Chapter -1

Introduction
1.1 Introduction

Operations Research (OR) is a discipline focussed on the application of information technology for informed decision-making. In other words, OR represents the decision making by seeking to understand and structure complex situations, and to utilize this understanding to predict system behavior and improve system performance. Much of the actual work is conducted by using analytical and numerical techniques to develop and manipulate mathematical models of organizational systems that are composed of people, solving techniques and methods applied in the pursuit of improved decision-making and efficiency. The methods of OR are very often used in inventory control, management science, industrial engineering, mathematics and economics etc. Operations researchers faced with a new problem must determine which of these techniques are most appropriate given the nature of the system, the goals for improvement, and constraints on time and computing power.

Work in operations research and management science may be characterized in one of three categories:

- Fundamental or foundational work takes place in three mathematical disciplines: Probability, Optimization and Dynamical systems theory.
- Modeling work is concerned with the construction of models, analyzing them tools, and assessing their effectiveness with data. This level is mainly instrumental, and driven mainly by statistics and econometrics.
- disciplines, attempts to use models to make a practical impact on real-world problems.

1.2 OR Activities

OR's role in both, the public and the private sectors is increasing rapidly. In general, OR addresses a wide variety of issues in transportation, inventory planning,
production planning, communication operations, computer operations, financial assets, risk management, revenue management and many other fields where improving business energy policy, defense, health care, water resource planning, design and operation of urban problem solving and decision-making that is useful in the management of organizations. In OR problems are: (1) decomposed into basic components and (2) solved via mathematical analysis. Some of the analytical methods used in OR include mathematical logic, in general be described involving three steps: (1) A set of potential solutions to a problem is identified and developed (the set may be rather large); (2) The alternatives derived in the first step are analyzed and reduced to a smaller set of solutions (the solutions have to be feasible and workable); (3) The alternatives derived in the second step are subjected to environment. Generally speaking, OR improves the effectiveness and the efficiency of an institution, hence some of the benefits offered by OR include:

- Decrease Cost or Investment
- Increase Revenue or Return on Investment
- Increase Market Share
- Manage and Reduce Risk
- Improve Quality
- Increase Throughput while Decreasing Delays
- Achieve Improved Utilization form Limited Resources
- Demonstrate Feasibility and Workability

1.3 Inventory

Inventory is the idle resource of an enterprise. Although idle, a certain amount of inventory is essential for smooth running of an organization. In a manufacturing or process industry, non-availability of a critical raw material may result in a closure of the plant. In a service-oriented organization such as a State Transport Corporation, absence of a spare
part when it is required will affect the maintenance of vehicles as a result of which the quality of service will deteriorate.

Inventory is any stored resource that is used to satisfy a current or a future need. Raw materials, work in process and finished goods are examples of inventory. Inventory levels for finished goods are a direct function of demand. When determining the demand for completed clothes dryers, for example, it is possible to use this information to determine how much sheet metal, paint, electric motors, switches and other raw materials and work in process are needed to produce the finished product.

Inventories represent a substantial portion of the total assets of a company, and considerable effort is required to control the inventories. Once inventory is physically present, it is essential that it be handled and stored at minimum cost while at the same time allowing production schedules to be met. Essentially the need for holding inventory arises because of a few definite reasons which may be:-

- Uncertainty conditions in availability.
- Advantage while purchasing lots.
- To cater to non-linear usage/consumption patterns.
- To counter-act lead times.
- To attend probable upward fluctuations in customer demands.
- Uncertainty conditions that might arise in national and international policies.

The game of inventory control aims to attain a healthy balance between cost of inventory and cost of not having it (cost of stock-out). The optimum solution is to find solution to:

- What quantities are required?
- When should we order / re-order?
- What quantity should be ordered?

The area of inventory control covers the following phases:

- Maintaining continuity of production or operation by ensuring continuous supply of standardized raw materials, etc.
• Providing satisfactory service to customers. For example, by ensuring a continuous supply of quality products round the year.

• Reducing the amount of working capital tied-up in inventories by proper planning and fixed up of the maximum, minimum and re-order levels, safety stock, and economic order quantity of each item of inventory.

• Ensuring that the laid down procedures are followed in of storage issue, inspection, recording, etc., in respect of all materials which come in store.

