Chapter-5
An Order-Level Inventory Model under Two-Level Storage System with Time Dependent Demand and Weibull Deterioration, Partial Backlogging under Inflation

Abstract
In this study, two-warehouse inventory models with Weibull distribution deterioration and partial backordering are developed. Here, it is assumed that the demand rate is linearly model. In addition, we consider the effect of inflation and apply the discounted cash flow in problem analysis. The discounted cash flow and optimization framework are presented to derive the optimal replenishment policy that minimizes the total present value cost per unit time. A numerical example and sensitivity analysis are presented to illustrate the model with the help of the software MATHEMATICA-5.2. Which conclude that when only rented or own warehouse is considered, the present value of the total relevant cost is higher than the case when two-warehouse is considered. At the end, particular cases are also provided.

• The paper based on the first section of this chapter has been communicated for the publication in the referred journal.
• The paper based on second section of this chapter has been published in the journal “Reflections des ERA journal of Mathematical Sciences”, Vol. 3(3), 2008, pp. 161-178. (Referenced Journal) ISSN: 0973-4587
5.1 Introduction

As inventory represents a very important part of any company’s financial assets, it is very much affected by the market’s response to various situations, especially inflation. In present day times inflation is a global phenomenon. Inflation can be defined as that state of disequilibrium in which an expansion of purchasing power tends to cause or is the effect of an increase in the price level. A period of prolonged, persistent and continuous inflation results in the economic, political, social and moral disruption of society. Today, inflation has become a permanent feature of the economy throughout the world. The basic assumption in the derivation of the classical EOQ model is that all the costs associated with the inventory system remain constant over time. Most of the inventory models developed so far does not include inflation as parameter of the system. This changing scenario in the world economy did not escape the attention of the inventory modelers.

Deterioration is the change, damage, decay, spoilage, evaporation, obsolescence, pilferage, and loss of utility or loss of marginal value of a commodity that results in decreasing usefulness from the original one. Most products such as medicine, blood, fish, alcohol, gasoline, vegetables and radioactive chemicals have finite shelf life, and start to deteriorate once they are replenished. In addition, for certain types of commodities, deterioration is usually observed during their normal storage period. In most of the inventory models unrealistically assume that during stock-out either all demand is replenishment, especially if the wait will be short, while others are more impatient and go elsewhere. The backlogging rate depends on the time to replenishment—the longer customers must wait, the greater the fraction of lost sales.

The classical inventory models usually assume that the available warehouse has unlimited capacity. In many practical situations, there exist many a factors like temporary price discounts making retailers buy a capacity of goods exceeding their own warehouse (OW). In this case, retailers will either rent other warehouses or rebuild a new warehouse.

Hence, an additional storage space known as rented warehouse (RW) is often required due to limited capacity of showroom facility. In recent years, various researchers have discussed a two-warehouse inventory system. This kind of system was first proposed by Hartely (1976). In this system, it was assumed that the holding cost in RW is greater than
that in OW. By assuming constant demand rate, Sarma (1987) developed a deterministic inventory model for a single deteriorating item with shortages and two levels of storage. Pakkala and Achary (1992) extended the two-warehouse inventory model for deteriorating items with finite replenishment rate and shortages. Besides, the ideas of time-varying demand for deteriorating items with two storage facilities were considered by Benkherouf (1997) and Bhunia and Maiti (1998).

Most researches in inventory do not consider the time-value of money. This is unrealistic, since the resource of an enterprise depends very much on time value of money and this is highly correlated to the return of investment. Therefore, taking into account the time-value of money should be critical especially when investment and forecasting are considered. Buzacott (1975) was the first author to include the concept of inflation in inventory modeling. He developed a minimum cost model for a single item inventory with inflation. Misra (1979) simultaneously considered both the inflation and the time-value of money for internal as well as external inflation rate, and analyzed the influence of interest rate and inflation rate on replenishment strategy. Chandra and Bahner (1985) extended the result in Misra’s (1979) model to allow for shortages. Sarkar and Pan (1994) assumed a finite replenishment model and analyzed the effects of inflation and time-value of money on order quantity in which shortages were allowed. Hariga (1995) extended the study to analyze the effects of inflation and time-value of money on an inventory model with time-dependent demand rate and shortages. Bose et al. (1995) developed an EOQ model for deteriorating items with linear time-dependent demand rate and shortages under inflation and time discounting. Van Delft and Vial (1996) proposed a simple economic order quantity model for inventory with a short and stochastic lifetime. Their approach was performed in the framework of the total discounted cost criterion. Moon and Lee (2000) discussed different types of inventory models with the effects of inflation. First time the effect of inflation in two-warehouse system was discussed by Yang (2004). In her study, she considered the constant rate of deterioration in both warehouses with constant demand rate and shortages were completely backlogged. Wee et al. (2005) developed two-warehouse inventory model with partial backordering and Weibull distribution deterioration under inflation. In their study, they assumed that the demand and backlogging rate are constant but is not always the case in real life. Dey et al. (2008) developed two storage inventory problems with inflation and time-value of money. Ghosh
and Chakrabarty (2009) suggested an order-level inventory model with two levels of storage for deteriorating items and shortages were fully backlogged. Recently, Jaggi and Verma (2010) developed a two-warehouse inventory model with linear trend in demand under the inflationary conditions. Shortage was allowed and completely backlogged.

It has been observed that most researchers on inventory models do not consider time-varying rate of deterioration, inflation and partial backordering simultaneously. Since this phenomenon is not uncommon in real life, the researchers should also incorporate them in his problem development. In this chapter, the researcher has considered a two-warehouse inventory system for deteriorating items. Here, shortages are allowed and partially backlogged. The holding cost at RW is higher as compared to OW. The rate of deterioration in both warehouses is different and follows a two-parameter Weibull distribution. Further, this study takes inflation and applies the discounted cash flow (DCF) approach for problem analysis. In this study, two types of models were developed and are; in model- 1, the demand rate is linearly increasing with time and shortages are partially backlogged with constant backlogging rate. In model-2, the demand rate exponentially increased with time and shortages are partially backlogged with exponential backlogging rate. The discounted cash flow and optimization framework are presented to derive the optimal replenishment policy that minimizes the total present value cost. A numerical example and sensitivity analysis are presented to illustrate all the models. When only rented or own warehouse is considered, the present value of the total relevant cost is higher than the case when two-warehouse is considered. From the sensitivity analysis, it was worked out that the total cost of the system is influenced by the deterioration rate, the inflation rate and the backordering ratio.

5.2 Assumptions and Notations

In developing the mathematical models of the inventory system for this chapter, the following common assumptions were used:

1. Shortages are partially backlogged.
2. Deterioration of the item follows a two-parameter Weibull distribution.
3. Deterioration occurs as soon as items are received into inventory.
4. There is no replacement or repair of deteriorating items during the period under consideration.
5. Product transactions are followed by instantaneous cash flow.
6. The holding costs in RW are higher than those in OW.
   7. The OW has a fixed capacity of \( W \) units and the RW has unlimited capacity.
8. Lead-time is zero and initial inventory level is zero.
9. The replenishment rate is infinite.

