Chapter-7
Two-Shop Inventory Models for Differential Items with Display Level and Price Dependent Demand under Inflationary and Fuzzy Environment

Abstract

In this study, we propose an inventory model for differential items, units of which are not in perfect conditions, sold from two-shops, primary and secondary shop, under one management is formulated with stock-dependent demand rate in inflationary and fuzzy
environment. Initially, items are purchased in lots and received at the primary shop with an infinite rate of replenishment, then perfect and defective units are separated. Only the perfect/good units are sold from the primary shop with a profit and its demand is a deterministic linear function of current stock level. The defective units spotted at the time of selling of the good products from the lot are transferred continuously to the adjacent secondary shop for sale at a reduced price and demand for these units is linearly proportional to the selling price. In both shops, shortages are allowed. In this study, there are three scenarios depending upon the time of occurrence of shortages at the shops. At the secondary shop, the time of shortages occurs: (1) exactly at the same time or (2) before or (3) after the time of shortages at the primary shop. In each case under each scenario, profit is maximized and optimum order quantities are evaluated using the computer algorithm based on a gradient method. Finally, a numerical example and sensitivity analysis is used to study the behavior of the model.

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### 7.1 Introduction

There are some items (e.g., fruits, vegetables, food grains, etc), which have a demand to some particular customers even after being partially deteriorated. This phenomenon is very common in the developing countries where majority of people live under poverty line. In business, the partially affected items are being immediately and continuously getting in contact with the spoiled ones. These damaged units are sold from the adjacent secondary shop. Here, the fresh/good units may be sold with a profit while the deteriorated ones are usually sold at a lower price, even incurring a loss, in such a way that the management makes a profit out of the total sales from the two shops.
The concept of two shops under a single management was introduced by Kar et al. (2001-a). In their study, they considered the stock-dependent demand for deteriorating items but they did not consider the shortage in the primary and secondary shop. Kar et al. (2001-b) also developed a deterministic inventory model of deteriorating items sold from two-shops under a single management. In that model, they considered shortage in the primary but not in secondary shop. Das and Maiti (2003) developed an inventory model of a differential item sold from two shops under single management with shortages and variable demand. Dey et al. (2006) presented an inventory of differential items selling from two shops under a single management with periodically increasing demand over a finite time-horizon. Mondal et al. (2007) presented a single period inventory model of a deteriorating item sold from two shops with shortage.

In the present day competitive market, the selling price of a product is one of the decisive factors in selecting the item for use though there is a market for some fashionable demand whereas reasonable or low price has the reverse effect. This argument is more appropriate for defective goods whose demand is always price dependent. Whitin (1955) first presented an inventory model considering the effect of price dependent demand. Later, Kunreuther and Richard (1977), Lee and Rosenblatt (1986), Mukherjee (1987), and Abad (1996) presented inventory of joint pricing and inventory planning. Recently, Maiti et al. (2009) and Banerjee and Sharma (2010) also presented inventory model with price dependent demand.
the inflationary effect on an inventory policy. Misra (1979), Ray and Chaudhuri (1997), and Sarkar et al. (2000) etc. developed an approach of modeling inventory by assuming a constant inflation rate. Yang (2004) considered inflation in the two warehouse inventory model. Singh et al. (2009) developed a two-warehouse inventory model for deteriorating items with shortages under inflation and time-value of money.

An inventory control problem is one of the very common problems in industrial engineering. Decision regarding how much and when to order are typical of every inventory problem. Almost research works on inventory control problem are solved by assuming vagueness or imprecise input data to crisp one. But, many variables in inventory control process may truly be fuzzy. Some components of the setup, carrying and shortage costs may not be known with certainty. Fuzzy set theory, originally introduced by Zadeh (1965), provided a framework for considering parameters that are vaguely or unclearly defined or whose values are imprecise or determined based on subjective beliefs of individuals. Petrovic et al. (1966) presented newsboy problem assuming that demand and backorder cost were fuzzy numbers. The application of fuzzy theory to inventory problem has been proposed by Kacprzyk and Staniewski (1982). Kaufamann and Gupta (1985) introduced theory and application to fuzzy arithmetic. Roy and Maiti (1995) presented a fuzzy inventory model with constraints. Roy and Maiti (1997) developed a fuzzy EOQ model with demand-dependent unit cost under limited storage capacity. Ishii and Konno (1998) introduced fuzziness of shortage cost explicitly into classical inventory problem. Chen and Hsieh (2000) established a fuzzy economic model to treat the inventory problem with all the parameters and variables as fuzzy numbers. Hsieh (2002) presented a fuzzy inventory model. Yao and Chiang (2003) presented an inventory model without backorder with fuzzy total cost and fuzzy storing cost defuzzified by centroid and signed distance. Dutta et al. (2005) developed a single-period inventory model with fuzzy demand. In that study, they have applied graded mean integration representation method to find the optimum order quantity. Chen and Chang (2008) presented an optimization of fuzzy inventory model. In their study, they used function principle as arithmetical operations of fuzzy total production inventory cost and also used the graded mean integration representation method to defuzzify the fuzzy total production and inventory cost. Mahata and Goswami (2009) presented a fuzzy inventory model for deteriorating items with the help of fuzzy numbers. In their study, a realistic inventory model with
imprecise inventory costs have been formulated for deteriorating items under inflation and shortages were allowed. For generality, they did not only provide the solution for the inventory system, where the inventory starts with no shortages; but also enumerated two possible shortage models in the fuzzy sense.

In the above cited references, most of the inventory models with two-shop system have been developed for differential items with or without shortages. But in all the above cited models, the researchers had ignored the effect of inflation and assumed all the cost parameters to be deterministic, whereas in the real life, it is not always so. So in order to two-shops in inflationary environment and all the cost parameters were assumed to be fuzzy in nature.

In this chapter, the researcher has developed two inventory models for both non-defective and defective units purchased in a lot and selling separately at two different shops under a single management with the assumption that the demand of the good units was stock dependent whereas the defective ones having price dependent demand only. The models thus developed are as follows and were related to: (1) in inflationary environment, and (2) in fuzzy environment. In primary shop, non-defective units were sold with a profit. And the defective units, which are continuously transferred to the adjacent secondary shop, were sold after some re-work or repair at a reduced price even incurring a loss. Here shortages were allowed in both shops. It was also assumed that the rate of replenishment of units for the primary shop is infinite. In the secondary shop, this sells only the defective units, which were continuously transferred from the primary shop with variable rates. The researcher considered three situations for dealing with defective units depending upon the coincidence of the time periods at two shops. In the present model, three scenarios were studied:

(1) The demand of defective units is so high that the shortages of defective units occur earlier than the shortages at the primary shop.

(2) At both shops shortages occur at the same time.

(3) At the secondary shop shortages occur after the occurrence of shortages at the primary shop.

In each scenario, the researcher had supposed that, to meet the shortages an amount of differential units were procured at the beginning of the subsequent scheduling period.
price for sorting. To meet the shortages of both good and defective units, the required amounts of differential units were purchased. In that case, differential units have been calculated on the basis of shortages of good units. The shortages of good units at the primary shop were met exactly, but for the secondary shop, the researcher had three cases for the defective units available from the above mentioned processes:

Case-a: shortages of defective units are also exactly met,

Case-b: shortages of defective units are more than the available amount. Here we suppose that the rest amount of shortages is lost, i.e., shortages are partially backordered and

Case-c: shortages are less than the available defective units. Here, to have the logistic advantages, we assumed that the excess amount is sold immediately at a much reduced price and the secondary shop starts from zero inventory at the beginning of the next scheduling period. It is assumed that there is sufficient demand of these units at a much more reduced price.

For the first two cases, the model formulation was the same as there was no penalty for lost sale and for third case, only revenue from excess defective units was added to the total cost. Thus, for each case under each scenario, profit was maximized and optimum order quantities were evaluated using the computer algorithm based on a gradient method (generalized reduced gradient method). The models were illustrated with numerical examples and sensitivity analyses were used to study the behavior of the models.

7.2 Assumptions and Notations

In order to developed inventory models of differential units for primary and secondary shops under a single management, the following common assumptions and notations were used:

7.2.1 Assumptions for the Primary Shop

1. Demand for good units is deterministic and function of current stock level

\[ D_1(q_1(t)) = a_1 + b_1 q_1(t) \]

where \( a_1, b_1 \) are positive constants.
2. Defective unit at any time $t$ is a fraction of on-hand inventory level and is 

$$\theta_{d_1}(t)$$

equal to $\theta_1$, where $\theta_1$ is constant and $0 < \theta_1 < 1$. These defective units are continuously transferred to the secondary shop.

3. To meet the shortages for good units, it is required to procure $S$ differential units out of which good and defective units are $(1 - \frac{S}{S_1})S$ and $S$ respectively.

4. Shortages are allowed and these are fully backlogged.

5. As shortages for good and defective units are met after sorting the differential units “in-no-time”, sorting cost for shortage units only is dependent on shortage quantity of differential units and is given by

$$C_s = kS^b$$

where $k$, $(k < 1)$ are positive constants. The continuous sorting cost of defective units at the time of selling of good units at the primary shop is included in the set-up cost as this job is done by the permanent employees of the management.

### 7.2.2 Assumptions for the Secondary Shop

1. In this shop, demand $D_2$ is dependent on the selling price, i.e., it is a function of mark-up price $\frac{M_2}{p_2}$, since $M_2 = M_0C_0$ and $D_2 = a_2 - b_2C_0M_2$.

2. Secondary shop sells only the defective units, which are received continuously from the primary shop at a variable rate $\omega(t)$ until the stock of differential items is exhausted in the primary shop. $S_2$ units are also received at the beginning of the next scheduling period to meet the shortages.

3. Shortages are allowed and fully backlogged.
In addition to above notations, the following notations were also considered:

### 7.2.3 Notations

- \( C_s \) = “in-no-time” sorting cost for shortage units only.
- \( C_v \) = unit cost of the differential item. 

