5.1 **INTRODUCTION**

The thermal instability of a fluid layer heated from below plays an important role in geophysics, oceanography, atmospheric physics etc., and has been investigated by many authors, e.g., **Benard** (1900), **Rayleigh** (1916), **Jeffrey’s** (1926). The problem of thermal convection in a Newtonian fluid is well studied in the literature. The effect of suspended (dust) particles was investigated by **Scanlon and Segel** (1973) and it was concluded that the critical Rayleigh number was reduced solely because the heat capacity of the fluid was supplemented by that of the particles.

The study of thermal convection was extended to the fluids with peculiar properties (i.e. viscoelastic, ferro-magnetic or electrically conducting fluids) by a number of research workers. **Sharma** (1975, 1976) studied the stability of a layer of viscoelastic fluid obeying Oldroyd equation, heated from below in the presence of a magnetic field, dust particles and rotation respectively. **Bhatia and Steiner** (1972) considered the effect of uniform rotation on the thermal instability of viscoelastic fluid and showed that rotation has a destabilizing influence, in contrast to the stabilizing effect on Newtonian fluid.

**Sharma** (1976) studied the thermal instability of a layer of **Oldroydian** (1958) fluid acted upon by a uniform rotation and found that rotation has destabilizing and stabilizing effects under certain conditions in contrast to a Maxwell fluid where the effect is destabilizing. The stability of flow of a single -
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component fluid through a porous medium taking into account the Darcy resistance has been considered by Lap wood (1948) and Wooding (1960).

The important conclusions of Sharma and Rajput (1993) are that the effect of suspended particles is to increase the instability of the layer and that the Oldroydian viscoelastic fluid behaves like a Newtonian Viscous fluid for stationary convection. Kochar and Jain (1979) examined the shear flow instability of a dusty gas under Michael’s assumptions (1965) and suggested improvement upon Howard’s semi-circle theorem by proving a semi-ellipse theorem for unstable case. They showed that dust particles have a stabilizing effect. Palaniswamy and Purushothom (1981) examined the shear flow instability of a stably stratified incompressible fluid laden with uniformly distributed fine dust particles. The effect of fine dust was found to increase the region of instability. Gupta and Agarwal (1988) reinvestigated the same problem. The conditions for stability and instability and the characterization of modes are some of the important results in their work. Bansal and Agrawal (1999) studied thermal instability of a compressible shear flow in the presence of a weak applied magnetic field. They obtained the sufficient conditions for stability and the bounds for arbitrary unstable modes. The effect of magnetic field was found to stabilize the system while velocity shear destabilizes it.

Goel and Agrawal (1998) have made on important contribution to the study of hydromagnetic thermal convection in a visco-elastic dusty fluid in a porous medium. Jaimala et.
al. (2010) studied the shear flow instability of a non-viscous dusty fluid in the presence of fine dust and uniform magnetic field. They showed that Miles’ criterion for stability and Howard’s semi circle theorem are extended for fine dust particles and weak magnetic field. In this chapter, we have investigated the thermal convection in a visco-elastic dusty fluid in an anisotropic porous medium and our emphases is mainly on the numerical computations of critical Rayleigh number under various physical situations.

5.2 FORMULATION OF THE PROBLEM AND PERTURBATION EQUATIONS

Consider the following constitutive relations due to Oldroyd for the viscoelastic fluid, namely

\[ T_{ij} = -p\delta_{ij} + \tau_{ij}, \]

\[ \left(1 + \lambda \frac{d}{dt}\right) \tau_{ij} = 2\mu \left(1 + \lambda_0 \frac{d}{dt}\right) \rho_{ij} \]

and \[ \rho_{ij} = \frac{1}{2} \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right), \]

where \( T_{ij}, \tau_{ij}, \rho_{ij}, \mu, \lambda, \lambda_0 (\lambda), p, \delta_{ij}, v_i, x_i \) and \( \frac{d}{dt} \) denote respectively the total stress tensor, the shear stress tensor, the rate of strain tensor, the viscosity of the fluid, the stress relaxation time, the strain retardation time, the pressure, the Kronecker
delta, the velocity vector, the position vector and the mobile operation. It is to be noted that \( \lambda_0 = 0 \) yields the Maxwellian fluid whereas \( \lambda_0 = \lambda = 0 \) gives the Newtonian viscous fluid.