The following notations were used throughout the chapter:

\( W \)  
Capacity of OW

\( \alpha \)  
Scale parameter of the deterioration rate in OW
Shape parameter of the deterioration rate in OW

\( g \)  
Scale parameter of the deterioration rate in RW, \( \alpha > g \)

\( h \)  
Shape parameter of the deterioration rate in RW

\( r \)  
Inflation rate

\( A \)  
Ordering cost per order ($/order)

\( H \)  
Holding cost per unit per unit time in OW ($/unit/unit time)

\( C_{r2} > C_{r2} \)

\( F \)  
Holding cost per unit per unit time in RW ($/unit/unit time),

\( s \)  
Shortage cost per unit time ($/unit/unit time)

\( \pi \)  
Shortage cost for lost sales per unit ($/unit)

\( C \)  
Item cost per unit ($/unit)

\( Q_0 \)  
The order quantity in OW

\( Q_r \)  
The order quantity in RW

\( I_r \)  
Maximum inventory level in RW

\( T_2 \)  
Time with positive inventory in RW

\( T_1 + T_2 \)  
Time with positive inventory in OW
\[ T_3 \]
Time when shortage occurs in OW

\[ T = T_1 + T_2 + T_3 \]
Length of the cycle,

\[ l_{0i}(t_i) \]
Inventory level in OW at time \( t_i \) of cycle \( i \) where \( i = 1, 2, 3 \)

\[ l_{f}(t_i) \]
Inventory level in RW at time \( t_i \) of cycle \( i \) where \( i = 1 \)

TUC The present value of the total relevant cost per unit time

The rate of deterioration is given as follows:

\[ t \]
Time to deterioration, \( t > 0 \)

\[ f(t) \]
Probability density function (p.d.f.) of product life

\[ F(t) \]
Cumulative distribution function (c.d.f.) of product life

\[ R(t) \]
Reliability

\[ Z(f) \]
Instantaneous rate of deterioration

\[ R(t) = 1 - F(t) \quad Z(t) = f(t)/R(t) \]

From the reliability theory, one has \( R(0) = 1 \), and

\[ R(t) = e^{-\alpha t} \]
The product life is assumed to follow a two-parameter Weibull distribution.

The researcher has assumed \( \alpha \) as scale parameter, \( \beta \) as shape parameter and \( \beta, \alpha > 0 \).

Then one has:

\[ f(t) = \alpha \beta t^{\beta-1} e^{-\alpha t^\beta} \]

\[ F(t) = \int_0^t f(t)dt = 1 - e^{-\alpha t^\beta} \]

\[ R(t) = 1 - F(t) = e^{-\alpha t^\beta} \]
\[ Z(t) = \frac{f(t)}{R(t)} = \frac{xy t^{r - a} e^{-at}}{e^{-at}} = xyt^{r-1} \]

The instantaneous rate of deterioration is used in the following model development.

When \( y > 1 \), deteriorating rate increases with time. When \( y < 1 \), deteriorating rate decreases with time. And when \( y = 1 \), deteriorating rate is constant. The two-parameter Weibull distribution reduces to the exponential distribution.

### 5.3 Inventory Model with Linearly Increasing Demand and Partial Backlogging Under the Effect of Inflation

Many Inventory models have been developed for the static environment where the product’s demand rate is assumed to be constant over the planning horizon. However, it is observed that in practical situations, constant demand can be justified only for the maturity phase of the product. Many products, such as clothes, fashion accessories, mobile phones, need to prove their worth before they are generally accepted. Hence, it is justifiable to approximate the demand for a product to be represented by a time dependent function during its growth stage. The inventory model with time-varying demand and deterioration has been considerably well studied and explored. However, in a realistic product life cycle, demand is increasing with time during the growth phase. In the present section the deterministic inventory model with linearly increasing demand is developed in which inventory is depleted not only by demand but also by deterioration. Shortages are allowed and partially backlogged.

Besides the earlier mentioned assumptions and notations, some other assumptions and notations for the proposed model are:

- Demand rate is known and is equal to \((a + bt)\), where \(a\) and \(b\) are constants.
• Shortages are allowed and backlogging rate is \( B \), the backlogging parameter is positive constant and \( 0 < B << 1 \).

5.3.1 Formulation and Solution of the Model

![Graphical representation of the OW inventory system](image)

Fig. 5.1 Graphical representation of the OW inventory system

The OW inventory system in Fig. 5.1 can be divided into three phases depicted by \( T_1 \) to \( T_3 \). For each replenishment, a portion of the replenished quantity is used to backorder shortage, while the rest enters the system. \( W \) units of items are stored in the OW and the rest is dispatched to the RW. The RW is therefore utilized only after OW is full, but stocks in RW are dispatched first. Stock in the RW depletes due to demand and deterioration until it reaches zero. During that time, the inventory in OW decreases due to deterioration only. The stock in OW depletes due to the combined effect of demand and deterioration during \( T_2 \) to \( T_3 \). During the time \( t_1 \), both warehouses are empty, and part of the shortage is backordered in the next replenishment.
Fig. 5.2 Graphical representation of the RW inventory system

The OW inventory system can be represented by the following differential equations:

\[ I'_{01}(t) = \beta I_{01}(t) - I_{01}(t-1) \]  
\[ \quad \; 0 \leq t \leq 1 \]  
\[ \quad \; \text{and} \quad \]  
\[ I_{02}(t_2) = - (a + \beta t) I_{02}(t_2-1) \]  
\[ \quad \; 0 \leq t_2 \leq 2 \]  
\[ I_{03}(t_3) = -B \left( a + b t_3 \right) \]  
\[ \quad \; 0 \leq t_3 \leq 3 \]

The first order differential equations can be solved using the boundary conditions,

\[ I_{01}(0) = W, \quad I_{01}(T_1) = I_{02}(0) = We^{-\alpha T_2}, \quad I_{03}(0) = 0 \]

and, one has

\[ I_{01}(t_1) = We^{-a t_1^2}, \]  
\[ \quad \; 0 \leq t_1 \leq T_1 \]  
\[ I_{02}(t_2) = \frac{We^{-a t_2^2} - \int_0^{t_2} (a + bu) e^{\alpha u^2} du}{e^{\alpha t_2^2}}, \]  
\[ \quad \; 0 \leq t_2 \leq T_2 \]  
\[ I_{03}(t_3) = -B \left( a t_3 + \frac{b}{2} t_3^2 \right), \]  
\[ \quad \; 0 \leq t_3 \leq T_3 \]

The RW inventory system can be represented by the following differential equation:

\[ I'_r(t_1) = - a + b t_1 \]  
\[ \quad \; - g h t_1^{l-1} I_r(t_1), \]  
\[ \quad \; 0 \leq t_1 \leq T_1 \]

The first order differential equation can be solved using the boundary condition,

\[ I_r(0) = I_r \]  

, one has
\[
I_c(t) = \frac{1 - \int_0^{t_1} (a + bu) e^{\theta u} \, du}{e^{\theta t_1}}, \quad 0 \leq t_1 \leq T_1
\]

\[
I_r = \int_0^{T_1} a + bu \, e^{\theta u} \, du = \int_0^{T_1} a \, e^{\theta u} \, du + \int_0^{T_1} bu \, e^{\theta u} \, du = a \left( T_1 + \frac{gT_1^{\beta+1}}{(h+1)^2} \right) + \left( \frac{T_1^2}{2} + \frac{gT_1^{\beta+2}}{h+2} \right)
\]

where

\[
... (5.9)
\]

(1) From fig. 5.1, replenishment is made at \( t = 0 \), the present worth ordering cost is

\[\text{OR} = A \]

\[
... (5.10)
\]

(2) Inventory occurs during \( T_1 \) and \( T_2 \) time periods. The OW present worth inventory cost is

\[
HO_{ow} = H \left\{ \int_0^{T_1} (t) \, e^{-rT} \, dt_1 + \int_0^{T_2} (t) \, e^{-rT} \, dt_2 \right\}
\]

\[
= HW \left( T_1 - \frac{rT_1}{2} \left( \frac{T_1^2}{\beta + 1} \right) \right) + W \left[ 1 - rT_1 \left( \frac{1}{\beta} \right) \right] + \frac{8Wr - a + ar}{2} T_2^2
\]

\[
+ \left( ar - \frac{b}{2} + \frac{br}{2} \right) \frac{T_3^2}{3} + \frac{a \beta T_2^{\beta+2}}{\beta + 1} + \frac{b \alpha \beta T_2^{\beta+3}}{(\beta + 2)(\beta + 3)}
\]

\[
... (5.11)
\]