For \( i^{th} \) \((i = 1, 2)\) shop,

- \( p_i \) = selling price per unit item and 

\[ p_i = M_i C_v \]

- \( H_i \) = holding cost per unit quantity per unit time.
- \( U_i \) = set-up cost per scheduling period.
- \( r \) = denotes the inflation rate.
- \( Q_i \) = optimum inventory level.

\[ M_i \geq 1 \quad \text{and} \quad 0 < M_i < M_2 \]

- \( M_i \) = mark-up price with \( i \) is decision variable.

- \( S_i \) = shortage level at the end of scheduling period.
- \( G_i \) = shortage cost per unit quantity per unit time.

- \( q_i(t) \) = inventory level at any time \( t \).

- \( NR_i \) = net revenue from \( i^{th} \) shop.

- \( TC_i \) = total cost at the \( i^{th} \) shop.

\( \pi \) = the total average profit from both the shops.

(Suffices \( i = 1 \) and 2 represent the parameters related to the primary and the secondary shops respectively).
7.3 Two-Shop Inventory Model in an Inflationary Environment

The literature survey revealed that no researcher has until now developed the two-shop inventory model with inflationary environment. The researcher has first time considered the effect of inflation in two-shops to make the model more realistic.

In most of the inventory models with two shops as mentioned above, the inflation was discarded. It has been done probably with the belief that the inflation would not influence the inventory policy to any significant degree. However, in the last several years purchasing power of money. As a result, while determining the optimal inventory policy, the effects of inflation cannot be ignored.

Clearly, there is need for the reformulation of the optimal inventory control policies taking into account the above mentioned factors. In the present study, a sincere attempt was made to study the situation when the differential item, units of which are not in perfect conditions, sold from two-shops- primary and secondary shop, under one management. The stock-dependent demand was taken into consideration. In order to make inflationary environment. The model has been developed for a finite planning horizon in which shortages were allowed.

7.3.1 Formulation and Solution of the Model

7.3.2 Primary Shop

In the present study, initially the inventory stock level of the system of differential units gradually declines mainly to meet up demand of goods units and the defective units, which are continuously transferred to the secondary shop up to \( t = t_1 \), and the stock level reaches zero at \( t = t_1 + t_2 \). After \( t = t_1 + t_2 \), shortages are allowed only for good units up to \( t = t_1 + t_2 + S_1 \) time. At time \( t = t_1 + t_2 + S_1 \), shortage level is for goods units and to meet this shortage, \( S \) units of differential units are procured and stored instantaneously. The pictorial
representation of the system is given in Fig. 7.1. The differential equations describing the
inventory level in the interval $0 \leq t \leq t_i$ is given by

$$
q'(t) = \begin{cases} 
-\theta q(t) - D_1 q_1(t), & 0 \leq t \leq t_i \\
- D_1 q(t), & t_i \leq t \leq t_i + t_2
\end{cases}
$$

... (7.1)

**Fig. 7.1 Instantaneous state of inventory for the primary shop**

$$
q_i(t) = Q_i - S_i \quad t_i \quad t_i + t_2
$$

With the conditions, $0$ and at $t = 0$, and $t_i$ respectively.

$$
D \equiv (q-t) = a_i + b_i q_i(t)
$$

Therefore, taking , the solutions of the above equations are

$$
q(t) = \begin{cases} 
\frac{a}{\alpha} \left[ e^{\alpha t} - 1 \right], & 0 \leq t \leq t_i \\
\frac{a}{b_i} \left[ e^{\beta t} - 1 \right], & t_i \leq t \leq t_i + t_2
\end{cases}
$$

where $\alpha = b_i + \theta$

... (7.2)

If be the total defective units during the interval $(0, t_i)$, then

$$
D \theta q_i(t) \int_0^t q_i(t) dt + \left[ \frac{b_i \alpha}{\alpha - \theta} \right] t_i + \theta
$$

... (7.3)
\[ Q_1 = q_1(0) = \frac{a_1}{\alpha} e^{\alpha \theta} - 1 \quad S_1 = q_1(t + t_1) = \frac{a_1}{b_1} \left( 1 - e^{b_1 \theta} \right) \]
\[ S = \frac{S}{1 - \theta} \quad \text{and} \quad \theta = \frac{1}{S} \]

The total cost for the primary shop is given by
\[ TC_1 = \text{Purchase cost} + \text{Holding cost} + \text{Shortage cost} + \text{Sorting cost} + \text{Setup cost} \]
\[ = C_q(Q + S) + H_1 \int_0^t \left( q(t) e^{-\eta t} dt + G_1 \frac{e^{-\eta t}}{r} - q(t) e^{-\eta t} dt + kS + U_1 \right) \]
\[ = C_q(Q + S)e^{-\eta t} + \frac{H_1 a_1}{\alpha} \left( \frac{1}{\alpha + r} \left( e^{\alpha t} - e^{-\eta t} \right) - \frac{1}{r} \left( 1 - e^{-\eta t} \right) \right) + \frac{G_1 a_1}{b_1} e^{\alpha t} \left( 1 - e^{-\eta t} \right) \]
\[ - \frac{e^{-\eta t}}{b_1 + r} \left. 1 - e^{-\eta (b_1 + t_1)} \right| + kS^\theta + U_1 \]

Net revenue cost for the primary shop is given by
\[ NR_1 = M_1 C_q \left( \frac{b_1}{\alpha} Q + 1 - \theta \right) S + \frac{a_1}{\alpha} \left( \frac{1}{\alpha} \left( 1 - e^{-\eta (b_1 + t_1)} \right) \right) \]

Here, we obtain the total cost and net revenue cost for the primary shop as given in the equations (7.5) and (7.6). These equations are used to calculate the total average profit in the section 7.3.4.

### 7.3.3 Secondary Shop

At the time of sale, the defective units are spotted at the primary shop and then transferred to the secondary shop, at \( t = 0 \), the amount of inventory in this shop at the beginning of the schedule is zero. It is assumed that initially the amount of defective units received from the primary shop is more than sufficient to meet the demand of defective units, i.e., \( \theta q_1(t) > D_2 \)

The inventory level is raised at a rate \( \theta q_1 - D_2 \) and after some time.
\[ \theta q_i(t) = D_2 \quad t = t_3 \quad 0 \leq t_3 \leq t_1 \quad t = t_3 \]
time it will be zero, i.e. \( t = t_3 \), (say). Hence, at this point, the process of building up of inventory will be stopped and stock attains its maximum level \( Q_2 \) at \( t = t_3 \), from the relation \( \theta q_i(t) = D_2 \) as in scenario-1.

\[
t_3 = t_3 - \frac{1}{\alpha} \log \left( 1 + \frac{\alpha D_2}{\alpha \theta} \right)
\]

\[ ... (7.7) \]

After \( t = t_3 \), the supply from the primary shop is short of the demand for defective units, \( \theta q_i(t) < D_2 \). \( D_2 - \theta q_i(t) \) i.e. and then to fulfill the demand, stock decreases at the rate upon the instants at which the stocks at the primary and the secondary shop are depleted

the number of defective units that are transferred from the primary shop. But, the available defective units \( S \) obtained out of the differential stock \( S \) may be less than, equal to or greater than the actual shortage \( S \) in the secondary shop. In these three cases, net revenue \( NR_2 \) can be written as follows:

\[
NR_2 = \begin{cases} 
M_2 C_0 \left[ \frac{b_i}{\alpha} \left( Q_1 + 1 - \theta \right) S + \frac{\alpha \theta D_2}{\alpha} \left[ \frac{1}{R} 1 - e^{-\theta} \right] \right] & \text{if } \theta S \leq S_2 \\
M_2 C_0 \left[ \frac{b_i}{\alpha} \left( Q_1 + 1 - \theta \right) S + \frac{\alpha \theta D_2}{\alpha} \left[ \frac{1}{R} 1 - e^{-\theta} \right] \right] + m_i C_0 \left( \theta S - S_2 \right) & \text{if } \theta S \geq S_2
\end{cases}
\]

\[ ... (7.8) \]

\[ m_i \quad (0 < m_i = M_2) \]

where, \( m_i \) is the pre-determined markup price for the excess amount, as the excess amount is sold immediately at a much reduced price and the secondary shop starts from zero inventory at the beginning of the scheduling period.

7.3.3.1 Scenario-1
In this scenario, the researcher has considered the situation when shortages at the secondary shop occur earlier than the occurrence of shortages at the primary shop.

![Diagram](image)

**Fig. 7.2 Instantaneous state of inventory of scenario-1**

According the assumptions, in this case, the amount of stock is zero initially; the defective units are sold in the adjacent secondary shop. Let the stock becomes zero at $t = t_3 + t_4$

and after that, shortages are allowed. But there will be some gradual decreasing supply of defective units from the primary shop up to $t = t_3$. So during $t = t_3$, $t_3 + t_4$, shortages increase at the rate $D_2 - \theta q_1(t)$ and attain shortages level $S_2$ at $t = t_3$. After $t = t_3$

, supply from the primary shop totally stops and shortage increases only due to demand up to $t = t_1 + t_4$, when shortage level is $S_2$.

