Chapter-6
A Fuzzy Inventory Model with Two-Storage Facilities under the Permissible Delay in Payment

Abstract
In this study, two-warehouse inventory model with the conditions of permissible delay in payments in fuzzy environment is developed. A rented warehouse is used when the ordering quantity exceeds the limited capacity of the owned warehouse. The holding cost at RW is higher the goods immediately after receiving the consignment. However, in practice, it is found that the supplier allows a certain fixed period to the customers to settle the account. During this fixed period no interest is charged by the supplier, but beyond this period interest is charged under the terms and conditions agreed upon and, moreover, interest can be earned on the revenue received during the credit period. We discuss four different cases for different realistic practical situations as (1) when the inventory system has both the warehouse facilities, (2) when the owned warehouse has large capacity to store the on-hand inventory, (3) when simple EOQ model of single storage system and (4) when one does not wish to take RW services and OW has maximum capacity. Finally, numerical example is given to illustrate feasibility of the discussed different cases.
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6.1 Introduction
Many researchers have developed a series of theories to find the optimal order policy so (crisp) inventory models. However, most of the earlier proposed methods had assumed that all the associated cost parameters and relevant data are exactly known and fixed.

imprecise. Thus, there are uncertainties in real world such as inventory costs usually exist with imprecise components. When uncertainty becomes a matter of debate, conventional approaches to treating uncertainty in inventory control focuses on probability theory. In this case, customer demand as one of the key parameters and source of uncertainty has been most often treated by a probability distribution. However, the probability-based approaches may not be sufficient enough to reflect all uncertainties that may arise in a real world inventory system. Modelers may face some difficulties while trying to build a valid model of an inventory system in which the related costs cannot be determined precisely. For example, costs may be dependent on some foreign monetary unit. In such a case, due to the change in the exchange rates, the costs are often not known precisely. Another source of uncertainty may arise because of the difficulty of determining exact cost components. In some cases trying to determine the precise values of such cost components may be very difficult and costly, if not impossible. For example, inventory-carrying cost is often dependent on some parameters like current interest rate and stock keeping the market price of the unit, which may not be known precisely.

The classical economic order quantity (EOQ) model assumes that the retailer's capital is unconstrained and the retailer must be paid for the items as soon as the items are received. However, in practice the
supplier usually offers the retailer a fixed delay period, which is called trade credit period for settling the accounts. Before the end of trade credit period, the retailer can sell the goods, accumulate revenue and earn interest. A higher interest is charged if the payment is not settled by the end of trade credit period. In the real world, the supplier often makes use of this policy to promote his commodities.

Till now, the algorithms have been developed for solving the inventory problems when inventory parameters like total floor space, set-up cost, holding cost, etc. are precisely known. In real life situation, these parameters may be uncertain. In competitive market, it is not possible to do the business with predefined fixed inventory parameters. at a later stage, to meet the sudden increase of demand or to avail the sudden fall in the price of the commodity, he/she is forced to augment some more space and capital as per the demand of the situation. Hence, in this case, cost determination is imprecise. Similar may be the case of storage area.

Fuzzy set theory, originally introduced by Zadeh (1965), provides a framework for considering parameters that are vaguely or unclearly defined or whose values are imprecise or determined based on subjective beliefs of individuals. Later on, Kaufmann and Gupta (1985) introduced fuzzy arithmetic operations. The application of fuzzy theory to inventory problem has been proposed by Kacprzyk and Staniewaski (1982). Chen (1985) proposed second function principle as the fuzzy numbers in the formula of fuzzy total cost, because it does not change the type of membership function of generalized fuzzy number after arithmetical operation. Park (1987) examined the EOQ formula in the fuzzy set theoretic perspective associating the fuzziness with the cost data. Here, inventory costs transformed to a fuzzy optimization problem. Instead of solving it, he gave an equivalent crisp expression of average inventory cost. Sarma and Naidu (1995) presented an EOQ model in which the two levels of storage were discussed and then they worked out a method of determining the EOQ. Roy and Maiti (1997) developed a fuzzy EOQ model where unit price varied inversely with the demand and setup cost increased with the increase of production. The problem is reduced to a fuzzy optimization problem associating fuzziness with the storage area and total expenditure. Mandal et al. (1998)
available space for inventory storage was limited, and holding cost and selling price were purchasing price dependent. The fuzzy environment is created making costs, purchasing price and storage area imprecise and vague to certain extent. Chen and Hsieh (1999) presented graded mean integration representation of generalized fuzzy numbers. Chen and Hsieh (2000) established a fuzzy inventory model to treat the inventory problem with all the parameters and variables, which are fuzzy numbers. Kao and Hsu (2002) present a single-period inventory model for the case of fuzzy demand. Yao and Chiang (2003) presented an inventory without backorder with fuzzy total cost and fuzzy storing cost defuzzified by centroid and signed distance. Dutta et al. (2005) developed a single-period inventory model with fuzzy random variable demand. In this study, they have applied graded mean integration representation method to find the optimum order quantity. Chen and Ouyang (2006) extended Jamal et al. (1997) by fuzzifying the carrying cost rate, interest paid rate and interest earned rate simultaneously, based on the interval-valued fuzzy numbers and triangular fuzzy numbers to fit the real world. Dutta et al. (2007) presented an inventory model for single-period products with reordering opportunities under fuzzy demand. Chen and Chang (2008) presented an optimization of fuzzy production inventory model. In this study, they have considered a fuzzy opportunity cost and trapezoidal fuzzy costs under crisp quantity or fuzzy quantity in order to extend the traditional inventory model to the fuzzy environment. They have used function principle as arithmetical operations of fuzzy total production inventory cost and also used the graded mean integration representation method to defuzzify the fuzzy total inventory cost. Vijayan and Kumaran (2009) developed a fuzzy economic order time models with random demand. In that model, they have used function principle as arithmetical operations of fuzzy total inventory cost and also used the graded mean integration representation method to defuzzify the fuzzy total inventory cost.

The above cited references revealed that no researcher has yet developed a model in which the effect of permissible delay of payment in fuzzy environment with two storage considering the effect of permissible delay in payment in fuzzy environment with two storage facilities to make a model more realistic.
This chapter incorporates the effect of permissible delay in payment for a system of two storage facilities. In this study, the researcher has proposed a two-warehouse inventory model with permissible delay under fuzzy environment, where demand rate, holding cost in RW and OW, ordering cost, capacity of OW, and purchase cost were taken to be fuzzy in nature. These fuzzy parameters have been represented by membership functions. The total cost function was defuzzified by using graded mean integration representation method. The researcher has discussed four different cases for different realistic practical situations as:

(1) when the inventory system has both the warehouse facilities,
(2) when the owned warehouse has too large a capacity to store the all on-hand inventory,
(3) when simple EOQ model, having single storage facility, and
(4) when one does not wish to take RW services and OW has maximum capacity.

Finally, numerical example is given to illustrate feasibility of the discussed cases.