(3) Shortages occur during \( T_3 \) time period. The OW present worth shortage cost is

\[
SC = s \left\{ \int_0^{T_3} \left[ -I_{o3}(t) \right] e^{-rT} \, dt_3 \right\} = \left\{ s \int_0^{T_3} B \left( at_3 + \frac{bt_3}{2} \right) e^{-rT} \, dt_3 \right\}
\]

\[
= sB \left[ -r - T_3 \right] \frac{T_3^2}{2} + \left[ \frac{b}{2} - ar - \frac{br}{2} \left( T_1 + T_2 \right) \right] \frac{T_3^3}{3} - \frac{br}{8} T_3^4
\]

\[
... (5.12)
\]

(4) Lost sales occur during \( T_3 \) time period. The OW present worth lost sale cost is

\[
LS = \pi \left\{ \int_0^{T_3} \left( 1 - B \right) \left( a + bt_3 \right) e^{-rT} \, dt_3 \right\}
\]

139
\[= \pi (1 - B) \left\{ a \left[ 1 - r \left( T_1 + T_2 \right) \right] T_1 \left\{ b - ar - br \left( T_1 + T_2 \right) \right\} \frac{T_1}{2} - \frac{br}{3} T_1^3 \right\} \]

\[= \pi (1 - B) \left\{ a \left[ 1 - r \left( T_1 + T_2 \right) \right] T_1 \left\{ b - ar - br \left( T_1 + T_2 \right) \right\} \frac{T_1}{2} - \frac{br}{3} T_1^3 \right\} \]

\[t = 0 \quad t = T_1 + T_2 + T_3 = T\]

(5) Replenishment occurs at \( t = 0 \) and \( t = T_1 + T_2 + T_3 = T \). The item cost includes loss due to deterioration as well as the cost of the item sold. The OW present worth item cost is

\[IT_0 = C \left\{ \int_0^T W + B \left( aT_3 + \frac{bT_3^2}{2} \right) e^{-r(T_1+T_2)} \right\} \]

\[= C \left\{ \int_0^T W + B \left( aT_3 + \frac{bT_3^2}{2} \right) e^{-r(T_1+T_2)} \right\} \]

(6) From Fig. 5.2, inventory occurs during \( T_3 \) time periods. The RW present worth inventory cost is

\[HO_{rw} = \int_0^{T_3} \{ I_1 \cdot t \} e^{-r_1 \cdot t} \, dt \]

\[= \int_0^{T_3} \left\{ a \left( \frac{T_3^2}{2} - \frac{rT_3^3}{6} + \frac{ghT_3^{h+2}}{h+2} \right) \right\} \left\{ \frac{4T_3^3}{3} - \frac{3rT_3^4}{8} + \frac{ghT_3^{h+3}}{h+1} \right\} \]

\[t = 0 \]

(7) Replenishment occurs at \( t = 0 \). The item cost therefore includes loses due to deterioration as well as the cost of the item sold. The RW present worth cost is

\[IT_w = CI_w = C \left\{ a \left( T_3 + \frac{gT_3^{h+1}}{h+1} \right) \right\} \left\{ \frac{T_3^2}{2} + \frac{gT_3^{h+2}}{h+2} \right\} \]

\[t = 0 \]

(8) The present value of the total relevant cost during the cycle is

\[TUC(T_1, T_2, T_3) = \frac{1}{T} \left\{ OR + HO_{ow} + HO_{rw} + SC + LS + IT_0 + IT_w \right\} \]
\[
\begin{align*}
&= \frac{1}{TH} \left[ A + HW \left( T_i - \frac{rT_i^2}{2} - \frac{a}{1 + T_i^{b_i}} \right) \right] \\
&+ H \left[ W - 1 - rT_i T_i^{\gamma} \right] + \frac{W - a + \alpha r}{2} T_i^{\gamma} \\
&+ \left( \frac{b}{2} + \frac{br}{2} \right) T_i^{\gamma} - \frac{W a T_i^{\gamma+1}}{\beta + 1} + \frac{a + b T_i^{\gamma+2}}{\beta + 2} T_i^{\gamma} + \frac{b a + b T_i^{\gamma+3}}{\beta + 3} \right] \\
&+ sB \left[ a - rT_i T_i^{\gamma} \right] + \left( \frac{b}{2} - \frac{a r}{2} T_i^{\gamma} + \frac{b r}{2} T_i^{\gamma} \right) \left( T_i^{\gamma} - \frac{b r T_i^{\gamma}}{8} \right] \\
&+ \pi \left[ 1 - B \right] \left( a - rT_i^{\gamma} + T_i^{\gamma} \right) + \left( b - a r - b r T_i^{\gamma} + T_i^{\gamma} \right) \left( T_i^{\gamma} - \frac{b r T_i^{\gamma}}{3} \right] \\
&+ C \left[ W + B \left( a T_i^{\gamma} + \frac{b T_i^{\gamma}}{2} \right) \left( 1 - r T_i^{\gamma} + T_i^{\gamma} \right) \right] \\
&+ F \left[ a \left( T_i^{\gamma} - \frac{r T_i^{\gamma}}{2} + \frac{gh T_i^{\gamma+2}}{(h+1)(h+2)} \right] \\
&- b \left( \frac{4 T_i^{\gamma}}{3} - \frac{3 T_i^{\gamma}}{8} + \frac{gh T_i^{\gamma+3}}{(h+1)(h+2)(h+3)} \right] \\
&+ C \left[ a \left( T_i + \frac{g T_i^{\gamma+1}}{h+1} \right) + \left( \frac{T_i^{2}}{2} + \frac{g T_i^{\gamma}}{h+2} \right] \\
&\ldots(5.17)
\end{align*}
\]

The optimization problem can be formulated as:

\[
\begin{align*}
\text{Minimize:} & \quad T_1, T_2, T_3 \\
\text{Subject to:} & \quad 0, 0, 0.
\end{align*}
\]
In the above expression, TUC is the objective function which we have to minimize,

\[ T_1, T_2, T_3, \]

which is the function of \( T_1 \), \( T_2 \) and \( T_3 \). To minimize the objective function, the solution methodology is presented in section 5.6.

5.3.2 **When the system has only rented warehouse (RW)**

![Graphical representation of the RW inventory system (one warehouse-rented)](image)

Fig. 5.3 Graphical representation of the RW inventory system (one warehouse-rented)

Fig. 5.3 shows the graphical representation of the RW inventory system having only rented warehouse. The RW inventory level function during \( T_1 \) and \( T_2 \) time periods are similar to equations (5.8) and (5.6). The ordering cost and holding cost of the system are similar to equations (5.10) and (5.15).