• Conserving materials by eliminating wastage in any form at any stage of the production, non-production operations. The wastage of materials may occur during transit, handling or manufacturing, etc.

• To take enough care to avail of the concessions available in purchasing materials.

• Ensuring that materials of requisite specification and quantity have been received in good condition.

• Establishing timing schedules, procedures for new orders.

• Formulating inventory receipts, issue and storage procedures and proper recording of all transactions.

• Keeping watch on show moving materials and materials likely to become obsolete; requisitioning materials where re-ordering level is reached.

1.3.1 Types of Inventory

An inventory is an idle resource that possesses economic value. It is an item that is stored or reserved for meeting future demand. Such items may be materials, machines, money, or even human beings. A convenient classification of the types of inventory is as follows:
• **Raw materials:** These are the basic materials not undergone any conversion but kept in stock for using in the production, since their receipt from the suppliers. Examples are steel (angles, channels, flats, etc.), copper (sheet, tube, etc.), lead, tin, cotton, rubber, timber, etc. These are required for the following purposes:

(i) For economical bulk purchasing.

(ii) To enable the changes in production rates.

(iii) To serve as buffer stock against delays in transportation, and

(iv) To meet the seasonal fluctuations.

• **Work-in-Progress inventories:** These are the materials in partially completed condition (semi-finished goods or goods in process) of manufacture. At the end of first operation, raw materials become work in process and remain in that classification until they become piece parts or finished goods. Usually the materials on the conveyors, trucks, etc. are considered work in process. These are provided for the following purposes:

(i) To provide economical lot production.

(ii) To enable the variety of products.

(iii) For replacement of wastages, and

(iv) To maintain uniform production even during the variations in sale.

• **Finished goods inventories:** These are the final products ready for delivery to the customer. They enter “finished goods” category at the point of final inspection. These are required for the following purposes:

(i) Variation of customer demand from period to period rules out planning flows of products into exactly match flows out.
(ii) Allow stabilization of the production level.

(iii) Allows the display of products to customers.

(iv) Promote sales.

- **Spare parts inventories**: These are the parts of main products, which are required for the following:

  (i) To provide after sales service to customers, and

  (ii) To utilize the machine fully by a buyer.

### 1.3.2 Inventory Control

Inventory control is the process of deciding what and how much of various items are to be kept in stock. It also determines the time and quantity of various items to be procured. The basic objective of inventory control is to reduce investment in inventories and ensuring that production process does not suffer at the same time. To attain various objectives, inventory control must: (i) determine items to be stocked, (ii) determine when and how much to replenish, (iii) keep suitable records, (iv) weed out obsolete. Inventory Control is the volitional break of the operative material flow; and thus deliberately composed stocks develop. Inventory Control needs a storage that means a room, building or area to store the item.
Fig. 1.1 The elementary storage transaction

The in-pouring items are called storage input, the outpouring items storage output. The graphical representation of elementary storage transaction is given above in fig.1.1.

Therefore inventory control contains all activities and considers all consequences, which are connected with the storage of items. On the one hand there is mere technical and logistical aspect of inventory control, for example the storage layout. While on the contrary there are general questions, which are related to the total stock of a company.

One of the most important decisions is about the quantity of inventories. Therefore a lot of mathematical models have been developed which are summarized under the concept of Inventory Control within the scope of Operations Research.

For the stock of a retail market the outflow is induced through customer demand and the replenishment is secured through orders. The stock disposal therefore consists of ordering the right quantity (lot size) at the right time. Less orders produce less order costs; but for a higher level of order quantity the storage costs rise. The advantage of a great inventory is that there is a high level of service and most customer requirements can be met. Problems of long term inventory control do not belong to this issue, because the order costs are considered global and not for each order. The storage costs and the service level.

Applications of inventory control are very wide and it is applicable to retail market, different industries, manufacturer system, production process etc. But also the inventories of industrial purchasing and selling are pliable to the models of inventory control. Subsequent to the inventory of finished items from industrial selling there is a system of distribution. The disposal of such hierarchical systems is in the domain of multi-echelon inventory control; that is an extension of real inventory control theory. The problems of inventory control are characterised below:

1. Several items are managed in one stock; this means that order, handling and storage need collaboration.
2. Demand and delivery time (of the order to the stock) are often stochastic or not known.