The fluid layer of finite depth \( d \), initially at rest, is finitely conducting (electrically and thermally both) in an anisotropic porous medium. It is acted upon by a horizontal magnetic field in the \( x \)-direction and the gravity acts in the negative \( z \)-direction so that \( \mathbf{g} = (0, 0, -g) \). Thus dust particles are assumed to be electrically non-conducting. Assuming the uniform size and spherical shape of dust particles and small relative velocities between the fluid the and the particles, the equations of motion and continuity for the fluid, through an anisotropic porous medium are

\[
\rho \left( 1 + \lambda \frac{\partial}{\partial t} \right) \frac{\partial \mathbf{u}}{\partial t} + \phi (\mathbf{u} \cdot \nabla) \mathbf{u} = \left( 1 + \lambda \frac{\partial}{\partial t} \right) \left[ -\nabla p + \rho \mathbf{g} + \frac{K \mathbf{N}}{\phi} (\mathbf{v} - \mathbf{u}) \right] \\
+ \left( 1 + \lambda_0 \frac{\partial}{\partial t} \right) \left[ \frac{\mu}{\phi} \nabla^2 \mathbf{u} - \frac{\mu}{k_x, k_y, k_z} \right] \mathbf{u},
\]

... (2)

and

\[ \nabla \cdot \mathbf{u} = 0, \]

... (3)

where \( \rho, \mu, p \) and \( \mathbf{u}(u_1, u_2, u_3) \) denote respectively the density, viscosity, pressure and seepage velocity of the fluid. \( \mathbf{v}(v_1, v_2, v_3) \) and \( \mathbf{N}(x, t) \) denote the seepage velocity and the number density of the particles respectively. \( \{k_x, k_y, k_z\} \) is the
anisotropic effect and \( \phi \) is medium porosity. \( K = 6\pi \mu r \) is the Stoke's resistance coefficient, where \( r \) is the particle radius. \( \eta \) is resistivity and \( \mu_c \) is the magnetic permeability.

If \( mN \) is the mass of the particles per unit volume, then the equations of motion and continuity with porosity corrections for the dust particles are

\[
\frac{mN}{\phi} \left[ \frac{\partial v}{\partial t} + \frac{1}{\phi} (v \nabla) v \right] = mNg + \frac{KN}{\phi} (u - v)
\]

... (6)

and

\[
\frac{\partial N}{\partial t} + \frac{1}{\phi} \nabla \cdot (Nv) = 0
\]

... (7)

Following Joseph (1976), the energy equation in permeable materials is obtained from an enthalpy balance over the fluid as

\[
\int_{v_f} \rho C_f \left[ \frac{\partial T}{\partial t} + \frac{1}{\phi} (u \nabla) T \right] \, dv_f = \int_{v_f} \nabla \cdot (k_f \nabla T) \, dv_f,
\]

... (8)

and over the dust particles as

\[
\int_{v_{pt}} mNC_{pt} \left[ \frac{\partial T}{\partial t} + \frac{1}{\phi} (v \nabla) T \right] \, dv_{pt} = \int_{v_{pt}} \nabla \cdot (k_{pt} \nabla T) \, dv_{pt},
\]

... (9)

where \( C_f, C_{pt}, k_f, k_{pt} \) and \( T \) denote respectively the heat capacity of the fluid at a constant pressure, heat capacity of the particles, thermal conductivity of the fluid, thermal conductivity of the particles and the temperature.
The integration is transferred to the common volume \( v = v_f \cup v_{pt} \) by the Jacobian relations

\[
\frac{dv_f}{dv} = \phi \quad \text{and} \quad \frac{dv_{pt}}{dv} = 1 - \phi.
\]

Putting the value of \( v_{pt} \) and \( v_f \) in equations (8) and (9) and on adding the resulting equations, after some simplifications, we obtain

\[
B \frac{\partial T}{\partial t} + \frac{1}{\phi} (u \nabla) T + \frac{1}{\phi} b (v \nabla) T = \kappa \nabla^2 T,
\]

...(10)

where \( k_m = k_f + k_{pt} \frac{(1-\phi)}{\phi} \), \( b = \frac{m N C_p (1-\phi)}{\rho c_f \phi} \),

\( B = 1 + b \) and \( \kappa = \frac{k_m}{\rho c_f} \) (the thermal diffusivity).

The use has also been made of the appropriate equation of state

given by

\[
\rho = \rho_0 \left[ 1 - \alpha (T - T_0) \right], \quad \text{...(11)}
\]

where \( \alpha \) is the coefficient of volume expansion.

**Basic State**

The basic state of which we wish to investigate the stability or otherwise, is characterized by
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\[ u = 0, \quad v = 0, \quad T = T_0 - \beta z, \quad \rho = \rho_0 (1 + \alpha \beta z), \]

and \[ p = p_0 - g \rho_0 \left( z + \frac{1}{2} \alpha \beta z \right)^2 \] ... (12)

and further, it is assumed that the number density \( N \) is constant, say \( N_0 \). The expression for density clearly shows that it will increase (decrease) in the vertically upward direction if the system in heated from below (above).