Shortages occur during \( T_1 \) time period. The RW present worth shortage cost is

\[
SC = s \left\{ \int_0^{T_1} B \left( at_1 + \frac{bt_1}{2} \right) e^{-r(T_1-t)} \, dt_1 \right\}
\]

\[
= sB \left[ a \left(1 - rT_1 \right) \frac{T_1^2}{2} + \left( \frac{b}{2} - ar - \frac{brT_1}{2} \right) \frac{T_1^3}{3} - \frac{brT_1^4}{8} \right]
\]

... (5.18)

Lost sales occur during \( T_2 \) time period. The RW present worth lost sales cost is

\[
LS = \pi \left\{ \int_0^{T_2} 1 - B \left( at_2 + bt_2 \right) e^{-r(T_2-t)} \, dt_2 \right\}
\]
\[ T = T_1 + T_2 \]

The order quantity in RW per order is

\[ Q_t = I_t + B \left( aT_2 + \frac{bT_1}{2} \right) = a \left( T_1 + \frac{gT_1^{br}}{h+1} \right) + \left( b \left( aT_2 + \frac{bT_1}{2} \right) \right) \]

\[ + \left( \frac{b^2}{2} - ar \right) \frac{T_1^3}{3} - \frac{brT_1^3}{8} \]

\[ + \pi \left( 1 - B \right) \left( aT_2 - rT_1 T_2 - \frac{rT_2^2}{2} \right) + \left( b \left( aT_2 + \frac{bT_1}{2} \right) \right) \]

\[ + \left( \frac{b^2}{2} - ar \right) \frac{T_1^3}{3} - \frac{brT_1^3}{8} \]

\[ ...(5.21) \]

Note that the total present value of the total relevant cost per unit time during the cycle is the sum of ordering cost, holding cost, shortage cost, lost sale cost and item cost.

\[ TUC_{r} \left( T_1, T_2 \right) \]

\[ \frac{1}{T} OR + HO_{d} + SC + LS + IT \]

\[ TUC_{r} \left( T_1, T_2 \right) = \frac{1}{T} \left\{ A + F \left[ a \left( T_1^2 - \frac{rT_1^3}{6} + \frac{gT_1^{br+2}}{h+1} \right) \right] - b \left( \frac{4T_1^3}{3} - \frac{3rT_1^4}{8} \right) \]

\[ + \frac{ghT_1^{br+3}}{h+1} \frac{T_1^3}{3} - \frac{brT_1^3}{8} \]

\[ + \pi \left( 1 - B \right) \left( aT_2 - rT_1 T_2 - \frac{rT_2^2}{2} \right) + \left( b \left( aT_2 + \frac{bT_1}{2} \right) \right) \]

\[ + \left( \frac{b^2}{2} - ar \right) \frac{T_1^3}{3} - \frac{brT_1^3}{8} \]

\[ ...(5.19) \]
\[ +C \left[ a \left( T_1 + \frac{gT_1^{h+1}}{h+1} \right) + \left( \frac{T_1^2}{2} + \frac{gT_1^{h+1}}{h+1} \right) + B \left( aT_2 + \frac{bT_2^2}{2} \right) \right] \]

...\(5.22\)

In the above expression, \( \text{TUC} \) is the objective function which we have to minimize, which is the function of \( T_1 \) and \( T_2 \). To minimize the objective function, the solution methodology is presented in section 5.6.

5.3.3 When the system has only own warehouse (OW)

![Graphical representation of OW inventory system](image)

\( \text{Fig. 5.4 Graphical representation of the OW inventory system having} \)
\( T_1 \quad T_2 \)

only own warehouse. The OW inventory level function during \( T_1 \) and \( T_2 \) time periods are similar to equations (5.8) and (5.6). The ordering cost, holding cost, shortage cost, and lost sale cost of the system are similar to equations (5.10), (5.15), (5.18) and (5.19). The order quantity in OW per order is

\[ Q_0 = W + B \left( aT_2 + \frac{bT_2^2}{2} \right) \]

...\(5.23\)

Replenishment occurs at \( t = 0 \) and \( t = T_1 + T_2 \). The item cost therefore includes loss due to deterioration as well as the cost of the item sold. The OW present worth item cost is

144
$$IT_r = C \left[ W + B \left( aT_2 + \frac{bT_2^3}{2} \right) e^{-a(T_1 + T_2)} \right] = C \left\{ W + \left\{ B \left( aT_2 + \frac{bT_2^3}{2} \left[ 1 - r \frac{R}{T_1 + T_2} \right] \right) \right\} \right\}$$

...(5.24)

$$T = T_1 + T_2$$

Note that the total present value of the total relevant cost per unit time during the cycle is the sum of ordering cost, holding cost, shortage cost, lost sale cost and item cost.

$$TUC_0(T_1, T_2) = \frac{1}{T} \left\{ OR + HO_{cwh} + SC + LS + IT_r \right\}$$

$$TUC_0(T_1, T_2) = \frac{1}{T} \left\{ A + H \left[ a \left( \frac{T_2^3}{2} - rT_2^3 + \frac{\alpha \beta T_2^{\beta+2}}{(\beta+1)(\beta+2)^2} \right) \right] 

- b \left( \frac{4T_1^3}{3} - \frac{3rT_1^4}{8} + \frac{\alpha \beta T_2^{\beta+3}}{(\beta+1)(\beta+2)(\beta+3)^2} \right) \right] 

+ sB \left[ a \left( 1 - rT_1 \right) T_2^2 \right] + \left( \frac{b}{2} - a \cdot \frac{brT_1^2}{2} \right) + \left( T_2^3 - rT_1T_2^2 \right) 

+ \pi \left( 1 - B \right) \left[ a \left( T_2 - rT_1T_2 \right) + \left( \frac{T_2^3}{2} - rT_1T_2^2 \right) \right] 

+ C \left[ W + B \left( aT_2 + \frac{bT_2^3}{2} \left[ 1 - r \frac{R}{T_1 + T_2} \right] \right) \right]$$

...(5.25)

In the above expression, $TUC_0$ is the objective function which we have to minimize, which is the function of $T_1$ and $T_2$. To minimize the objective function, the solution methodology has been presented in section 5.6.

5.4 Inventory Model with Exponentially Increasing Demand and Partial Backlogging Under the Effect of Inflation

Many Inventory models have been developed for the static environment where the product’s demand rate was assumed to be constant over the planning horizon. However, in a realistic product life cycle, demand increases with time during the growth phase. In the
present section the deterministic inventory model with exponentially increasing demand is developed in which inventory is depleted not only by demand but also by deterioration. Shortages are allowed and partially backlogged.

Besides the earlier mentioned assumptions and notations, some other assumptions and notations for the proposed model are:

- Demand rate is known and is equal to $a e^{b t}$, where $a$ and $b$ are constants.

- Shortages are allowed and backlogging rate is $e^{-h t}$, when inventory is in shortage. The backlogging parameter is a positive constant and $0 < h << 1$.

### 5.4.1 Formulation and Solution of the Model

The OW inventory system in Fig. 5.1 can be divided into three phases depicted by $T_1$ to $T_3$. For each replenishment, a portion of the replenished quantity is used to backorder shortage, while the rest enters the system. $W$ units of items are stored in the OW and the rest is dispatched to the RW. The RW is therefore utilized only after OW is full, but stocks in RW are dispatched first. Stock in the RW depletes due to demand and deterioration until it reaches zero. During that time, the inventory in OW decreases due to deterioration only. The stock in OW depletes due to the combined effect of demand and deterioration during time $T_2$. During the time $T_2$, both warehouses are empty, and part of the shortage is backordered in the next replenishment.