3. Not only the disposal of costs has to be considered, but also non-monetary and non-quantitative aspects.

1.3.3 Reasons for Keeping Stock

There are three basic reasons for keeping an inventory:

1. Every stage, requires that you maintain certain amount of inventory to use in this "lead time".

2. Uncertainty - Inventories are maintained as buffers to meet uncertainties in demand, supply and movements of goods.

3. Economies of scale - Ideal condition of "one unit at a time at a place where user needs it, when he needs it" principle tends to incur lots of costs in terms of logistics. So bulk buying, movement and storing brings in economies of scale, thus inventory.

All these stock reasons can be applied to any owner or product stage.

1.3.4 Economic Order Quantity (EOQ) Model

The importance of effective inventory management is directly related to the size of the inventory. Effective management of inventory is essential to the objective of maximization of shareholders wealth. To control the investment in inventory, the financial manager must solve two interrelated problems: (i) the order quantity problem, and (ii) the optimum level of inventory in order to minimize the costs attached with different inventory levels. Average level of inventory to a great extent, depends upon the number of units procured in one lot and then the speed at which these units are used or sold. The average level can be optimized by careful analysis of quantity ordered, the carrying cost of each unit and the annual requirement of different items.
The Economic Order Quantity (EOQ) model attempts to determine the orders size that will minimize the total inventory costs. It assumes that the total inventory cost = Total carrying cost + Total ordering cost. The EOQ model as a technique of inventory management defines three parameters for any inventory item.

(i) Minimum level of inventory of that item depending upon the usage rate of that item, time lag in procuring that item and unforeseen circumstances, if any.

(ii) The re-order level of the item, at which next order for that item must be placed to avoid any chance of a stock-out, and

(iii) The re-order quantity for which each order must be placed.

The EOQ model attempts to determine quantity to be ordered at a time so as to optimize the cost of carrying and holding inventory and also ensuring availability of that item whenever needed. The most economic size of the order is determined by considering the cost of carrying inventory, it’s purchasing, and it’s ordering costs and usage rate. The graphical representation of the EOQ model is given below in fig. 1.2.
1.3.5 Inventory Modeling

An inventory control problem can be solved by using several methods starting from trial-and-error methods to mathematical and simulation models. Mathematical models help in deriving certain rules which may suggest how to minimize the total (or cost in case of probabilistic demand. In the present study only deterministic inventory control models were used and also the derivation of economic order quantity (EOQ) for a given inventory situation was done.

1.3.6 Steps of Inventory Model Building

The steps to build up a suitable inventory model and then to derive decision rules are as follows:

- Collect the data regarding the pattern of demand, the replenishment policy, planning horizon, relevant inventory costs, etc.
- Examine the inventory situation carefully, listing characteristics and assumptions concerning the situation.
- Develop the total annual relevant cost equation in narrative form as follows:

\[
\text{Total annual relevant cost} = \text{Cost of the item} + \text{Procurement costs} + \text{Carrying costs (Cycle stocks} + \text{Safety stocks)}
\]
+ Stock-out costs (Cost/Sales Back order)

- Transform the total annual cost equation from narrative into the shorthand logic of mathematics.

- Optimize the cost equation by finding the optimum for how much to order (order quantity), when to re-order (re-order point) and the total relevant cost.

The inventory situations can be classified as either deterministic (variables are known with certainty) or stochastic (variables are probabilistic).

1.4 Warehouse System

A warehouse is a location with adequate facilities where volume shipments are received from a production centre, broken order or orders, and shipped to the customer’s

is the creation of a differential advantage for the firm. This advantage accrues from achieving a lower overall distribution cost and or obtaining service advantage in a market area. Given the service level which is desired to be achieved by a company, every warehouse component must have the least total cost. In other words, a warehouse should be added or eliminated only when, as a result of this action, the total cost of distribution is reduced.