### Linearized Perturbation Equations

The basic state given by (12) is slightly perturbed and let \( \delta \rho', \delta \rho', \delta N', \quad u = (u'_1, u'_2, u'_3), \quad v = (v'_1, v'_2, v'_3) \) and \( \theta' \) respectively be the perturbations in pressure, density, number density, velocity of fluid, velocity of dust particles and the temperature. The equations are linearized in perturbations within the framework of classical linear theory of stability and the elimination of various perturbation quantities is made in favour of \( u_3' \) and \( \theta' \). Then fluid equations become

\[
\left( \frac{\partial}{\partial t} - \eta \nabla^2 \right) \left[ 1 + \lambda_0 \frac{\partial}{\partial \xi} \left( \frac{m}{K} \frac{\partial}{\partial t} + 1 + \frac{KN_0}{\rho_0} \right) \frac{1}{v} \frac{\partial}{\partial t} \right] \nabla^2 u_3' \\
+ \left( 1 + \lambda_0 \frac{\partial}{\partial \xi} \right) \left( \frac{m}{K} \frac{\partial}{\partial t} + 1 \left( \frac{v}{k_x,k_y,k_z} \nabla^2 \right) \right) \nabla^2 \theta' \\
= \left( 1 + \lambda_0 \frac{\partial}{\partial \xi} \right) \left( \frac{m}{K} \frac{\partial}{\partial t} + 1 \left( \nabla^2 \frac{\partial}{\partial t} - \eta \nabla^2 \right) \right) \frac{\alpha \beta \phi}{\rho_0}, \quad ...(13)
\]
and \( B \left( \frac{\partial}{\partial t} - \kappa \nabla^2 + \frac{m}{K} \frac{\partial}{\partial t} + B \right) \theta' = \beta \left( \frac{m}{K} \frac{\partial}{\partial t} + B \right) u'_3 \), \( \ldots(14) \)

In case of two free and isothermal boundary surfaces, the boundary conditions appropriate for the problem are

\[ u'_3 = \frac{\partial^2 u'_3}{\partial z^2} = 0, \quad \theta' = 0 \atop \text{at } z = 0 \text{ and } z = 1. \]
\( \ldots(15) \)

### 5.3 Normal Mode Analysis and Eigen-Value Problem

Analysing the disturbances into normal modes, we assume that the perturbation quantities are of the form

\[ (u'_3, \theta') = \left[ W(z) \Theta(z) \right] \exp[ik_x x \atop ik_y y \atop n] \]
\( \ldots(16) \)

Equations (13) and (14) on using (16), in non-dimensional form, become

\[
\left\{ \begin{array}{l}
\left( D^2 - a^2 p_1 \sigma \right) (1 + \lambda' \sigma) L_1 + (1 + \lambda_0' \sigma) L_2 \left( R_e \left( D^2 R_{p1}^2 - a^2 R_{D}^{-1} \right) W \right. \\
\left. - \left( D^2 - a^2 \right)^2 W \right) = -N_R \phi a^2 L_2 (1 + \lambda' \sigma) \left( D^2 - a^2 - p_1 \sigma \right) \Theta \\
\end{array} \right. \]
\( \ldots(17) \)

and \( \left( D^2 - a^2 - B \sigma \right) L_2 \Theta = -\frac{1}{\phi} (B + \tau \sigma) W \), \( \ldots(18) \)
where the physical variables have been scaled using

\[ d, \frac{d^2}{\kappa}, \frac{\kappa}{d}, \frac{\rho \nu \kappa}{d^2}, \text{ and } \beta d \] as the length, time, velocity, pressure and temperature scale factors

respectively and

\[ p_1 = \frac{\kappa}{\eta}, \quad \sigma = \frac{\nu d^2}{\kappa}, \quad \lambda' = \frac{\lambda \kappa}{d^2}, \]

\[ L_1 = N_p^{-1} \left( \tau \sigma^2 + F \sigma \right), \quad F = 1 + f. \]

\[ \tau = \frac{m \kappa}{Kd^2}, \quad \lambda'_0 = \frac{\lambda_0 \kappa}{d^2}, \quad L_2 = \tau \sigma + 1 \]

\[ N_p = \frac{\nu}{\kappa} \]

is the Prandtl number,

\[ R_e = \frac{U_0 \rho d}{\mu} \]

is the Reynolds number,

\[ R_D = \frac{k U_0 \rho d}{\mu \phi} \]

is the Darcy - Reynolds number,

\[ R_{Dz} = \frac{k_z U_0 \rho}{\mu \phi d} \]

is the Darcy - Reynolds number in z-direction,

\[ N_R = \frac{g \alpha \beta d^4}{\nu \kappa} \]

is the Rayleigh number,
\[ B = 1 + b, \quad a = kd, \quad b = \frac{f C_{pt}(1-\phi)}{\phi C_{pt}}. \]

