to obtain conduction heat transfer rate. The proportionality constant 'k' is called the thermal conductivity of the material. The thermal conductivity k for homogeneous materials which are Isotropic is a thermodynamic property of the material, and is a function of pressure and temperature. Thermal conductivity is only little independent upon the pressure for solids and for liquids as well, if they are not near the critical point. Also, the thermal most gases are essentially independent of the pressure of the gases which are not pressures near standard atmospheric. The value of thermal conductivity k increases with temperature for gases and non-metallic solids, and decreases with temperature for most liquids except water.

Example of conduction heat transfer is legion the exposed of a metal spoon suddenly immersed in a cup of hot coffee will eventually be warned due to the conduction of energy through the spoon. On a winter day there is significant energy loss from a heated room to the outside air. This loss is principally due to conduction heat transfer through the wall that separates the room air from outside air.

CONVECTION

Heat can be carried from one place to another by movement of a fluid. This process is Called convection. The process of convection is heat transfer between a solid surface and the bounding fluid convection may occur naturally may be stimulated. Free convection is a process in which the fluid motion is due to density gradients within the fluid, usually caused by the temperature differences between the solid surface and adjacent fluid.
Convection is a mechanism in which heat flows are transferred between a fluid and a solid surface because of motion of fluid particles to the solid surface when there exists a temperature gradient.

Convection heat transfer may be classified into Forced convection heat transfer and Free or Natural convection heat transfer.

If the heat transfer between the fluid and the solid surface occurs by fluid motion induced by external agencies or forces then the mode of heat transfer is termed as "Forced convection" in all types of heat. Exchanges, nuclear reactors, air conditioning apparatus are by forced convection.

A free convection flow field is a self-sustained flow driven by the presence of temperature gradients. (As opposed to a forced convection flow where external means are used to provide the flow). As the result of the temperature difference, the density field is not uniform. Buoyancy will induce a flow of current due to the gravitational field and the variation in the density field. In general, a free convection heat transfer is usually much smaller compared to a forced convection heat transfer.

Free convection results from the buoyancy forces on the fluid, whether it is gas or liquid, when its density in the neighborhood of the heat transfer surface is reduced as a result of the heating process. Free convection flow also arises when a heated object is placed in a fluid, otherwise at rest, the density of which varies with temperature. Heat is transferred from the surface of the object to the fluid layers in its neighborhood. The density decreases, when a normal fluid is convicted to temperature increase, it causes these layers to rise and create the free convection flow which now transfer away from the object. Physically such flow is described by stating that is caused by bodily force. The example of a practical device which transfers heat by free convection is a radiator used for heating a room.

Free convection flow occurs frequently in Nature. It occurs not only due to the temperature difference, but also due to concentration difference or combination of these two. For example, its atmospheric flows there exist differences in the H2O concentration and hence the flow is affected by such concentration difference. Flows in bodies of water are driven is affected by such concentration difference. Flows in bodies of water are driven through the comparable effects upon the density of
temperature, concentration of dissolved materials and suspended particular matter may transport process exist in nature and in industrial applications in which the simultaneous heat and mass transfer occur as a result of combined buoyancy effects of diffusion of chemical species.

RADIATION

Heat can be transported through space in the absence of any intervening material. This process is called radiation. The process of radiation is a familiar process of heat transfer. When two objects are placed at different temperatures apart from a finite distance in a perfect vacuum, a net energy transfer occurs from the higher temperature object to the lower temperature object, even though there is no medium between the two objects to support heat transfer and this net energy transfer process is called thermal radiation. This is the mechanism whereby the sun transmits heat to the earth. The heat radiated is proportional to the fourth power of absolute temperature, which is known as the Stefan-Boltzmann Law. Thus the radiant transfer increases as temperature increases.

1.5 MASS TRANSFER

Mass transfer is defined as the transfer of matter by virtue of species concentration difference in a system. The difference in concentration provides a driving force for the transfer of mass. Mass transfer always occurs in the direction of redacting concentration gradient.