The concept of a distribution warehouse or a distribution centre is vastly different from the earlier concept of a godown for storage. The godown is merely a dumping place. Godowns are maintained merely for storage of surplus place. The earlier concept, which led to the establishment of warehouses, was based on the need for ensuring a continuous, uninterrupted supply of goods in the market area for the following:

1. Ensuring protection against delays and uncertainties in transportation arising from a variety of factors;
2. Eliminating lack of sophistication in production control and consequent uncertainties in the availabilities of product at the desired time and place;

3. Providing for adjustment between the time of production and the time of use because production and use can be synchronised;

4. Serving as a reservoir of goods, receiving surplus goods when production exceeds demand and releasing in anticipated.

From the foregoing, it is obvious that earlier a warehouse considered a necessary evil which was to be tolerated, but which did little to provide a differential advantage. The modern distribution centre or distribution warehouse is a pivot in the physical distribution system. According to this system, movement is the primary objective of a warehouse. As are converted into outputs (outputs shipments representing orders of customers). This conversion takes place without consuming too much time. The goods may be received over a period of time from different places, combined or broken down into each individual customer’s orders, and dispatched to the next point in the distribution channel without their coming rest within the confines of the distribution centre. Because of the usual and often inevitable lack of co-ordination between inbound and outbound goods, storage facilities of a temporary nature must be provided for in the scheme. However, the distribution centre continues to be a dynamic location in which flow is accentuating and where storage, with its static connotation, is a facilitating function of secondary importance. Conceptually, the distribution centre is not unlike a retail store in its interactions with its customers.

A warehouse is a commercial building for storage of goods. Warehouses are used by manufacturers, importers, exporters, wholesalers, transport businesses, customs, etc. have loading docks to load and unload goods from trucks. Sometimes warehouses load and unload goods directly from railways, airports, or seaports. They often have cranes and forklifts for moving goods, which are usually placed on ISO standard pallets loaded into pallet racks.
1.5 Requirement of Two-Warehouses

In the busy markets like supermarket, municipality market etc. the storage area of items is limited. When an attractive price discount for bulk purchase is available or the cost of procuring goods is higher than the other inventory related cost or demand of items is very high or there are some problems in frequent procurement, management decide to purchase a large amount of items at a time. These items cannot be accommodated in the existing storehouse (viz. the Own Warehouse, OW) located at busy market place. In this situation, for storing the excess items, one additional warehouse (viz. rented warehouse, RW) is hired on rental basis, which may be located little away from it. We assume that the rent (holding cost for the item) in RW is greater than OW and hence the items are stored first in OW and only excess stock is stored in RW, which are emptied first by transporting the stocks from RW to OW in a continuous release pattern/ bulk release pattern for

is beneficiary only when rented warehouse is nearby own warehouse. In that case no transportation is required. Bulk release is beneficiary when rented warehouse is far away from own warehouse. In that case transportation cost is imposed.

1.6 Relevant Costs of Inventory

The costs incurred in operating an inventory system play major role in determining what the operating doctrine should be. The following types of cost are considered;

(i) Inventory Carrying Cost

It is defined as the cost of holding material inside and outside the stores. It is associated with the level of inventories. Components of inventory carrying cost or holding cost are:

• Cost of money or capital tied up in inventories e. g., interest on the locked up capital.
- Cost of storage space and staff e. g., rent and other maintenance costs including part of salary of stores personnel.
- Inventory maintenance cost e. g., security, stock checking record maintenance and monitoring of inventory.
- Cost of maintaining inventory records.
- Damage and deterioration cost e. g., cost of damage in storage and quality losses.
- Pilferage cost e. g., cost of missing items due to various causes.
- Obsolescence cost e. g., certain items would become obsolete or outdated and related costs.
- Insurance cost e. g., premium to be paid while in storage.

Maintenance of inventory means storage costs. These include expenditure incurred on inventory staff, expenditure on providing various facilities like heating, lighting, floor space, shelves and racks, bins and containers, material handling equipment and other provisions for safe and proper storage of items. These costs generally depend upon the volume to value ratio of an item.

(ii) Ordering Cost

This cost is associated with the placement of an order for the acquisition of inventories. It is determined on the basis of expenses incurred in the purchase and accounting department. In case, a firm produces its own inventory instead of purchasing from an outside source, production set up costs are analogous to ordering costs. Some of the components of cost under this category are:

- Man power cost e. g., money spent in sending enquiries, receiving quotations, comparing etc.
- Finalizing orders and placing orders, expediting and follow up cost of an order.
- Transportation cost and stationary cost.
- Inspection cost and cost of settlement for payment.
- Requisition cost of handling of invoices, stationary, payments, etc.
(iii) **Shortage Cost**

These costs are associated with either a delay in meeting demand or the inability to meet it at all. There, shortage costs or stock-out costs are usually interpreted in two ways proportional to quantity that is short as well as the delay time. They represent loss of goodwill and cost of idle equipment. In case the unfulfilled demand is lost (no backlog case), these costs become proportional to only the quantity that is short. These results in cancelled orders, lost sales, profit and even the business itself.