Finally, if \( \Theta \) is eliminated from equations (17) and (18), we get one equation in \( W \) alone, namely,

\[
\left( D^2 - a^2 - B\sigma \right) \left( D^2 - a^2 - p_1\sigma \right) \left( 1 + \lambda \sigma \right) L_1 \left( D^2 - a^2 \right) W
+ \left( 1 + \lambda_0' \sigma \right) L_2 \left[ R_e \left( R_D^{-1} D^2 - a^2 R_D^{-1} \right) W - \left( D^2 - a^2 \right)^2 \right] W

= \frac{N_R}{a^2} (1 + \lambda' \sigma) \left( D^2 - a^2 - p_1\sigma \right) (B + \tau \sigma) W. \quad \text{...(19)}
\]

The boundary conditions (11) and (12) transform to

\[
W = 0, \quad D^2 W = 0, \quad \Theta = 0 \quad \text{at} \quad z = 0 \quad \text{and} \quad z = 1. \quad \text{...(20)}
\]

Using equation (20) it can be shown that all the even – order derivatives of \( W \) must vanish for \( z = 0, \) and \( z = 1, \) and hence the proper solution of equation (19), characterizing the lowest mode is taken as

\[ W = AS\sin \pi z. \]

Equation (19) now yields

\[
\left( \pi^4 + a^2 \pi^2 + B\sigma \pi^2 + a^2 \pi^2 + a^4 + B\sigma a^2 \right) (1 + \lambda' \sigma) L_1
\]

\[
+(1 + \lambda_0' \sigma) L_2 R_e R_D^{-1} \left( \pi^4 + \pi^2 a^2 + B\sigma \pi^2 \right)
\]

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\begin{align*}
\left[(1 + \lambda_0')\sigma\right]L_2a^2R_eR_{Dz}^{-1}\left[\pi^4 + \pi^2 + B\sigma\right] \\
-\left[(1 + \lambda_0')\sigma\right]L_2\left[-\pi^6 - a^4\pi^2 - 2a^2\pi^4 - a^2\pi^4 - a^6 \right] \\
-2a^4\pi^2 - B\sigma\left[\pi^4 + a^4 + 2\pi^2a^2\right] = N_Ra^2(1 + \lambda'\sigma)(B + \tau\sigma),
\end{align*}

or
\[
\frac{N_R}{\pi^2} = \left(1 + x + \frac{B\sigma}{\pi^2}\right)\left[\left[l + x\right]\left[l + \lambda'\sigma\right]L_1 + \left[l + \lambda'\hat{\psi}\right]L_2 \right]
\]

\[
\frac{\left[R_e\left(R_{Dz}^{-1} + xR_{Dz}^{-1}\right) + \pi^2\left(1 + x\right)^2\right]}{xB} = x(1 + \lambda'\sigma)(B + \tau\sigma),
\]

at \(\sigma = 0, L_1 = 0, L_2 = 1\) and \(\frac{a^2}{\pi^2} = x^2\)

or
\[
N_{R_1} = \frac{(1 + x)\left[R_e\left(R_{Dz}^{-1} + xR_{Dz}^{-1}\right) + \pi^2\left(1 + x\right)^2\right]}{xB} \quad \text{where} \quad \frac{N_R}{\pi^2} = N_{R_1}.
\]

The necessary condition \(\frac{dN_{R_1}}{dx} = 0\) for the existence of critical Rayleigh number leads to
\[
\frac{dN_{R_1}}{dx} = \frac{1}{B}\left\{\left(1 + x\right)x^{-1}\left[R_eR_{Dz}^{-1} + 2\pi^2\left(1 + x\right)\right] \right. \\
+ \left. \left[R_e\left(R_{Dz}^{-1} + xR_{Dz}^{-1}\right) + \pi^2\left(1 + x\right)^2\right]\left(1 + x\right)\left(-\frac{1}{x^2}\right) + x^{-1}(0 + 1) \right\} = 0
\]
\[
1 + \frac{x}{x} \left\{ R_e R_{D_z}^{-1} + 2\pi^2 (1 + x) \right\} + \left\{ R_e \left[ R_D^{-1} + x R_{D_z}^{-1} \right] \right\} \\
+ \pi^2 \left(1 + x^2 + 2x\right) \left\{ \frac{-1 - x + x}{x^2} \right\} = 0,
\]

\[
\Rightarrow 2\pi^2 x^3 + \left(3\pi^2 + R_e R_{D_z}^{-1}\right) x^2 - \left(\pi^2 + R_e R_D^{-1}\right) = 0. \quad \text{...}(21)
\]