6.2 Assumptions and Notations

The following assumptions were used to develop the mathematical model in this chapter:

1. The demand rate is fuzzy variable.
2. Holding cost at RW, holding cost at OW, ordering cost per order, purchase cost is fuzzy variable.
3. Capacity to store number of units in OW is also fuzzy variable.
4. Interest that can be earned on the sales revenue of units sold during the permissible delay period is less than Interest charges per rupee per year.
5. Lead-time is zero.
6. During the time period, when the account is not settled, generated sales revenue is deposited in an interest bearing account. At the end of this period the account is settled and interest charges are payable on the items in stock.
7. The time horizon is infinite.
8. Storage capacity of OW is W and that of RW is infinite. If the order quantity exceeds W, then excess units are kept in RW.

The following notations were used in the construction of mathematical model:
\[ Q = \text{Ordering quantity.} \]

\[ \hat{D} = (d_1, d_2, d_3, d_4) \]

\[ \hat{F} = \text{Fuzzy unit stock holding cost at RW excluding interest charges,} \]

\[ (f_1, f_2, f_3, f_4) \]

\[ H = \text{Fuzzy unit stock holding cost at OW excluding interest charges,} \]

\[ (h_1, h_2, h_3, h_4) \]

\[ \hat{A} = (a_1, a_2, a_3, a_4) \]

\[ \hat{W} = \text{Capacity to store number of units in OW fuzzy type,} \]

\[ (w_1, w_2, w_3, w_4) \]

\[ \hat{C} = \text{Fuzzy purchase cost per unit,} \]

\[ (c_1, c_2, c_3, c_4) \]

\[ I_c = \text{Interest charges per rupee per year.} \]

\[ I_e = \text{Interest that can be earned on the sales revenue of units sold during the} \]

\[ \{I_e < I_c\} \]

\[ T^* = \text{Permissible delay period in settling the accounts.} \]

\[ T = \text{Cycle time.} \]

### 6.3 Fuzzy Sets, Membership Function, Defuzzifying Approach and Arithmetical Operations

#### 6.3.1 Fuzzy Sets

A fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership (characteristic) function which assigns to each object a grade of membership ranging between zero and one. Let \( X \) denote a space of objects. Then a fuzzy set in \( X \) is a set of ordered pairs:
\[ A = \{ x, \mu_A(x) \}, \ x \in X \]

\[ \mu_A(x) \]

where \( \mu_A(x) \) is termed “the grade of the membership of \( x \) in \( A \).” For simplicity, \( \mu_A(x) \) is a number in the interval \([0, 1]\), with the grades of unity and zero respectively, full membership and non-membership in the fuzzy set. An object (point) \( P \) contained in a set \( Q \) (class) is an element of \( Q \) \( (P \subset Q) \).

### 6.3.2 Membership Function

![Membership Function Diagram](image)

**Fig. 6.1 Membership function for triangle number**

At the outset it would be prudent introduce the concept of membership function. There are different shapes of membership function in the inventory control such as the triangle and trapezoid. The shapes of the triangle membership function and the trapezoid membership function are shown in Fig. 6.1 and 6.2.

![Membership Function Diagram](image)
Fig. 6.2 Membership function for trapezoid number

\( \tilde{A} \) is assumed as a fuzzy number. If \( \tilde{A} \) is a triangle number, \( \tilde{A} \) can be represented as

\[
\tilde{A} = [k_1, k_2, k_3, k_4] \quad \text{subject to the constraint } 0 < k_1, k_2, k_3, k_4
\]

While \( \tilde{A} \) is a trapezoid fuzzy number, \( \tilde{A} = [k_1, k_2, k_3, k_4] \) is subject to the constraint that \( 0 < k_1, k_2, k_3, k_4 \).

Membership function of the triangle and trapezoid fuzzy numbers can be defined as follows:

\[
\mu_{\tilde{A}}(x) = \begin{cases} 
0 & x < k_1, x > k_3 \\
L(x) = \frac{x - k_1}{k_2 - k_1} & k_1 \leq x < k_2 \\
R(x) = \frac{k_3 - x}{k_3 - k_2} & k_2 \leq x < k_3 
\end{cases}
\]

\[
\mu_{\tilde{A}}(x) = \begin{cases} 
0 & x < k_1, x > k_2 \\
L(x) = \frac{x - k_1}{k_2 - k_1} & k_1 \leq x < k_2 \\
1 & k_2 \leq x < k_3 \\
R(x) = \frac{k_3 - x}{k_3 - k_2} & k_2 \leq x < k_3 
\end{cases}
\]

where \( \mu_{\tilde{A}}(x) \) is a membership function.

6.3.3 Graded Mean Integration Representation Method

In this study, generalized fuzzy number \( \tilde{A} \) was denoted in Fig. 6.1 as \( \tilde{A} = (c, a, b, d, \omega_{\tilde{A}})_{L,R} \) \( \omega_{\tilde{A}} \in (c, a, b, d)_{L,R} \).

When \( \omega_{\tilde{A}} = 1 \), we simplify the notation as \( \tilde{A} \). Chen and Hsieh (1999) introduced the graded mean integration representation method of generalized fuzzy number based on the integral value of graded mean \( h \)-level of generalized fuzzy number. Its meaning is as follows:
Let $L^{-1}$ and $R^{-1}$ are inverse function of $L$ and $R$ respectively, then the graded mean $h$-level value of generalized fuzzy number $\tilde{\mathcal{A}}_{(c,a,b,d;W_{A})_{LR}}$ is

$$h \cdot \frac{L^{-1}(h) + R^{-1}(h)}{2}$$

Fig. 6.3 The graded mean $h$-level of generalized fuzzy number $\tilde{A} = (c, a, b, d, W_{A})_{LR}$ is determined by

$$\tilde{A} = \frac{1}{2} \int_{a}^{b} h \cdot \frac{L^{-1}(A) + R^{-1}(h)}{2} dh \int_{0}^{h} h dh$$

its' corresponding graded mean integration representation is

$$\tilde{B} = (c, a, b, d; W_{B})_{LR}$$

denoted as

$$\tilde{B} = \frac{1}{6} \int_{0}^{h} \frac{h(c + d + a - c - d + b)h}{6} dh \int_{0}^{h} h dh = \frac{c + 2a + 2b + d}{6}$$
where \( a, b, c, d \) are any real numbers.