The differential equations governing the instantaneous state of inventory $q_2(t)$ are
\[ q_2(t) = \begin{cases} 
\theta q_1(t) - D_2, & 0 \leq t \leq t_1 \\
\theta q_1(t) - D_2, & t_1 \leq t \leq t_1 + t_3 \\
\theta q_1(t) - D_2, & t_1 + t_3 \leq t \leq t_1 \end{cases} \]

With the boundary conditions

\[ q_2(t) = \begin{cases} 
0 & \text{at } t = 0 \text{ and } t = t_3 + t_4 \\
-S_2' \text{ and } -S_2, & \text{at } t = t_1 \text{ and } t = t_1 + t_2 
\end{cases} \]

The solutions of the above equations are

\[ q_2(t) = \begin{cases} 
\{ \alpha \theta + D_2 \} t Y + e^{\theta t} - 1 - e^{-\theta t} & 0 \leq t \leq t_1 \\
\alpha \theta + D_2 \{ t_1 + t_3 - t \} - Ye^{\theta t} - e^{-\theta t}/(e^{\theta t}) & t_1 \leq t \leq t_1 + t_3 \\
\alpha \theta + D_2 \{ t_1 + t_3 - t \} - Ye^{\theta t} - e^{-\theta t}/(e^{\theta t}) & t_1 + t_3 \leq t \leq t_1 \\
-S_2 + D_2 \{ t_1 + t_2 - t \} & t \leq t \leq t_1 + t_2 
\end{cases} \]

...(7.9)

Since \( q_2(t) \) is continuous at \( t = t_1 \) and \( t = t_3 \) and \( t = t_4 \), one can get an equation related by \( t = t_1 \) and \( t = t_3 \) as follows

\[ \{ \alpha \theta + D_2 \} t Y + e^{\theta t} - 1 - e^{-\theta t} = 0 \]

\[ \text{where } Y = \theta \theta / \alpha \theta \]

...(7.10)

Inventory level at \( t = t_3 \) and the shortage levels at \( t = t_1, t_1 + t_2 \) are respectively given by

\[ Q_2 = -\alpha \theta + D_2 \{ t_1 + e^{\theta t} - 1 - e^{-\theta t} \} = \alpha \theta + D_2 \{ t_1 + e^{\theta t} - 1 - e^{-\theta t} \} \]

...(7.11)

\[ S_2' = \alpha \theta + D_2 \{ t_1 + Y \} - Y e^{\theta t} - 1 \]

...(7.12)

\[ S_2 = \alpha \theta + D_2 \{ t_1 + Y \} - Y e^{\theta t} - 1 + D_2 t_2 \]

...(7.13)

The available deteriorated units \( S \) obtained out of the differential stock \( S \) may be less than, equal to or greater than the actual shortages \( S \) in the secondary shop. In these cases, the relations between \( t_1 \) and \( t_2 \) can be written as follows:
\[ S_2 = \theta S \quad t_1 \quad t_2 \]

**Case-1a:** When \( S_2 > \theta S \), and \( t_1, t_2 \) are related by

\[ \alpha Y t_1 + D \cdot (t_1 + t_2) - \theta \left( \frac{Q}{\alpha} + S_2 \right) = 0 \]

\[ \ldots (7.14) \]

**Case-1b:** When \( S_2 > \theta S \) and \( t_1, t_2, m_2 \) satisfying the inequation

\[ \alpha Y t_1 + D \cdot (t_1 + t_2) - \theta \left( \frac{Q}{\alpha} + S_2 \right) > 0 \]

\[ \ldots (7.15) \]

**Case-1c:** When \( S_2 < \theta S \), and \( t_1, t_2, m_2 \) satisfying the inequation

\[ \alpha Y t_1 + D \cdot (t_1 + t_2) - \theta \left( \frac{Q}{\alpha} + S_2 \right) < 0 \]

\[ \ldots (7.16) \]

Therefore total cost in this scenario is given by

\[ TC_1 = H_2 \left( \sum q_1 t \int e^{-\gamma t} dt + \int \frac{r}{\alpha (t_1 + t_2)} e^{-\gamma t} dt \right) + G_2 \left( \int \frac{r}{\alpha (t_1 + t_2)} e^{-\gamma t} dt + \int \left( q_2 t \right) e^{-\gamma t} dt \right) + U_2 \]

\[ + \frac{Y + D}{2} \left( t_1 + t_2 \right) \left( \frac{r}{\alpha + r} \right) e^{-rn} \left( 1 - e^{-rn} \right) \left( 1 - e^{-rn} \right) \]

\[ + \frac{Y + D}{2} \left( t_1 + t_2 \right) \left( \frac{r}{\alpha + r} \right) e^{-rn} \left( 1 - e^{-rn} \right) \left( 1 - e^{-rn} \right) \]

\[ + \frac{Y + D}{2} \left( t_1 + t_2 \right) \left( \frac{r}{\alpha + r} \right) e^{-rn} \left( 1 - e^{-rn} \right) \left( 1 - e^{-rn} \right) \]

\[ \ldots (7.17) \]

In this scenario, we obtain the total cost for the secondary shop as given in the shortages at the primary shop. This equation is used to calculate the total average profit in the section 7.3.4.
7.3.3.2 Scenario-2

In the present scenario, the researcher has assumed the situation when shortages at
both the shops occur exactly at the same time. Stock is exhausted at \( t = t_1 \), and then
shortages are allowed. As the shortages at both shops occur at the same time, to meet

\[ D_2 \quad t = t_1 + t_2 \]

demand of deteriorated units shortages increase at the rate \( S_2 \) up to when

shortage level is \( S_2 \).

Inventory

\[ \begin{align*}
Q_2 & \quad | \ 
\end{align*} \]

\[ \begin{align*}
0 & \quad t_3 \\
t_1 & \quad t_{1+t2}
\end{align*} \]

\[ \begin{align*}
S_2 & \quad \text{Time}
\end{align*} \]

Fig. 7.3 Instantaneous state of inventory of scenario-2

The differential equations governing the instantaneous state of inventory \( q_2(t) \) is given by

\[
q_2(t) = \begin{cases} 
0 & \text{at } t = 0 \text{ and } t = t_1 \\
\frac{\partial q(t)}{\partial t} = \begin{cases} 
D_2 - \frac{D_2}{t_3}, & 0 \leq t \leq t_3 \\
-\frac{D_2}{t}, & t_3 < t \leq t_1 + t_2 \\
\end{cases} & \text{respectively}
\end{cases}
\]

With the boundary conditions

\[
q_2(t) = \begin{cases} 
0 & \text{at } t = 0 \text{ and } t = t_1 \\
Q_2 \text{ and } -S_2, & \text{at } t = t_3 \text{ and } t = t_1 + t_2
\end{cases}
\]

The solutions of the above equations are
\[
q_2(t) = \begin{cases} 
- \alpha Y + D_2 \{ t + Ye^{1-e^{-\alpha t}} \}, & 0 \leq t \leq t_i \\
\alpha Y + D_2 \{ t - t_i \} + Ye^{\alpha t_i - 1} - e^{-\alpha t_i}, & t_i \leq t \leq t_i + t_1 \\
-D_2 \{ t - t_i \}, & t_i + t_1 \leq t 
\end{cases}
\]...

(7.18)

Since \( t = t_i \) is continuous at \( t = t_1 \), one can get from the following equation

\[
\{ \alpha Y + D_2 \} t_i + Y(1 - e^{\alpha t_i}) = 0
\]...

(7.19)

\[
t = t_3 \quad t = t_i + t_2
\] Inventory level at \( t = t_3 \) and the shortage level at \( t = t_i + t_2 \) are respectively given by

\[
Q_2 = -\{ \alpha Y + D_2 \} t_3 + Ye^{\alpha t_i - 1} - e^{-\alpha t_i} = \{ \alpha Y + D_2 \} t_i - t_i + Y - e^{-\alpha t_i}
\]...

(7.20)

\[
S_2 = D_2 t_2
\]...

(7.21)

Case-2a: When \( S_2 = 0 \)

\[
D_2 t_2 - \frac{a_2}{b_2 1 - e^{\nu_2}} (1 - e^{\nu_2}) = 0
\]...

(7.22)

\[
S_2 > 0 \quad S_2 < 0 \quad t_2 \quad m_2
\]

Case-2b & 2c: When \( S_2 > 0 \) or \( S_2 < 0 \)

\[
D_2 t_2 - \frac{a_2}{\nu_2 1 - e^{\nu_2}} (1 - e^{\nu_2}) > 0 \quad \text{(or} < 0)\]

\[t_3 + t_4 = t_i\]

One can get the above results directly from the scenario-1 by putting

\[
S_2 = 0
\] and

Therefore the total cost in this scenario is given by
\[ TC_2 = H_2 \left( \int_0^q q_e(t)e^{-rt}dt + \int_0^q q_s(t)e^{-s(t)}dt \right) + G_2 \left( \int_0^q q_e(t) dt \right) e^{-st}dt + U_2 \]

\[ TC_2 = H_2 \left\{ -\left( \alpha Y + D_2 \right) \left[ \frac{-t e^{-rt}}{r} + \frac{t}{r^2} \left( 1 - e^{-rt} \right) \right] + Ye^{-rt} \left[ \frac{1}{r} \left( 1 - e^{-rt} \right) - \frac{1}{(\alpha + r)} \left( 1 - e^{-s(t)} \right) \right] \right. \]

\[ + \frac{\alpha Y + D_2}{r} \left\{ e^{-rt} - e^{-s(t)} - \frac{\alpha Y + D_2}{r} \left[ \frac{1}{r} \left( 1 - e^{-rt} \right) - \frac{1}{(\alpha + r)} \left( 1 - e^{-s(t)} \right) \right] \right\} + \frac{Y D_2}{\alpha + r} \left\{ -e^{-s(t)} + e^{-rt} \left( 1 - e^{-s(t)} \right) \right\} + U_2 \]

\[ \ldots (7.24) \]

In this scenario, we obtain the total cost for the secondary shop as given in the equation (7.24), when shortages at both the shops occur exactly at the same time. This equation is used to calculate the total average profit in the section 7.3.4.

### 7.3.3.3 Scenario-3

In the present scenario, the investigator has measured the situation when shortages at the primary shop occur before the occurrence of shortages at the secondary shop. Here

\[ t = t_1 \]

the stock is not exhausted at \( t = t_1 \), through the supply of defective units stop at \( t = t_1 + t_5 \). So,

\[ t = t_1 \]

the stock at that time decreases due to demand after \( t = t_1 \) and becomes zero at \( t = t_1 + t_5 \). Then the shortages at this shop increases at a rate \( D_2 \) up to \( t = t_1 + t_2 \) when shortages level \( S_2 \) is.