The involvement and application of mass transfer process go to greater lengths in numerous fields of science, engineering and technology. Mass transfer operations quite often occur in the fields of electrical engineering, civil engineering, aeronautics, metallurgy, environmental engineering, refrigeration, air conditioning, biological and industrial process. The study of geophysics, astronomy, meteorology, agricultural oceanography and food processing, demands the knowledge of heat and mass transfer.

Mass transfer broadly occurs in biological, chemical, physical and engineering fields. It involves in biological functions or process like respiratory mechanisms, oxygenation or purification of blood, kidney functions, osmosis and assimilation of food and drugs. Evaporation of clouds, smoke formation, dispersion of fog, the distribution of temperature and moisture over agricultural fields and groves of fruit
trees, damages of crops due to freezing and pollution of the environment are some of the mass transfer phenomena found in nature.

Mass transfer applications are widely found in chemical engineering processes like distillation, absorption of gases, interaction of solids and liquids from the mixtures and chromatography processes like air humidification, cooling water, ion exchange involves mass transfer.

Mass transfer occurs through two mechanisms

Diffusion mass transfer
Convective mass transfer

**Diffusion Mass Transfer**

In diffusion, mass transfer of matter occurs through movement of molecules or species or particles of one component to another. Diffusion mass transfer may occur either due to concentration gradient or temperature gradient or pressure gradient or pressure diffusion.

**Convective Mass Transfer**

Convective mass transfer is a mechanism in which mass transfer is transferred between the fluid and the solid surface as a result of movement of matter from the fluid to the solid surface or fluid.

Convective mass transfer is again classified into
- Natural or free convective mass transfer
- Forced convection mass transfer

In natural convective mass transfer, the transfer of mass occurs through the motion of species due to the density differences resulting from temperature or concentration differences or mixture of varying composition.

**1.6 MAGNETO HYDRODYNAMICS**

Magnetohydrodynamics (MHD) (magnetofluid dynamics or hydromagnetics) is the study of the magnetic properties of electrically conducting fluids. Examples of such magneto-fluids include plasmas, liquid metals, and salt water or electrolytes. The
The fundamental concept behind MHD is that magnetic fields can induce currents in a moving conductive fluid, which in turn polarizes the fluid and reciprocally changes the magnetic field itself. The set of equations that describe MHD are a combination of the Navier-Stokes equations of fluid dynamics and Maxwell's equations of electromagnetism. These differential equations must be solved simultaneously, either analytically or numerically.

The study of magneto hydrodynamics with heat and mass transfer in the presence of radiation and diffusion has attracted the attention of a large number of scholars due to diverse applications. In astrophysics and geophysics, it is applied to study the stellar and solar structures, radio propagation through the ionosphere etc. In engineering it has vast applications like in MHD pumps, MHD bearings, etc. The phenomenon of mass transfer is also very common in the theory of stellar structure and observable effects are detectable on the solar surface. In free convection flow the study of the effects of magnetic field play a major role in liquid metals, electrolytes and ionized gases. In power engineering, the thermal physics of hydro magnetic problems with mass transfer have enormous applications. Radioactive flows are encountered in many industrial and environmental processes. E.g. heating and cooling chambers, fossil fuel combustion energy processes, evaporation from large open water reservoirs, astrophysical flows, solar power technology and space vehicle re-entry.

The study of magneto hydrodynamics of viscous conducting fluids play a significant role during the recent times, owing to its practical interest and abundant applications in astrophysical and geophysical phenomenon. Astrophysicists and Geophysicists released soon after the advent of special relativity theory that electromagnetic fluid interactions were of great importance in stellar and planetary processes. The main impetus to the engineering approach to electromagnetic fluid interaction studies has come from the concept of the MHD direct conversion generation, propulsion studies of radio propagation in the ionosphere, and controlled nuclear fusion. The study of flow problems of electrically conducting fluid,
particularly of ionized gases is currently receiving considerable interest. Such studies have made for many years in connection with astrophysical and geophysical such as sun spot theory, the motion of the interstellar gas, etc. In recent years, some Engineering problems need the studies of the flow of an electrically conducting fluid.