Since the product of the roots is positive and the sum of the roots is negative, therefore either one root is positive and two roots are negative, or one root is positive and two roots are complex with negative real parts. Since \(x\) is real and positive, therefore we consider only the positive roots of equation (21).

**Table-1**

<table>
<thead>
<tr>
<th>(X)</th>
<th>(N_{RI} (B = 1.1))</th>
<th>(N_{RI} (B = 2.1))</th>
<th>(N_{RI} (B = 3.1))</th>
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<tr>
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Fig:1. Rayleigh number ($N_{Ri}$) vs. wave number ($x$)
for $B = 1.1$, $B=2.1$, $B=3.1$.

Table-2

<table>
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<th>$R_{Dz}$</th>
<th>$N_{Ri}$ ($B = 1.1$)</th>
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<th>$N_{Ri}$ ($B = 3.1$)</th>
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</tr>
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<td>21.567</td>
</tr>
</tbody>
</table>
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**Fig : 2. Rayleigh number ($N_{R_i}$) vs. Darcy-Reynolds number ($R_{D_z}$)**

for $B=1.1$, $B=2.1$, $B=3.1$

---

**Table-3**

<table>
<thead>
<tr>
<th>$R_D$</th>
<th>$N_{R_i} \ (B = 1.1)$</th>
<th>$N_{R_i} \ (B = 2.1)$</th>
<th>$N_{R_i} \ (B = 3.1)$</th>
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<tr>
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<td>61.869</td>
<td>32.407</td>
<td>21.954</td>
</tr>
</tbody>
</table>
**Fig: 3. Rayleigh number ($N_{R_i}$) vs. Darcy-Reynolds number ($R_D$) for $B=1.1$, $B=2.1$, $B=3.1$.**

<table>
<thead>
<tr>
<th>$R_D$</th>
<th>$N_{R_i}$ ($x = 0.5$)</th>
<th>$N_{R_i}$ ($x = 1.0$)</th>
<th>$N_{R_i}$ ($x = 1.5$)</th>
<th>$N_{R_i}$ ($x = 2.0$)</th>
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**Table 5**

<table>
<thead>
<tr>
<th>$R_D$</th>
<th>$N_{Ri}$ ($x = 0.5$)</th>
<th>$N_{Ri}$ ($x = 1.0$)</th>
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5.4 NUMERICAL RESULTS AND DISCUSSION

We have made numerical calculations for evaluating $N_{R_i}$ (Rayleigh number) for different values of the parameters and the results are shown in Figs : 1-5 along with the tables. The following conclusions are made with the help of these figures.

Fig :1 and Tables :1 show the variation of $N_{R_i}$ (Rayleigh number) with $x$(wave number). It follows that for a given $B$, $N_{R_i}$
(Rayleigh number) increases as $x$ (wave number) increases though the increase in $N_{R_i}$ (Rayleigh number) is very large.

Figs : 2-3 and Tables : 2-3 show the variation of $N_{R_i}$ (Rayleigh number) with $R_D$ (Darcy-Reynolds number), $R_{Dz}$ (Darcy-Reynolds number in z-direction). It follows that for a given $B$, $N_{R_i}$ (Rayleigh number) increases as $R_D$ (Darcy-Reynolds number), $R_{Dz}$ (Darcy-Reynolds number in z-direction) increases though the increase in $N_{R_i}$ (Rayleigh number) is very small.

Figs : 4-5 and Tables : 4-5 show the variation of $N_{R_i}$ (Rayleigh number) with $R_D$ (Darcy-Reynolds number), $R_{Dz}$ (Darcy-Reynolds number in z-direction). It follows that for a given $x$ (wave number), $N_{R_i}$ (Rayleigh number) increases as $R_D$ (Darcy-Reynolds number), $R_{Dz}$ (Darcy-Reynolds number in z-direction) increases though the increase in $N_{R_i}$ (Rayleigh number) is very small.

$N_{R_i}$ (Rayleigh number) increase as $x$ (wave number), $R_D$ (Darcy-Reynolds number), $R_{Dz}$ (Darcy-Reynolds number in z-direction), increase and hence anisotropic permeability has stabilizing effect.