**Inventory**

![Diagram showing inventory levels over time with specific points labeled Q2, Q', t1, t1+t5, t1+t2, and S2.](attachment:image.png)
The differential equations governing the instantaneous state of inventory are

\[ q_2(t) = \begin{cases} 
\theta q_1(t) - D_z, & 0 \leq t \leq t_1 \\
\theta q_1(t) - D_z, & t_1 \leq t \leq t_1 + t_2 \\
-D_z, & t_1 \leq t \leq t_1 + t_5 \\
-D_z, & t_1 + t_5 \leq t \leq t_1 + t_2 
\end{cases} \]

With the boundary conditions

\[ q_2(t) = \begin{cases} 
0, & \text{at } t = 0 \text{ and } t = t_1 + t_2 \\
Q_2, Q_3 \text{ and } -S_2, & \text{at } t = t_1, t = t_1 + t_5 \text{ and } t = t_1 + t_2 
\end{cases} \]

The solutions of the above equations are

\[ q_2(t) = \begin{cases} 
\left(-\alpha Y + D_z\right) t + Ye^{s_2} \left(1 - e^{-t}\right) e^{s_2}, & 0 \leq t \leq t_1 \\
Q_2 + \alpha Y + D_z \left(t_1 - t\right) + Ye^{s_2} \left(1 - e^{-t}\right) e^{s_2}, & t_1 \leq t \leq t_1 + t_5 \\
D_z \left(t_1 + t_5 - t\right), & t_1 \leq t \leq t_1 + t_2 \\
-D_z \left(t - t_1 - t_5\right), & t_1 + t_5 \leq t \leq t_1 + t_2 
\end{cases} \]

\[ Q_2 = D_z t_1 \quad \text{...(7.25)} \]

Using the continuity of at \( t = t_5 \), one can get a relation between \( t_1 \) and \( t_5 \) as follows:

\[ D_z \left(t_1 + t_5\right) + \alpha Y t_1 + Ye^{s_2} = 0 \quad \text{...(7.26)} \]

Inventory level at \( t = t_3 \) and the shortages level at \( t = t_1 + t_2 \) are respectively given by
\[ Q_2 = \left[ \alpha Y + D_j \right] t_2 + Ye^{\alpha t} - 1 - e^{-\alpha t} = D_j t_2 + \left[ \alpha Y + D_j \right] t_1 - t_2 + Y - 1 - e^{{-\alpha t_2 - t_1}} \]

...(7.27)

\[ S_2 = D_j (t_2 - t_1) \]

...(7.28)

\[ S_2 = \begin{cases} t_2 & S_2 < \Theta S \\ t_1 & S_2 > \Theta S \end{cases} \]

Case-3a: When \( S_2 \) are related by

\[ D_j (t_2 - t_1) - \Theta S = 0 \]

...(7.29)

Case-3b & 3c: When \( S_2 \) or \( \Theta S \) then and satisfying

\[ D_j (t_2 - \Theta S) > 0 \quad (or \ < 0) \]

...(7.30)

Therefore the total cost in this scenario is given by

\[ TC_2 = H \left\{ \left( \int_{t_2}^{t_2} \alpha Y + D_j \left[ -t \frac{e^{-\alpha t}}{r} + \frac{1}{r^2} (1 - e^{-\alpha t}) \right] + \frac{\gamma}{r} \left[ \frac{1}{r} \left( 1 - e^{-\alpha t} \right) - \frac{1}{(\alpha + r)} e^{-\alpha t} \right] \right) \right. \]

\[ + Q_j \left( t_2 - t_1 \right) + \frac{\alpha Y + D_j}{r} \left[ e^{-\alpha t} - e^{-\alpha t_2} \right] - \frac{\alpha Y + D_j}{\gamma} \left[ \frac{1}{(\alpha + r)} e^{-\alpha t} + \frac{1}{(\alpha + r)} e^{-\alpha t_2} \right] \]

\[ + \frac{\gamma}{r} (e^{-\alpha t_2} - e^{-\alpha t_1}) + \frac{\gamma}{r} (e^{-\alpha t_2} - e^{-\alpha t_1}) - \frac{D_j}{r} \left( e^{-\alpha t_2} - 1 \right) + G D_j \left( e^{-\alpha t_2} - 1 - e^{-\alpha t_1} \right) + U_j \]

...(7.31)

In this scenario, we obtain the total cost for the secondary shop as given in the equation (7.31), when shortages at the primary shop occur before the occurrence of shortages at the secondary shop. This equation is used to calculate the total average profit in the section 7.3.4.

7.3.4 Profit for Management
If \( \pi \) be the total average profit out of proceeds from both the shops then

\[
\pi = \frac{1}{t_1 + t_2} \sum_{i=1}^{n} (NR_i - TC_i)
\]  

... (7.32)

where net revenue \( NR_i \) and the total cost \( TC_i \) for the primary shop is given by the equations (7.5) and (7.6) respectively. Also, for the secondary shop net revenue \( NR_j \) and the total cost \( TC_j \) for different scenarios and cases are shown in equations (7.8) (7.17) (7.24) and (7.31). Thus nine different problems may arise. Hence, the objective is to maximize the total average profit given by (7.32) with the appropriate constraints for different scenarios and cases.

### 7.4 Two-Shop Inventory Model in Fuzzy Environment

A through survey of the literature enabled the researcher to find that no study has been conducted until now involving two-shops under fuzzy environment, that make it more particular in the real life situation. Here, the researcher has made an attempt to introduce such a unique combination with stock-dependent demand rate and shortages in fuzzy environment.

Till now, models have been developed for solving the two-shop inventory problems when inventory parameters like sorting cost, unit cost, set-up cost, mark-up price, holding cost, etc. are precisely known. In real life situation, these parameters may be uncertain. In competitive market, it is not possible to do the business with predefined fixed inventory parameters. Initially, a decision maker may start with some fixed space and fixed holding cost, set-up cost etc., but at a later stage, to meet the sudden increase of demand or to avail the sudden fall in the price of the commodity, he/she is forced to augment some more space and capital as per demand of the situation. Hence, in this case, cost determination is imprecise. In the present study, it was assumed that the holding cost, shortage cost, set-up, sorting cost, unit cost and mark-up price are fuzzy in nature. Here, the fuzzy cost was worked out with the help of the model-1.

#### 7.4.1 Formulation and Solution of the Model

#### 7.4.2 Primary Shop
Here the researcher has discussed the fuzzy inventory model with the fuzzy parameters of

\[
\begin{align*}
\mathcal{C}_0 &= d, d_i, d_i, d_i, \quad \mathcal{H}_0 = h, h_i, h_i, h_i, \quad \mathcal{G}_0 = g, g_i, g_i, g_i, \quad \mathcal{F}_0 = c, c_i, c_i, c_i, \\
\mathcal{E} &= l, l_i, l_i, l_i, \quad \mathcal{F}_1 = u, u_i, u_i, u_i, \quad \mathcal{H}_1 = i, i_i, i_i, i_i, \quad \mathcal{G}_1 = f, f_i, f_i, f_i, \quad \mathcal{F}_2 = v, v_i, v_i, v_i,
\end{align*}
\]

\[
\mathcal{F}_3 = r, r_i, r_i, r_i, \quad \mathcal{F}_4 = m, m_i, m_i, m_i
\]

The following fuzzy costs are given by

\[
\mathcal{C}_0 \otimes (Q, \oplus S)
\]

(1) Fuzzy purchasing cost is

\[
\mathcal{H}_0 \otimes Q_i \in (\mathcal{G}_1 \otimes (Q \otimes S)) \otimes \mathcal{F}_1
\]

(2) Fuzzy holding cost is

\[
\mathcal{G}_1 \otimes (Q \otimes S) \in (\mathcal{G}_1 \otimes (Q \otimes S)) \otimes \mathcal{F}_1
\]

(3) Fuzzy shortage cost is

\[
\mathcal{G}_1 \otimes (Q \otimes S)
\]

(4) Fuzzy sorting cost is

\[
\mathcal{G}_1 \otimes (Q \otimes S)
\]

(5) Fuzzy setup cost is

\[
\mathcal{F}_3 \otimes (Q \otimes S)
\]

Total fuzzy cost for the primary shop is given by

\[
T \mathcal{C}_0 = (\mathcal{C}_0 \otimes (Q \otimes S)) \oplus ((\mathcal{H}_0 \otimes (Q \otimes S)) \otimes \mathcal{F}_1) \oplus ((\mathcal{G}_1 \otimes (Q \otimes S)) \otimes \mathcal{F}_1) 
\]

\[
\oplus \mathcal{G}_1 \otimes \mathcal{F}_1 
\]

...(7.33)

By second function principal, one has

\[
T \mathcal{C}_0 = \left( \begin{array}{c}
\mathcal{C}_0 \otimes (Q + S) + \frac{h_2}{\alpha} (Q - d, t_i, + \frac{g_2}{b_i} (d, t_i, -(1 - \theta) S) + l_e + u_i, c_i, (Q + S) + \frac{h_2}{\alpha} (Q - d, t_i, + \frac{g_2}{b_i} (d, t_i, -(1 - \theta) S) + l_e + u_i, \\
\{ d, t_i, -(1 - \theta) S) + l_e + u_i, c_i, (Q + S) + \frac{h_2}{\alpha} (Q - d, t_i, + \frac{g_2}{b_i} (d, t_i, -(1 - \theta) S) + l_e + u_i, \\
c_i, (Q + S) + \frac{h_2}{\alpha} (Q - d, t_i, + \frac{g_2}{b_i} (d, t_i, -(1 - \theta) S) + l_e + u_i, \end{array} \right)
\]

...(7.34)