(iv) **Cost of Operating the Information Processing System**

As stock levels change, some one must update records whether by hand or by computer. Where the inventory levels are not recorded daily, the operating cost is incurred in obtaining accurate physical counts of inventories.

(v) **Purchase Price**

The cost of the product is the amount paid to the supplier (purchase price) for the product received or the direct production cost if manufactured. In the situation of fluctuating prices, planning for inventory depends on the average price (generally considered as fixed price). Thus, the cost/price factor is of great significance in cases where price discounts can be obtained or when large production runs results into lower production cost.

(vi) **Selling Price**

In some cases, the demands may be influenced by the quantity stocked, and, thus, the inventory model will be based on a profit maximization criterion. This includes the revenue from selling the product. The unit selling price may be constant or variable, depending upon the quantity discount.

1.7 **Inventory Terminology**

(i) **Demand**

The demand of a product is the number of units taken from its inventory and can be categorized according to its size and pattern. Demand size is the quantity required to
satisfy the demand for inventory. When the demand size is known, the system is referred

Sometimes it depends on various parameters namely time, stock, price etc. The demand
may be characterized as:

(i) Constant
(ii) Time dependent
(iii) Stock dependent
(iv) Price dependent
(v) Time and on hand inventory level dependent
(vi) Probabilistic with known or unknown distribution.

(ii) Re-order Level

The re-order level is the level of the inventory at which the fresh order for that item
must be placed to procure fresh supply. The re-order level depends upon:

(i) Length of time between the placement of an order and receiving the supply, and

(ii) The usage rate of the item. The inventory is constantly being used up. This
is true regardless of the type of inventory. Raw materials and work-in-
progress inventories are being used in the production while the finished
goods are being sold regularly. The rate at which the inventory is being
used up is called the usage rate.

(iii) Partial Backlogging

A supply chain system in which a supplier prepares for the selling season by
building stock levels prior to the beginning of the season and shortages realized at the
beginning of the season are represented as mixtures of backorders and lost sales. The
many real-life situations and practical experiences reveal that some but not all customers
will wait for backlogged items during a shortage period, such as for fashionable
commodities or high-tech products with short product life cycle. The longer the waiting
time is, the smaller the backlogging rate would be. According to such phenomenon, taking
the backlogging rate into account is necessary.

(iv) Delivery Lag or Lead Time

The time between the requisition for an item and its receipt is known as delivery
lag or lead time. In general, the lead time may be deterministic or probabilistic having the
following four components:

(a) Administrative lead time – can be fixed in nature,
(b) Supplier’s lead time – can be fixed in nature,
(c) Transportation lead time – cannot be fixed, and
(d) Inspection lead time – cannot be fixed.

(v) Safety Stock

Safety stock is a term used by inventory specialists to describe a level of extra
stock that is maintained below the cycle stock to buffer against stock outs. Safety Stock
(also called Buffer Stock) exists to counter uncertainties in supply and demand. Safety
(shortfall in raw material or packaging). By having an adequate amount of safety stock on
hand, a company can meet a sales demand which exceeds the demand they forecasted
without altering their production plan. It is held when an organization cannot accurately
predict demand and/or lead time for the product. It serves as an insurance against stock
outs.

(vi) Quantity Discount and Order Quantity

Form of an economic order quantity (EOQ) model that takes into account quantity
quantity discounts are offered, the buyer must weigh the potential benefits of reduced
purchase price and fewer orders against the increase in carrying costs caused by higher average inventories. Hence, the buyer's goal in this case is to select the order quantity that will minimize total costs, where total cost is the sum of carrying cost, ordering cost, and purchase cost.

It is a common business practice for pricing schedule to display economies of scale with prices decreasing as lot size increases. Such pricing schedules offer discounts based on the quantity ordered in a single lot. This encourages the retailers to order in larger lots to take advantage of price discounts. This adds to the average inventory and flow time in a supply chain. Unlike the EOQ model, the purchase cost now becomes an important criterion in determining the optimal order size and the corresponding total annual inventory cost.