Greif, Habit and Lin [14] obtained an exact solution for the problem of laminar convective flow in a vertical heated channel in the optically thin limit. The fluid does not absorb its own emitted radiation, which means that there is no self absorption, but the fluid does absorb radiation emitted by the boundaries.

Oscillating Flow

Oscillation is a periodic function between two things. In the broadest sense, oscillation can occur in anything from a person’s decision making process to tides and the pendulum of a clock.

The transient flow of complex fluids in small channels is of fundamental interest for many technical applications like ink jet printing. A rapid switching between flow and no flow of a non-Newtonian fluid is required, and often very small channel geometrics are involved especially the usually low-Reynolds numbers in such flows and the fact, that the dimensions of the channels and macro molecules in the fluid can be on the same order of magnitude, can lead to effects unseen in macroscopic systems. We aim to shed light on bulk phenomena as well as on the behavior of single molecules in constricted oscillatory flows. With oscillating fluid, we can study both frequency dependent effects and long time effects that would require none practically long channels to be observed in steady flow.

1.7 POROUS MEDIUM

The study of the flow of fluids through porous media, has great significance in the present century because of their occurrence in nature and newly emerging fields such as petroleum industry, Polymer industry and its wide applications in physiology. The petroleum industries are showing good interest in the study of the heat and mass transfer of the flows of oil through porous rocks.

The porous media is a continuous solid phase with many void spaces or pores on it. The pores may be effective or ineffective depending on the interconnection of
pores. The porous media with no interconnected pores is impermeable and with interconnected pores are permeable. Clothes, paper, sponges, wicks, and gravel are a few examples of porous media.

To study the seepage of water in river beds, one requires the knowledge of the fluid flow through porous media, which is necessary to tap ground water from the underground reservoirs to overcome the crisis created by the failure of monsoons in the rain fed areas. Flow through porous media is also useful to the paper industry because in the paper making process, felts are used to carry the paper sheet through its many stages of formation and drying.

DARCY LAW

Although Darcy’s law, an expression of conservation of momentum, was determined experimentally by Henry Darcy [10], it has since been derived from the Navies-Stokes equations via homogenization.

One application of Darcy’s law is to water flow through an aquifer, Darcy’s law along with the equation of conservation of mass are equivalent to the ground water flow equation, one of the basic relationships of hydrogeology. Darcy’s law is also used to describe oil, water and gas flows through petroleum reservoirs.

In 1856 Darcy made experiments with natural sand. He found that the total volume of running water through the sand is proportional to the loss of pressure. Darcy described a binary model consisting of a rigid porous solid and a liquid in motion and formulated the following law, the governing of the fluid flow in a porous medium.

This law meaningfully defines the permeability in terms of measurable quantities. If horizontal clear flow of an incompressible fluid is established through a sample of porous material of length in the direction of the flow and cross sectional area then the permeability of the material is defined as

\[ k = \frac{q \mu}{A(\Delta P/L)} \]  \hspace{1cm} (1.7.1)

Here \( q \) is the fluid flow rate in volume per unit time. It is the viscosity of the fluid and \( P \) is the pressure difference across the length of spectrum.
1.8. BASIC EQUATIONS IN VECTOR FORM

The investigation of any liquid motion involves solving a set of non-linear partial differential equations called the Fundamental equations of fluid dynamics. The fundamental equations governing any flow phenomena are stated below:

**Equation of continuity (Conservation of mass)**

Mass can neither be created nor be destroyed.