OW inventory system can be represented by the following differential equations:

\[
I'_{a_1} = \alpha \beta \gamma - I \frac{\beta}{t} \frac{\beta}{t_1} \frac{\beta}{t_2}, \quad t \geq t_1 \leq 1 \quad \cdots (5.26)
\]

\[
I'_{a_2} (t_2) = -ae^{b t} \beta t \quad t \geq t_2 \leq t_3 \quad \cdots (5.27)
\]

\[
I'_{a_3} (t_3) = e^{-h t} ae^{b t} = -ae^{b t} \beta \gamma, \quad 0 \leq t_3 \leq T_3 \quad \cdots (5.28)
\]
The first order differential equations can be solved using the boundary conditions,

\[ I_{01}(0) = W, \quad I_{01}(T_1) = I_{02}(0) = We^{-\alpha T_1}, \quad I_{03}(0) = 0 \]

and, one has

\[ I_{01}(t_1) = We^{-\alpha T_1}, \quad 0 \leq t_1 \leq T_1 \]  

\[ I_{02}(t_2) = \frac{We^{-\alpha T_1} - a \int_{0}^{t_2} e^{\beta u} du}{e^{\alpha T_1}}, \quad 0 \leq t_2 \leq T_2 \]  

\[ I_{03}(t_3) = \frac{a}{\lambda} \left( 1 - e^{(\lambda \gamma) t_3} \right), \quad 0 \leq t_3 \leq T_3 \]

The RW inventory system can be represented by the following differential equation:

\[ I_r(t_1) = -ae^{\beta t_1} - gh(t_1 - I_r(t_1)), \quad 0 \leq t_1 \leq T_1 \]

The first order differential equation can be solved using the boundary condition,

\[ I_r(0) = I_r \]

one has

\[ I_r - a \int_{0}^{t_1} e^{\beta u} du \]

\[ I_r(t_1) = \frac{I_r - a \int_{0}^{t_1} e^{\beta u} du}{e^{\alpha T_1}}, \quad 0 \leq t_1 \leq T_1 \]

\[ I_r = a \int_{0}^{T_1} e^{\beta u} du = a \left( T_1 + \frac{bT_1^2}{2} + \frac{gT_1^{h+1}}{h+1} \right) \]

where

\[ t_1 \]

(1) From Fig 5.1, replenishment is made at \( t = 0 \), the present worth ordering cost is

\[ \text{OR} = A \]  

(2) Inventory occurs during \( T_1 \) and \( T_2 \) time periods. The OW present worth inventory cost is
\[
HO_{ow} = H \left\{ \int_{0}^{T_1} I_{w1}(t_1) e^{-rt_1} dt_1 + \int_{0}^{T_1} I_{w2}(t_2) e^{-r(T_1-2t_2)} dt_2 \right\} \\
= HW \left[ T_1 - \frac{aT_1^{\beta+1}}{\beta+1} - \frac{rT_1^2}{2} \right] + H \left[ W \left( T - rt - T_1 \right) \right]_2 \\
+ \left( \frac{-W-a+arT_1}{2} \right) T_2^2 + \left( \frac{ar-ab}{2} + \frac{ab\alpha}{3} \right) T_2^3 + \frac{abrT_2^4}{8} \\
- \frac{W\alpha T_1^{\beta+1}}{\beta+1} + \frac{a\alpha T_2^{\beta+2}}{\beta+2} \right) + \frac{ab\alpha T_3^{\beta+3}}{2(\beta+3)} \right] \\
\]

(5.36)

3. Shortages occur during \( T_3 \) time period. The OW present worth shortage cost is

\[
SC = s \left\{ \int_{0}^{T_1} \left[ -I_{w1}(t_1) e^{-r(T_1+t_1)} \right] dt_1 = s a \left[ \frac{(1-rT_1-rT_2) T_2^2 - rT_3^3}{2} \right] \right\} \\
\]

(5.37)

4. Lost sales occur during \( T_3 \) time period. The OW present worth lost sale cost is

\[
LS = \pi \left\{ \int_{0}^{T_1} \left[ (1-e^{-\lambda t}) \ e^{-r(T_1+t_1)} \right] dt_1 \right\} \\
= \pi \alpha \left[ \frac{(1-rT_1-rT_2) T_2^2 - rT_3^3}{2} \right] \\
\]

(5.38)

5. Replenishment occurs at \( t = 0 \) and \( t = T_1 + T_2 + T_3 = T \). The item cost includes loss due to deterioration as well as the cost of the item sold. The OW present worth item cost is

\[
IT_0 = C \left( W - \frac{a}{b-\lambda} \left[ 1-e^{b-\lambda T_1} \right] e^{-r(T_1+T_2+T_3)} \right) \\
= C \left(W + aT_1 \left[ 1 - rT_1 + T_2 + T_3 \right] \right) \\
\]

(5.39)

6. From fig. 5.2, inventory occurs during \( T_1 \) time periods. The RW present worth inventory cost is
\[ \text{HO}_{\text{GW}} = F \left\{ \frac{\alpha}{\delta} \right\} \int_{0}^{T_1} \left[ e^{-r \gamma_1} \right] \right\] = F a \left[ \frac{T_1^2}{2} + \frac{bT_1^3}{3} - \frac{rT_1^3}{6} + \left( \frac{ghT_1^{h+2}}{h+1} \right) \frac{T_1^{h+1}}{h+2} - \frac{bqT_1^{h+3}}{h+3} \right]

(7) Replenishment occurs at \( t = 0 \). The item cost therefore includes loses due to deterioration as well as the cost of the item sold. The RW present worth cost is

\[ IT_1 = CT_1 = C \left( \int_{0}^{T_1} \left( e^{p \gamma_1} \right) \right) = Ca \left[ T_1 + \frac{bT_1^2}{2} + \frac{gT_1^{h+1}}{h+1} \right]

(5.41)

(8) The present value of the total relevant cost during the cycle is

\[ \text{TUC} \left( T_1, T_2, T_3 \right) = \frac{1}{T} \left[ OR + \{ \text{HO}_{\text{GW}} + \text{HO}_{\text{GW}} \} + \text{SC} + \text{LS} + IT_2 + IT_3 \right] \]

\[ \text{TUC} \left( T_1, T_2, T_3 \right) = \frac{1}{T} \left[ A + HW \left( T_1 - \frac{aT_1^{\beta+1}}{\beta+1} - \frac{rT_1^3}{2} \right) \right. \]

\[ + H \left[ \frac{\alpha \gamma_1 - T_1}{2} \right] + \left[ -W - a + arT_1 \right] \frac{T_1^2}{2} \]

\[ + \left( ar - \frac{ab}{2} + \frac{ab \alpha}{2} \right) \frac{T_1^3}{2} + \frac{abrT_1^3}{8} - \frac{W \alpha T_1^{\beta+1}}{(\beta+1)} \]

\[ + \left( a \alpha \beta T_1^{\beta+2} + \frac{ab \alpha T_1^{\beta+3}}{(\beta+2)} \right) \]

\[ + s \alpha \left[ \frac{(1 - r T_1 - r T_2) T_1}{2} - \frac{r T_1^3}{3} \right] \]

\[ + \pi \alpha \left[ \frac{(1 - r T_1 - r T_2) T_1}{2} - \frac{r T_1^3}{3} \right] \]
\[ +C \left[ W + aT_3 \left( 1 - r \left( T_1 + T_2 + T_3 \right) \right) \right] \\
+ Fa \left[ \frac{T_1^2}{2} + \frac{bT_1^3}{3} - \frac{rT_1^3}{6} - \frac{brT_1^5}{30} + \frac{ghT_1^{h+1}}{(h+1)(h+2)} \right] + Ca \left( \frac{bT_1^2}{2} + \frac{gT_1^{h+1}}{h+1} \right) \]  
\[ \ldots (5.42) \]

The optimization problem can be formulated as:

\[
\begin{align*}
\text{Minimize} & : & & \text{T U C ( } & & ( \text{ )}) \\
\text{Subject to} & : & & 0, & & 0, & & 0 \\
\end{align*}
\]

In the above expression, TUC is the objective function which we have to minimize, which is the function of \( T_1, T_2, T_3 \). To minimize the objective function, the solution methodology is presented in section 5.6.

