Chapter 3

Signal Analysis using Cross-correlation Technique and Cross-wavelet Transform

3.1 Introduction

Mathematical operation which converts a function or sequence from one domain into another is refereed as transform. Transform is applied to functions or sequences to extract further information, which were not readily available in their raw format. Most of real life signals in the raw format are in time domain. As distinct information are hidden in frequency content of signals, time domain representation is not suitable for many signal processing applications. Hence, for further processing, the raw signals are required to be transferred into other domains, viz. frequency domain, time-frequency domain, etc. A transform helps to extract hidden information and the choice of transformation depends on the application. In addition to revealing hidden information, the transform makes an equation easier to solve compared to that of their raw format (e.g. Laplace transform converts a differential equation to an algebraic equation which is easier to solve). Moreover, as the transform provide data compression/reduction, transformed signal or sequence requires less storage space than that for original. In the present study, hidden information from acquired transformer winding current (time domain signal) had been extracted by transferring the signal into time-frequency domain using cross-wavelet transform (XWT).

3.2 Wavelet Transform

As stated earlier, in most of the signal processing applications, knowledge of the frequency content of the signal has significant role to play. There are number of transformations that may be employed to covert the signal from time domain into frequency domain, among which Fourier
transform (FT) is probably the most popular. The frequency spectrum obtained from FT reveals how much of each frequency is present in the signal, but it will not provide any information about at what time which particular frequency exists. Hence, FT is only suitable for signals whose frequency content will not change with time, i.e. for stationary signals. But, to analyze and process non-stationary signals (signals whose frequency content changes with time) both time and frequency information is needed simultaneously. As FT only reveals frequency content of the signals, it is not suitable for analyzing such non-stationary signals.

To overcome this drawback of FT, Short Time Fourier Transform (STFT) was introduced as an alternative [51]-[53]. In STFT, the signal to be transformed is divided into small segments of particular window width. Within each window, the signal is considered to be stationary and the FT is performed on that windowed signal. STFT reveals both time and frequency information simultaneously. Depending on the nature and type of the signal being processed, the width of the window is chosen and is maintained as constant for that signal analysis. But, the fixed window size of STFT leads to resolution problem. Narrow window in STFT gives good time resolution and poor frequency resolution, while wide window gives good frequency resolution and poor time resolution. Furthermore, wide windows may violate the condition of stationarity. If the frequency components are well separated from each other in original signal, then, there is a possibility of sacrificing frequency resolution for getting good time resolution and vice-versa. Therefore, flexible window width may be the alternative to the resolution problem of STFT. As Wavelet Transform (WT) has flexible window width, it may be an alternative to STFT. WT has evolved considerably in the last three decades and provides a better time-frequency representation of the signal than STFT [53]-[55].

### 3.3 Continuous Wavelet Transform (CWT)

CWT was developed as an alternative to STFT to overcome the resolution problem. The wavelet analysis is done in a similar way to the STFT analysis. The signal is multiplied with a function, similar to the window function in STFT, and the transform is computed separately for different segment of time-domain signal. The CWT of a signal \( f(t) \) is defined as follows [51]-[55].
Diagnosis of Dynamic Insulation Fault in Transformers Subjected to Impulse Voltage Applications

\[ CWT(\tau, s) = \Psi(\tau, s) = \frac{1}{\sqrt{|s|}} \int_{-\infty}^{\infty} f(t) \psi^{*} \left( \frac{t-\tau}{s} \right) \, dt \] (3.1)

As may be seen from the above equation, transformed signal is a function of two variables, \textit{translation} (\( \tau \)) and \textit{scale} (\( s \)). \( \psi(t) \) is the transforming function, and is referred as \textit{mother wavelet}. \( f(t) \) is the signal to be processed and \( \frac{1}{\sqrt{|s|}} \) is the normalization factor. \textit{Wavelet} means small wave of finite length. The term \textit{mother} implies that the functions that are used in the transformation process are derived from one main function. In other words, the \textit{mother wavelet} is the prototype for generating the other window functions. There are different types of mother wavelets, viz. Haar, Daubechies, Bi-orthogonal, Symlet, Gaussian, Morlet, Mexican Hat, Shannon, etc, which may be used for analysis. Typical shape of Morlet mother wavelet is shown in Fig. 3.1.

The term \textit{translation} is related to the location of the window (mother wavelet) in the signal. This corresponds to time information in the transform domain, as the window is shifted through the signal. The \textit{scale} parameter, related to inverse of frequency, is similar to the scale in maps. As in the case of maps, a high scale corresponds to a non-detailed global view of the signal and low scale corresponds to a detailed view of the
signal. Similarly in terms of frequency, low frequency (high scale) corresponds to the global information of the signal, whereas high frequency (low scale) corresponds to the detailed information of the signal. Moreover, scaling is a mathematical operation which either dilates or compresses a signal. Large scale corresponds to dilated and small scale corresponds to compressed view of the signals. The scale, \( s > 1 \) and \( s < 1 \) means dilated and compressed view of the signal, respectively.

### 3.4 Computation of CWT

In CWT, all the wavelet functions (mother wavelet or window) used in the transform are derived from the mother wavelet through dilation or compression processes. Similar to window in STFT, wavelet function (mother wavelet) is shifted throughout the signal being transformed. Time and frequency information of the wavelet transform are provided by translation and scale of the mother wavelet, respectively. The wavelet series is obtained by discretizing CWT. This aids in computation of CWT using computers and is obtained by sampling the time-scale plane. The sampling rate can be changed according to scale without violating Nyquist criterion, i.e., the minimum sampling rate that allows reconstruction of original signal in \( 2f \text{ Hz} \) (\( f \)- highest frequency of the signal in Hertz). Therefore, higher is the value of scale, lower is sampling rate and thus it requires lower number of calculations.