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{q}) = 0 \]  

(1.8.1)

where, \( \mathbf{q} \) is the velocity vector of the fluid, \( \frac{\partial \rho}{\partial t} \) is the rate of increase of the density in the control volume, \( \nabla \cdot (\rho \mathbf{q}) \) is the rate of mass flux passing out of the control surface per unit volume.

**Equation of momentum (Conservation of momentum)**

According to Newton’s law of motion, the total force acting on a fluid mass enclosed in an arbitrary volume fixed in space is equal to the time rate of change of linear momentum.

\[ \frac{\partial \mathbf{q}}{\partial t} + (\mathbf{q} \cdot \nabla) \mathbf{q} = -\nabla p + \rho \nabla^2 \mathbf{q} - \mu \nabla \times (\mathbf{q} \times \mathbf{B}) + \frac{\partial \mathbf{B}}{\partial t} \mathbf{J} + \nabla \times (\rho \mathbf{q}) \mathbf{J} + \nabla \cdot (\rho \mathbf{q}) \mathbf{J} - \nabla \times (\rho \mathbf{q}) \mathbf{J} - \nabla \cdot (\rho \mathbf{q}) \mathbf{J} 
\]  

(1.8.2)

where, \( \mathbf{B} \) is a magnetic field vector, \( \mathbf{J} \) is the Current density vector, \( \mathbf{B} \times \mathbf{J} \) is the Lorentz force, \( \mathbf{p} \) is the pressure of the liquid, \( \mathbf{q} \) is the velocity vector of the fluid, \( \frac{\partial \mathbf{q}}{\partial t} \) is an unsteady acceleration, \( (\mathbf{q} \cdot \nabla) \mathbf{q} \) is the convective acceleration, \( \frac{\partial \mathbf{q}}{\partial t} + (\mathbf{q} \cdot \nabla) \mathbf{q} \) is the inertia force, \( \nabla \times (\rho \mathbf{q}) \mathbf{J} \) is the viscous force.

This momentum equation is called as the Navier-Stokes equation.
Equation of energy (Conservation of energy)

According to the first law of thermodynamics the energy added to a closed system increases the internal energy per unit mass of the fluid.

\[ \frac{\partial \tilde{T}}{\partial t} + (q \nabla) \tilde{T} = \frac{K}{\rho c_r} \nabla^2 \tilde{T} + \phi + \frac{j^2}{\sigma} \]  

(1.8.3)

where, \( j \) is the Current density, \( \tilde{T} \) is the temperature of the fluid, \( \phi \) is the viscous energy dissipation, \( \frac{j^2}{\sigma} \) is the ohmic dissipation function,

Equation of species diffusion

\[ \frac{\partial \tilde{C}}{\partial t} + (q \nabla) \tilde{C} = D \nabla^2 \tilde{C}, \]

(1.8.4)

where, \( \tilde{C} \) is the Species diffusion, \( q \) is the velocity vector of the fluid, \( D \) is the Species diffusion coefficient

The above four equations are the fundamental equations of the fluid dynamics.

Now, we shall consider the mathematical formulation which forms the basis for the specific problems investigated in the thesis.

Under gravitational body force: (Cartesian - coordinate)

When the body force consists of only gravitational force, then in Cartesian coordinate system, the above governing equations of an unsteady, viscous, incompressible and free convective fluid past a vertical plate are given by

\[ \frac{\partial \nu}{\partial y} = 0 \]  

(1.8.5)

\[ \rho \frac{\partial u}{\partial t} = - \frac{\partial p}{\partial x} - \rho g + \mu \frac{\partial^2 u}{\partial y^2} \]  

(1.8.6)

\[ \rho c_r \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q}{\partial y} \]  

(1.8.7)
Substituting equation (1.12) in equation (1.9), we obtain

\[ \frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial y^2} \]  

(1.8.8)

Here, we note that

\[ - \frac{\partial p}{\partial x} - \rho \alpha g = 0 \]  

(1.8.9)