### 5.4.2 When the system has only rented warehouse (RW)

Fig. 5.3 shows the graphical representation of the RW inventory system having

\( T_1 \) \( T_2 \)

only rented warehouse. The RW inventory level function during \( T_1 \) and \( T_2 \) time periods are similar to equations (5.33) and (5.31). The ordering cost and holding cost of the system are similar to equations (5.35) and (5.40).

**Shortages occur during \( T_2 \) time period.** The RW present worth shortage cost is

\[ SC = s \left\{ \int_0^{T_2} \left( \frac{a}{b-\lambda} \left( 1 - e^{-\lambda t} \right) \right) e^{-\gamma t} dt \right\} = sa \left( \frac{1-rT_1}{2} - \frac{rT_2^3}{3} \right) \]  
\[ \ldots (5.43) \]

Lost sales occur during \( T_2 \) time period. The RW present worth lost sales cost is

\[ LS = \pi \left\{ \int_0^{T_2} \left( 1 - e^{-\lambda t} \right) ae^{-\gamma t} dt \right\} = \pi \phi \lambda \left( \frac{1-rT_1}{2} - \frac{rT_2^3}{3} \right) \]  
\[ \ldots (5.44) \]
Replenishment occurs at \( t = 0 \) and \( t = T_1 + T_2 = T \). The item cost therefore includes loss due to deterioration as well as the cost of the item sold. The RW present worth item cost is:

\[
IT_r = C \left[ \frac{a}{b - \lambda} - 1 - e^{-\lambda T_1} \right] e^{-r(T_1 + T_2)}
\]

\[
= Ca \left[ \frac{T_1 + \frac{bT_2^2}{2} + \frac{qT_1^{h+1}}{h+1}}{1 - r(T_1 + T_2)} \right]
\]

... (5.45)

The order quantity in RW per order is

\[
Q_r = I_r - \frac{a}{b - \lambda} - 1 - e^{-\lambda T_1} = a \left( T_1 + \frac{bT_2^2}{2} + \frac{qT_1^{h+1}}{h+1} \right) + aT_2
\]

... (5.46)

\[
T = T_1 + T_2
\]

Noting that the total present value of the total relevant cost per unit time during the cycle is the sum of ordering cost, holding cost, shortage cost, lost sale cost and item cost.

\[
TUC_r(T_1, T_2) = \frac{1}{T} \{ OR + HO_{rw} + SC + LS + IT_r \}
\]

\[
TUC_r(T_1, T_2) = \frac{1}{T} \left\{ A + Fa \left[ \frac{T_1^2}{2} + \frac{bT_1^3}{3} + \frac{brT_1^4}{6} + \frac{ghT_1^{h+2}}{h+1} + \frac{gT_1^{h+3}}{h+1} \left( h+2 \right) \right] - \frac{bgT_1^{h+3}}{h+1} \left( h+3 \right) \right\}
\]

\[
+Fa \left[ \frac{1-rT_1}{2} - \frac{rT_1^2}{3} \right] + \pi q \left\{ \frac{1-rT_1}{2} - \frac{rT_1^2}{h+1} \right\}
\]

\[
+Ca \left[ \frac{T_1}{2} + \frac{\frac{T_1}{h+1}}{2} + T_2 (1 - \frac{T_1}{T_2}) \right] \right\}
\]

... (5.47)

In the above expression, \( TUC_r \) is the objective function which we have to minimize, which is the function of \( T_1 \) and \( T_2 \). To minimize the objective function, the solution methodology is presented in section 5.6.
5.4.3 When the system has only own warehouse (OW)

Fig. 5.4 shows the graphical representation of the OW inventory system having only own warehouse. The OW inventory level function during $T_1$ and $T_2$ time periods are similar to equations (5.33) and (5.31). The ordering cost, holding cost, shortage cost, and lost sale cost of the system are similar to equations (5.35), (5.40), (5.43) and (5.44). The order quantity in OW per order is

$$Q_0 = W - \frac{a}{b-\lambda} \left[ 1 - e^{-(b-\lambda)T_2} \right] = W + aT_2$$

...(5.48)

Replenishment occurs at $t = 0$ and $t = T_1 + T_2$. The item cost therefore includes loss due to deterioration as well as the cost of the item sold. The OW present worth item cost is

$$IT_1 = C \left[ W - \frac{a}{b-\lambda} \left( 1 - e^{b-\lambda T_2} \right) e^{-\lambda T_1} \right]$$

$$= C \ (W + aT_2 \left[ 1 - r(T_1 + T_2) \right])$$

...(5.49)

$$T = T_1 + T_2$$

Noting that $T$, the total present value of the total relevant cost per unit time during the cycle is the sum of ordering cost, holding cost, shortage cost, lost sale cost and item cost.

$$TUC_0(T_1, T_2) = \frac{1}{T} \left( OR + HO_{\text{OW}} + SC + LS + IT_1 \right)$$

$$TUC_0(T_1, T_2) = \frac{1}{T} \left( A + H_T \left( \frac{T_1^2}{2} + \frac{bT_1^3}{3} - \frac{rT_1^3}{6} - \frac{brT_1^3}{8} \right) \right.$$  

$$\left. + \frac{\alpha \beta T_1^{\beta+2}}{\beta + 1) \beta + 2} - \frac{b\alpha T_1^{\beta+3}}{\beta + 1) \beta + 3} \right)$$

152
\[ + sa \left[ \frac{1-rT_i}{2} \left( \frac{rT_i}{3} \right) \right] + \pi a \left( \frac{1-rT_i}{2} \left( \frac{rT_i}{3} \right) \right) \]

\[ + C \left[ W + aT_i \left( 1 - r \frac{T_i}{T_i + T_2} \right) \right] \] 

...(5.50)

In the above expression, TUC \( T_i \) is the objective function which we have to minimize, which is the function of \( T_i \) and \( T_2 \). To minimize the objective function, the solution methodology is presented in section 5.6.

5.5 Particular Case

In the model 1, if we put \( b = 0 \), the demand rate reduces in the constant demand rate which was discussed by Wee et al. (2005) and we find, the total present value of the total relevant cost per unit time during the cycle for two-warehouse system is reducing as:

\[ TUC \left( T_i, T_2, T_3 \right) = \frac{1}{T} \left[ OR + HO_{Ow} + HO_{Rw} + SC + LSR + IT_i + IT_i \right] \]

\[ TUC \left( T_i, T_2, T_3 \right) = \frac{1}{T_b} \left[ \frac{A + HW \left( T_i - \frac{rT_iT_i}{2} + 1 \right)}{1 + \frac{1}{T_i}} \right] \]

\[ + H \left[ W \left( 1 - rT_i \frac{T_i}{T_i} \right) \right] + \frac{-Wr - a + ar}{2} \left( T_2 \right) \]

\[ + \frac{arT_i^3}{3} \left( \frac{W}{\beta + 1} + \frac{aT_i^3}{(\beta + 1)(\beta + 2)} \right) \]

\[ + sBa \left[ \frac{(1 - rT_i - rT_2) T_i^2}{2} - \frac{rT_i^3}{3} \right] \]

\[ + \pi \left[ 1 - B \right] a \left[ \left( 1 - r \frac{T_i}{T_i + T_2} \right) T_2 - \frac{rT_2^3}{2} \right] \]

\[ + C \left[ W + Ba \left( 1 - r \frac{T_i}{T_i + T_2} \right) \right] \]
\[ + Fa \left( \frac{T_2^2}{2} - \frac{rT_2^3}{6} + \frac{ghT_1^{h+2}}{h+1 \{ h+2 \}^2} \right) + Ca \left( T_1 + \frac{gT_1^{h+1}}{h+1} \right) \]