### 3.5 Discrete Wavelet Transform

The terms approximation and details are mostly used in DWT. Approximation refers high-scale, low frequency components and the term details refer low-scale, high frequency components. The original signal, \( S \), passes through two filters, viz. low pass and high pass filters, and emerge as two output signals. Unfortunately, this operation doubles the number of digital data compared to that of original signal \( S \). For example, if the signal \( CS \) consists of 2500 samples of data, corresponding approximation and details signals will have 1250 samples each, i.e. 2500 in total. There exists a subtler way to perform the decomposition using wavelets, by keeping only one sample out of every successive two samples in each of 1250 samples to get complete information, which is known as down-sampling. Down sampling produces two sequences, viz. \( CA_1 \) and \( CD_1 \)
Fig. 3.2. First stage of DWT of transformer winding current with down-sampling [49]
each having 625 samples (data). These sequences $cA_1$ and $cD_1$ are obtained by convolving the signal $CS$ with low-pass and high-pass filters, respectively. For multi stage (level) DWT, the next step is to split the approximations co-efficient, $cA_1$, into two parts, viz. $cA_2$ and $cD_2$, and so on, using the same scheme. In general, for a given signal $CS$ of length $N$ (data points), the DWT consist of $\log_2 N$ stages at most. The first stage of DWT of transformer winding current along with down sampling is shown in Fig. 3.2 [49]. In practice, suitable number of stage is selected based on the nature of the signal, or on the basis of requirements.

### 3.6 Cross-wavelet Transform (XWT)

Cross-wavelet transform is a mathematical operation, which may be considered as an extension of wavelet analysis. XWT gives a measure of correlation between two time series signals. Moreover, the mathematical operations of XWT are similar to CWT. Resultant cross-wavelet spectrum of two signals indicates the regions in time-frequency domain, where the two signals are having high common power. Mathematically, the XWT of two signals, $x(t)$ and $y(t)$, is referred as [56][57]

$$W^{xy}(s, \tau) = \frac{1}{k_\psi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W^x(a, b) W^y\left(\frac{a-b}{s}, \frac{b-\tau}{s}\right) \frac{da db}{a^2}$$

(3.2)

where, $W^x(s, \tau)$ and $W^y(s, \tau)$ are the wavelet transform of $x(t)$ and $y(t)$, respectively. These WT are obtained with the help of particular mother wavelet. However, one may choose different mother wavelet based on the nature of the signal being processed. $k_\psi$, is a constant, whose value is $k_\psi = \int_{-\infty}^{\infty} (|\Psi(\omega)|^2/|\omega|) d\omega < \infty$. The cross-wavelet spectrum is plotted using the magnitude of $W^{xy}$ and the phase angle, $\phi = \tan^{-1}(\Im\{W^{xy}\}/\Re\{W^{xy}\})$. The symbols, $\Im\{W^{xy}\}$ and $\Re\{W^{xy}\}$ refer to the real and imaginary parts of $W^{xy}$, respectively. The cross-wavelet spectrum of acquired winding currents of healthy and faulty insulation is shown in Fig. 3.3[58]. The ‘U’ shaped black colour indicates the “cone of influence”, which shows that the regions edge effect due to zero padding are significant. Black arrows in Fig.3.3, shows the phase angle at time-frequency space. Arrows pointing towards right and left indicates, ‘in-phase” and “anti-phase” conditions, respectively.
Cross-correlation is a mathematical technique, which measures the levels of similarity between two signals being correlated [59][60]. Cross-correlation of two signals having finite energy, viz. \( x(t) \) and \( y(t) \), is defined by [59][60]

\[
\hat{A}_{xy}(m) = \begin{cases} 
\sum_{n=0}^{W-1} x_{n+m} y_n & m \geq 0 \\
\hat{A}_{yx}(-m) & m < 0
\end{cases}
\]  

(3.3)

where, \( m = -W, -1, 0, 1, ..., +W \) represents time shift parameter and the subscript \( xy \) represents the signals correlated. If, \( W \) is the finite number of samples of each of the signals, then the resultant cross-correlation
sequence will have $2^W-1$ number of samples. The cross-correlogram of two signals (e.g. transformer winding current of healthy and faulty insulation), $x(t)$ and $y(t)$, is shown in Fig.3.4.

![Fig.3.4.Cross-correlogram of two time series signals](image)

### 3.8 Applications of XWT and CCL

As XWT has the ability to compare two signals in time-frequency domain, it has been successfully applied in many fields, viz. biomedical signal processing, fault identification, pattern classification, etc.[56][57][59]-[61]. In the present study, for identification of impulse fault characteristics, both cross-wavelet and cross-correlation techniques are used for feature extraction. The location and type of insulation failures have significant effect in these time-frequency spectra of acquired winding currents of healthy as well as faulty insulation. Hence, the
spectrum gives important information of impulse fault characteristics, which may be used for fault identification.

3.9 Conclusions

This chapter discusses the basics along with the required mathematical equations of wavelet transform and cross-correlation and their spectra. As the spectrum contains possible information of fault characteristics, typical features are extracted for accurate identification of insulation failure. The type and number of extracted features and performance of extracted features on fault identification are discussed in subsequent Chapters.