Substituting equation (1.12) in equation (1.9), we obtain

\[ \rho \frac{\partial u}{\partial t} = -(\rho - \rho_0)g + \mu \frac{\partial^2 u}{\partial y^2} \]  

(1.8.10)

For small temperature and concentration differences the density \( \rho \) in the equation (1.8.7) can be considered constant except for the term \( (\rho - \rho_0) \). Bousinessq first introduced this approximation, since the flow is driven differences due to both temperature and concentration difference expressing the effect of buoyancy force through volumetric coefficients, the density differences can be expressed as

\[ (\rho - \rho_0) = -\rho \left[ \beta'(T - T_0) + \beta''(C - C_0) \right] \]  

(1.8.11)

In view of equation (1.8.8), the equation (1.8.7) can be written as

\[ \frac{\partial u}{\partial t} = g \beta'(T - T_0) + g \beta''(C - C_0) + \nu \frac{\partial^2 u}{\partial y^2} \]  

(1.8.12)

Where \( \nu \) is the kinematic viscosity of the fluid. The equations (1.8.5), (1.8.7), (1.8.8) and (1.8.12) represent the governing equations for the flow under consideration.

**Under magnetic body force**

Suppose the fluid is electrically conducting, and a uniform transverse magnetic field of strength \( B_0 \) is applied, then the interaction between the motion and the magnetic field can be described by Maxwell's equations. As with most problems involving conductors, Maxwell's displacement currents are ignored, the electric currents are regarded as flowing in closed circuits. Assuming that the velocity of flow is too small compared to the velocity of light, i.e. the relativistic effects are ignored. The system of Maxwell's equations can be written in the form:
\[ \nabla \times \vec{B} = \mu \vec{J} \]
\[ \nabla \vec{J} = 0 \]
\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \]  
(1.8.13)
\[ \nabla \vec{B} = 0 \]

Ohm's law can be written in the form
\[ \vec{J} = \sigma (\vec{E} + \vec{q} \times \vec{B}) \]  
(1.8.14)

Where \( \vec{B} \) is the magnetic induction intensity, \( \vec{E} \) is the electric field intensity, \( \vec{J} \) is the electrical current density, \( \mu \) is the magnetic permeability and \( \sigma \) is the electrical conductivity. In the equation of motion, the body force term \( \vec{J} \times \vec{B} \) per unit volume is added. This body force represents the coupling between the magnetic field and fluid motion, which is called Lorentz force.

The induced magnetic field is neglected under the assumption that the magnetic Reynold's number is small. This is a rather important case for some practical engineering problems where the conductivity is not large in the absence of an externally applied and with negligible effects of polarization of the ionized gas. It has been taken \( \vec{E} = 0 \) i.e. in the absence of convection outside the boundary layer, \( \vec{B} = B_0 \) and \( \nabla \times \vec{B} = \mu \vec{J} = 0 \). Then equation (1.8.14) leads to \( \dot{j} = \sigma (\vec{q} \times \vec{B}) \). Thus Lorentz force becomes \( \vec{J} \times \vec{B} = \sigma (\vec{q} \times \vec{B}) \times \vec{B} \). In what follows, the induced magnetic field will be neglected. This is justified, if the magnetic Reynold's number is small. Hence, to get a better approximation, the Lorentz force can be replaced by
\[ (\vec{q} \times B_0) \times B_0 = -\sigma B_0^2 \vec{q} \].

Now, equation (1.8.12) takes the form
\[ \frac{\partial u}{\partial t} = g \beta (T - T_w) + g \beta' (C - C_w) + \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2 u}{\rho} \]  
(1.8.15)

The above equations (1.8.7), (1.8.8) and (1.8.15) represent the governing equations for the hydro magnetic case.
1.9. NON-DIMENSIONAL PARAMETERS

Non dimensional parameters help us to understand the physical significance of a particular phenomenon associated with the problem. The basic equations are made dimensionless using certain dependent and independent characteristic values. Some of the dimensionless parameters used in thesis are explained below.