Now we defuzzify the fuzzy cost, using graded mean integration representation method, the result is
\[ P(\mathcal{R}(\mathcal{B})_{c}) = \frac{1}{6} \left( c_i Q_i + S + \frac{h_i}{\alpha} (Q_i - d_{i1}) + \frac{g_i}{b_i} (d_{i2} - 1 - \theta) S + l_i - u_i + 2c_i(Q_i + S) + \frac{2h_i}{\alpha} (Q_i - d_{i1}) + \frac{2g_i}{b_i} d_{i2} - (1 - \theta) S \right) + 2l_i + 2u_i + c_i (Q_i + S) + \frac{h_i}{\alpha} (Q_i - d_{i1}) + \frac{g_i}{b_i} d_{i2} - (1 - \theta) S + l_i - u_i \]  

(7.35)

Fuzzy net revenue cost for the primary shop is given by

\[ Z \ K^\mathcal{R} = \left[ \left( \mathcal{M} \otimes \mathcal{O}^\mathcal{R} \right) \otimes \left( (h_i \otimes Q_i) \mathcal{O} \alpha \right) \oplus \left( 1 - \theta \right) \mathcal{O} S \right] \oplus \left( \mathcal{E} \otimes \theta \right) \otimes \left( i_j \mathcal{O} \alpha \right) \]  

(7.36)

By second function principal, one has

\[ N^\mathcal{R} = \begin{pmatrix} m_{i1} \left[ \frac{h_i}{\alpha} Q_i + (1 - \theta) S + \frac{a_i \theta_i}{\alpha} \right], m_{i2} \left[ \frac{h_i}{\alpha} Q_i + (1 - \theta) S + \frac{a_i \theta_i}{\alpha} \right] \end{pmatrix} \]  

(7.37)

Now we defuzzify the fuzzy cost, using graded mean integration representation method, the result is

\[ P(\mathcal{R}_n^\mathcal{R}) = \frac{1}{6} \left( m_{i1} \left[ \frac{h_i}{\alpha} Q_i + (1 - \theta) S + \frac{a_i \theta_i}{\alpha} \right] + 2m_{i2} \left[ \frac{h_i}{\alpha} Q_i + (1 - \theta) S + \frac{a_i \theta_i}{\alpha} \right] + 2m_{i3} \left[ \frac{h_i}{\alpha} Q_i + (1 - \theta) S + \frac{a_i \theta_i}{\alpha} \right] + m_{i4} \left[ \frac{h_i}{\alpha} Q_i + (1 - \theta) S + \frac{a_i \theta_i}{\alpha} \right] \]  

(7.38)

Here, we obtain the total cost and net revenue cost for the primary shop when the cost and demand parameters are fuzzy in nature with the help of second function principal and graded mean integration representation method, which are given as in the equations (7.35) and (7.38). These equations are used to calculate the total average profit in the section 7.4.3.

### 7.4.3 Secondary Shop

In these three cases, net revenue NR_2 can be written as follows:
\[ \Theta S \leq S_2 \]

**Case-1: If**

Fuzzy net revenue cost for the primary shop is given by

\[
NR^p = \left[ M_{\alpha} \otimes \Theta^{[0]} \otimes \left( (\Theta \otimes \alpha \otimes Q \otimes (\theta \otimes e \otimes (\theta \otimes t_i) )) \right) \right] + (\Theta \otimes S) 
\]

...(7.39)

By second function principal, one has

\[
NR^p = \left[ r_c \left\{ \frac{\theta}{\alpha} (Q_i - d_i + e \otimes t_i) \right\} , r_c \left\{ \frac{\theta}{\alpha} (Q_i - d_i + e \otimes t_i) \right\} , \Theta S \right]
\]

\[
+ r_c \left\{ \frac{\theta}{\alpha} (Q_i - d_i + e \otimes t_i) \right\} , r_c \left\{ \frac{\theta}{\alpha} (Q_i - d_i + e \otimes t_i) \right\} , \Theta S \right]
\]

...(7.40)

Now we defuzzify the fuzzy cost, using graded mean integration representation method, the result is

\[
P \cdot NR^p = \frac{1}{6} \left[ r_c \left\{ \frac{\theta}{\alpha} (Q_i - d_i + e \otimes t_i) \right\} + 2 r_c \left\{ \frac{\theta}{\alpha} (Q_i - d_i + e \otimes t_i) \right\} + \Theta S \right]
\]

\[
+ 2 r_c \left\{ \frac{\theta}{\alpha} (Q_i - d_i + e \otimes t_i) \right\} , r_c \left\{ \frac{\theta}{\alpha} (Q_i - d_i + e \otimes t_i) \right\} , \Theta S \right]
\]

...(7.41)

\[ \Theta S \geq S_2 \]

**Case-2: If**

Fuzzy net revenue cost for the primary shop is given by

\[
NR^p = \left[ M_{\alpha} \otimes \Theta^{[0]} \otimes \left( (\Theta \otimes \alpha \otimes Q \otimes (\theta \otimes e \otimes (\theta \otimes t_i) )) \right) \right] + (\Theta \otimes S) \otimes \left\{ m_i \otimes \Theta \otimes ((\theta \otimes S) \otimes e \otimes S_i) \right\}
\]

...(7.42)

By second function principal, one has
\[ NR_2 = \left\{ r_1 \left( \frac{\theta}{\alpha} (Q - d_{t_1}) + \theta S \right) + m_1 c_1 (\theta S - S) + r_2 \left( \frac{\theta}{\alpha} (Q - d_{t_2}) + \theta S + m_2 c_2 (\theta S - S) \right) \right\} \]

Now we defuzzify the fuzzy cost, using graded mean integration representation method, the result is

\[ P(NR_2) = \frac{1}{6} \left\{ r_1 \left( \frac{\theta}{\alpha} (Q - d_{t_1}) + \theta S \right) + m_1 c_1 (\theta S - S) + 2 r_2 \left( \frac{\theta}{\alpha} (Q - d_{t_2}) + \theta S + m_2 c_2 (\theta S - S) \right) \right\} \]

\[ + 2 r_3 \left( \frac{\theta}{\alpha} (Q - d_{t_3}) + \theta S \right) + 2 m_3 c_3 (\theta S - S) + r_4 \left( \frac{\theta}{\alpha} (Q - d_{t_4}) + \theta S + m_4 c_4 (\theta S - S) \right) \]

\[ \cdots \quad (7.44) \]

Similarly, we obtain the net revenue cost for the secondary shop when the cost and demand parameters are fuzzy in nature with the help of second function principal and graded mean integration representation method, which are given as in the equations (7.41) and (7.44) for the situation when stock-level of defective units is less then or equal to the shortage level.

### 7.4.3.1 Scenario-1: When shortages at the secondary shop occurs earlier than the occurrence of shortages at the primary shop

The following fuzzy costs are given by

(1) Fuzzy holding cost is

\[ \left(\left( \left\{ \left( \left( Y \otimes (1 \otimes \alpha \otimes t_1) \right) \right| e \right\} \left( \left( (\alpha \otimes Y) \otimes D_{t_2} \right) \otimes t_2 \right) \otimes t_3 \right) \otimes 2 \otimes (\alpha) \right) \right) \]

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(2) Fuzzy shortage cost is

\[
\left( \mathcal{G} \otimes \left( e \otimes Y \otimes \left( \delta \otimes (1 \oplus (e \otimes t_i)) \right) \right) \otimes \left( \alpha \otimes Y \otimes D_i \right) \otimes \left( \left( t_i \otimes t_j \right) \otimes \left( t_i \right) \right) \right) \otimes \left( \left( e \otimes t_i \right) \left( e \otimes t_i \right) \right) \otimes 2
\]

\[
\oplus \left( \mathcal{G} \otimes \left( Y \otimes \left( t_i \otimes t_j \right) \otimes \left( t_i \otimes \left( \alpha \otimes Y \otimes D_i \right) \right) \right) \otimes \left( \alpha \otimes Y \otimes t_i \right) \otimes \left( D_i \otimes t_j \right) \otimes 2 \right) \right) \right)
\]

(3) Fuzzy setup cost is

Therefore the total fuzzy cost in this scenario is given by

\[
T \mathcal{C}_2 = \left( \left( \mathcal{F} \otimes \left( 2 \otimes (Y \otimes \left( \delta \otimes \left( \alpha \otimes t_i \right) \right) \right) \right) \otimes \left( \alpha \otimes Y \otimes D_i \right) \otimes \left( \left( \left( t_i \otimes t_j \right) \otimes \left( e \otimes 2 \otimes \alpha \right) \right) \right) \right) \otimes \left( \left( t_i \otimes t_j \right) \otimes \left( \alpha \otimes \left( Y \otimes \left( e \otimes t_i \right) \otimes \left( e \otimes t_i \right) \right) \right) \right) \otimes \left( \alpha \otimes Y \otimes \left( t_i \otimes t_j \right) \right) \otimes \left( D_i \otimes t_j \right) \otimes 2 \right) \right) \right)
\]

(7.45)

By second function principal, one has

\[
T \mathcal{C}_2 = \left( \frac{i}{2} \left[ 2 Y (1 + \alpha t_i) - \alpha Y + D_i \right] \left( t_i + t_i - \frac{2}{\alpha} \right) \right) \left( t_i + t_i + \frac{f_t}{2} \left( -2 Y (1 + \alpha t_i) + \alpha Y + D_i \right) \right) + \frac{f_t}{2} \left( -2 Y (1 + \alpha t_i) + \alpha Y + D_i \right) + v,
\]

\[
\left( t_i + t_i + \frac{f_t}{2} \left( -2 Y (1 + \alpha t_i) + \alpha Y + D_i \right) \right) + v,
\]

\[
\left( t_i + t_i + \frac{f_t}{2} \left( -2 Y (1 + \alpha t_i) + \alpha Y + D_i \right) \right) + v,
\]

\[
\left( t_i + t_i + \frac{f_t}{2} \left( -2 Y (1 + \alpha t_i) + \alpha Y + D_i \right) \right) + v,
\]

\[
\left( t_i + t_i + \frac{f_t}{2} \left( -2 Y (1 + \alpha t_i) + \alpha Y + D_i \right) \right) + v,
\]

\[
(7.45)
\]