### 6.3.4 Properties of Second Function Principle

Chen (1985) proposed second function principal to be as the fuzzy arithmetical operations between generalized trapezoidal fuzzy numbers. Because it does not change the type of membership function of generalized fuzzy number after arithmetical operations. It reduces the trouble and tediousness of operations. Furthermore, Chen already proved the properties of fuzzy arithmetical operations under second function principle. Here some properties of the fuzzy arithmetical operations have been described as follows:

\[
\begin{align*}
\mathcal{R}_1 &= c_1, a_1, b_1, d_1 \\
\mathcal{R}_2 &= c_2, a_2, b_2, d_2 \\
\end{align*}
\]

Suppose \( \mathcal{R}_1 \) and \( \mathcal{R}_2 \) are two generalized trapezoidal fuzzy numbers. Then

1. The addition of \( \mathcal{A}^1 \) and \( \mathcal{A}^2 \) is \( \mathcal{A}^1 \oplus \mathcal{A}^2 = c_1 + c_2, a_1 + a_2, b_1 + b_2, d_1 + d_2 \)

2. The multiplication of \( \mathcal{A}^1 \) and \( \mathcal{A}^2 \) is \( \mathcal{A}^1 \otimes \mathcal{A}^2 = (c_1 c_2, a_1 a_2, b_1 b_2, d_1 d_2) \)

3. Then the subtraction of \( \mathcal{A}^1 \) and \( \mathcal{A}^2 \) is \( \mathcal{A}^1 \ominus \mathcal{A}^2 = (c_1 - d_2, a_1 - b_2, b_1 - a_2, d_1 - c_2) \)

4. \( 1/\mathcal{A}^2 = \left( \frac{1}{d_2}, \frac{1}{b_2}, \frac{1}{a_2}, \frac{1}{c_2} \right) \)

where \( c_1, a_1, b_1, d_1 \) and \( c_2, a_2, b_2, d_2 \) are all positive real numbers. If \( c_1, a_1, b_1, d_1 \) and \( c_2, a_2, b_2, d_2 \) are all non-zero positive real numbers, then the division of \( \mathcal{R}_1 \) and \( \mathcal{R}_2 \) is

\[
\mathcal{R}_1 / \mathcal{R}_2 = \left( \frac{c_1}{d_2}, \frac{a_1}{b_2}, \frac{b_1}{a_2}, \frac{d_1}{c_2} \right)
\]

### 6.4 Construction of Cost Function
Initially, the system is buying quantity $Q = DT$ in bulk order, which is larger than the capacity $W$ of OW. Consequently $W$ units are stored at OW and the excess $(DT - W)$ cost, fuzzy holding cost and interest earned and paid. Two cases may arise here, viz. (1) $T^* \leq T$ and (2) $T^* > T$. It can easily be seen from Fig. 4 that per unit time cost consists of the following fuzzy costs.

i) Fuzzy cost of placing an order per unit time is $\tilde{K'} \otimes T$

ii) Total fuzzy holding cost at RW per unit time is

$$\left[ \tilde{H_0} \otimes \tilde{T} \otimes \tilde{H} \right] = \tilde{H_1} \otimes \tilde{T} + \tilde{H_2} \otimes T$$

iii) Total fuzzy holding cost at OW per unit time is

$$\tilde{H_1} \otimes \tilde{W} \left[ \tilde{1} \otimes \tilde{W} \right] = \tilde{H_3} \otimes \tilde{T}$$

iv) Fuzzy interest earned per unit time is

$$\tilde{D'} \otimes C \otimes \left( I_e \left( T^* \right) / 2T \right), \text{ if } T^* \leq T$$

and

$$\tilde{D'} \otimes C \otimes \left[ I_e \left( T - T^*/2 \right) \right], \text{ if } T^* > T$$

v) Fuzzy interest payable per unit time is

$$\tilde{D'} \otimes C \otimes \left( I_e \left( T - T^*/2 \right) \right), \text{ if } T^* \leq T$$

and is zero, if $T^* > T$.

Note that the interest earned should be subtracted from other fuzzy costs in order to get the net total fuzzy costs per unit time.
6.4.1 Case-I: Determination of Order Quantity When $T^* \leq T$ i.e., when the permissible delay period is less than the cycle length

The retailer can accumulate revenue and earn interest after his/her customer pays for the amount of purchasing cost to the retailer until the end of the trade credit period offered by the supplier. That is, the retailer can accumulate revenue and earn interest during the period 0 to $T$ with rate $I$, under the condition of trade credit. Consequently, after the permissible delay period he/she has to pay interest for the remaining cycle length. That is, the retailer has to pay some interest during the period $T^*$ to $T$ with rate $I$.

Here, the researcher has discussed the fuzzy inventory model with fuzzy parameters of annum fuzzy demand

$$\mu = [d_1, d_2, d_3, d_4]$$

fuzzy holding cost

$$\lambda = [h_1, h_2, h_3, h_4]$$

at RW, fuzzy holding cost

$$\lambda = [h_1, h_2, h_3, h_4]$$

at OW, fuzzy ordering cost

$$\mu = [a_1, a_2, a_3, a_4]$$

fuzzy purchase cost per order

$$\mu = [c_1, c_2, c_3, c_4]$$

and fuzzy capacity to store number of units

$$\mu = [w_1, w_2, w_3, w_4]$$

in OW. In this case, the total fuzzy cost per unit time is given by

$$\mu_T = \text{Fuzzy cost of placing an order} + \text{Total fuzzy holding cost at RW}$$

$$+ \text{Total fuzzy holding cost at OW} + \text{Fuzzy interest payable}$$

$$- \text{Fuzzy interest earned}$$
\[ \mathcal{J}_o(T) = \mathcal{J}_o(T) + \left[ \mathcal{J}_o(T) \cap \mathcal{J}_o(T) \right] \Theta \left[ \mathcal{J}_o(T) \cap \mathcal{J}_o(T) \right] \Theta \left[ \mathcal{J}_o(T) \cap \mathcal{J}_o(T) \right] \]

\[ + \mathcal{J}_o(T) \Theta \left( \frac{L_c T}{T} \right) \]

\[ \mathcal{J}_o(T) = \left[ \mathcal{J}_o(T) \cap \mathcal{J}_o(T) \right] \Theta \left[ \mathcal{J}_o(T) \cap \mathcal{J}_o(T) \right] \Theta \left[ \mathcal{J}_o(T) \cap \mathcal{J}_o(T) \right] \]

\[ \Theta \left[ \mathcal{J}_o(T) \cap \mathcal{J}_o(T) \right] \Theta \left[ \mathcal{J}_o(T) \cap \mathcal{J}_o(T) \right] \Theta \left[ \mathcal{J}_o(T) \cap \mathcal{J}_o(T) \right] \]

\[ ... (6.1) \]

By second function principal, one has

\[ \left( \frac{1}{2d_1T} \left[ 2a_1d_1 + \left( f_i - h_i \right) w^2 + d_1c_i T^{2} - L_c - L_c \right] \frac{d_1 f_i T}{2} + \frac{d_1 c_i T L_c}{2} \left[ \left( f_i - h_i \right) w_i + d_1c_i L_c \right] \right)^T, \]

\[ \frac{1}{2d_1T} \left[ 2a_1d_1 + \left( f_i - h_i \right) w^2 + d_1c_i T^{2} \left( L_c - L_c \right) \right] \frac{d_1 f_i T}{2} + \frac{d_1 c_i T L_c}{2} \left[ \left( f_i - h_i \right) w_i + d_1c_i L_c \right] \right)^T, \]

\[ \frac{1}{2d_1T} \left[ 2a_1d_1 + \left( f_i - h_i \right) w^2 + d_1c_i T^{2} \left( L_c - L_c \right) \right] \frac{d_1 f_i T}{2} + \frac{d_1 c_i T L_c}{2} \left[ \left( f_i - h_i \right) w_i + d_1c_i L_c \right] \right)^T, \]

\[ \frac{1}{2d_1T} \left[ 2a_1d_1 + \left( f_i - h_i \right) w^2 + d_1c_i T^{2} \left( L_c - L_c \right) \right] \frac{d_1 f_i T}{2} + \frac{d_1 c_i T L_c}{2} \left[ \left( f_i - h_i \right) w_i + d_1c_i L_c \right] \right)^T, \]