Sometimes, manufacturers use trade promotions to increase sales by offering a discounted price over a pre specified period of time over which the discount is effective. In some cases, the manufacturer may require some specific actions from the retailer to qualify for the discount, such as, displays, advertising, promotion, and so on. Trade promotions are quite common in the consumer packaged goods industry, with manufacturers promoting different things at different times of the year. The goal of the trade promotions is to influence retailers to act in a way that helps the manufacturer achieve its goal.

(vii) Time Horizon

This refers to the planning period over which inventory is to be controlled. The planning period may be finite or infinite. In general, inventory planning is carried out on annual basis. If the time period is long, the time value of cost should be taken into consideration using proper discount factors. On the other hand, it is also possible that the item can be stored only for a limited time period due to perishability or obsolescence. The model should, therefore, minimize the cost over the specific period of time.

(viii) Deterioration

Deterioration occurs for most products in the real world. Deterioration means that a product fails to regularly implement its function. Within the deterioration the deteriorating properties of inventory may fall into three categories:
(1) Direct spoilage, e.g., vegetable, fruit, and fresh food, etc.;
(2) Physical depletion, e.g., gasoline and alcohol, etc.;
(3) Deterioration such as radiation changing, negative spoiling, and loss of efficacy in inventory, e.g., electronic components and medicine.

From another point of view, deterioration can also be classified by the time-value or the products life of inventory.

(1) Utility Constant: Its utility does not change significantly as time passes within its valid usage period, e.g., liquid medicine.
(2) Utility Increasing: Its utility increases as time passes, e.g., some alcoholic drinks.
(3) Utility Decreasing: Its utility decreases as time passes, e.g., vegetables, fruits, and fresh foods, etc.

(ix) Trade Credit

The traditional economic order quantity (EOQ) model focuses on the buyer’s view and makes several assumptions, for example, no stock-outs, fixed demand rate, unlimited store space, zero lead time and must be paid for the items as soon as the items were received. But we know these assumptions are rarely met in real-life situation. For instance, in most business transactions, the supplier would allow a specified credit period (say, 30 days) to the retailer for payment without penalty to stimulate the demand of his/her products. This credit term in financial management is denoted as “net 30.” Before the end of the trade credit period, the retailer can sell the goods and accumulate revenue and earn interest. A higher interest is charged if the payment is not settled by the end of the trade credit period. Therefore, it makes economic sense for the retailer to delay the settlement of the replenishment account up to the last moment of the permissible period allowed by the supplier. So the assumption that the retailer must be paid for the items as soon as the items were received is debatable. The effect of supplier’s trade credit policy on inventory problem has received the attention of many researchers.

(x) Inflation

In economics, inflation is a rise in the general level of prices of goods and services in an economy over a period of time. When the price level rises, each unit of currency
power of money – a loss of real value in the internal medium of exchange and unit of account in the economy. A chief measure of price inflation is the inflation rate, the annualized percentage change in a general price index (normally the Consumer Price Index) over time.

Inflation can have many effects that can simultaneously have positive and negative effects on an economy. Negative effects of inflation include a decrease in the real value of money and other monetary items over time; uncertainty about future inflation may discourage investment and saving, or may lead to reductions in investment of productive capital and increase savings in non-producing assets, e.g. selling stocks and buying gold. This can reduce overall economic productivity rates, as the capital required to retool companies becomes more elusive or expensive. High inflation may lead to shortages of goods if consumers begin hoarding out of concern that prices will increase in the future. Positive effects include a mitigation of economic recessions, and debt relief by reducing the real level of debt.

Economists generally agree that high rates of inflation and hyperinflation are caused by an excessive growth of the money supply. Views on which factors determine low to moderate rates of inflation are more varied. Low or moderate inflation may be attributed to fluctuations in real demand for goods and services, or changes in available supplies such as during scarcities, as well as to growth in the money supply. However, the consensus view is that a long sustained period of inflation is caused by money supply growing faster than the rate of economic growth.