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\[ t_1 + t_3 + t_4 \left( t_1 - t_3 - t_4 \right) + f_1 \left[ \alpha Y \left( t_1 - t_3 - t_4 \right) + t_3 \left( \alpha Y + D_3 \right) t_1 - \alpha Y t_3 + \frac{D t_3}{2} \right] + v_1 \]

\[ t_1 + t_3 + t_4 \left( t_1 - t_3 - t_4 \right) + f_2 \left[ \alpha Y \left( t_1 - t_3 - t_4 \right) + t_3 \left( \alpha Y + D_3 \right) t_1 - \alpha Y t_3 + \frac{D t_3}{2} \right] + v_2 \]

(7.46)

Now we defuzzify the fuzzy cost, using graded mean integration representation method, the result is

\[ P \ T_e^{opt} = \frac{1}{6} \left[ \frac{2}{1+\alpha t} \left( \alpha Y + D_2 \right) \left( t_1 + t_3 - \frac{2}{\alpha} \right) \right] \left( t_1 + t_3 + \frac{1}{2} \left[ -2Y \left( 1+\alpha t \right) + \left( \alpha Y + D_3 \right) \right] \right) \]

\[ t_1 + t_3 + t_4 \left( t_1 - t_3 - t_4 \right) + f_1 \left[ \alpha Y \left( t_1 - t_3 - t_4 \right) + t_3 \left( \alpha Y + D_3 \right) t_1 - \alpha Y t_3 + \frac{D t_3}{2} \right] + v_1 \]

\[ + f_2 \left[ 2Y \left( 1+\alpha t \right) - \left( \alpha Y + D_2 \right) \left( t_1 + t_3 - \frac{2}{\alpha} \right) \right] \left( t_1 + t_3 + \frac{1}{2} \left[ -2Y \left( 1+\alpha t \right) + \left( \alpha Y + D_3 \right) \right] \right) + 2v_2 \]

\[ + f_3 \left[ 2Y \left( 1+\alpha t \right) - \alpha Y + D_2 \right) \left( t_1 + t_3 - \frac{2}{\alpha} \right) \right] \left( t_1 + t_3 + \frac{1}{2} \left[ -2Y \left( 1+\alpha t \right) + \left( \alpha Y + D_3 \right) \right] \right) + 2v_3 \]

(7.47)

In this scenario, we obtain the total cost for the secondary shop when the cost and demand parameters are fuzzy in nature with the help of second function principal and graded mean integration representation method for the situation when shortages at the
secondary shop occurs earlier than the occurrence of shortages at the primary shop, which is given as in the equation (7.47). This equation is used to calculate the total average profit in the section 7.4.3.

### 7.4.3.2 Scenario-2: When shortages at both the shops occur exactly at the same time

The following fuzzy costs are given by

\[
\left( \bigoplus_{2}^{Y} \left( \bigotimes Y \bigotimes D_{2} \bigotimes t_{2} \bigotimes 2\bigotimes \alpha \right) \bigotimes t_{2} \right) \bigotimes 2
\]

(1) Fuzzy holding cost is

\[
\left( \bigoplus_{2}^{Y} \bigotimes D_{2} \bigotimes t_{2} \bigotimes t_{2} \bigotimes 2 \right)
\]

(2) Fuzzy shortage cost is

\[\mathcal{U}^{2} \]

(3) Fuzzy setup cost is

Therefore the total fuzzy cost in this scenario is given by

\[
T^{\%} = \left( \bigoplus_{2}^{Y} \left( \bigotimes Y \bigotimes D_{2} \bigotimes t_{2} \bigotimes 2\bigotimes \alpha \right) \bigotimes t_{2} \right) \bigotimes 2 \bigoplus \left( \bigotimes \bigotimes D_{2} \bigotimes t_{2} \bigotimes t_{2} \bigotimes 2 \right)
\]

By second function principal, one has

\[
T^{\%} = \left( \frac{i}{2} \left[ Y + D_{2} \left( t_{2} + \frac{2}{\alpha} \right) + 2Y \right] t_{2} + \frac{fD_{2}t_{2}^{2}}{2} + v, \right] Y + D_{2} \left( t_{2} + \frac{2}{\alpha} \right) + \frac{fD_{2}t_{2}^{2}}{2} + v, \right] t_{2} + \frac{fD_{2}t_{2}^{2}}{2} + v, \right]
\]

\[
Y + D_{2} \left( t_{2} + \frac{2}{\alpha} \right) + 2Y \right] t_{2} + \frac{fD_{2}t_{2}^{2}}{2} + v, \right] Y + D_{2} \left( t_{2} + \frac{2}{\alpha} \right) + \frac{fD_{2}t_{2}^{2}}{2} + v, \right] t_{2} + \frac{fD_{2}t_{2}^{2}}{2} + v, \right]
\]

\[
Y + D_{2} \left( t_{2} + \frac{2}{\alpha} \right) + 2Y \right] t_{2} + \frac{fD_{2}t_{2}^{2}}{2} + v, \right] Y + D_{2} \left( t_{2} + \frac{2}{\alpha} \right) + \frac{fD_{2}t_{2}^{2}}{2} + v, \right] t_{2} + \frac{fD_{2}t_{2}^{2}}{2} + v, \right]
\]

(7.49)

Now we defuzzify the fuzzy cost, using graded mean integration representation method, the result is

216
\[ P \, T(\theta) = \frac{1}{6} \left( \frac{\alpha^2}{2} \left( Y + \frac{2}{\alpha^2} \right) + 2Y \right) t_1 + \frac{\alpha}{2} D x t_2 + v_x + \left( \frac{\alpha}{2} \right) \left( Y + \frac{2}{\alpha^2} \right) t_1 + f, D \]

\[ t_1 + 2v_1 + v_1 \left( \frac{\alpha Y + D_0}{\alpha} \right) \left( t_1 + \frac{2}{\alpha^2} \right) t_1 + f, D t_2 + v_2 + \left( \frac{\alpha Y + D_0}{\alpha^2} \right) t_1 + \frac{2}{\alpha^2} \right) Y \]

\[ t_1 + \frac{\alpha}{2} D x t_2 + v_x \]

\[ \ldots(7.50) \]

In this scenario, we obtain the total cost for the secondary shop when the cost and demand parameters are fuzzy in nature with the help of second function principal and graded mean integration representation method for the situation when shortages at both the shops occur exactly at the same time, which is given as in the equation (7.50). This equation is used to calculate the total average profit in the section 7.4.3.

### 7.4.3.3 Scenario-3: When shortages at the secondary shop occur after the occurrence of shortages at the primary shop

The following fuzzy costs are given by

1. Fuzzy holding cost is
   \[
   \left\{ \left( \theta_1 \left( \alpha \otimes Y \right) \otimes t_1 \right) \otimes t_2 \right\} \otimes \left( \theta_2 \left( D_1 \otimes t_1 \otimes t_1 \right) \right) \otimes \left( t_1 \otimes t_2 \otimes e, \left( 2 \otimes \alpha \right) \right) \right| \otimes 2 \right\}
   \]

2. Fuzzy shortage cost is
   \[ t_2 \]

3. Fuzzy setup cost is
   \[ t_2 \]

Therefore the total fuzzy cost in this scenario is given by

\[
TC_2 = \left\{ \left( \theta_1 \left( \alpha \otimes Y \right) \otimes t_1 \right) \otimes t_2 \right\} \otimes \left( \theta_2 \left( D_1 \otimes t_1 \otimes t_1 \right) \right) \otimes \left( \left( t_1 \otimes t_2 \otimes e, \left( 2 \otimes \alpha \right) \right) \right) \right| \otimes 2 \right\} \otimes \theta_2
\]

\[
\left\{ \left( t_1 \otimes t_2 \otimes e, t_1 \right) \otimes \left( \theta_2 \left( D_1 \otimes t_1 \otimes t_1 \right) \right) \otimes \left( \left( t_1 \otimes t_2 \otimes e, \left( 2 \otimes \alpha \right) \right) \right) \right| \otimes 2 \right\} \otimes \theta_2
\]

\[ \ldots(7.51) \]
By second function principal, one has
\[ p \{ \text{Cost} \} = \left( \frac{1}{2} \right) \left[ Y_i^2 + D_i \cdot t_i + t_i \left( t_i + t_i - \frac{2}{\alpha_i^2} \right) + \frac{F}{2} D_i \cdot (t_i - t_i)^2 \right] + \left( \frac{F}{2} D_i \cdot (t_i - t_i)^2 \right)
\]
\[ + \left[ Y_i^2 + D_i \cdot t_i + t_i \left( t_i + t_i - \frac{2}{\alpha_i^2} \right) + \frac{F}{2} D_i \cdot (t_i - t_i)^2 \right] + v_i \left( \frac{F}{2} D_i \cdot (t_i - t_i)^2 \right) + v_i \]
\[ = \left( \frac{1}{2} \right) \left[ Y_i^2 + D_i \cdot t_i + t_i \left( t_i + t_i - \frac{2}{\alpha_i^2} \right) + \frac{F}{2} D_i \cdot (t_i - t_i)^2 \right] + v_i \left( \frac{F}{2} D_i \cdot (t_i - t_i)^2 \right) + v_i \]
\[ \ldots \text{(7.52)} \]

Now we defuzzify the fuzzy cost, using graded mean integration representation method, the result is
\[ p \{ \text{Cost} \} = \left( \frac{1}{6} \right) \left[ Y_i^2 + D_i \cdot t_i + t_i \left( t_i + t_i - \frac{2}{\alpha_i^2} \right) + \frac{F}{2} D_i \cdot (t_i - t_i)^2 \right] + v_i \left( \frac{F}{2} D_i \cdot (t_i - t_i)^2 \right) + v_i \]
\[ + \left[ Y_i^2 + D_i \cdot t_i + t_i \left( t_i + t_i - \frac{2}{\alpha_i^2} \right) + \frac{F}{2} D_i \cdot (t_i - t_i)^2 \right] + 3v_i + \frac{F}{2} \]
\[ \left[ Y_i^2 + D_i \cdot (t_i + t_i) \left( t_i + t_i - \frac{2}{\alpha_i^2} \right) + \frac{F}{2} D_i \cdot (t_i - t_i)^2 \right] + v_i \left( \frac{F}{2} D_i \cdot (t_i - t_i)^2 \right) + v_i \]
\[ \ldots \text{(7.53)} \]

In this scenario, we obtain the total cost for the secondary shop when the cost and demand parameters are fuzzy in nature with the help of second function principal and graded mean integration representation method for the situation when shortages at the secondary shop occur after the occurrence of shortages at the primary shop, which is given as in the equation (7.53). This equation is used to calculate the total average profit in the section 7.4.3.