... (6.2)

Now we defuzzify the total cost per unit time, using graded mean integration representation method, the result is

\[ \mathcal{P} \mathcal{J}_o(T) = \frac{1}{6} \left( \frac{1}{2d_1T} \left[ 2a_1d_1 + \left( f_i - h_i \right) w^2 + d_1c_i T^{2} \left( L_c - L_c \right) \right] \frac{d_1 f_i T}{2} + \frac{d_1 c_i T L_c}{2} \left[ \left( f_i - h_i \right) w_i + d_1c_i L_c \right] \right) \]

\[ + \frac{1}{d_1T} \left[ 2a_1d_1 + \left( f_i - h_i \right) w^2 + d_1c_i T^{2} \left( L_c - L_c \right) \right] \frac{d_1 f_i T}{2} + \frac{d_1 c_i T L_c}{2} \left[ \left( f_i - h_i \right) w_i + d_1c_i L_c \right] \right)^T \]
\[ + \frac{1}{d_3} \left[ 2a_i d_4 + \left( f_i - h_i \right) w_i^2 + d^2_i c_3 T^{2} \left( f_i - h_i \right) \right] + d_3 f_j T + d_i c_j T \left( 2 \left( f_i - h_i \right) w_i + d_i c_i I \right) \]

\[ + \frac{1}{2d_4} \left[ 2a_i d_4 + \left( f_i - h_i \right) w_i^2 + d^2_i c_3 T^{2} \left( f_i - h_i \right) \right] + d_4 \left( \frac{T}{2} \right) + d_i c_i T \left( 2 \left( f_i - h_i \right) w_i + d_i c_i I \right) \]

\[ \frac{1}{2} \]

For minimum total cost per unit time, the optimum value of \( T = T_{10} \) will be solution

\[
dP \left( T_{10} \right) / dT = 0
\]

of the \( T_{10} \), which gives

\[
\left( \frac{1}{T^2} \right) \left[ \frac{1}{2d_4} \left[ 2a_i d_4 + \left( f_i - h_i \right) w_i^2 + d^2_i c_3 T^{2} \left( f_i - h_i \right) \right] - \frac{1}{d_3} \left[ 2a_i d_4 + \left( f_i - h_i \right) w_i^2 + d^2_i c_3 T^{2} \left( f_i - h_i \right) \right] \right]
\]

\[
- \frac{1}{d_2} \left[ 2a_i d_4 + \left( f_i - h_i \right) w_i^2 + d^2_i c_3 T^{2} \left( f_i - h_i \right) \right]
\]

\[
\frac{d_4 f_j T + d_i c_j T}{2} = 0
\]

\[
T_{10} = \sqrt[1/2]{\left[ \frac{1}{d_4} \left[ 2a_i d_4 + \left( f_i - h_i \right) w_i^2 + d^2_i c_3 T^{2} \left( f_i - h_i \right) \right] + \frac{2}{d_3} \left( 2a_i d_4 + \left( f_i - h_i \right) w_i^2 + d^2_i c_3 T^{2} \left( f_i - h_i \right) \right) \right]}
\]

\[
+ \frac{2}{d_2} \left[ 2a_i d_4 + \left( f_i - h_i \right) w_i^2 + d^2_i c_3 T^{2} \left( f_i - h_i \right) \right] + \frac{1}{d_2} \left[ 2a_i d_4 + \left( f_i - h_i \right) w_i^2 + d^2_i c_3 T^{2} \left( f_i - h_i \right) \right]
\]

\[
\frac{d_4 f_j T + d_i c_j T}{2}
\]

\[ \left[ d_4 \left( f_i + c_j I \right) + 2d_3 \left( f_i + c_i I \right) + 2d_2 \left( f_i + c_j I \right) + d_2 \left( f_i + c_j I \right) \right]
\]

\[ \frac{1}{2} \]

\[
P \left( T_{10} \right) = \frac{1}{6} \left[ \frac{1}{d_4} \left[ 2a_i d_4 + \left( f_i - h_i \right) w_i^2 + d^2_i c_3 T^{2} \left( f_i - h_i \right) \right] + \frac{2}{d_3} \left( 2a_i d_4 + \left( f_i - h_i \right) w_i^2 + d^2_i c_3 T^{2} \left( f_i - h_i \right) \right) \right]
\]

And minimum total cost per unit time is

\[
P \left( T_{10} \right) = \frac{1}{6} \left[ \frac{1}{d_4} \left[ 2a_i d_4 + \left( f_i - h_i \right) w_i^2 + d^2_i c_3 T^{2} \left( f_i - h_i \right) \right] + \frac{2}{d_3} \left( 2a_i d_4 + \left( f_i - h_i \right) w_i^2 + d^2_i c_3 T^{2} \left( f_i - h_i \right) \right) \right]
\]

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\[ + \frac{2}{d_2} \left[ 2a d_4 + f_3 - h_1 \right] w_1^2 + d_2 c T^2 (I_c - I_c) + \frac{b}{d_1} 2a d_4 + \left( f_4 - h_2 \right) w_2^2 + d_3 c T^2 (I_c - I_c) \right] \]
\[
\left( d_1 \left( f_1 + c_1 I_c \right) + 2d_2 \left( f_2 + c_2 I_c \right) + d_3 \left( f_3 + c_3 I_c \right) \right)^{1/2}
\]
\[- \left( \left( f_4 - h_1 \right) w_1 + d_4 c_4 I_c T \right) + \left( \left( f_4 - h_2 \right) w_2 + d_4 c_4 I_c T \right) \]
\[
+ 2 \left( f_2 - h_1 \right) w_1 + d_4 c_4 I_c T \right] \left( f_1 - h_1 \right) w_1 + d_4 c_4 I_c T \right) \}
\]

(6.5)

Then the optimum order quantity is
\[ Q_{\text{OPT}}(T_{10}) = \sqrt{\frac{1}{6} d_1 T_{10} + 2d_2 T_{10} + 2d_3 T_{10} + 2d_4 T_{10}} \]

... (6.6)

**Special Case-1:** When costs, demand and capacity to store number of units in OW are real
\[ d_1 = d_2 = d_3 = d_4 = D, \quad f_1 = f_2 = f_3 = f_4 = F, \quad h_1 = h_2 = h_3 = h_4 = H, \]
numbers, they are
\[ a_1 = a_2 = a_3 = a_4 = A, \quad c_1 = c_2 = c_3 = c_4 = C, \quad w_1 = w_2 = w_3 = w_4 = W \]

Then the optimum is
\[ T = T_{10} \]

\[ T_{10} = \left[ 2AD + F - H \right] W^2 + D^2 c T^2 (I_c - I_c) \left( \left( F + C \right) \right)^{1/2} \]

... (6.7)

Then optimum order quantity is
\[ Q_{\text{OPT}}(T_{10}) = \left[ 2AD + \left( F - H \right) W^2 + D^2 c T^2 (I_c - I_c) \right] \left( \left( F + C \right) \right)^{1/2} \]

... (6.8)

And minimum cost \( Z_i(T_{10}) \) per unit is
\[ Z_i(T_{10}) = \left[ 2AD + \left( F - H \right) W^2 + D^2 c T^2 (I_c - I_c) \right] \left( F + C \right) \]
\[ - \left[ \left( F - H \right) W + DC I_c T \right] \]

... (6.9)
Thus when all the fuzzy parameters are taken in crisp environment then above equations are same as given by Shah and Shah (1992).