Skin Friction

When the boundary layer equations are investigated the velocity distribution can be obtained and the position of the point of separation can be determined, this in turn permits us to calculate the viscous drag and is known as skin friction. The shearing stress as the wall is given by

$$\tau_0 = \mu \frac{\partial u}{\partial y} \bigg|_{y=0} \tag{1.9.1}$$

Nusselt Number

The ratio of the conductive thermal resistance to the convective thermal resistance of the fluid is called Nusselt number.

$$Nu = \frac{hL}{\kappa} \tag{1.9.2}$$

Sherwood Number

The dimensionless quantity $Sh$ is known as Sherwood number and it is defined

$$Sh = \frac{-h}{(c_w-c_\infty)} \frac{\partial \theta}{\partial y} \bigg|_{y=0} \tag{1.9.3}$$

Prandtl number

It is an important dimensional parameter dealing with the properties of a fluid. It is defined as ratio of fluid velocity boundary layer thickness to the fluid temperature boundary layer thickness

$$Pr = \frac{\mu c_p}{\kappa} \tag{1.9.4}$$
Schmidt number

The ratio of molecular diffusivity or momentum to the mass molecular diffusivity is given by Schmidt number. It plays a major role in convective mass transfer.

\[ Sc = \frac{v}{D} \quad (1.9.5) \]

Hartmann number

The dimensionless quantity denoted by \( M \) is known as the Hartmann number. It was first introduced by Hartmann in 1930, in the study of plane Poiseuille flow of an electrically conducting fluid in the presence of transverse magnetic field. It is defined as the ratio of magnetic force to viscous force.

\[ M = \frac{\text{Magnetic force}}{\text{Viscous force}} = \frac{\rho_0 v \sqrt{\sigma}}{\nu_0 \sqrt{\mu}} \quad (1.9.6) \]

Thermal Grashof number

It plays a significant role in free convection heat and mass transfer. The ratio of the product of the internal force and the buoyancy force to the square of viscous force in the convection flow system is interpreted as in free convection is analogous to Reynolds number in forced convection.

\[ Gr = \frac{\rho g \beta v^2 (T_w - T_c)}{\nu_0^3 \mu} \quad (1.9.7) \]

Modified Grashof number

Another non-dimensional parameter usually occurs in natural convection in natural convection problem is modified Grashof number which is defined as the ratio of the species buoyancy force to the viscous force to the hydromagnetic force.

\[ Gm = \frac{\rho g \beta v^2 (\gamma_w - \gamma_c)}{\nu_0^3 \mu} \quad (1.9.8) \]

Reynolds Number

In fluid mechanics, the Reynolds number \( Re \) is a dimensionless number that gives a measure of the ratio of inertial forces to viscous forces and consequently
quantifies the relative importance of these two types of forces for given flow conditions. The concept was introduced by George Gabriel Stokes in 1851, but the Reynolds number is named after Osborne Reynolds (1842–1912), who popularized its use in 1883.

The dimensionless number $Re = \frac{Lu_0}{v}$ is known as local Reynolds number. Where $L$ is characteristic length, $v$ is kinematic viscosity and $u_0$ is initial velocity. The above quantity has dimension $\frac{Lu}{v}$. It can be easily shown that

$$Re = \frac{\text{inertia force}}{\text{viscous force}}$$  \hspace{1cm} (1.9.9)

Hence Reynolds measures the relative change of inertia force per unit change of viscous force.

Radiation Parameter

$$N = \frac{KK^*}{4\sigma T_w^3}$$  \hspace{1cm} (1.9.10)

Here $K$ stands for thermal conductivity, $K^*$ is the absorption coefficient, $\sigma$ means constant of Stefan-Boltzmann and $T_w$ shows the temperature away from the wall.