### 7.4.4 Profit for Management

If \( \mathcal{R} \) be the total average profit out of proceeds from both the shops then
\[ \mathcal{R} = \frac{1}{t_i + t_i} \sum_{j=1}^{n} \left( P \cdot NR_j - P \cdot TC_j \right) \]
\[ \ldots \text{(7.54)} \]

where net revenue \( NR_i \) and the total cost \( TC_i \) for the primary shop is given by the equations (7.38) and (7.35) respectively. Also, for the secondary shop net revenue \( NR_2 \) and the total
cost TC_2 for different scenarios and cases are shown in the equations (7.41) (7.47) (7.50) and (7.53). Thus nine different problems may arise. Hence, the objective is to maximize the total average profit given by (7.54) with the appropriate constraints for different scenarios and cases.

7.5 Solution Procedure: Generalized Reduced Gradient (GRG) Method

The researcher has used GRG method to find the optimal solution of the problem mentioned earlier. The GRG method is a method for solving problems with linear constraint only. Let us consider the non-linear problems as:

\[
\begin{align*}
\text{Maximize} & \quad f(X) \\
\text{Subject to} & \quad h_j(X) < 0, \quad j = 1, 2, \ldots, m \\
& \quad l_k(X) = 0, \quad k = 1, 2, \ldots, 1 \\
& \quad X = [x_1, x_2, \ldots, x_n]^T \\
& \quad x_j^L \leq x_j \leq x_j^U, \quad i = 1, 2, \ldots, n \\
\end{align*}
\]

By adding a non-negative slack variable to each of the inequality constraints, the above problem can be reduced to the form,

\[
\begin{align*}
\text{Maximize} & \quad f(X) \\
\text{Subject to} & \quad h_j(X) + s^*_j = 0, \quad j = 1, 2, \ldots, m \\
& \quad l_k(X) = 0, \quad k = 1, 2, \ldots, 1 \\
& \quad x_j^L \leq x_j \leq x_j^U, \quad i = 1, 2, \ldots, n \\
& \quad s^*_j \geq 0, \quad j = 1, 2, \ldots, m \\
\end{align*}
\]

\[
(n + m) \quad \begin{pmatrix} x_1, x_2, \ldots, x_n, x_{n+1}, \ldots, x_{n+m} \end{pmatrix}
\]

with \( n + m \) variables. The problem can be rewritten in a general form as:

219
Maximize

\[ f(X) \]

Subject to

\[ g_j(X) = 0, \quad j = 1, 2, \ldots, m + 1 \]

\[ X = [x_1, x_2, \ldots, x_i, x_{i+1}, \ldots, x_{m+i}]^T \]

\[ x_i^{lb} \leq x_i \leq x_i^{ub}, \quad i = 1, 2, \ldots, n + m \]

where the lower and upper bounds on the slack variables, \( x_i \) \((i = n+1, n+2, \ldots, n+m)\) are taken as zero and a sufficiently large number respectively.

This method is based on the idea of elimination of variables using the equality constraints. Here, \((n - m)\) design variables can be classified into two sets, the first set is of \((n-1)\) design or independent variables and the other set is of \((m + 1)\) state or dependent variables and where the design variables are completely independent and the state variables are dependent on the design variables used to satisfy the constraints

\[ g_j(X) < 0, \quad j = 1, 2, \ldots, m + 1 \]

To determine the search direction, GRG is calculated in terms of above two sets of variables. Geometrically, the reduced gradient can be desired as a projection of the original \(n\)-dimensional gradient into the \((n-m)\) dimensional feasible region described by the design variables.

Now, to find the local maximum objective along the search direction, any one of the one-dimensional maximization procedures may be used. Here, the researcher has used the quadratic interpolation method for finding the optimal step length.

Now, GRG method is used to optimize the profit functions given by (6.54) for different scenarios and cases subject to the appropriate constraints. To illustrate it explicitly, the problem of case 1b of scenario 1 can be mathematically written as

Maximize

\[ \mathcal{R}(t_1, t_2, t_3, m_1) \]

Subject to

\[ \{ \alpha Y + D_2 \} (t_1 + t_3) - Ye^{\alpha Y} (1 - e^{-\alpha Y}) = 0 \]
\[ \alpha Y_t + D \left( t_1 + t_2 \right) - \theta \left( \frac{Q}{\alpha} + S \right) > 0 \]

\[ t_1, t_2, t_3, m_2 \]

Here, \( t_1, t_2, t_3 \), and \( m_2 \) are the decision variables and using their optimum values, the optimum order quantity and shortage level at the primary shop and then corresponding quantities at the secondary shop and the optimum selling price of the defective units are obtained.

### 7.6 Numerical Examples

#### 7.6.1 Numerical Example for the Model \(-1\)

The following numerical data has been used to analyze the model:

\[
\begin{align*}
C_0 & \quad H_1 & \quad G_1 & \quad U_1 & \quad H_2 & \quad G_2 & \quad U_2 & \quad b_1 & \quad b_2 \\
= & \quad 1.1, & = 1.5, & = 2.0, & = 70, & = 1.2, & = 1.8, & = 35, & = 0.15, & = 2.50, \\
\beta & \quad = 2, & \quad \theta & \quad = 0.40, & \quad \gamma & \quad = 0.30, & \quad r & \quad = 0.2. & \text{In addition to the above}
\end{align*}
\]

Table 7.1 Different parametric values for different cases

<table>
<thead>
<tr>
<th></th>
<th>Scenario-1</th>
<th>Scenario-2</th>
<th>Scenario-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_1 )</td>
<td>75</td>
<td>50</td>
<td>20</td>
</tr>
<tr>
<td>( a_2 )</td>
<td>32</td>
<td>50</td>
<td>20</td>
</tr>
<tr>
<td>( M_1 )</td>
<td>5.5</td>
<td>5.5</td>
<td>5.5</td>
</tr>
<tr>
<td>( m'_2 )</td>
<td>0.12</td>
<td>0.40</td>
<td>0.40</td>
</tr>
</tbody>
</table>

To solve the highly nonlinear equation in the profit function, the mathematical software MATHEMATICA-5.2 was used. Applying the above solution procedure the computational results showed the following optimal values:

1. Profit for the scenario-1 is 2223.26.
2. Profit for the scenario-2 is 1645.26.
3. Profit for the scenario-3 is 608.58.

Table 7.2 Comparison of policy by varying inflation rate

<table>
<thead>
<tr>
<th>Inflation rate ((r))</th>
<th>Profit for the Scenario-1</th>
<th>Profit for the Scenario-2</th>
<th>Profit for the Scenario-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>2223.26</td>
<td>1645.26</td>
<td>608.58</td>
</tr>
<tr>
<td>0.4</td>
<td>1233.55</td>
<td>1002.21</td>
<td>346.83</td>
</tr>
<tr>
<td>0.6</td>
<td>895.441</td>
<td>750.41</td>
<td>243.42</td>
</tr>
<tr>
<td>0.8</td>
<td>602.17</td>
<td>182.41</td>
<td>182.41</td>
</tr>
<tr>
<td>$G_t$</td>
<td>Profit for the Scenario-1</td>
<td>Profit for the Scenario-2</td>
<td>Profit for the Scenario-3</td>
</tr>
<tr>
<td>-------</td>
<td>---------------------------</td>
<td>---------------------------</td>
<td>--------------------------</td>
</tr>
<tr>
<td>2.5</td>
<td>2516.97</td>
<td>1772.89</td>
<td>662.35</td>
</tr>
<tr>
<td>3.0</td>
<td>2663.83</td>
<td>1836.70</td>
<td>689.24</td>
</tr>
<tr>
<td>3.5</td>
<td>2810.68</td>
<td>1900.52</td>
<td>716.13</td>
</tr>
<tr>
<td>4.0</td>
<td>2957.54</td>
<td>1964.33</td>
<td>743.02</td>
</tr>
<tr>
<td>4.5</td>
<td>3104.39</td>
<td>2028.15</td>
<td>769.91</td>
</tr>
<tr>
<td>5.0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 7.5 Graphical representation of comparison of the profit function

Table 7.3 Comparison of policy by varying holding cost in primary shop
Fig. 7.6 Graphical representation of comparison of the profit function

Table 7.4 Comparison of policy by varying holding cost in secondary shop

<table>
<thead>
<tr>
<th>G2</th>
<th>Profit for the Scenario-1</th>
<th>Profit for the Scenario-2</th>
<th>Profit for the Scenario-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2235.17</td>
<td>1641.07</td>
<td>605.37</td>
</tr>
<tr>
<td>4</td>
<td>2354.29</td>
<td>1599.21</td>
<td>573.29</td>
</tr>
<tr>
<td>6</td>
<td>2473.40</td>
<td>1557.34</td>
<td>541.21</td>
</tr>
<tr>
<td>8</td>
<td>2592.52</td>
<td>1515.48</td>
<td>509.13</td>
</tr>
</tbody>
</table>

Fig. 7.7 Graphical representation of comparison of the profit function

The main observations drawn from the numerical examples are as follows:

1. Inflation. Total profit decreases as inflation rate \( r \) increases in all scenarios,
because as the inflation rate \( r \) increases, the total inventory cost increases. The profit is maximum for the scenario-1. So, it is very important to take consideration of inflation in model otherwise model will not represent the realistic situation.