1.8 Two-Shop System

In the case of some goods, especially fruits, vegetables, etc. which is purchased in lots and deteriorates continuously with time, a retailer first separates the deteriorated units from the fresh/good ones. Otherwise the good units will be affected by the spoiled ones. As the items deteriorate continuously, the units considered fresh at a particular point of
the market whereas the spoiled ones have a less market and are sold at a much lower price. Due to these facts, the retailer sells the items separately from two counters/shops at different prices and tries to make a profit out of these two proceeds. This is a very common phenomenon in the case of fruits like mango, orange, apple, etc. and vegetables like potato, onions, etc. This realistic phenomenon is very often observed in the developing countries where some people are very rich and others live below poverty line.

In a manufacturing system, all units cannot be produced exactly as per the required specifications. Amongst the products, those which are within the allowable limits of violations of the specifications are termed non-defective units and others as defective are produced and sold together in a lot. A retailer continuously separates the non-defective and defective units from the lot at the time of sale. Non-defective units are sold with a price after some rework, even incurring a loss, from the adjacent shop counter called the secondary shop, so that management ultimately makes a profit out of these two proceeds.

1.9 Fuzzy System

Analysis of an inventory control system is one of the outstanding subjects in operations research and industrial engineering. In most of the inventory models that had been proposed in the early literature, the associated costs are represented as real numbers, although the real world inventory costs usually exist with imprecise components. When inventory control focus on probability theory. In this case, customer demand as one of the key parameters and source of uncertainty have been most often treated by a probability distribution. However, the probability-based approaches may not be sufficient enough to reflect all uncertainties that may arise in a real world inventory system. Modelers may face some difficulties while trying to build a valid model of an inventory system, in which the related costs cannot be determined precisely. For example, costs may be dependent on some foreign monetary unit. In such a case, due to a change in the exchange rates, the difficulty of determining exact cost components. In some cases trying to determine the precise values of such cost components may be very difficult and costly, if not impossible.
For example, inventory-carrying cost is often dependent on some parameters like current interest rate and stock keeping unit’s market price, which may not be known precisely. Since some uncertainty within inventory systems cannot be considered appropriately using concepts of probability theory, fuzzy set theory has been used in modeling of inventory systems since 1965s.

1.10 Mathematical Modeling

A mathematical model uses mathematical language to describe a system. The process of developing a mathematical model is termed mathematical modeling (also modeling). Mathematical models are used not only in the natural sciences (such as physics, biology, earth science, meteorology) and engineering disciplines, but also in the social sciences (such as economics, psychology, sociology and political science); physicists, engineers, computer scientists, and economists use mathematical models most extensively. Eykhoff (1974) defined a mathematical model as 'a representation of the essential aspects of an existing system (or a system to be constructed) which presents knowledge of that system in usable form'.

A model is defined as idealized representation or an abstraction of some real-life system, whether such system refers to a problem, process, operation, object or events. The objective of the model is to provide a means for analyzing the behavior of the system for the purpose of improving its performance or, if the system is not in the existence, to define the ideal structure of this future system indicating the functional relationships among its

problem can be changed to a logical structure that is amendable to formal analysis. Such a model specifies the decision alternatives and their anticipated consequences for all possible events that may occur, indicates the relevant data for analyzing the alternatives, and leads to meaningful and informative managerial conclusions. In short, modeling is a means of providing a clear structural frame-work to the problem for purpose of understanding and dealing with reality.

Mathematical models are those which employ a set of symbols (i.e., letters, numbers, etc.) and functions to represent the decision variable and their relationships to describe the behavior of the system. The symbols used are generally mathematical or
logical in character. They are by far, the most widely employed in inventory because of the
great deal of complexity associated with an organization. A mathematical model consists
of a set of equation which defines and specifies the relationship and interactions among
various elements of decision problem under study. The solution of the problem is then
obtained by applying well-developed mathematical techniques to the model.

The advantages of mathematical models are:

- They are precise, abstract and general.
- Transformation of a model from a verbal to mathematical form makes far
greater clarification of existing and future relationships and interactions among
variables.
- Being logical, mathematical models are more objective.

It is a known fact that about 2/3 of the investment in industries is tied up in the
form of inventories. It is almost important to release part of this locked-up capital for other
functions of the industries. Mathematical inventory models are used to determine optimal
timing and quantities for orders of resources and what quantities of a product should be
stored. The basic or objective of an inventory model is to minimize the negative cost
trade-off associated with inventories.