Magnetic Parameter

The ratio of magnetic force and force due to density of the fluid is known as the Magnetic parameter.

$$Mm = \frac{\sigma B_0}{pc}$$  \hspace{1cm} (1.9.11)

Heat Source/ Sink Parameter

It is the dimensionless parameter defined by

$$S = \frac{Q}{pc_p}$$  \hspace{1cm} (1.9.12)

Where $Q$ represents the heat source when $Q > 0$ the heat sink when $Q < 0$. 

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Chemical Reaction parameter

The dimensionless parameter is defined as \( Kr = \frac{Rv}{\nu_0} \) \hspace{1cm} (1.9.13)

(i) \( Kr > 0 \) for the destructive reaction.

(ii) \( Kr = 0 \) for no reaction.

(iii) \( Kr < 0 \) for the generative reaction.

Peclet Number

The product of the Reynolds number and the prandtl number is peclet number.

The dimensionless parameter is defined as \( Pe = \frac{LuMc}{\nu k} \) \hspace{1cm} (1.9.14)

1.10. REVIEW OF LITERATURE

Hydro magnetic incompressible viscous flows have many important engineering applications such as magneto hydrodynamic power generators and the cooling of reactors besides its application to problems occured in geophysics and engineering etc. The first exact solution of the Navier - Stokes equation is given by Stokes [36] which is connected with the flow of a viscous incompressible fluid past an infinite horizontal plate oscillating in its own plane in an infinite mass of stationary fluid. This is also known as Rayleigh's problem in the literature. England and Emery [13] have studied the thermal radiation effects of optically thin gray – gas bounded by a stationary vertical plate.

Instead of horizontal plate, if an infinite isothermal vertical is given an impulsive motion, the flow is affected by the free convection currents which exist due to the temperature difference between the plate temperature and that of fluid far away from the plate. This was studied by Soundalgekar [33] who presented an exact solution for free convection effects on the Stokes problem for an infinite vertical plate. Free convection effects on the flow of an incompressible viscous fluid past an infinite vertical isothermal plate impulsively moving in its own plane was studied by Soundalgekar [31] and it is observed that the velocity decreases due to more heating of the plate and increases due to more cooling of the plate.
The effects of transversely applied magnetic field, to the flow of an electrically conducting fluid past an impulsively started infinite isothermal vertical plate by Soundalgekar et al [34]. MHD effects on impulsively started vertical plate with variable temperature in the presence of transverse magnetic field were considered by Soundalgekar et al [35]. The dimensionless governing equations were solved using Laplace-transform technique. The influence of magnetic field in flow past an impulsively started vertical plate was investigated by Raptis et al [25]. On the other hand, the study of heat generation or absorption of moving fluids is important in problems dealings with chemical reactions dissociating fluids. Possible heat generation effects may alter the temperature distribution and consequently, the particle deposition rate in nuclear reactors, electronic chips and semi-conductor wafers. Since some fluids can also emit and absorb thermal radiation, it is of interest to study the effects of magnetic field on the temperature distribution and heat transfer when the fluid is not only an electrical conductor but also when it is capable of emitting and absorbing radiation. Hence, heat transfer by thermal radiation is of greater importance.

Radiative and free convection effects on the oscillation flow past a vertical plate studied by Mansour [20]. Soundalgekar and Takhar [32] have considered radioactive free convective flow of an optically thin gray-gas past a semi-infinite vertical plate. Development of oscillatory Asymmetric Recirculating flow in transient laminar opposing mixed convection in a Symmetrically heated vertical channel was investigated by Lin et al. [18]. Mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction studied by Das et al. [12]. The dimensionless governing equations were solved by the usual Laplace Transform technique. Das et al [11] has analyzed radiation effects on flow past an impulsively started infinite isothermal vertical plate. The governing equations were solved by the Laplace-transform technique. Radiation effects on mixed convection along an isothermal vertical plate were studied by Hossain and Takhar [16].