And for the rented warehouse is

\[ TUC_r (T_1, T_2) = \frac{1}{T} \left\{ A + Fa \left( \frac{T_2^2}{2} - \frac{rT_2^3}{6} + \frac{ghT_1^{h+2}}{h+1 \{ h+2 \}^2} \right) \right. \]

\[ + sBa \left( \frac{-rT_1^2 T_2}{2} - \frac{rT_2^3}{3} \right) + \pi(1 - B) a \left( T_2 - rT_1 T_2 - \frac{T_2^2}{2} \right) \]

\[ + Ca \left\{ T_1 + \frac{gT_1^{h+1}}{h+1} \right\} + BT \left\{ 1 - \frac{r(T_1 + T_2)^2}{12} \right\} \]

(5.52)

And for the own warehouse is

\[ TUC_o (T_1, T_2) = \frac{1}{T} \left\{ A + Ha \left( \frac{T_2^2}{2} - \frac{rT_2^3}{6} + \frac{\alpha \beta T_1^{h+2}}{(\beta + 1) \{ \beta + 2 \}^2} \right) \right. \]

\[ + sBa \left( \frac{(1-rT_1) T_2}{2} - \frac{rT_2^3}{3} \right) + \pi(1 - B) a \left( T_2 - rT_1 T_2 - \frac{T_2^2}{2} \right) \]

\[ + Ca \left\{ T_1 + \frac{gT_1^{h+1}}{h+1} \right\} + BT \left\{ 1 - \frac{r(T_1 + T_2)^2}{12} \right\} \]

(5.53)

The equations (5.51), (5.52) and (5.53) are same as those given by Wee et al. (2005).

5.6 Solution Procedure

To derive the optimal solutions, the following classical optimization technique was used.

**Step-1**: Take the partial derivatives of TUC \( T_1, T_2, T_3 \) with respect to \( T_1, T_2, T_3 \) and equating the results to zero. The necessary conditions for optimality are

\[ \frac{\partial TUC}{\partial T_1} (T_1, T_2, T_3) = 0, \quad \frac{\partial TUC}{\partial T_2} (T_1, T_2, T_3) = 0, \quad \frac{\partial TUC}{\partial T_3} (T_1, T_2, T_3) = 0 \]

and
Step 2: The simultaneous equations above can be solved for $T_1^*, T_2^*$ and $T_3^*$. 

Step 3: With $T_1^*, T_2^*$ and $T_3^*$ found in step 2, derive $\text{TUC} = T_1^*, T_2^*, T_3^*$.

5.7 Numerical Example

Optimal replenishment policy to minimize the total present value cost is derived by using the methodology given in the preceding section. The following parameters are assumed: $a = 400, b = 0.05$, ordering cost = 100, holding cost in OW = 2, holding cost in RW = 25, shortage cost = 25, lost sale cost = 10, item cost = 10, inflation rate = 0.06, own warehouse capacity = 100, fraction backordered = 0.8 and the deterioration parameter $\alpha = 0.05, \beta = 1.8, \gamma = 0.02, \delta = 1.8$.

<table>
<thead>
<tr>
<th>Model</th>
<th>Demand</th>
<th>Warehouse</th>
<th>$T_1$</th>
<th>$T_2$</th>
<th>$T_3$</th>
<th>TUC</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Exponential increasing demand rate, $\alpha e^{\beta t}$ i.e.,</td>
<td>Two-warehouse</td>
<td>4.31</td>
<td>3.41</td>
<td>-</td>
<td>1006.31</td>
</tr>
<tr>
<td></td>
<td></td>
<td>One warehouse (rented)</td>
<td>0.04</td>
<td>0.11</td>
<td>-</td>
<td>2112.41</td>
</tr>
<tr>
<td></td>
<td></td>
<td>One warehouse (own)</td>
<td>5.52</td>
<td>3.67</td>
<td>-</td>
<td>3378.63</td>
</tr>
<tr>
<td>II</td>
<td>Linearly increasing demand rate, $\alpha + bt$ i.e.,</td>
<td>Two-warehouse</td>
<td>0.67</td>
<td>4.11</td>
<td>-</td>
<td>1786.38</td>
</tr>
<tr>
<td></td>
<td></td>
<td>One warehouse (rented)</td>
<td>0.58</td>
<td>0.12</td>
<td>-</td>
<td>3755.68</td>
</tr>
<tr>
<td></td>
<td></td>
<td>One warehouse (own)</td>
<td>4.23</td>
<td>-</td>
<td>-</td>
<td>5302.62</td>
</tr>
<tr>
<td>III</td>
<td>Constant demand rate ($a$)</td>
<td>Two-warehouse</td>
<td>0.82</td>
<td>9.49</td>
<td>5.06</td>
<td>2506.38</td>
</tr>
<tr>
<td></td>
<td></td>
<td>One warehouse (rented)</td>
<td>0.13</td>
<td>-</td>
<td>-</td>
<td>4912.22</td>
</tr>
<tr>
<td></td>
<td></td>
<td>One warehouse (own)</td>
<td>8.52</td>
<td>5.76</td>
<td>-</td>
<td>6078.92</td>
</tr>
</tbody>
</table>

The main observations drawn from the numerical example are as follows:

For Model-1:

(1) From Table 5.1, when all the given conditions and constraints are satisfied, the optimal solution is found. In this example, the minimal value of the total
present value cost per unit time is $1006.31, while the respective optimal values of \( T^*_1, T^*_2, T^*_3 \) and \( T^* \) are 0.53, 4.31, 3.41 and 8.25 respectively.

(2) When there is only rented warehouse, the minimal value of the total present value cost per unit time is $2112.41, while the respective optimal period of positive and negative inventory level are 0.04 and 0.11 respectively.

(3) When there is only own warehouse with fixed capacity \( W \) units, the minimal value of the total present value cost per unit time is $3378.63, while the respective optimal period of positive and negative inventory level are 5.52 and 3.67 respectively. The system has no space to store excess unit and its TUC is higher than our example due to holding cost and shortage cost.

For Model-2:

(1) From Table 5.1, when all the given conditions and constraints are satisfied, the optimal solution is found. In this example, the minimal value of the total present value cost per unit time is $1786.38, while the respective optimal values of \( T^*_1, T^*_2, T^*_3 \) and \( T^* \) are 0.67, 6.42, 4.11 and 11.20 respectively.

(2) When there is only rented warehouse, the minimal value of the total present value cost per unit time is $3755.68, while the respective optimal period of positive and negative inventory level are 0.58 and 0.12 respectively.

(3) When there is only own warehouse with fixed capacity \( W \) units, the minimal value of the total present value cost per unit time is $5302.62, while the respective optimal period of positive and negative inventory level are 6.27 and 4.23 respectively. The system has no space to store excess unit and its TUC is higher than our example due to holding cost and shortage cost.

For Model-3:

(1) From Table 5.1, when all the given conditions and constraints are satisfied, the optimal solution is found. In this example, the minimal value of the total present value cost per unit time is $2506.38, while the respective optimal values of \( T^*_1, T^*_2, T^*_3 \) and \( T^* \) are 0.82, 9.49, 5.06 and 15.30 respectively.
(2) When there is only rented warehouse, the minimal value of the total present value cost per unit time is $4912.22, while the respective optimal period of positive and negative inventory level are 0.09 and 0.13 respectively.

(3) When there is only own warehouse with fixed capacity \( W \) units, the minimal value of the total present value cost per unit time is $6078.92, while the respective optimal period of positive and negative inventory level are 8.52 and 5.76 respectively. The system has no space to store excess unit and its TUC is higher than our example due to holding cost and shortage cost.