**Special Case-2:** When we take holding cost in both the warehouses to be same \((H = F)\), optimum values of \(T\)

\[
T_{10} = \left[ \frac{1}{d_1} \left[ 2a_1d_1 + d_1^2c_1T^{r_2} (I - I_c) \right] + \frac{2}{d_3} \left[ 2a_2d_2 + d_2^2c_2T^{r_2} (I - I_c) \right] \right]^{1/2}
\]

\[
+ \frac{2}{d_2} \left[ 2a_3d_3 + d_3^2c_3T^{r_2} (I - I_c) \right] + \frac{1}{d_4} \left[ 2a_4d_4 + d_4^2c_4T^{r_2} (I - I_c) \right] \right) \Bigg]^{1/2}
\]

\[
\left( d_1(h_1 + c_1I_c) + 2d_2(h_2 + c_2I_c) + 2d_3(h_3 + c_3I_c) + d_4(h_4 + c_4I_c) \right) \Bigg]^{1/2}
\]

\[
\tilde{Q}_1(T_{10}) = \tilde{B} \otimes T_{10} = \frac{1}{6} \left( d_1T_{10} + 2d_2T_{10} + 2d_3T_{10} + d_4T_{10} \right)
\]

where \(T_{10}\) is given above.

When all costs are real numbers, one has

\[
T_{10} = \left[ \left[ 2AD + D^2CT^{r_2} I_c - I_c \right] \right]^{1/2}
\]

\[
Q_1(T_{10}) = \left[ 2AD + D^2CT^{r_2} I_c - I_c \right] \right]^{1/2}
\]

\[
Z_1(T_{10}) = \left[ 2AD + D^2CT^{r_2} (I_c - I_c) \right] \right]^{1/2} - DCI^*T^*
\]

and \(\ldots\) (6.13)
When all the parameters are real numbers and the holding cost of both warehouses are the same then the above results are same as given by Goyal (1985).

**Special Case-3:** When we take holding cost in both the warehouses is same, i.e., \( H = F \), \( I_c = I_s = 0 \), then optimum values of \( T \) is

\[
T_{10} = \left[ \frac{2a_1d_1a + 2a_2d_2 + 2a_3d_3 + 2a_4d_4}{d_1h_1 + 2d_2h_2 + 2d_3h_3 + d_4h_4} \right]^{1/2}
\]

...(6.14)

\[
P(\bar{Z}_1, T_{10}) = \frac{1}{6} \left[ \left( \frac{1}{d_1} 2a_1d_1 + \frac{2}{d_2} 2a_2d_2 + \frac{1}{d_3} 2a_3d_3 + \frac{1}{d_4} 2a_4d_4 \right) \right]
\]

\[
= \left( d_1h_1 + 2d_2h_2 + 2d_3h_3 + d_4h_4 \right)^{1/2}
\]

...(6.15)

\[
Q_1(T_{10}) = 2^{\frac{3}{2}} \left( T_{10} \right) = \frac{1}{6} \left( d_1T_{10} + 2d_2T_{10} + 2d_3T_{10} + d_4T_{10} \right)
\]

and

\[
T_{10}
\]

where \( T_{10} \) is given above.

When all costs real numbers, one has

\[
T_{10} = \left( \frac{2A}{DH} \right)^{1/2} \quad Q_1(T_{10}) = \left( \frac{2AD}{H} \right)^{1/2} \quad Z_1(T_{10}) = \left( \frac{2ADH}{I_c} \right) \quad Z_1(T_{10}) = \left( \frac{2AD}{I_c} \right)
\]

...(6.16)

These results are similar to those of single storage EOQ model, i.e., these equations are same as those of classical EOQ model of Naddor (1966).

**Special Case-4:** Suppose that we do not wish to use RW at all and then we order \( \frac{W}{W} \) units per replenishment, i.e., we take

\[
DT_i = \frac{W}{W} \quad T_i = \frac{W}{D}
\]

where \( T_i \) is cycle time. In this case the total cost per unit time due to OW is

\[
Z_i(T_i) = \left[ 2AD \otimes \tilde{D} \otimes \tilde{D} \otimes \tilde{C} \otimes T^{\frac{3}{2}} \right] \left( I_c - I_s \right) \quad \Phi \left[ 2^{\frac{W}{W}} \right]
\]

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\[ \Theta \left[ \left( \widetilde{W} \otimes I_{L} \right) + \left( \widetilde{W} \otimes \widetilde{C} \otimes I_{T} \right) \right] \otimes \left( D \otimes \widetilde{C} \otimes I_{T} \right) \]

...(6.17)

By second function principle, one has

\[ Z_{i}^{*} = T_{i}^{*} = \left( \frac{1}{2W_{1}} \left[ 2a_{i}d_{i} + d_{i}c_{i}T^{*2} \left( I_{i} - I_{e} \right) + \frac{w_{i}}{2} \left( h_{i} + c_{i}I_{e} \right) - d_{i}c_{i}I_{e}T_{i}^{*} \right] + \right. \]

\[ + \frac{1}{2W_{2}} \left[ 2a_{i}d_{i} + d_{i}c_{i}T_{i}^{*} \left( I_{i} - I_{e} \right) + \frac{w_{i}}{2} \left( h_{i} + c_{i}I_{e} \right) - d_{i}c_{i}I_{i}T_{i}^{*} \right] \]

\[ + \frac{1}{2W_{3}} \left[ 2a_{i}d_{i} + d_{i}c_{i}T^{*} \left( I_{i} - I_{e} \right) + \frac{w_{i}}{2} \left( h_{i} + c_{i}I_{e} \right) - d_{i}c_{i}I_{i}T_{i}^{*} \right] \]

\[ \left. + \frac{1}{2W_{4}} \left[ 2a_{i}d_{i} + d_{i}c_{i}T^{*2} \left( I_{i} - I_{e} \right) + \frac{w_{i}}{2} \left( h_{i} + c_{i}I_{e} \right) - d_{i}c_{i}I_{i}T_{i}^{*} \right] \right) \]

...(6.18)