2. It is evident from the Table 7.3; that the changes in the holding cost \( g_1 \) for the primary shop do have a significant effect in the total profit. The total profit for the scenario-1 is more in comparison to the scenario-2 and 3.

3. It can be concluded from the Table 7.4; that the changes in the holding cost \( g_2 \) ( ) for the secondary shop do have a significant effect in the total profit. The total profit for the scenario-1 is more in comparison to the scenario-2 and 3.

Finally, the profit is maximum for the scenario-1 with respect to all the parameters when the shortages at secondary shop occur earlier than the occurrence of shortages at the primary shop.

### 7.6.2 Sensitivity Analysis

The change in the values of parameters may commence due to uncertainties in any decision-making situation. In order to examine the implications of these changes, the sensitivity analysis is of great help in decision-making. Using the numerical example done. The results of the sensitivity analysis are summarized in Table 6.5. The following inferences can be made:

**Table 7.5 Effects of changing various parameters on the profit**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Percentage change in the Parameter</th>
<th>Profit for the Scenario -1</th>
<th>Percentage change in the Profit for the Scenario -1</th>
<th>Profit for the Scenario -2</th>
<th>Percentage change in the Profit for the Scenario -2</th>
<th>Profit for the Scenario -3</th>
<th>Percentage change in the Profit for the Scenario -3</th>
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</thead>
<tbody>
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<td>( C_0 )</td>
<td>-20.00</td>
<td>1977.02</td>
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<td>1397.88</td>
<td>-15.03</td>
<td>504.03</td>
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<td></td>
<td>-10.00</td>
<td>2100.14</td>
<td>-5.33</td>
<td>1521.57</td>
<td>-7.51</td>
<td>556.30</td>
<td>-8.58</td>
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<td></td>
<td>10.00</td>
<td>2346.38</td>
<td>5.33</td>
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<td>7.51</td>
<td>660.85</td>
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<td>11.07</td>
<td>1892.63</td>
<td>15.03</td>
<td>713.13</td>
<td>17.17</td>
</tr>
<tr>
<td>---</td>
<td>-----</td>
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<td>-------</td>
<td>---------</td>
<td>-------</td>
<td>--------</td>
<td>-------</td>
</tr>
<tr>
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<td>-0.59</td>
<td>1639.16</td>
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<td>1635.48</td>
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<td>603.69</td>
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<td>-1.02</td>
<td>1680.16</td>
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<td>624.96</td>
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</tr>
<tr>
<td>$b_2$</td>
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<td>0.00</td>
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<td>1645.26</td>
<td>-0.001</td>
<td>600.05</td>
<td>0.608</td>
</tr>
<tr>
<td></td>
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<td>0.00</td>
<td>1645.26</td>
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<td>600.05</td>
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<td>-4.06</td>
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<td>2175.66</td>
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<td>1611.86</td>
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<td>592.84</td>
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<td>1679.66</td>
<td>2.02</td>
<td>624.32</td>
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<td>-10</td>
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<td>9.84</td>
<td>1780.87</td>
<td>8.24</td>
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<td>1438.89</td>
<td>-12.54</td>
<td>524.82</td>
<td>-13.76</td>
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</tbody>
</table>

The main observations drawn from the sensitivity analysis (table 7.5) are as follows:

1. As the value of unit cost ($C_0$) increases, the total profit in all the three scenarios also increases.
2. As the value of holding cost ($H_1$) for the primary shop increases, the total profit in all the three scenarios also increases.
3. As the value of shortage cost ($G_1$) for the primary shop increases, the total profit in all the three scenarios also increases.
4. As the value of holding cost ($H_2$) for the secondary shop increases, the total profit in all the three scenarios also increases.
5. As the value of shortage cost \((C_2)\) for the secondary shop increases, the total profit for the scenario-1 increases, whereas the total profit for the scenario-2 and scenario-3 decreases.

\[
b_1
\]

6. As the value of \((b_2)\) for the primary shop increases, the total profit for the scenario-1 increases, whereas the total profit for the scenario-2 and scenario-3 decreases.

7. The effect of changes in \(b_2\) on the profit for all the three scenarios is not significant.

8. The total profit in all the three scenarios is highly sensible with respect to and the rate of inflation \((r)\). As the value of \((r)\) increases, the total profit in all three scenarios also increases. Whereas the total profit decreases with the increase in inflation rate.

### 7.6.3 Numerical Example of the Model-2

To illustrate the model, the researcher consider \(k = 2, \quad \bar{\bar{c}}_0 = (1.045, 1.1, 1.1, 1.21),\)

\[
b_1 = 0.15, \quad b_2 = 2.50, \quad \bar{h}_1 = (1.425, 1.5, 1.5, 1.65), \quad \bar{h}_0 = (1.14, 1.2, 1.2, 1.32), \quad \bar{\bar{c}}_1 = (66.5, 70, 70, 77), \quad \bar{\bar{c}}_2 = \bar{\bar{c}}_1 = (33.25, 35, 35, 38.5), \quad \bar{\bar{c}}_2 = (1.9, 2.0, 2.0, 2.2), \quad \bar{\bar{c}}_2 = (1.71, 1.8, 1.8, 1.98), \quad \beta = 0.30, \quad \beta = 0.40.
\]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Scenario-1</th>
<th>Scenario-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x_0)</td>
<td>(71.25, 75, 75, 82.5)</td>
<td>(47.5, 50, 50, 55)</td>
</tr>
<tr>
<td>(a_1)</td>
<td>32</td>
<td>30</td>
</tr>
<tr>
<td>(\bar{h}_1)</td>
<td>(6.65, 7, 7, 7.7)</td>
<td></td>
</tr>
<tr>
<td>(m_2)</td>
<td>0.12</td>
<td>0.40</td>
</tr>
</tbody>
</table>

To solve, the highly nonlinear equation in the profit function the mathematical software MATHEMATICA-5.2 was used. Applying the above solution procedure, the computational results showed the following optimal values:

1. Profit for the scenario-1 = 268.531
2. Profit for the scenario-2 = 28.9958
3. Profit for the scenario-3 = 53.0071

### 7.6.4 Sensitivity Analysis

Using the numerical example given in the preceding section, the sensitivity analysis with respect to holding cost for both the shops has been done. The results of the sensitivity analysis are summarized in Table 6.7. The following inferences can be made.

#### Table 7.7 Variation in holding cost in both shops

<table>
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<tr>
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<th>-10%</th>
<th>10%</th>
<th>20%</th>
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<tr>
<td>Profit for the</td>
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<td>220.374</td>
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<td>78.472</td>
<td>40.7544</td>
<td>4.2574</td>
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<td>Profit for the</td>
<td>61.179</td>
<td>56.8482</td>
<td>44.8566</td>
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<tr>
<td>scenario-3</td>
<td></td>
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</tr>
</tbody>
</table>

![Profit for the scenario-3](image)

**Fig. 7.8 Graphical representation of the profit function**

The main observations drawn from the sensitivity analysis (table 7.7) are as follows:

1. From the table 7.7, it is clear that profit function is highly sensitive with respect to holding cost.
2. When the shortages at the secondary shop of defective units occur earlier than the shortages at the primary shop, then the total profit is maximized.

3. When the shortages occur at the same time at the primary and the secondary shops, then the total profit gets minimized.

4. Occurrence of shortages at the primary shop, then the total profit is minimum from scenario-2 and maximum from scenario-1.

Decision maker will prefer scenario-1, since in this case profit is maximized.

7.7 Conclusion

In this study, two inventory models have been framed for differential items sold from two shops under the single management in which one was in inflationary environment and the other in fuzzy environment. The demand rate for the primary shop shortages were allowed and fully backlogged. Here, three different scenarios were presented for the secondary shop. Retailers purchased the items in a lot from wholesalers and sold the fresh (non-defective) and deteriorated (defective) ones in separate shop. The main objective of the study was to maximize the retailer’s total inventory profit by considering both environments. Sensitivity analysis with respect to various parameters has been carried out. With the help of numerical examples and sensitivity analysis, the researcher concluded that the profit was maximum for the scenario-1, i.e., when the shortages at the secondary shop of defective units occurred earlier than the occurrence of shortages at the primary shop, then the profit was maximum. The total profit in all the three scenarios was highly sensible with respect to the deterioration rate (\( d \)) and the rate of inflation (\( i \)). As the value of the deterioration rate (\( d \)) increases, the total profit in all three scenarios is also increased, whereas total profit decreased with the increase of inflation rate (\( i \)). These results are applicable for the products like shoes, ready-made garments, fruits, etc., where the product is not sold immediately. So, the cost of these types of products is affected by inflation and deterioration. The extent to which inflation has affected the business world is clearly elucidated through the sensitivity analysis, where the effect of inflation is visibly shown over the total profit.
Similarly, the total profit in all the three scenarios was highly sensible with respect to the holding cost for both the shops in fuzzy environment. With the help of numerical example and sensitivity analysis, the investigator concluded that the profit for the scenario-1 is also maximum in comparison to the scenario-2 and 3 in fuzzy environment.

Further studies in future can still elaborate the present work by incorporating more realistic assumptions such as, probabilistic demand, non-instantaneous deterioration rate with a two-parameter Weibull distribution and the fuzzy parameters in the present model.

References