Kumari and Nath [17] studied the development of the asymmetric flow of a viscous electrically conducting fluid in the forward stagnation point region of a two-dimensional body and cover a stretching surface were set into an impulsive motion form the rest. The governing equations were solved using finite difference scheme. Transient MHD heat and mass transfer flow with thermal diffusion in a rotating
system was studied by Alam and Sattar [6]. Raptis and Peridikis [26] have studied the effects of radiation and free convection flow of on moving plate. Makinde and Mhorie[19] studied the Heat transfer to MHD oscillatory flow in a channel filled with porous medium. Alam et al [2] investigated the Dufour and Soret effects on unsteady free convection and mass transfer flow past an impulsively started infinite vertical plate embedded in a porous medium under the influence of transverse magnetic field. They observed that large Darcy number leads to the increase of the velocity and decrease of the temperature as well as concentration of the flow fluid with in the boundary layer.

Alam and Rahman [5] examined Dufour and Soret effects on mixed free-forced convective and mass transfer flow of a viscous incompressible fluid over an isothermal semi-infinite vertical porous flat plate embedded in a porous medium. They found in their study interestingly, the flow field is appreciably influenced by the Dufour and Soret effects. Again Alam et al. [3,4] studied Dufour and Soret effects on steady/unsteady free convective heat and mass transfer flow past a semi-infinite vertical porous plate in a porous medium. The governing non-linear partial differential equations were solved numerically by applying Nachtsheim-Swigert shooting iteration technique together with sixth-order Runge-Kutta integration technique. They examined that velocity profiles decrease with the increase of permeability parameter while it increases with increase of free convection currents. Muthucumaraswamy et al [21] studied magnetic field and radiation effects on moving isothermal vertical plate with variable mass diffusion.


Hamza et al. [15] presented Unsteady heat transfer to MHD oscillatory flow through a porous medium under slip condition. Singh [29] investigated Exact solution

The contributions of various authors of heat and mass transfer with/without oscillatory flow is indispensable. Inspired by their intransigent endeavors I feel greater appetite to continue their work.
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GENERAL SUMMARY OF THE PRESENT WORK

In this thesis an attempt is made to study the hydromagnetic mixed convective oscillatory flows through porous media. It is divided into six chapters including the introduction chapter. Chapter 1 is introductory which is highlighting the importance of flows. It is also presents the basic definitions and fluid equations satisfied by the flows examined.

In the second chapter an oscillatory magneto hydrodynamic mixed convection flow of a Newtonian fluid through a porous medium bounded by two parallel vertical porous plates is considered. The effects of thermal diffusion, homogeneous chemical reaction and thermal radiation are taken into account. A magnetic field of uniform strength Br is applied perpendicular to the plate. The expressions for velocity, temperature and concentration are obtained by solving the non-dimensional governing equations which uses a regular perturbation technique. With the aid of these the expressions for Skin friction, Nusselt number and Sherwood number are derived. The effects of various material parameters on the velocity, temperature, concentration, skin friction, the rate of heat and mass transfer coefficients are discussed and analysed in graphs and tables.

In the third chapter an unsteady MHD periodic flow of a viscous incompressible fluid through a porous medium in a vertical channel under influence of a magnetic field is discussed. The two vertical porous infinite plates are subjected to constant injection and suction velocity. The pressure gradient in the channel and the temperature of the one of the plates varies periodically with time. The expressions for velocity, temperature and concentration are obtained by solving the non-dimensional governing equations by a regular perturbation technique. The expressions for Skin friction, Nusselt number and Sherwood number are also derived. The effects of various emerging parameters on the velocity, temperature, concentration, skin friction, the rate of heat and mass transfer coefficients have been discussed and analysed in detail through graphs and tables.

In the fourth chapter the effects of mass transfer on unsteady MHD mixed convective periodic flow of a viscous incompressible and electrically conducting fluid through porous medium in an inclined channel with horizontal. The lower stationary plate and the upper plate in unsteady periodic motion are subjected to a