Now we defuzzify the total cost per unit time, using graded mean integration representation method, the result is

\[ P = \sum_{i} Z_{i} = \left( \frac{1}{2W_{1}} \left[ 2a_{i}d_{i} + d_{i}c_{i}T^{*2} \left( I_{i} - I_{e} \right) + \frac{w_{i}}{2} \left( h_{i} + c_{i}I_{e} \right) - d_{i}c_{i}I_{e}T_{i}^{*} \right] + \right. \]

\[ + \frac{1}{2W_{2}} \left[ 2a_{i}d_{i} + d_{i}c_{i}T_{i}^{*} \left( I_{i} - I_{e} \right) + \frac{w_{i}}{2} \left( h_{i} + c_{i}I_{e} \right) - d_{i}c_{i}I_{i}T_{i}^{*} \right] \]

\[ + \frac{1}{2W_{3}} \left[ 2a_{i}d_{i} + d_{i}c_{i}T^{*} \left( I_{i} - I_{e} \right) + \frac{w_{i}}{2} \left( h_{i} + c_{i}I_{e} \right) - d_{i}c_{i}I_{i}T_{i}^{*} \right] \]

\[ + \frac{1}{2W_{4}} \left[ 2a_{i}d_{i} + d_{i}c_{i}T^{*2} \left( I_{i} - I_{e} \right) + \frac{w_{i}}{2} \left( h_{i} + c_{i}I_{e} \right) - d_{i}c_{i}I_{i}T_{i}^{*} \right] \]

...(6.19)

If the cost in (6.5) is less than (6.19), therefore it is better to hire RW services.

### 6.4.2 Case-II: Determination of Order Quality When $T^{*} > T$ i.e., when the permissible delay period is greater than the cycle length

In this situation, since the permissible delay period is greater than the cycle length retailer only earn the interest. The retailer starts selling products at time 0 and receiving the money at time $T$. Consequently, the retailer accumulates, sells revenue in an account that earns $I_{e}$ per
dollar per year starting from T to T*. In this case interest charges are not paid for the items kept in stock. The total fuzzy cost per unit time is given by

\[
\begin{align*}
\hat{c}_2(T) = & \left[ 2 \beta \otimes \beta \otimes \left( \hat{P} \otimes \hat{H}_2 \right) \otimes \hat{W} \otimes \hat{W} \phi \left[ \beta \otimes \beta \otimes 2I \right] \otimes \left[ \beta \otimes \beta \otimes I \otimes I \otimes I \right] \right] \\
& \Theta \left[ \beta \otimes \beta \otimes \hat{W} \otimes \hat{W} \phi \left[ \beta \otimes \beta \otimes \beta \otimes \beta \otimes I \otimes I \right] \right] \end{align*}
\]

...(6.20)

Now we defuzzify the total cost per unit time using graded mean integration representation method the result is

\[
P\left( \hat{Z}_2, T \right) = \frac{1}{6} \left( \frac{1}{T} \left[ \frac{1}{2d_z} \left[ 2a_z d_i + \left( f_i - h_i \right) w_i \right] + \frac{T}{2} \left[ 2a_z d_i + \left( f_i - h_i \right) w_i \right] \phi \left[ \beta \otimes \beta \otimes 2I \right] \otimes \left[ \beta \otimes \beta \otimes I \otimes I \otimes I \right] \right] \\
+ \frac{1}{d_z} \left[ 2a_z d_i + \left( f_i - h_i \right) w_i \right] + \frac{T}{2d_i} \left[ 2w_i d_i + \left( f_i - h_i \right) w_i \right] \phi \left[ \beta \otimes \beta \otimes \beta \otimes \beta \otimes I \otimes I \right] \right) \\
+ T \left[ \frac{d_i}{f + c I} \frac{d_i}{f + c I} \right] \\
- \left[ \left( f_i - h_i \right) w_i + d_i c I \right] \\
+ 2 \left[ \left( f_i - h_i \right) w_i + d_i c I \right] + \left( f_i - h_i \right) w_i + d_i c I \right] \\
+ 2 \left[ \left( f_i - h_i \right) w_i + d_i c I \right] + \left( f_i - h_i \right) w_i + d_i c I \right] \\
\right)
\]

...(6.21)

For minimum total cost per unit time, the optimum value of \( \frac{\partial P}{\partial T} \) will be the solution of:

\[
\begin{align*}
\frac{\partial P}{\partial T} = & \left( \frac{-1}{T^2} \left[ \frac{1}{2d_z} \left[ 2a_z d_i + \left( f_i - h_i \right) w_i \right] + \frac{T}{2} \left[ 2a_z d_i + \left( f_i - h_i \right) w_i \right] \phi \left[ \beta \otimes \beta \otimes 2I \right] \otimes \left[ \beta \otimes \beta \otimes I \otimes I \otimes I \right] \right] \\
& + \frac{1}{d_z} \left[ 2a_z d_i + \left( f_i - h_i \right) w_i \right] + \frac{T}{2d_i} \left[ 2w_i d_i + \left( f_i - h_i \right) w_i \right] \phi \left[ \beta \otimes \beta \otimes \beta \otimes \beta \otimes I \otimes I \right] \right) \\
& + T \left[ \frac{d_i}{f + c I} \frac{d_i}{f + c I} \right] \\
& - \left[ \left( f_i - h_i \right) w_i + d_i c I \right] \\
& + 2 \left[ \left( f_i - h_i \right) w_i + d_i c I \right] + \left( f_i - h_i \right) w_i + d_i c I \right] \\
\right) = 0
\]

...
\[
\tilde{T}_{20} = \left[ \frac{1}{2d_s} \left[ 2a_d d_1 + f_1 - h_d \right] w_i^2 + \frac{1}{d_s} 2a_d d_2 + (f_2 - h_d) \right] \frac{1}{\sqrt{2}} \\
+ \frac{1}{d_s} \left[ 2a_d d_3 + (f_3 - h_d) \right] w_i^2 + \frac{1}{2d_s} 2a_d d_4 + (f_4 - h_d) \right] \frac{1}{\sqrt{2}} \\
\frac{1}{d_s} \left[ \frac{1}{2} \left( f_1 + c_l I_c \right) + d_2 \left( f_2 + c_l I_c \right) + d_3 \left( f_3 + c_l I_c \right) + \frac{1}{2} (f_4 + c_l I_c) \right]
\]

The optimum order quantity

\[
Q_{20} \left( \tilde{T}_{20} \right) = \left[ \frac{1}{2d_s} \left[ 2a_d d_1 + f_1 - h_d \right] w_i^2 + \frac{1}{d_s} 2a_d d_2 + (f_2 - h_d) \right] \frac{1}{\sqrt{2}} \\
+ \frac{1}{d_s} \left[ 2a_d d_3 + (f_3 - h_d) \right] w_i^2 + \frac{1}{2d_s} 2a_d d_4 + (f_4 - h_d) \right] \frac{1}{\sqrt{2}} \\
\frac{1}{d_s} \left[ \frac{1}{2} \left( d_1 \left( f_1 + c_l I_c \right) + 2d_2 \left( f_2 + c_l I_c \right) + 2d_3 \left( f_3 + c_l I_c \right) + d_4 \left( f_4 + c_l I_c \right) \right) \right]
\]

\[P \left( Z_2 (T_{20}) \right) \]