5.8 Sensitivity Analysis

In order to study how the parameters affect the optimal solution, the sensitivity analysis is carried out with respect to the various parameters. The results of the sensitivity analysis are presented in table 5.2 and figure 6.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>%</th>
<th>( T_1 )</th>
<th>( T_2 )</th>
<th>( T_3 )</th>
<th>TUC</th>
<th>PCI (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W )</td>
<td>20</td>
<td>0.52</td>
<td>4.35</td>
<td>3.38</td>
<td>1015.31</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.52</td>
<td>4.32</td>
<td>3.40</td>
<td>1010.06</td>
<td>0.37</td>
</tr>
<tr>
<td></td>
<td>-10</td>
<td>4.20</td>
<td>3.43</td>
<td>1002.82</td>
<td>-0.34</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-20</td>
<td>4.26</td>
<td>3.44</td>
<td>998.74</td>
<td>-0.72</td>
<td></td>
</tr>
<tr>
<td>( B )</td>
<td>20</td>
<td>0.60</td>
<td>4.45</td>
<td>3.29</td>
<td>1176.21</td>
<td>16.88</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.42</td>
<td>4.29</td>
<td>3.38</td>
<td>1098.03</td>
<td>9.11</td>
</tr>
<tr>
<td></td>
<td>-10</td>
<td>0.34</td>
<td>4.20</td>
<td>3.61</td>
<td>840.80</td>
<td>-16.44</td>
</tr>
<tr>
<td></td>
<td>-20</td>
<td>0.23</td>
<td>3.67</td>
<td>2.97</td>
<td>384.75</td>
<td>-61.76</td>
</tr>
<tr>
<td>( r )</td>
<td>20</td>
<td>0.23</td>
<td>4.01</td>
<td>3.48</td>
<td>658.76</td>
<td>-34.53</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.92</td>
<td>4.90</td>
<td>5.03</td>
<td>1349.81</td>
<td>34.14</td>
</tr>
<tr>
<td></td>
<td>-20</td>
<td>4.57</td>
<td>5.98</td>
<td>1643.78</td>
<td>63.34</td>
<td></td>
</tr>
<tr>
<td>( \alpha )</td>
<td>20</td>
<td>0.60</td>
<td>4.04</td>
<td>3.58</td>
<td>1921.46</td>
<td>90.94</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.57</td>
<td>3.49</td>
<td>1435.53</td>
<td>42.65</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-10</td>
<td>0.48</td>
<td>4.67</td>
<td>3.20</td>
<td>800.69</td>
<td>-20.43</td>
</tr>
<tr>
<td></td>
<td>-20</td>
<td>0.43</td>
<td>5.32</td>
<td>3.18</td>
<td>506.23</td>
<td>-49.69</td>
</tr>
<tr>
<td>( \beta )</td>
<td>20</td>
<td>2.46</td>
<td>4.25</td>
<td>4.25</td>
<td>2365.46</td>
<td>135.06</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.63</td>
<td>3.21</td>
<td>3.80</td>
<td>1686.47</td>
<td>67.58</td>
</tr>
<tr>
<td></td>
<td>-10</td>
<td>0.38</td>
<td>4.67</td>
<td>3.03</td>
<td>754.87</td>
<td>-24.98</td>
</tr>
<tr>
<td></td>
<td>-20</td>
<td>0.27</td>
<td>5.09</td>
<td>2.68</td>
<td>456.23</td>
<td>-54.66</td>
</tr>
<tr>
<td>( g )</td>
<td>20</td>
<td>4.31</td>
<td>3.41</td>
<td>1006.96</td>
<td>0.06</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>10</td>
<td>0.53</td>
<td>4.31</td>
<td>3.41</td>
<td>1006.75</td>
<td>0.04</td>
</tr>
<tr>
<td>----</td>
<td>-----</td>
<td>------</td>
<td>------</td>
<td>------</td>
<td>---------</td>
<td>------</td>
</tr>
<tr>
<td></td>
<td>-10</td>
<td>0.53</td>
<td>4.31</td>
<td>3.41</td>
<td>1005.86</td>
<td>-0.04</td>
</tr>
<tr>
<td></td>
<td>-20</td>
<td>0.54</td>
<td>4.31</td>
<td>3.41</td>
<td>1005.68</td>
<td>-0.06</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>0.53</td>
<td>4.31</td>
<td>3.41</td>
<td>1006.96</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.53</td>
<td>4.31</td>
<td>3.41</td>
<td>1006.75</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>-10</td>
<td>0.53</td>
<td>4.31</td>
<td>3.41</td>
<td>1005.86</td>
<td>-0.04</td>
</tr>
<tr>
<td></td>
<td>-20</td>
<td>0.53</td>
<td>4.31</td>
<td>3.41</td>
<td>1005.68</td>
<td>-0.06</td>
</tr>
</tbody>
</table>

Fig. 5.5 Percentage change in the total cost w. r. to the parameters W, B and r

Fig. 5.6 Percentage change in the total cost w. r. to the parameters α, β, g and h

The main observations drawn from the sensitivity analysis are as follows:

1. The value of PCI is the most sensitive to the shape parameter (β) of the deterioration rate and the inflation rate (r). When β increases by 20%, the
value of PCI increases by over 135%. When \( r \) increases by 20%, the value of PCI decreases by over 61%.

(2) The values of PCI are quite sensitive to the parameters \( \alpha \), and not so sensitive to the parameters \( h, g \) and \( W \).

(3) The parameters \( \alpha, \beta, g, h, W \) and \( B \) increase proportional to the value of PCI. The PCI is inversely proportional to the parameters \( r \).

(4) A high parameter value of \( \beta \) results in a high value of \( T_1 \) and \( T_2 \). A high parameter value of \( r \) results in a small value of \( T_1 \), \( T_2 \) and \( T_3 \).

5.9 Conclusion

In this chapter, an inventory model is presented to determine the optimal replenishment cycle for two-warehouse inventory problem under inflation, varying rate of deterioration and partial backordering. The model assumes limited warehouse’s capacity of the distributors. Here, shortages are allowed and partially backlogged. The holding cost at RW is higher as compared to OW. The rate of deterioration in both warehouses is different and follows a two-parameter Weibull distribution. The DCF approach permits a proper recognition of the financial implication of the lost sale in inventory analysis. Some items such as fashionable goods, luxury items, and electronic products are easily identifiable with such kind of a setup. Here two types of models have been developed: Model-1, in which the demand rate is linearly increasing with time and shortages are allowed and partially backlogged with constant backlogging rate. In model-2, the demand rate is exponentially increasing with time and shortages are allowed and partially backlogged with exponential backlogging rate. The discounted cash flow (DCF) and classical optimization technique are used to derive the optimal replenishment policy. A numerical example and sensitivity analysis are implemented to illustrate the model with the help of MATHEMATICA-5.2. When there is only rented or own warehouse in the inventory system, the total present values of the total relevant cost per unit time are higher than the two-warehouse model. From the numerical example, we could finally conclude that the model-1 is more profitable in comparison to the model-2 and model-3 for two-warehouse inventory system. As the backlogging and deterioration rate increases, the total cost of the system also increases. But as the inflation rate increases, the total cost of the system decreases. From the sensitivity analysis, it is evident that the deterioration rate, the
inflation and the backordering rate affects the total cost of the system. In order to optimize the system, the decision maker must develop the most economical replenishment strategy.

The extent to which inflation and partial backlogging has affected the business world, is clearly elucidated through the sensitivity analysis for two-warehouse system where the effect of inflation and partial backlogging is visibly shown over the total system cost with two-warehouses. There is ample scope for further extension of the present research study in fuzzy environments, trade credits and in situations of stock-dependent demand.

References