And minimum cost per unit time is

\[
P \left( Z_2 (T_{20}) \right) = \frac{1}{6} \left[ \frac{1}{d_s} \left[ 2a_d d_1 + f_1 - h_d \right] w_i^2 + \frac{1}{d_s} 2a_d d_2 + (f_2 - h_d) \right] \frac{1}{\sqrt{2}} \\
+ \frac{1}{d_s} \left[ 2a_d d_3 + (f_3 - h_d) \right] w_i^2 + \frac{1}{2d_s} 2a_d d_4 + (f_4 - h_d) \right] \frac{1}{\sqrt{2}} \\
- \left[ 2 (f_3 - h_d) w_i + d_3 c_l I_c \right] + \frac{1}{2} (f_3 - h_d) w_i + d_3 c_l I_c \right] \\
+ 2 \left( f_3 - h_d \right) w_i + d_3 c_l I_c \right] \left( f_3 - h_d \right) w_i + d_3 c_l I_c \right] \right] \frac{1}{\sqrt{2}} \\
\]

\[\ldots(6.24)\]

**Special Case-1:** When costs, demand and capacity to store number of units in OW are real

\[d_1 = d_2 = d_3 = d_4 = D, f_1 = f_2 = f_3 = f_4 = F, h_1 = h_2 = h_3 = h_4 = H,\]

numbers, they are

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\[ a_1 = a_2 = a_3 = a_4 = A, \quad c_1 = c_2 = c_3 = c_2 = C, \quad w_1 = w_2 = w_3 = w_4 = W \]

then the optimum of

\[ T = T_2 \]

is

\[ T_{20} = \left[ 2AD + (F - H)W \right]\left[ D^2 \left( F + C \right) \right]^{1/2} \]

...(6.25)

Then optimum order quantity is

\[ Q_2(T_{20}) = DT_{20} = \left[ 2AD + (F - H)W \right] \left[ D^2 \left( F + C \right) \right]^{1/2} \]

...(6.26)

And minimum cost \( Z_2(T_{20}) \) per unit is

\[ Z_2(T_{20}) = \left[ 2AD + (F - H)W \right] \left[ D^2 \left( F + C \right) \right]^{1/2} \left[ F - H \right] W + DCF \]

...(6.27)

Thus when all the fuzzy parameters are taken in crisp environment then above equations are same as given by Shah and Shah (1992).

**Special Case-2:** When we take holding cost in both warehouses are same (\( H = F \)), optimum values of \( T \) is

\[ \tilde{T}_{20} = \left[ \frac{1}{2d_4} \cdot 2a_1 d_1 + \frac{1}{d_5} \cdot 2a_2 d_2 + \frac{1}{d_3} \cdot \left[ 2a_3 d_3 + \frac{1}{2d_1} \cdot 2a_4 d_4 \right] \right] \left[ \frac{d_1}{2} \cdot h + c_1 I_1 + d_2 \cdot h_2 + c_2 I_2 + d_3 \cdot \left[ h_3 + c_3 I_3 \right] + \frac{d_4}{2} \cdot \left[ h_4 + c_4 I_4 \right] \right]^{1/2} \]

...(6.28)

\[ Q_2 \left( \tilde{T}_{20} \right) = \left[ \frac{1}{2d_4} \cdot 2a_1 d_1 + \frac{1}{d_5} \cdot 2a_2 d_2 + \frac{1}{d_3} \cdot 2a_3 d_3 + \frac{1}{2d_1} \cdot 2a_4 d_4 \right] \]

...(6.29)

\[ P \left( \tilde{Z}_2(T_{20}) \right) = \frac{1}{6} \left\{ \left[ \frac{1}{d_4} \cdot 2a_1 d_1 + \frac{2}{d_5} \cdot 2a_2 d_2 + \frac{2}{d_3} \cdot 2a_3 d_3 + \frac{1}{d_1} \cdot 2a_4 d_4 \right] \right\}^{1/2} \]

\[ \left[ d_1 c_1 I_1 \right] + \left[ 2d_2 c_2 I_2 \right] + \left[ d_3 c_3 I_3 \right] + \left[ 2d_4 c_4 I_4 \right] \]

...(6.30)
When all costs real numbers, one has
\[ T_{20} = \left[ \frac{2A}{D(\lambda + CL_c)} \right]^{1/2}, \quad Q_2 \left( T_{20} \right) = \left[ 2AD/(\lambda + CL_c) \right]^{1/2} \]
and
\[ Z_2(T_{20}) = \left[ 2AD(H + CL_c) \right]^{1/2} - DCL_c T' \]

...(6.31)

When all the parameters are real numbers and the holding cost of both warehouses are same then the above results are same as given by Goyal (1985).

**Special Case-3:** When we take holding cost in both warehouses as same i.e. \( H = F, L_c = 0 \) and \( T' = 0 \), optimum values of \( T \) is

\[ \tilde{T}_{20} = \left[ \frac{1}{2d_z} \left[ \frac{2a_i d_1}{d_z} + \frac{1}{d_z} \left[ \frac{2a_i d_1}{d_z} + \frac{1}{d_z} \left[ \frac{2a_i d_1}{d_z} + \frac{1}{2d_z} \left[ \frac{2a_i d_1}{d_z} \right] \right] \right] \right] \]

...(6.32)

\[ Q_2 \left( \tilde{T}_{20} \right) = \left[ \frac{1}{2d_z} \left[ \frac{2a_i d_1}{d_z} + \frac{1}{d_z} \left[ \frac{2a_i d_1}{d_z} + \frac{1}{d_z} \left[ \frac{2a_i d_1}{d_z} + \frac{1}{d_z} \left[ \frac{2a_i d_1}{d_z} \right] \right] \right] \right] \]

...(6.33)

\[ P \left( \tilde{T}_{20} \right) = \frac{1}{6} \left\{ \left[ \frac{1}{d_z} \left[ \frac{2a_i d_1}{d_z} + \frac{1}{d_z} \left[ \frac{2a_i d_1}{d_z} + \frac{2}{d_z} \left[ \frac{2a_i d_1}{d_z} + \frac{1}{d_z} \left[ \frac{2a_i d_1}{d_z} \right] \right] \right] \right] \right\}^{1/2} \]

...(6.34)

When all costs real numbers, one has
\[ T_{30} = \left[ \frac{2A/ DH}{1/2}, \quad Q_2 \left( T_{30} \right) = \left[ \frac{2AD}{H} \right]^{1/2} \]
\[ Z_2 \left( T_{30} \right) = 2ADH \]

and

...(6.35)

These results are similar to those of single storage EOQ model i.e. these equations are same as those of classical EOQ model of Naddor (1966).
\[ DT_2 = W \quad \quad T'_2 = W/D \]

**Special Case-4:** If we do not wish to use RW, then taking cycle time. In this case the total cost per unit time due to OW is

\[ Z'_2(T'_2) = \left[ (L \otimes B^T \otimes \hat{W}^T) \otimes (\hat{W} \otimes C \otimes I_i / 2) \right] \Theta \left( D \otimes C \otimes I_i T'_2 \right) \]

By second function principle, one has

\[ Z'_2(T'_2) = \left( a_1 d_1 + \frac{w_1}{2} h_i + c_i I_i \right) - d_1 c_i I_i T'_2^*, \quad a_2 d_2 + \frac{w_2}{2} h_i + c_i I_i \right) - d_2 c_i I_i T'_2^*, \]

\[ \frac{a_3 d_3 + w_3}{2} h_i + c_i I_i \right) - d_2 c_i I_i T'_2^*, \quad a_4 d_4 + \frac{w_4}{2} h_i + c_i I_i \right) - d_4 c_i I_i T'_2^* \right) \]

...(6.36)

\[ \frac{a_1 d_1 + w_1}{2} h_i + c_i I_i \right) - d_1 c_i I_i T'_2^* + 2 a_2 d_2 + w_2 \left( h_i + c_i I_i \right) - 2 d_2 c_i I_i T'_2^* \]

\[ + 2 a_3 d_3 + w_3 \left( h_i + c_i I_i \right) - 2 d_3 c_i I_i T'_2^* + \frac{a_4 d_4}{w_4} \left( h_i + c_i I_i \right) - d_4 c_i I_i T'_2^* \right) \]

...(6.37)

Now we defuzzify the total cost per unit time, using graded mean integration representation method, the result is

\[
\begin{align*}
P(Z'_2(T'_2)) &= \left( a_1 d_1 + \frac{w_1}{2} h_i + c_i I_i \right) - d_1 c_i I_i T'_2^* + 2 a_2 d_2 + \frac{w_2}{2} h_i + c_i I_i \right) - 2 d_2 c_i I_i T'_2^* \\
&\quad + 2 a_3 d_3 + \frac{w_3}{2} \left( h_i + c_i I_i \right) - 2 d_3 c_i I_i T'_2^* + \frac{a_4 d_4}{w_4} \left( h_i + c_i I_i \right) - d_4 c_i I_i T'_2^* \right) \\
\end{align*}
\]

...(6.38)

If the cost in (6.38) is less than (6.24), therefore it is better to hire RW services.

### 6.5 Numerical Example

Let us consider the inventory system that has:

- Holding cost in RW = Rs. 2.5, Holding cost in OW = Rs. 1.5,
- Purchasing cost = Rs. 20, \( l_c = 0.30 \), \( l = 0.15 \),
- Capacity to store number of units in OW = 1000 units,
- Ordering cost = 500, Demand rate = 15000 units per annum.
- Values of time interval, order quantity and total minimum cost with prescribed permissible delay period and increasing order cost are given by:
Table 6.1 When $T^* = 0.0634 < T$

<table>
<thead>
<tr>
<th>A</th>
<th>$T_{1_0}$</th>
<th>$D_{T_{10}}$</th>
<th>$Z_2(T_{10})$</th>
<th>$T_{10}$</th>
<th>$Z_1'(T_2')$</th>
<th>Ratio $Z_2(T_{10}) / Z_1'(T_2')$</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>0.0989</td>
<td>1483.50</td>
<td>5905.98</td>
<td>0.0667</td>
<td>6900.60</td>
<td>0.8559</td>
</tr>
<tr>
<td>1000</td>
<td>0.1327</td>
<td>1991.54</td>
<td>10222.15</td>
<td>0.0667</td>
<td>14400.60</td>
<td>0.7098</td>
</tr>
<tr>
<td>1500</td>
<td>0.1595</td>
<td>2392.50</td>
<td>13642.52</td>
<td>0.0667</td>
<td>21900.60</td>
<td>0.6229</td>
</tr>
</tbody>
</table>

Fig. 6.5 Variation of total cost for two storage facilities when $D_{T_{10}} = W$ with ordering cost when $T^* < T$

Table 6.2 When $T^* = 0.2536 > T$

<table>
<thead>
<tr>
<th>A</th>
<th>$T_{1_0}$</th>
<th>$D_{T_{10}}$</th>
<th>$Z_2(T_{10})$</th>
<th>$T_{10}$</th>
<th>$Z_2'(T_2')$</th>
<th>Ratio $Z_2(T_{10}) / Z_2'(T_2')$</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>0.1454</td>
<td>2181.00</td>
<td>5985.68</td>
<td>0.0667</td>
<td>6748.50</td>
<td>0.0615</td>
</tr>
<tr>
<td>1000</td>
<td>0.1702</td>
<td>3585.68</td>
<td>14248.50</td>
<td>0.0667</td>
<td>14248.50</td>
<td>0.2516</td>
</tr>
<tr>
<td>1500</td>
<td>0.1918</td>
<td>2877.00</td>
<td>6347.17</td>
<td>0.0667</td>
<td>21748.50</td>
<td>0.2958</td>
</tr>
</tbody>
</table>

Fig. 6.6 Variation of total cost for two storage facilities when $D_{T_{10}} = W$ with ordering cost when $T^* > T$

6.6 Observations
Table 6.1 / Fig. 6.5 showed values of $T_{10}$, $DT_{10}$, $Z_4 (T_{10})$ for two storage facilities and when $DT_{10} = W$, for ordering cost variation. For a system with two storage facilities, it was found that the $Q_{10} = DT_{10}$ and $Z_4 (T_{10})$ increased as ‘A’ increased. Moreover, it was

then that where RW was not used, which suggested that whenever necessary it was economical to hire RW. Similar observations were also be made when $T^* > T$. But comparing the two cases the investigator found that the total minimum cost when $T^* > T$ was very low as compared to the case when $T^* < T$.

6.7 Conclusion

In this chapter, the researcher has proposed a policy for a two-warehouse inventory system with fuzzy inventory costs under the conditions of permissible delay in payments. Here, the investigator obtained the membership function of the total fuzzy inventory cost when the demand quantity, unit holding cost, ordering cost, purchasing cost and the capacity to store items were fuzzy numbers. In the present study, the researcher considered the permissible delay in payments, and its effect on optimal ordering quantity of two-warehouse facilities models for realistic inventory system. Four different cases for different realistic practical situations are discussed as follows; (1) when the inventory system has both the warehouse facilities, (2) when the owned warehouse has large capacity to store the on-hand inventory, (3) when simple EOQ model of single storage system and (4) when one does not wish to take RW services and OW has maximum capacity.

Finally, numerical example was solved to illustrate the feasibility of different cases observed. It was observed that an EOQ model with permissible delay period with single total optimum cost for the system with two storage facilities was less as compared to single storage facilities.

When all the costs parameters are real numbers, in this situation the above model reduced to the model given by Shah and Shah (1992). For the conditions, when the holding cost was same in the both warehouses, the model reduced to the model given by Goyal (1985). Again, when we took the holding cost in both warehouses as same and did
not consider the effect of permissible delay in payment; the model reduced to the model
given by Naddor (1966).

Thus, the proposed model incorporates some realistic features that are likely to be
associated with some kinds of inventory. The model is very useful in the retail business
such as electronic components, fashionable clothes, domestic goods and other products
which are more likely with the characteristics above. The model is best fit for the real life
situations in which the demand is not certain and the single storage facilities is insufficient
to accommodate the order level and as a result an additional warehouse is used.

For future research, it should be interesting and a challenge to consider the factors
of shortage, inflation, deterioration, stock-dependent demand rate and two stage interest
payable criterions under fuzzy environment.

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